$U(1)$s in F-theory: Keeping it smooth and rational

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Based on work in collaboration
with Craig Lawrie and Jin-Mann Wong `1504.05593`
and with Sven Krippendorf and Jin-Mann Wong `to appear`

Goal

Determine universal, distinguishing characteristics of F-theory models, with distinct phenomenological signatures.

F-theory model building based on lots of examples: local and by now also global, with semi-realistic properties.

Challenge:
Combined package of realistic spectra, flavor, susy breaking, moduli stabilization, etc all into one framework, and genericity of such features.

Strategy:
Ask questions of universal nature: find characteristics that can be comprehensively understood and constrain the phenomenology

Setup

Constraining 4d $N = 1$ SUSY $SU(5)$ F-theory GUTs using additional symmetries: $U(1)$s and discrete.

1. Symmetries:
   What continuous and discrete symmetries are both geometrically consistent within F-theory and phenomenologically sound?

2. Anomalies:
   Spectra consistent with hypercharge flux (GUT breaking) induced anomalies

3. Flavor:
   Realistic quark sector Yukawa textures from distribution of matter, and using Froggatt-Nielsen type mechanism

Input: what are possible $U(1)$ symmetries in F-theory?

Summary

General characterization of global ways of realizing $U(1)$ symmetries and possible matter charges in F-theory [Lawrie, SSN, Wong]

$\Rightarrow$ Model-independent, superset of charges for GUTs

$\Rightarrow$ All charged matter and GUT-Singlet $U(1)$-charges

$\Rightarrow$ Classification of possible Higgsings for $U(1)$s to discrete symmetries

Phenomenological Implications:

Combined system of F-theory $U(1)$ charges, phenomenological consistency and anomaly cancellation has solutions with realistic flavor texture

$\Rightarrow$ Pheno: Sven Krippendorf's talk

[Krippendorf, SSN, Wong]

I. Components in F-theory GUT model building

1. Uses of Symmetries

- Suppress unwanted couplings: Proton decay
- Forbid tree-level μ-term
- Flavor: $U(1)$s for Froggatt-Nielsen

Rapid Proton Decay

Protect model from Proton Decay: half-life $> 10^{36}$ years.

- Dim 4: B/L-violating operators (R-parity violating)

$$W_{\text{dim 4}} = \lambda^{(4)}_{ija} \bar{\mathbf{5}}_i \bar{\mathbf{5}}_j \mathbf{10}_a \supset \lambda^0_{ija} L_i L_j \bar{e}_a + \lambda^1_{ija} \bar{d}_i L_j Q_a + \lambda^2_{ija} \bar{d}_i \bar{d}_j \bar{u}_a$$

$$\sqrt{\lambda^1 \lambda^2} \leq \left(\frac{M_{SUSY}}{\text{TeV}} \right) 10^{-12}$$

- Dim 5:

$$\begin{aligned} W_{\text{dim5}} &= \delta^{(5)}_{abci} \mathbf{10}_a \mathbf{10}_b \mathbf{10}_c \bar{\mathbf{5}}_i \\ &\supset \delta^1_{abci} Q_a Q_b Q_c L_i + \delta^2_{abci} \bar{u}_a \bar{u}_b \bar{e}_c \bar{d}_i + \delta^3_{abci} Q_a \bar{u}_b \bar{e}_c L_i \end{aligned}$$

$$\delta^1_{112i} \leq 16\pi^2 \left(\frac{M_{SUSY}}{M^2_{GUT}} \right) \qquad i = 1, 2$$

$\Rightarrow$ $U(1)$s or discrete symmetries Γ to control spectrum

2. Anomalies

F_Y GUT breaking[a] generates chiral spectrum
$\Rightarrow$ In presence of $U(1)$s: Require $G^2_{MSSM} \times U(1)$ and $U(1)_Y \times U(1) \times U(1)'$ anomaly cancellation

[Dudas Palti], [Marsano, Saulina, SS-N], [Marsano], [Palti]

$\Rightarrow$ Compatibility constraints between charges and F_Y restriction N:

$$\mathbf{10}_a : \quad \begin{cases} (\mathbf{3},\mathbf{2})_{1/6} : & M_a \\ (\bar{\mathbf{3}},\mathbf{1})_{-2/3} : & M_a - N_a \\ (\mathbf{1},\mathbf{1})_1 : & M_a + N_a \end{cases} \qquad \bar{\mathbf{5}}_i : \quad \begin{cases} (\bar{\mathbf{3}},\mathbf{1})_{1/3} : & M_i \\ (\bar{\mathbf{1}},\mathbf{2})_{-1/2} : & M_i + N_i \end{cases}$$

For example, with $q = U(1)$-charges:

$$\sum_i q_i^\alpha N_i + \sum_a q_a^\alpha N_a = 0 \,.$$

$\Rightarrow$ Constraints on M, N and $U(1)$ charges.

[a] Wilson lines generate always have chiral exotics. [Donagi, Wijnholt], [Marsano, Clemens, Pantev, Raby, Tseng]

3. Flavor and Froggatt-Nielsen

Long History of Flavor in F-theory: [Font, Ibañez, Heckman, Vafa, Dudas, Palti, Marchesano, Aparicio, Uranga, Regalado, Zoccarato, King, Leontaris, Ross, Hayashi, Kawano, Tsuchiya, Watari, ...]

$U(1)$s to generate flavor textures, Froggatt-Nielsen (FN) type. Tree-level Yukawas + subleading terms from $U(1)$-charged singlets $\epsilon = \frac{\langle S \rangle}{\Lambda}$. Consistent with $SU(5)$ GUT e.g. [Dreiner, Thormeier]

$$Y_u \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^4 \\ \epsilon^6 & \epsilon^4 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix} , \quad Y_d \sim \begin{pmatrix} \epsilon^4 & \epsilon^4 & \epsilon^4 \\ \epsilon^2 & \epsilon^2 & \epsilon^2 \\ 1 & 1 & 1 \end{pmatrix} .$$

For local F-theory GUTs: no realistic FN models from E_8 [Dudas, Palti].

Why reconsider now?

New insights and general understanding of $U(1)$s in F-theory.

New insights from Geometry

Idea of this Program:

1. Phenomenological constraints on Symmetries, 2. Anomalies, and 3. Realistic Flavor combined with global, geometric consistencies imply constraints on resulting 4d EFT.

F-theory/String theory input:

Constraints on F-theory compactification geometries for GUTs with extra $U(1)$s. What type of $U(1)$ charges can be realized?

$\Rightarrow$ This talk.

GUTs with extra $U(1)$s

- Toric Constructions with extra $U(1)$s.
  [Morrison, Park][Braun, Grimm, Keitel][Mayrhofer, Palti, Weigand][Cvetic, Klever, Piragua], [Morrison, Taylor]...
- All toric hypersurfaces: [Klever, Pena, Piragua, Oehlmann, Reuter]
- Multiple **10** matter loci:
  [Mayrhofer, Palti, Weigand], [Kuentzler, SSN], [Lawrie, Sacco],[Braun, Grimm, Keitel]
- Preliminary Pheno: [Krippendorf, Pena, Oehlmann, Ruehle]
- Systematic approach: Tate-like forms, however limited by ability to factor polynomials of UFD... [Kuentzler, SSN][Lawrie, Sacco]

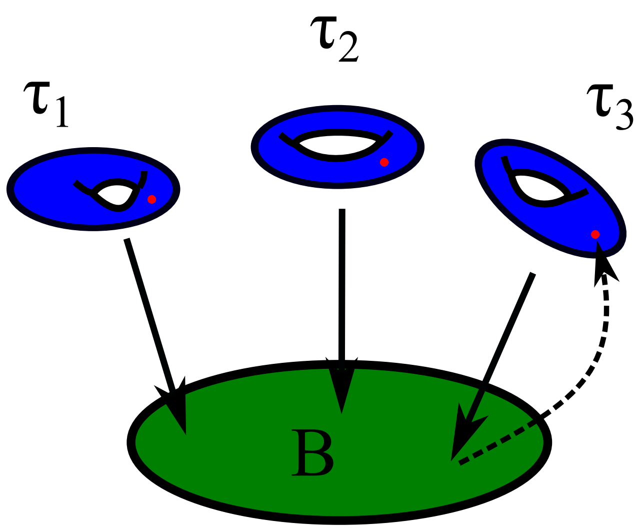
Goal: Find general way to constrain $U(1)$s from first principles

[Lawrie, SSN, Wong]

I. Non-Abelian Gauge Groups in F-theory

F-theory and Elliptic Fibrations

4d vacua: Elliptically fibered Calabi-Yau, $\tau = C_0 + ie^{-\phi}$ axio-dilaton of IIB:



$\Rightarrow$ $\mathbb{E}_\tau$ fibers = Tori $\mathbb{C}/\mathbb{Z} \oplus \tau\mathbb{Z}$ with marked point O (elliptic curve, with O = origin) with complex structure τ

$\Rightarrow$ Exists "zero section" σ_0: $B \longrightarrow \mathbb{E}_\tau : b \mapsto O$

$\Rightarrow$ For such there is always a Weierstrass form with $O = [0, 1, 1]$

$$y^2 = x^3 + fxw^4 + gw^6 \qquad [w, x, y] \in \mathbb{P}(1, 2, 3)$$

4d gauge bosons from F-theory

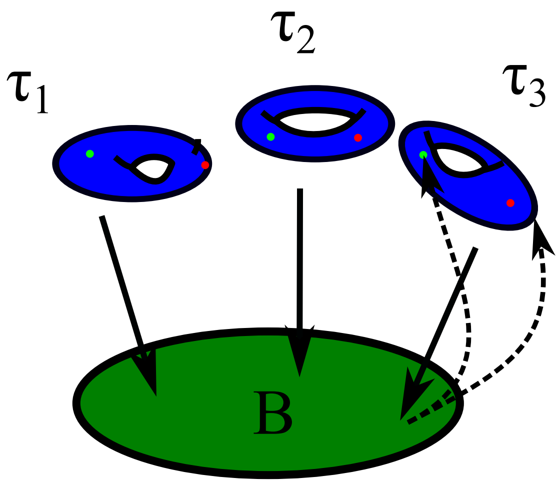
Reduce M-theory 3-form along $(1,1)$ forms $\omega^{(1,1)}$ in fiber:

$$C_3 = \omega^{(1,1)} \wedge A$$

$\Rightarrow$ abelian gauge potentials A. Two types

1. ω from special fibers (ADE like singularities) $\Rightarrow$ GUT gauge bosons
2. ω from rational sections $\Rightarrow$ $U(1)$s [Morrison, Vafa]

   Mathematically: maps from base to fiber: $\sigma: \quad B \longrightarrow \mathbb{E}_\tau\colon b \mapsto P$ with
   P a rational solution to $y^2 = x^3 + fxw^4 + gw^6$, $P \neq O$



$(1,1)$ Forms and Singular Fibers

[Kodaira]: $\exists$ "Singular fibers", which are $\mathbb{P}^1$s intersecting in affine ADE Dynkin diagram $\Rightarrow$ $\omega^{(1,1)}$ from volume form of $\mathbb{P}^1$

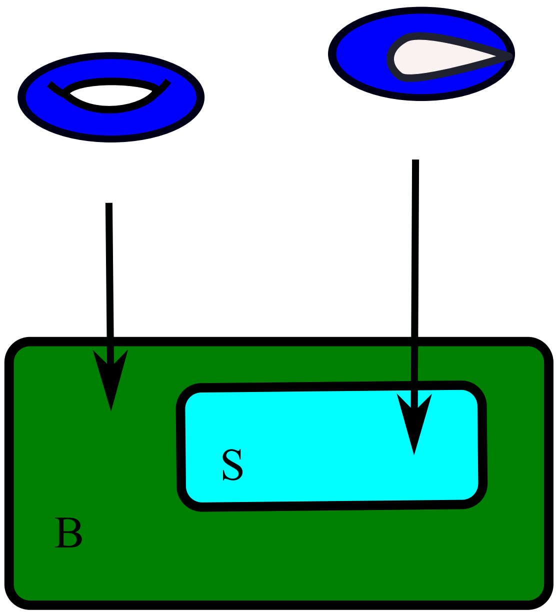
- Kodaira fibers from resolutions of singular fibrations
- Elliptic curve is $y^2 = x^3 + fxw^4 + gw^6$ singular if

$$\Delta = 4f^3 + 27g^2 = 0$$

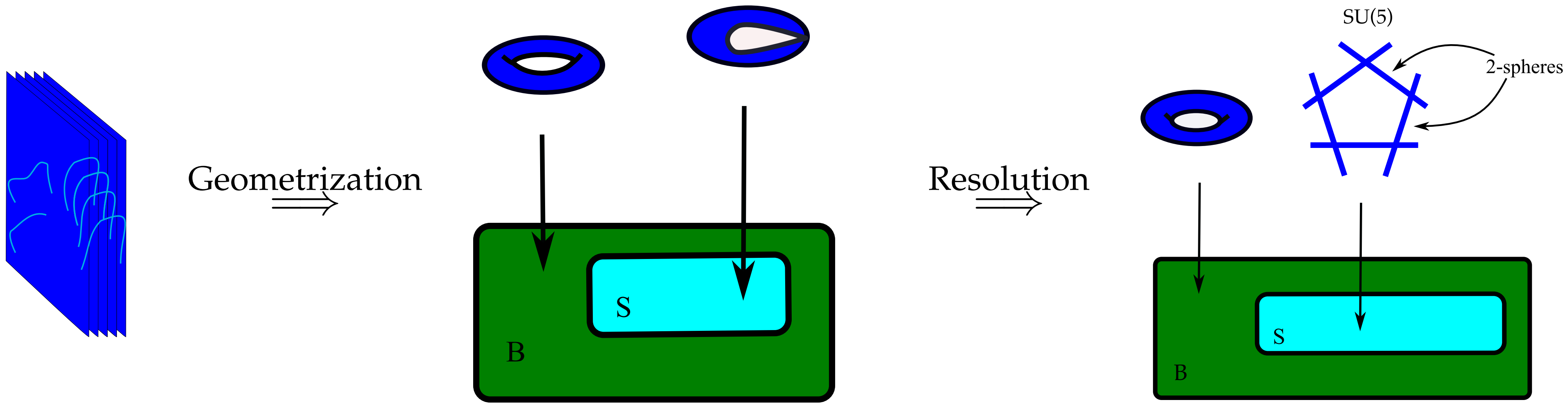
  Here Δ depends on base:

$$\Delta(z) = O(z^n) \quad \Leftrightarrow \quad z = 0 \text{ is surface } S \subset B$$

- Physics:
  Syncs with 7-branes intuition in IIB, which sources F_9 and $\tau \sim \log(x - x_0)$ undergoes monodromy $SL_2\mathbb{Z}$



Gauge theory from Singular Fibers

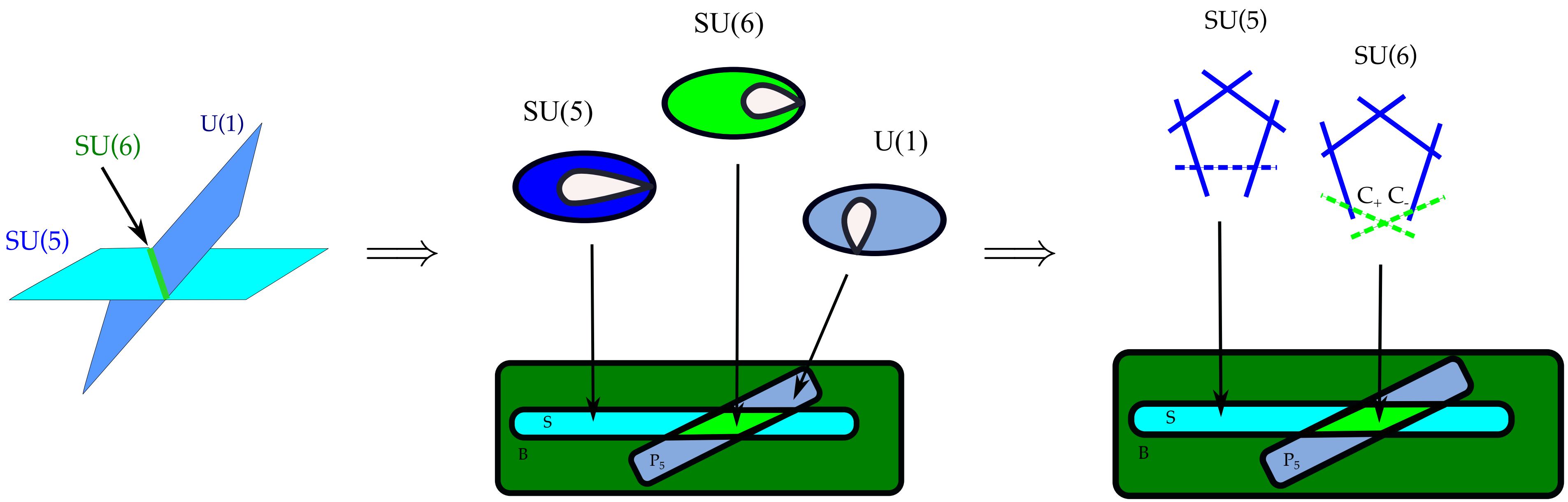


- Resolution of singularities:
  Trees of $\mathbb{P}^1$s, intersecting in Affine $SU(5)$ Dynkin diagram

  $\mathbb{P}^1 = S^2 =$ curves in resolved fiber $\overset{1:1}{\longleftrightarrow}$ simple roots of gauge group $SU(5)$

- M/F-theory:
  Gauge bosons from $C_3 = A_i \wedge \omega_i^{(1,1)}$ and wrapped M2

Matter

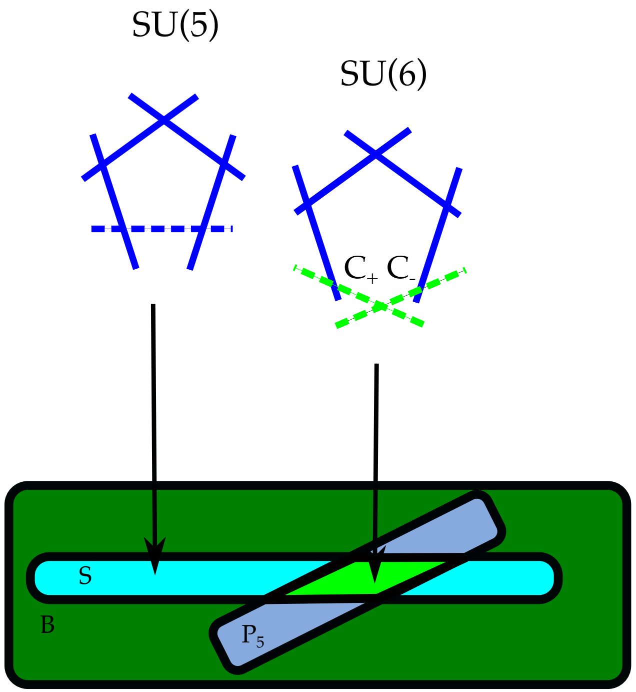


- Matter is localized along codimension 2 loci Σ: Singularity worsens

$$\Delta = P_{\mathbf{5}} z^5 + O(z^6)$$

- Matter determined by fiber type along codim 2:

$$z = P_5 = 0: \ SU(6) \to SU(5) \times U(1): \qquad \mathbf{35} \to \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_6 \oplus \overline{\mathbf{5}}_{-6}$$



Geometry:

$\mathbb{P}^1$ associated to root α splits into "weights" of $\bar{\mathbf{5}}$

$$\mathbb{P}^1_\alpha \quad \longrightarrow \quad C_+ + C_-$$

M/F-theory picture:

Wrapped M2-branes give matter transforming in representation of $SU(5)$

$\Rightarrow$ Classification of posssible codim 2 fibers?

Classification of Singular Fibers

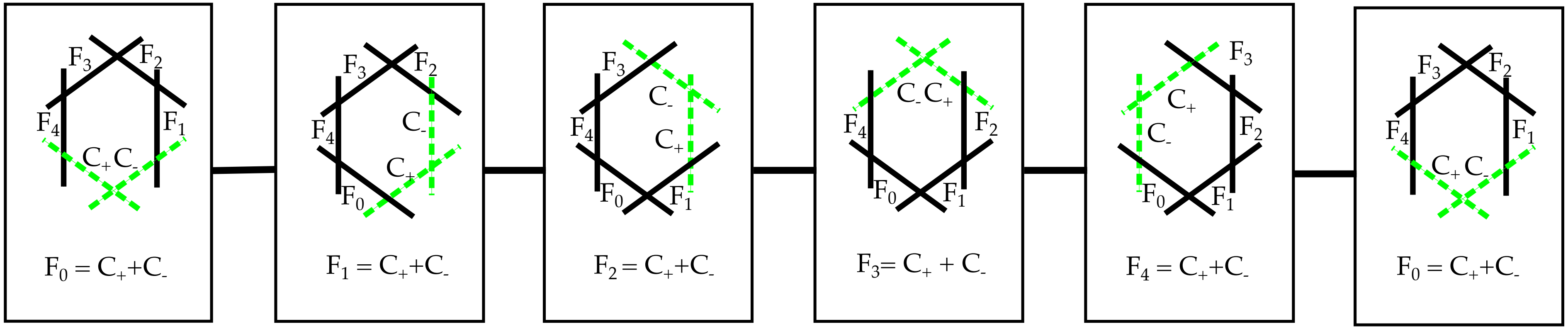
- Codim 1: Classic Algebraic Geometry [Kodaira][Néron]: Lie algebra $\mathfrak{g}$

$$\boxed{\text{Singular Fiber Codim 1}} \quad \longleftrightarrow \quad \boxed{\text{(Decorated) affine Dynkin diagram of } \mathfrak{g}}$$

- Codim 2: **R**= representation of $\mathfrak{g}$ [Hayashi, Lawrie, Morrison, SSN]

$$\boxed{\text{Singular Fiber Codim 2}} \quad \longleftrightarrow \quad \boxed{\text{Box Graph = Decorated rep graph of } \mathbf{R}}$$

Tool: Coulomb phases of $3d$ $N = 2$ susy gauge theories.



NB: known also now for other matter and higher rank

II. Abelian Gauge Groups in F-theory

Mordell-Weil group and $U(1)$s

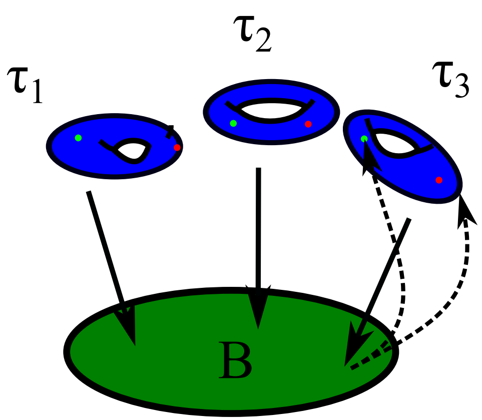
$U(1)$s arise from additional $(1,1)$-forms in fibration

$$C_3 = A \wedge \omega^{(1,1)}$$

(1,1)-forms in elliptic fibration:

- Kodaira singular fiber ($\Rightarrow$ GUT gauge bosons)
- Rational sections of fibration (″rational solutions to the elliptic curve equation″ or ″marked points″)

$U(1)$s $\leftrightarrow$ rational sections



Math fun facts:

- Elliptic curves have group laws: can add points on curves $p \boxplus q$
- The rational points on an elliptic curve form a free abelian group

$$\text{Mordell-Weil group} \cong \mathbb{Z}^n \oplus \Gamma$$

- Rational points:

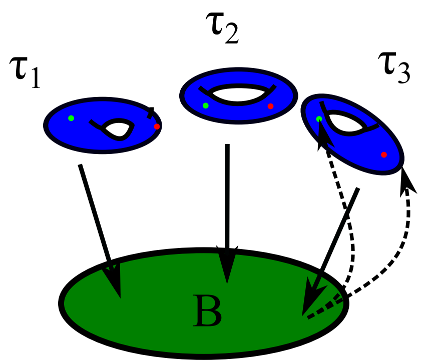
$$y^2 = x^3 + f x w^4 + g w^6 \qquad \sigma_0 : \quad w = 0, \; x = y = 1$$

$\Rightarrow$ Recall: Weierstrass generically has only one marked point "origin"

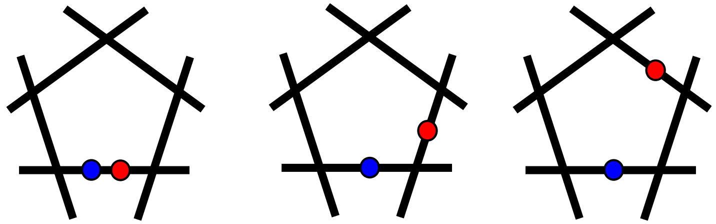
$$y\left(y + bx^2\right) = wP(x, y, w) \qquad \begin{cases} \sigma_0 : & w = 0, \; y = 0 \\ \sigma_1 : & w = 0, \; y = -bx^2 \end{cases}$$

$\Rightarrow$ σ_0 is the origin and σ_1 generates Mordell-Weil=$\mathbb{Z}$

Elliptic fibrations with rational sections



<u>Codim 1:</u> $SU(5)$ singular fiber with σ_0 and σ_1 intersecting one of the $\mathbb{P}^1$s:



<u>Codim 2:</u>

- $\mathbb{P}^1 \to C^+ + C^-$ with $C^\pm$ weights of matter representation.
- $U(1)$ charge: σ_1 intersected with $C^\pm$
- Question: what can σ_0 and σ_1 do in codim 2?
  $\Rightarrow$ Universal characterization of $U(1)$s in F-theory

Strategy

[Lawrie, SSN, Wong]

Fibers in codim 2 (Box graphs)

$\searrow$

Constraining all possible U(1) charges

$\nearrow$

General properties of sections

Constraining rational sections in codim 2: CY3 and CY4

[Lawrie, SSN, Wong]

- Compatibility codim 1 and codim 2: $\sigma \cdot F = 1$ etc.
- New effect: sections can contain $\mathbb{P}^1$s of fiber $\Rightarrow$ "wrapping"

1. Constraints on normal bundle of rational curves C: If $C \subset \sigma \subset Y$, and σ and Y smooth, with σ divisor:

$$0 \to N_{C/\sigma} \to N_{C/Y} \to N_{\sigma/Y}|_C \to 0$$

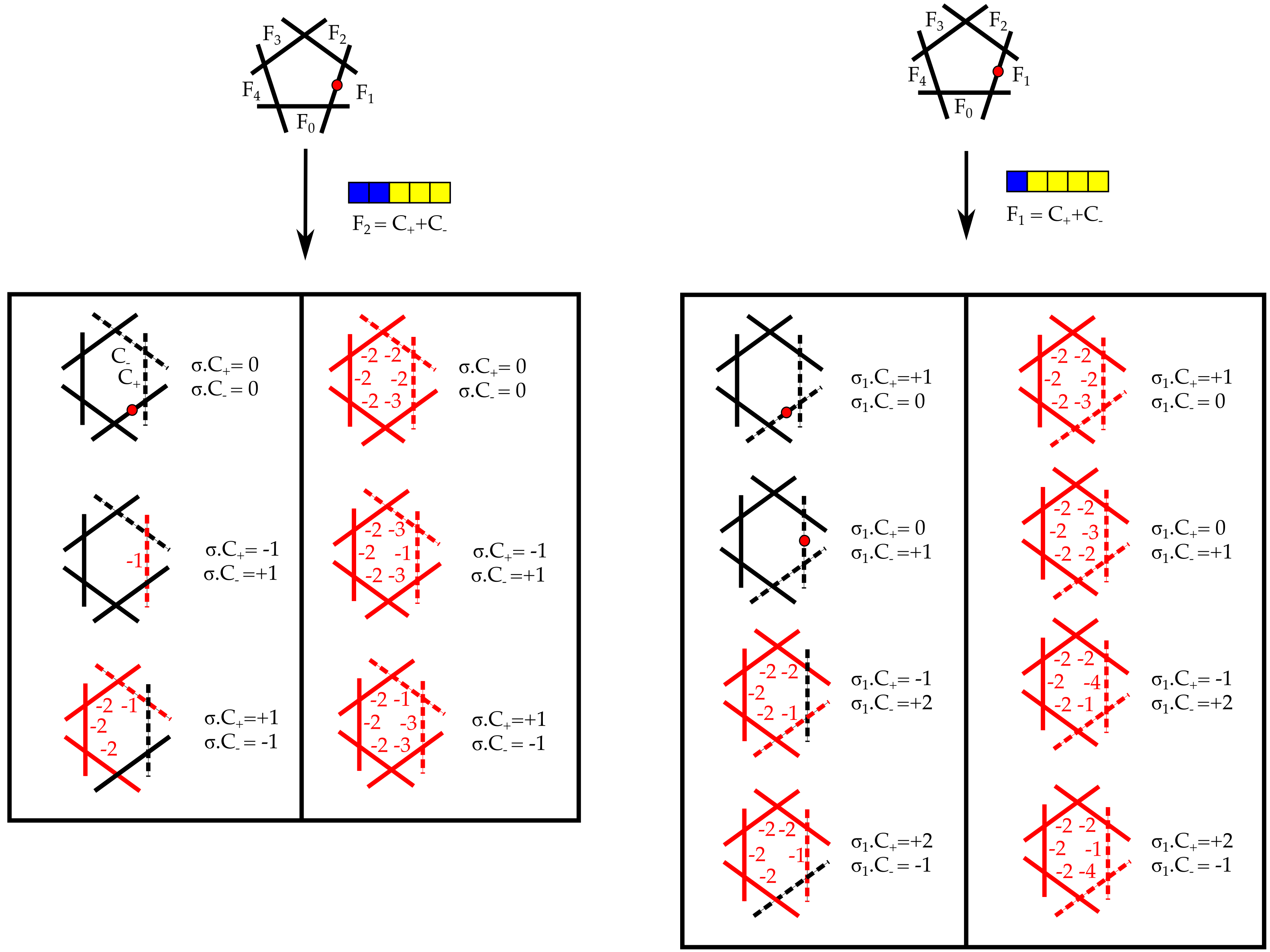
2. Connecting normal bundle to charge:

$$\sigma \cdot_Y C = -2 - \deg N_{C/\sigma}$$

3. Know $N_{C/Y}$ from codim 2 fibers/box graphs
   $\Rightarrow$ determine all possible embeddings of $N_{C/\sigma}$

Key assumption: σ is smooth.

Codim 2 Fibers: $SU(5) \longrightarrow SU(6)$

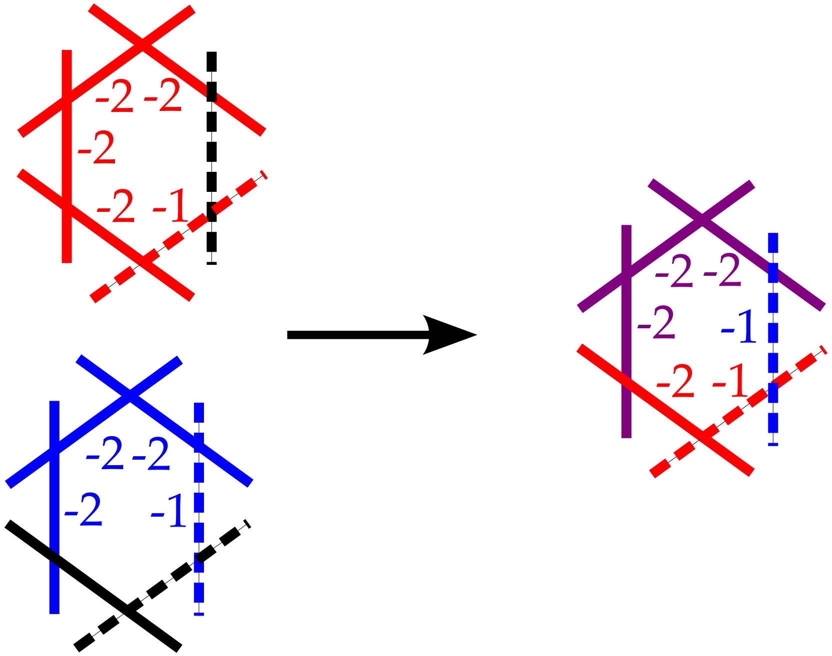


$U(1)$ charges

The $U(1)$ charge obtained from intersecting with (Shioda)

$$S = 5\sigma_1 - 5\sigma_0 + S_f \; ,$$

S_f ensures that roots of $SU(5)$ remain uncharged under $U(1)$.

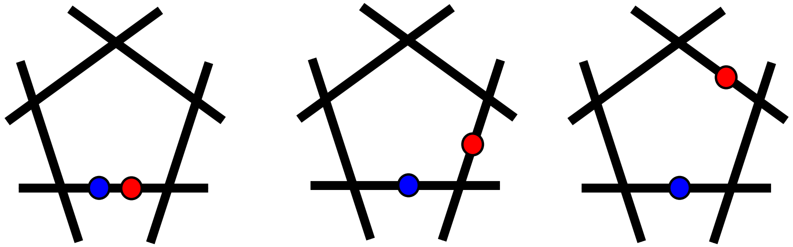


$S.C_{+}$=-14

$S.C_{-}$=+14

Complete set of charges

CY three- AND four-fold charges with smooth rational sections are constrained to be as follows:

$I_5^{(01)}$, $I_5^{(0|1)}$, $I_5^{(0||1)}$: 

$$I_5^{(01)}: \quad \{-15, -10, -5, 0, +5, +10, +15\}$$

$\bar{\mathbf{5}}$ $U(1)$ charges:

$$I_5^{(0|1)}: \quad \{-14, -9, -4, +1, +6, +11\}$$

$$I_5^{(0||1)}: \quad \{-13, -8, -3, +2, +7, +12\}\ .$$

Similar analysis for I_1^* yields all possible charges for **10** matter.

$$I_5^{(01)}: \quad \{\mp 15, \mp 10, \mp 5, 0, \pm 5, \pm 10, \pm 15\}$$

10 $U(1)$ charges:

$$I_5^{(0|1)}: \quad \{\mp 13, \mp 8, \mp 3, \pm 2, \pm 7, \pm 12\}$$

$$I_5^{(0||1)}: \quad \{\mp 11, \mp 6, \mp 1, \pm 4, \pm 9\}\ .$$

$U(1)$ charges of GUT-singlets

Similar analysis for $U(1)$-charged GUT singlets: key to break to discrete symmetries $\Gamma \subset U(1)$.

Realizes all the KK-charges $\sigma_0 \cdot C$ for these singlets as well $\Rightarrow$ all elements of Tate-Shafarevich, see also [Mayrhofer, Palti, Till, Weigand], [Cvetic, Donagi, Klever, Piragua, Poretschkin] for charge 2 and 3.

For singlets for CY3: exists criterion for contractibility of rational curves [Reid, Laufer] $N_{C/Y}$ has degree $(0,-2), (1,1), (-3,1)$ (For CY4, we determine all possibilities, but don't impose contractability)

[Lawrie, SSN, Wong]

$$U(1) \text{ charges of GUT singlets in } \quad \begin{cases} I_5^{(01)} \in \{0, \pm 5, \pm 10, \pm 15, \pm 20, \pm 25, \pm 30\} \\ I_5^{(0|1)} \in \{0, \pm 5, \pm 10, \pm 15, \pm 20, \pm 25\} \\ I_5^{(0||1)} \in \{0, \pm 5, \pm 10, \pm 15, \pm 20, \pm 25\}\ . \end{cases}$$

Discussion

We determined the most general $U(1)^n$ charges for $SU(5)$ models with $\bar{\mathbf{5}}$ and $\mathbf{10}$ matter.

1. Physics: [Krippendorf, SSN, Wong]
   Combine with 1. Phenomenological constraints on Symmetries,
   2. Anomalies and 3. Realistic Flavor
   Spoiler alert: gives realistic flavor $\Rightarrow$ Sven Krippendorf's talk
2. Validity: Smooth versus singular sections
   Singular divisors also contribute to $(1,1)$ forms. Normal bundle exact sequence does not hold. What replaces it, and what are constraints?
3. Global Patching:
   What are the global compatibility conditions between the fibers?
4. Geometric Construction:
   Algebraic realization of the new fibers, with higher charges and section wrapping?

Organizers:

Riccardo Argurio (ULB)
Marcus Berg (Karlstad)
Matteo Bertolini (SISSA)
Gabriele Honecker (Mainz)
Enrico Pajer (Utrecht)
Diederik Roest (Groningen)
Sakura Schafer-Nameki (KCL)

Lecturers at the ``School for Methods in String Theory and Applications in Particle Physics and Cosmology'':

- Cosmology:
  Daniel Baumann (DAMTP), Enrico Pajer (Utrecht)
- Phenomenology:
  David Shih (Rutgers), Florian Staub (CERN)
- Geometry:
  David Morrison (UCSB), Andreas Braun (Oxford)
- Effective Actions:
  Mariana Grana (Saclay), Hagen Triendl (CERN)

Speakers at the Workshops include:

Steve Abel (Durham)
Paolo Creminelli (ICTP)
Raphael Flauger (Carnegie Mellon)
Jim Halverson (KITP)
Liam McAllister (Cornell)
Hiranya Peiris (UCL)
Fernando Quevedo (ICTP/DAMTP)
Matt Reece (Harvard)
Roberto Valandro (ICTP)
Irene Valenzuela (Madrid)
Giovanni Villadoro (ICTP)
Timo Weigand (Heidelberg)

Details and Registration:
http://www.mth.kcl.ac.uk/~ss299/GGI

Thank you