Towards dimensional oxidation of four-dimensional non-geometric type IIB action

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([arXiv:1306.2761] with R. Blumenhagen, X. Gao and D. Herschmann), ([arXiv:1501.07248] with X. Gao), And [arXiv:1505.00544].

Motivation

It is quite remarkable that the non-geometric 4D effective potentials could be studied via merely knowing the forms of Kähler and super-potentials and without having a full understanding of their ten-dimensional origin.

Toroidal setups have been utilized as toolkits for many purposes. Can they help for knowing 10D non-geometric action ?

The purpose of this talk is: trying to zoom into the above window through a bottom-up approach.

For attractions on cosmological applications, recall talks by Blumenhagen, Plauschinn and Fuchs.

Plan of the talk

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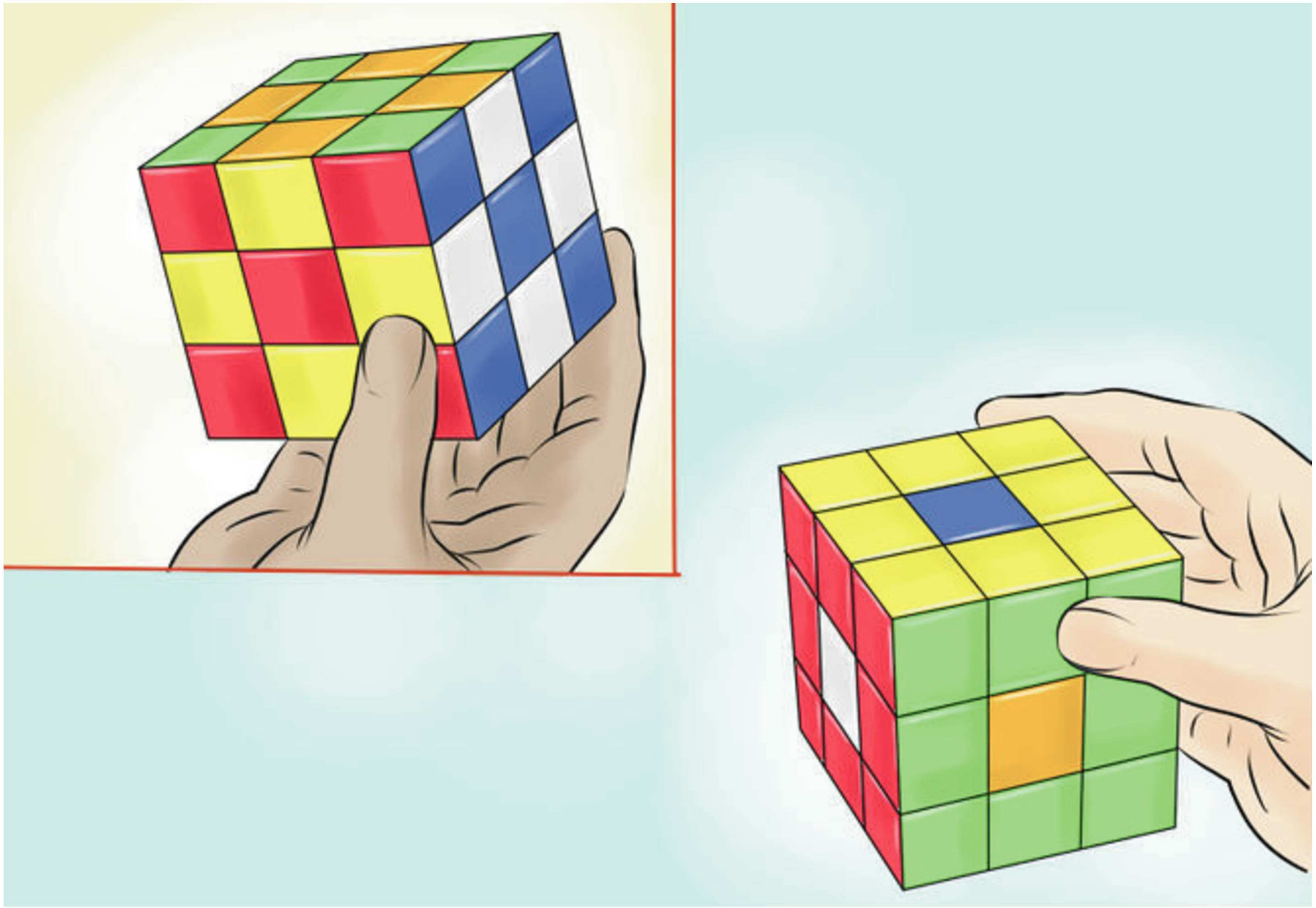

Outline and summary

To begin with, the idea is to consider type IIB flux-compactification on toroidal orientifolds, and follow the steps:

- Given the expression of Kähler potential K and the superpotential W, compute the effective four dimensional scalar potential.
- Invoke some special flux combinations guided by the previous literature.
- Utilizing the new flux combinations, rearrange the full scalar potential into a 'suitable' form.
- Dimensional oxidation

This analysis will be done in an iterative manner from the point of view of including all the standard F_3 and H_3 along with the other (non-)geometric fluxes.

Relevant Ingredients of type IIB orietifolds

We consider type IIB $\hookrightarrow CY_3/\mathcal{O}$. The field ingredients relevant for this talk are:

- Depending on the choice of orientifold involution, the following complexified fields are part of the low energy spectrum [Grimm+Louis'04]

$$\tau = C_0 + i\,e^{-\phi}\;, \quad G^a = c^a + \tau\, b^a\;; \quad a \in h^{1,1}_-,$$
$$T_\alpha = \left(\rho_\alpha + \hat\kappa_{\alpha ab} c^a b^b + \frac{1}{2}\,\tau\,\hat\kappa_{\alpha ab} b^a\, b^b\right) - \frac{i}{2} e^{-\phi}\,\kappa_{\alpha\beta\gamma} t^\beta t^\gamma\;; \quad \alpha \in h^{1,1}_+\,.$$

  which can also be motivated via considering a complex multi-form of even degree Φ_c^{even} defined as [Grana+Louis+Waldram'06; Benmachiche+Grimm'06],

$$\Phi_c^{even} := e^{B_2}\wedge C_{RR} + i\,e^{-\phi} Re(e^{B_2 + i\,J}) \equiv \tau + G^a\;\nu_a + T_\alpha\;\tilde\mu^\alpha\;,$$

- Generic form of Kähler potential K and the superpotential W at tree level are:

$$K = -\ln\left(-i\,(\tau - \overline{\tau})\right) - \ln\left(-i\int_X \Omega_3 \wedge \bar\Omega_3\right) - 2\ln\mathcal{V}_E$$
$$W = \int_X \left[F + \mathcal{D}\Phi_c^{even}\right]_3 \wedge \Omega_3 = \int_X \left[F + \tau\, H + \omega_a G^a + \hat{Q}^\alpha\, T_\alpha\right]_3 \wedge \Omega_3.$$

  where $\mathcal{D} = d + H\wedge . - \omega \triangleleft . + Q \triangleright . - R\bullet$

- The $\mathcal{N} = 1$, F-term scalar potential: $V = e^K\,\left(K^{i\bar{j}}\,(D_i W)(\overline{D_{\bar j}}\,\overline{W}) - 3\,|W|^2\right)$.

Some ingredients of Type IIB $\hookrightarrow \mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-orientifold

- The two $\mathbb{Z}_2$ actions along with the holomorphic involution are defined as:

$$\theta : (z^1, z^2, z^3) \to (-z^1, -z^2, z^3)\,,\ \overline{\theta} : (z^1, z^2, z^3) \to (z^1, -z^2, -z^3)$$
$$I_6 : (z^1, z^2, z^3) \to (-z^1, -z^2, -z^3)\,,$$

- The complex coordinates z_i's on $T^6 = T^2 \times T^2 \times T^2$ are defined as

$$z^1 = x^1 + U_1 x^2,\ z^2 = x^3 + U_2 x^4,\ z^3 = x^5 + U_3 x^6; \qquad U_i = u_i + i\, v_i$$

- Now choosing the following basis of closed three-forms as $\alpha_0 = dx^1 \wedge dx^3 \wedge dx^5\,, \beta^0 = dx^2 \wedge dx^4 \wedge dx^6$ etc. with $\int \alpha_I \wedge \beta^J = \delta_I{}^J$, the holomorphic three-form $\Omega_3 = dz^1 \wedge dz^2 \wedge dz^3$ becomes:

$$\Omega_3 = \alpha_0 + \ (U_1\beta^1 + U_2\beta^2 + U_3\beta^3) + U_1U_2U_3\beta^0 + U_2U_3\alpha_1 + U_1U_3\alpha_2 + U_1U_2\alpha_3$$

- The (string frame) internal metric g_{ij} is known:

$$g_{11} = \frac{t_1}{u_1}\,,\ g_{12} = -\frac{t_1 v_1}{u_1} = g_{21}\,,\ g_{22} = \frac{t_1(u_1^2 + v_1^2)}{u_1}\,,\ g_{33} = \frac{t_2}{u_2}\,,\ g_{34} = -\frac{t_2 v_2}{u_2} = g_{43}\,,$$
$$g_{44} = \frac{t_2(u_2^2 + v_2^2)}{u_2}\,,\ g_{55} = \frac{t_3}{u_3}\,,\ g_{56} = -\frac{t_3 v_3}{u_3} = g_{65}\,,\ g_{66} = \frac{t_3(u_3^2 + v_3^2)}{u_3}\,.$$

- Explicit Flux/moduli components are known.
- Expressins for K/W are known in terms of Flux/moduli components.

Step 1: Inclusion of standard H_3 and F_3 fluxes only

The total F-term scalar potential can be rearranged into the following form [Taylor+Vafa' 99; Blumenhagen+Lüst+Taylor'03]:

$$\mathbf{V_F} = \mathbf{V}_{\mathcal{H}\mathcal{H}} + \mathbf{V}_{\mathcal{F}\mathcal{F}} + \mathbf{V}_{\mathcal{H}\mathcal{F}}$$

where

$$\mathbf{V}_{\mathcal{H}\mathcal{H}} = \frac{s}{4\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{H}}_{ijk}\,\overline{\mathcal{H}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right]$$

$$\mathbf{V}_{\mathcal{F}\mathcal{F}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{F}}_{ijk}\,\overline{\mathcal{F}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right]$$

$$\mathbf{V}_{\mathcal{H}\mathcal{F}} = \frac{1}{4\,\mathcal{V}_E}\left[(-\mathbf{2})\times\left(\frac{1}{3!}\times\frac{1}{3!}\,\overline{\mathcal{H}}_{ijk}\,\mathcal{E}_E^{ijklmn}\,\overline{\mathcal{F}}_{lmn}\right)\right] \equiv \text{Tadpoles}$$

- Orbits: $\mathcal{H}_{ijk} = H_{ijk},\ \mathcal{F}_{ijk} = F_{ijk} + C_0\ H_{ijk}$; $\mathcal{E}_E^{ijklmn} = \frac{\epsilon^{ijklmn}}{\sqrt{|g_{ij}^E|}} = \frac{\epsilon^{ijklmn}}{\mathcal{V}_E}$.

Dimensional oxidation: With this 'suitable' rearrangement of F-term scalar potential, we can anticipate the well known 10D piece in string-frame given as,

$$S = \frac{1}{2}\int d^{10}x\,\sqrt{-g}\,\left(\mathcal{L}_{\mathcal{H}\mathcal{H}} + \mathcal{L}_{\mathcal{F}\mathcal{F}}\right)$$

$$\mathcal{L}_{\mathcal{H}\mathcal{H}} = -\frac{e^{-2\phi}}{2}\left[\frac{1}{3!}\overline{\mathcal{H}}_{ijk}\,\overline{\mathcal{H}}_{i'j'k'}\,g^{ii'}\,g^{jj'}\,g^{kk'}\right],\quad \mathcal{L}_{\mathcal{F}\mathcal{F}} = -\frac{1}{2}\left[\frac{1}{3!}\overline{\mathcal{F}}_{ijk}\,\overline{\mathcal{F}}_{i'j'k'}\,g^{ii'}\,g^{jj'}\,g^{kk'}\right]$$

Step 2: Inclusion of (H_3, F_3) along with non-geometric Q-flux

Motivated by [Villadoro + Zwirner'05]: inclusion of Q-flux on top of standard (H_3, F_3) fluxes to seek for 10D action [Blumenhagen+Gao+Herschmann + PS'13]

$$\mathbf{V_F} = \mathbf{V}_{\mathcal{H}\mathcal{H}} + \mathbf{V}_{\mathcal{F}\mathcal{F}} + \mathbf{V}_{\mathcal{Q}\mathcal{Q}} + \mathbf{V}_{\mathcal{H}\mathcal{Q}} + \mathbf{V}_{\mathcal{H}\mathcal{F}} + \mathbf{V}_{\mathcal{F}\mathcal{Q}} + \ldots\ldots$$

$$\mathbf{V}_{\mathcal{H}\mathcal{H}} = \frac{s}{4\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{H}}_{ijk}\,\overline{\mathcal{H}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right]$$

$$\mathbf{V}_{\mathcal{F}\mathcal{F}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{F}}_{ijk}\,\overline{\mathcal{F}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right]$$

$$\mathbf{V}_{\mathcal{Q}\mathcal{Q}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[3\times\left(\frac{1}{3!}\,\overline{\mathcal{Q}}_k{}^{ij}\,\overline{\mathcal{Q}}_{k'}{}^{i'j'}\,g^E_{ii'}g^E_{jj'}g_E^{kk'}\right) + 2\times\left(\frac{1}{2!}\overline{\mathcal{Q}}_m{}^{ni}\,\overline{\mathcal{Q}}_n{}^{mi'}\,g^E_{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{H}\mathcal{Q}} = \frac{1}{4\,\mathcal{V}_E}\left[(+\mathbf{2})\times\left(\frac{1}{2!}\overline{\mathcal{H}}_{mni}\,\overline{\mathcal{Q}}_{i'}{}^{mn}\,g_E^{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{H}\mathcal{F}} = \frac{1}{4\,\mathcal{V}_E}\left[(+\mathbf{2})\times\left(\frac{1}{3!}\times\frac{1}{3!}\,\overline{\mathcal{H}}_{ijk}\,\mathcal{E}_E^{ijklmn}\,\overline{\mathcal{F}}_{lmn}\right)\right] \equiv \text{Tadpole}$$

$$\mathbf{V}_{\mathcal{F}\mathcal{Q}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[(-\mathbf{2})\times\left(\frac{1}{2!}\times\frac{1}{2!}\,\overline{\mathcal{Q}}_i{}^{j'k'}\,\overline{\mathcal{F}}_{j'k'j}\;\;\tau^E_{klmn}\;\;\mathcal{E}_E^{ijklmn}\right)\right] \equiv \text{Tadpole}$$

BIs: $Q_k{}^{[\bar{i}\,\bar{j}}Q_n{}^{\bar{l}]k} = 0 = Q_{[\bar{k}}{}^{ij}H_{\overline{lm}]j}$. The new flux orbits are:

$$\mathcal{H}_{ijk} = h_{ijk}\;, \qquad \mathcal{F}_{ijk} = f_{ijk} + C_0\;\,h_{ijk}, \qquad \mathcal{Q}^{ij}_k = Q^{ij}_k,$$

where $$h_{ijk} = H_{ijk}\;, \qquad f_{ijk} = \left(F_{ijk} + \frac{3}{2}\,Q_{[\underline{i}}{}^{lm}C^{(4)}_{lm\underline{jk}]}\right).$$

Modular completion

The four dimensional scalar potential generically have an S-duality invariance following from the underlying ten-dimensional type IIB supergravity.

- This corresponds to a $SL(2,\mathbb{Z})$ transformation of various moduli/fluxes as under [Aldazabal+Camara+Font+Ibanez'06; Grimm'07; Guarino+Weatherill'08],

$$\tau \to \frac{a\tau+b}{c\tau+d}\,, \quad G^a \to \frac{G^a}{c\tau+d}, \quad T_\alpha \to T_\alpha - \frac{c}{c\tau+d}\left(\frac{1}{2}\hat{\kappa}_{\alpha ab}\, G^a\, G^b\right).$$

$$\begin{pmatrix} C_2 \\ B_2 \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} C_2 \\ B_2 \end{pmatrix}, \quad \begin{pmatrix} F \\ H \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} F \\ H \end{pmatrix}, \quad \begin{pmatrix} Q \\ P \end{pmatrix} \to \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} Q \\ P \end{pmatrix}.$$

$$\Longrightarrow e^K \to |c\,\tau+d|^2\, e^K\,, \qquad W \to \frac{W}{c\,\tau+d}; \quad \text{wehre} \quad ad-bc=1\,, \quad a,\ b,\ c,\ d \in \mathbb{Z}.$$

- For the current purpose, we consider a simplified version:

$$\tau \to -\frac{1}{\tau},\ G^a \to -\frac{G^a}{\tau},\ T_\alpha \to T_\alpha - \frac{1}{2}\,\frac{\hat{\kappa}_{\alpha ab}G^a G^b}{\tau}$$
$$H \to F,\ F \to -H,\ Q \to -P,\ P \to Q,\ \omega \to \omega,\ R \to R\,.$$

- This leads to a modular completed form of superpotential is [Blumenhagen+ Font+Fuchs + Herschmann + Plauschinn + Sekiguchi + Wolf'15],

$$W = \int_X \left[(F+\tau\, H) + \omega_a\, G^a + \left(\hat{Q}^\alpha + \tau\, \hat{P}^\alpha\right) T_\alpha - \hat{P}^\alpha \left(\frac{1}{2}\hat{\kappa}_{\alpha ab} G^a G^b\right)\right]_3 \wedge \Omega_3.$$

Step 3: Inclusion of S-dual pairs of (H_3, F_3) and (P, Q)-fluxes

$$\mathbf{V_F} = \mathbf{V}_{\mathcal{H}\mathcal{H}} + \mathbf{V}_{\mathcal{F}\mathcal{F}} + \mathbf{V}_{\mathcal{Q}\mathcal{Q}} + \mathbf{V}_{\mathcal{P}\mathcal{P}} + \mathbf{V}_{\mathcal{H}\mathcal{Q}} + \mathbf{V}_{\mathcal{F}\mathcal{P}} + \mathbf{V}_{\mathcal{Q}\mathcal{P}} + \mathbf{V}_{\mathcal{H}\mathcal{F}} + \mathbf{V}_{\mathcal{F}\mathcal{Q}} + \mathbf{V}_{\mathcal{H}\mathcal{P}} + \ldots\ldots$$

$$\mathbf{V}_{\mathcal{H}\mathcal{H}} = \frac{s}{4\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{H}}_{ijk}\,\overline{\mathcal{H}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right] \qquad [\mathrm{Gao}+\mathrm{PS}'15]$$

$$\mathbf{V}_{\mathcal{F}\mathcal{F}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[\frac{1}{3!}\overline{\mathcal{F}}_{ijk}\,\overline{\mathcal{F}}_{i'j'k'}\,g_E^{ii'}\,g_E^{jj'}\,g_E^{kk'}\right]$$

$$\mathbf{V}_{\mathcal{Q}\mathcal{Q}} = \frac{1}{4\,s\,\mathcal{V}_E}\left[3\times\left(\frac{1}{3!}\,\overline{\mathcal{Q}}_k{}^{ij}\,\overline{\mathcal{Q}}_{k'}{}^{i'j'}\;g^E_{ii'}g^E_{jj'}g_E^{kk'}\right) + 2\times\left(\frac{1}{2!}\overline{\mathcal{Q}}_m{}^{ni}\,\overline{\mathcal{Q}}_n{}^{mi'}\;g^E_{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{P}\mathcal{P}} = \frac{s}{4\,\mathcal{V}_E}\left[3\times\left(\frac{1}{3!}\,\overline{\mathcal{P}}_k{}^{ij}\,\overline{\mathcal{P}}_{k'}{}^{i'j'}\;g^E_{ii'}g^E_{jj'}g_E^{kk'}\right) + 2\times\left(\frac{1}{2!}\overline{\mathcal{P}}_m{}^{ni}\,\overline{\mathcal{P}}_n{}^{mi'}\;g^E_{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{H}\mathcal{Q}} = \frac{1}{4\,\mathcal{V}_E}\left[(+\mathbf{2})\times\left(\frac{1}{2!}\overline{\mathcal{H}}_{mni}\,\overline{\mathcal{Q}}_{i'}{}^{mn}\;g_E^{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{F}\mathcal{P}} = \frac{1}{4\,\mathcal{V}_E}\left[(-\mathbf{2})\times\left(\frac{1}{2!}\overline{\mathcal{F}}_{mni}\,\overline{\mathcal{P}}_{i'}{}^{mn}\;g_E^{ii'}\right)\right]$$

$$\mathbf{V}_{\mathcal{Q}\mathcal{P}} = \frac{1}{4\,\mathcal{V}_E}\left[(+\mathbf{2})\times\left(\frac{1}{3!}\;(3\,\overline{\mathcal{P}}_n{}^{l'm'}\;g^E_{l'l}\,g^E_{m'm})\right)\;\mathcal{E}_E^{ijklmn}\;\left(\frac{1}{3!}\;(3\,\overline{\mathcal{Q}}_k{}^{i'j'}\;g^E_{i'i}g^E_{j'j})\right)\right]$$

$$\mathbf{V}_{\mathcal{H}\mathcal{F}} = \frac{1}{4\,\mathcal{V}_E}\left[(+\mathbf{2})\times\left(\frac{1}{3!}\;\times\;\frac{1}{3!}\;\overline{\mathcal{H}}_{ijk}\;\mathcal{E}_E^{ijklmn}\;\overline{\mathcal{F}}_{lmn}\right)\right] \equiv \text{Tadpole}$$

$$\mathbf{V}_{\mathcal{F}\mathcal{Q}} = \frac{1}{4\,s\,\mathcal{V}_E}\;\left[(-\mathbf{2})\times\left(\frac{1}{2!}\;\times\;\frac{1}{2!}\;\overline{\mathcal{Q}}_i{}^{j'k'}\,\overline{\mathcal{F}}_{j'k'j}\;\;\tau^E_{klmn}\;\;\mathcal{E}_E^{ijklmn}\;\right)\right] \equiv \text{Tadpole}$$

$$\mathbf{V}_{\mathcal{H}\mathcal{P}} = \frac{s}{4\,\mathcal{V}_E}\;\left[(-\mathbf{2})\times\left(\frac{1}{2!}\;\times\;\frac{1}{2!}\,\overline{\mathcal{P}}_i{}^{j'k'}\,\overline{\mathcal{H}}_{j'k'j}\;\;\tau^E_{klmn}\;\;\mathcal{E}_E^{ijklmn}\;\right)\right] \equiv \text{Tadpole}.$$

Step 3: .....continued

The new generalized flux orbits becomes: [Gao+PS'15]

$$\mathcal{H}_{ijk} = h_{ijk} \ , \qquad \mathcal{Q}^{ij}_k = q^{ij}_k + C_0 \ p^{ij}_k \ ,$$

$$\mathcal{F}_{ijk} = f_{ijk} + C_0 \ h_{ijk} \ , \qquad \mathcal{P}^{ij}_k = p^{ij}_k \ ,$$

where $$h_{ijk} = \left(H_{ijk} + \frac{3}{2} P_{[\underline{i}}{}^{lm} C^{(4)}_{lm\underline{jk}]} \right) \ , \quad f_{ijk} = \left(F_{ijk} + \frac{3}{2} Q_{[\underline{i}}{}^{lm} C^{(4)}_{lm\underline{jk}]} \right)$$

$$q^{ij}_k = Q^{ij}_k, \qquad p^{ij}_k = P^{ij}_k$$

Observations to appreciate:

- The modular completion of flux orbits is manifest.
- The topological piece in the rearrangement reproduces the 3/7-brane tadpoles (subject to satisfying certainties BIs).
- In the usual flux orbits (H, F, Q, P), we know that tadpole terms are: $HF = D3$, $FQ/HP = D7/NS7$, and $(HQ + FP) = I_7$ as C_8 eight form triplet $(C^{(8)}$, $\tilde{C}^{(8)}$, $C'^{(8)})$ transforms as $C^{(8)} \to -\tilde{C}^{(8)}$, $\tilde{C}^{(8)} \to -C^{(8)}$, $C'^{(8)} \to -C'^{(8)}$.
- Without using flux orbits, its hard to guess the 4D tadpole terms of especially for I_7 as it involves a pre-factor C_0/s.
- All the three types of 7-brane tadpoles are clubbed into $(\mathbf{V}_{\mathcal{FQ}} + \mathbf{V}_{\mathcal{HP}})$ pieces (in the new rearrangement) subject to satisfying a set of BIs.
- The story so far has been without odd moduli !

Counting of terms for searching the rearrangement

Fluxes turned-on	$\mathbf{V_F}$	$\mathbf{V}_{\rm kin}$	$\mathbf{V_F} - \mathbf{V}_{\rm kin}$	$\mathbf{V_D} = V_D + BIs$	$(\mathbf{V_F} + \mathbf{V_D}) - \mathbf{V}_{\rm kin}$ (to be removed by BIs)
H	152	152	0	0=0+0	0
F	76	76	0	0=0+0	0
Q	1059	891	168	48=0+48	120
P	2118	1782	336	96=0+96	240
H, F	361	353	8	8=8+0	0
H, Q	1814	1478	336	**96=0+96**	240
H, P	3068	2684	384	144=48+96	240
F, Q	1534	1342	192	72=24+48	120
F, P	2797	2293	504	**144=0+144**	360
Q, P	6897	4857	2040	312=24+288	1728
H, F, Q	2422	2054	368	128=32+96	240
H, F, P	3880	3320	560	200=56+144	360
H, Q, P	8450	6194	2256	408=72+336	1848
F, Q, P	7975	5743	2232	384=48+336	1848
H, F, Q, P	9661	7205	2456	488=152+336	1968

Step 4: Comments on including all fluxes [PS'15]

Now, let us reconsider the three-form appearing in modular completed superpotential W

$$\left[(F+\tau H)+\omega_a G^a+\left(\hat{Q}^\alpha+\tau\hat{P}^\alpha\right)T_\alpha-\hat{P}^\alpha\left(\frac{1}{2}\hat{\kappa}_{\alpha ab}G^aG^b\right)\right]_3$$

$$=\left[\left(\mathcal{F}^k+s\,\hat{\mathcal{P}}^{\alpha\,k}\sigma_\alpha\right)+i\,\left(s\,\mathcal{H}^k-\hat{\mathcal{Q}}^{\alpha\,k}\sigma_\alpha\right)\right]A_k+\left[\left(\mathcal{F}_k+s\,\hat{\mathcal{P}}^{\alpha}{}_k\sigma_\alpha\right)+i\,\left(s\,\mathcal{H}_k-\hat{\mathcal{Q}}^{\alpha}{}_k\sigma_\alpha\right)\right]B^k$$

where $\mathcal{H}^k=\mathbf{h}^k\,,\quad \hat{\mathcal{Q}}^{\alpha k}=\hat{\mathbf{q}}^{\alpha\mathbf{k}}+C_0\,\hat{\mathbf{p}}^{\alpha\mathbf{k}}\,,\quad \mathcal{F}^k=\mathbf{f}^k+C_0\;\mathbf{h}^k\,,\quad \hat{\mathcal{P}}^{\alpha k}=\hat{\mathbf{p}}^{\alpha k}\,,$

$$\mathbf{h}^k=\left[H^k+(\omega_a{}^kb^a)+\hat{Q}^{\alpha k}\left(\frac{1}{2}\,\hat{\kappa}_{\alpha ab}b^ab^b\right)\right]+\left[\hat{P}^{\alpha k}\left(\tilde{\rho}_\alpha-\frac{1}{2}\hat{\kappa}_{\alpha ab}c^ab^b\right)\right]$$

$$\mathbf{f}^k=\left[F^k+(\omega_a{}^kc^a)-\hat{P}^{\alpha k}\left(\frac{1}{2}\,\hat{\kappa}_{\alpha ab}c^ac^b\right)\right]+\left[\hat{Q}^{\alpha k}\left(\tilde{\rho}_\alpha+\frac{1}{2}\hat{\kappa}_{\alpha ab}c^ab^b\right)\right]$$

$$\mho_{\mathbf{a}}{}^{\mathbf{k}}\equiv\omega_{\mathbf{a}}{}^{\mathbf{k}}=\omega_a{}^k+\hat{Q}^{\alpha k}(\hat{d}^{-1})^\delta_\alpha\hat{k}_{\delta ab}b^b-\hat{P}^{\alpha k}(\hat{d}^{-1})^\delta_\alpha\hat{k}_{\delta ab}\,c^b$$

$$\hat{\mathbf{q}}^{\alpha\mathbf{k}}=\hat{Q}^{\alpha k},\quad \hat{\mathbf{p}}^{\alpha\mathbf{k}}=\hat{P}^{\alpha k}\,.$$

Flux components with even-index $K\in h^{2,1}_+$ are: $\mathcal{R}^K\equiv\mathbf{r}^K=\dfrac{(1+|\tau|^2)}{s}\,R^K$

$$\hat{\mho}_\alpha{}^{\mathbf{K}}\equiv\hat{\omega}_\alpha{}^{\mathbf{K}}=\hat{\omega}_\alpha{}^K-(d^{-1})_b{}^a\,\hat{k}_{\alpha ac}\left(Q^{bK}b^c-P^{bK}c^c\right)+f^{-1}\,R^K\left(\frac{1}{2}\hat{k}_{\alpha ab}\,b^a\,b^b+\frac{1}{2}\hat{k}_{\alpha ab}\,c^a\,c^b\right)$$

$$\mathbf{q}^{aK}=Q^{aK}-f^{-1}R^K\,b^a\,,\;\mathbf{p}^{aK}=P^{aK}+f^{-1}R^K\,c^a\,.$$

Thus: $h^k\to f^k, f^k\to -h^k, \hat{\mathbf{q}}^{\alpha\mathbf{k}}\to-\hat{\mathbf{p}}^{\alpha\mathbf{k}}, \hat{\mathbf{p}}^{\alpha\mathbf{k}}\to\hat{\mathbf{q}}^{\alpha\mathbf{k}}, \mathbf{q}^{aK}\to-\mathbf{p}^{aK}, \mathbf{p}^{aK}\to\mathbf{q}^{aK}$

while all even/odd components of new geometric flux $(\omega_{\mathbf{a}}{}^{\mathbf{k}},\hat{\omega}_\alpha{}^{\mathbf{K}})$ and non-geometric flux $\mathbf{r}^K$ are self S-daul.

Summary and Outlook

- In the context of type IIB flux-compactification on a toroidal orietifold, we have presented some 'suitable' rearrangement of the four dimensional scalar potential which helps in dimensional oxidation of various pieces into ten dimensions.
- This has been done in an iterative manner as:

  Without odd moduli: Type IIB $\hookrightarrow \mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$-orientifold

  - Step 1: include H_3 and, F_3 fluxes.
  - Step 2: include H_3, F_3 and Q fluxes.
  - Step 3: Modular completion: include (H_3, F_3) and (Q, P) fluxes.
  - Step 4: Comments on all fluxes (with odd moduli).
- In the process, we have invoked a modular completed version of the new generalized flux orbits.
- It will be desired to test the new generalized flux orbits via taking a more fundamental route such as DFT compactification on CY.