Natural Inflation and the Weak Gravity Conjecture

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String Pheno '15 - Madrid

Axions & Large field inflation

- Natural inflation is one of the most fruitful frameworks for large field inflation

Freese et. al '90

$$V(\phi) = \sum_k c_k \, e^{-km} \left[1 - \cos\left(\frac{k\phi}{f} \right) \right]$$

- Controlled, slow-roll inflation: $e^{-m} \ll 1 \,, \quad f > M_p$

$$f \cdot m > M_p$$

- Approximately exact global symmetry !

- Not found in (perturbative) string theory

Banks et al. '03
Svrcek, Witten '06

The Weak Gravity Conjecture

Arkani-Hamed et al. '06

- The conjecture:

"Gravity is the Weakest Force"

- = For every long range gauge field there exists a particle of charge q and mass m, s.t.

$$\frac{q}{m} M_P \geq \text{“1”}$$

- Required to avoid large number of BH remnants: ~~CEB~~
- Black holes should be allowed to emit their charge:

$$\text{“1”} \equiv M_p \left(\frac{Q}{M} \right)_{Ext}$$

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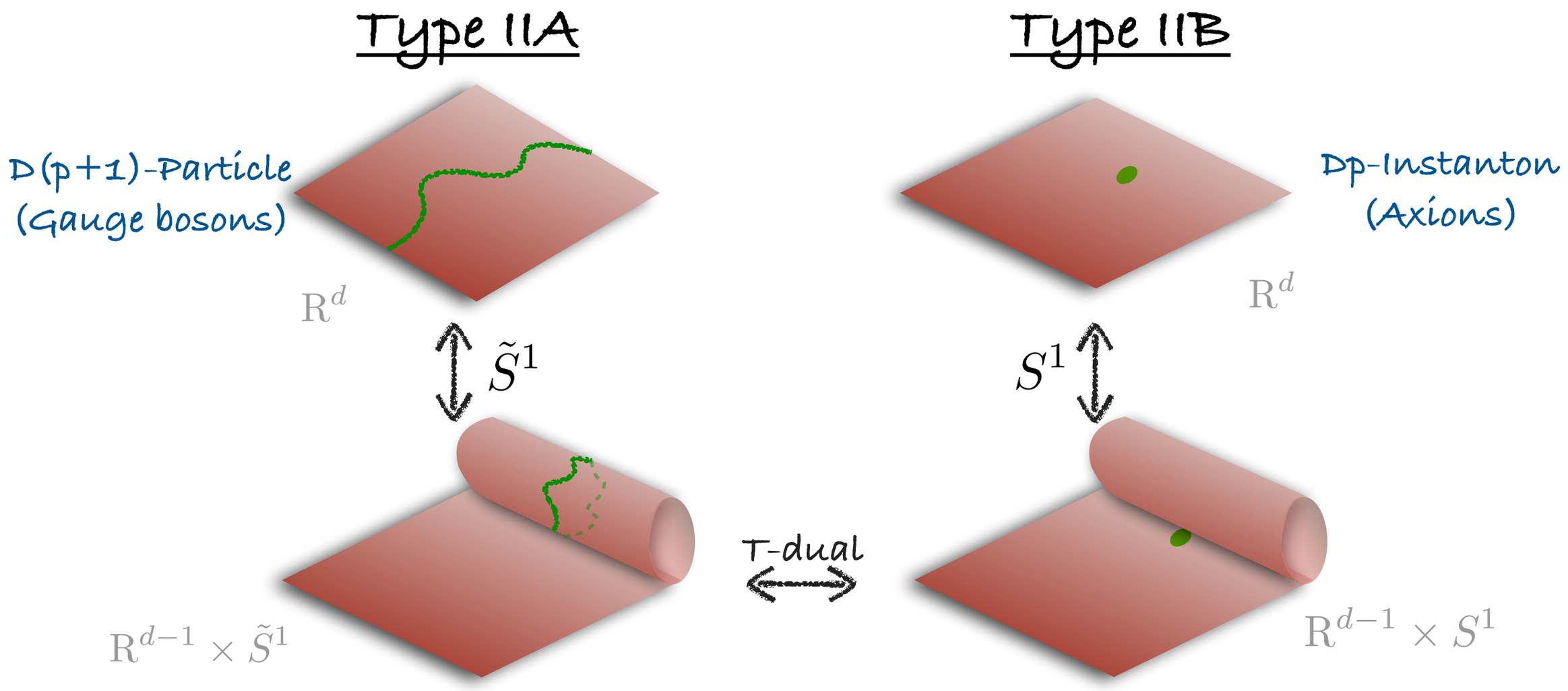
WGC and Axions

Brown et al. '15

- T-duality provides a connection between instantons and particles



WGC

$$\frac{q}{m} \geq 1$$

$$f \cdot m \leq 1$$

$$e^{-S_{inst}} = e^{-m+i\phi/f}$$

Multiple axions/ Multiple U(1)'s

Notice: ongoing debate. See related results in

Montero, Uranga, Valenzuela: 1503.03886

Bachlechner, Long, McAllister: 1503.07853

WGC and axions

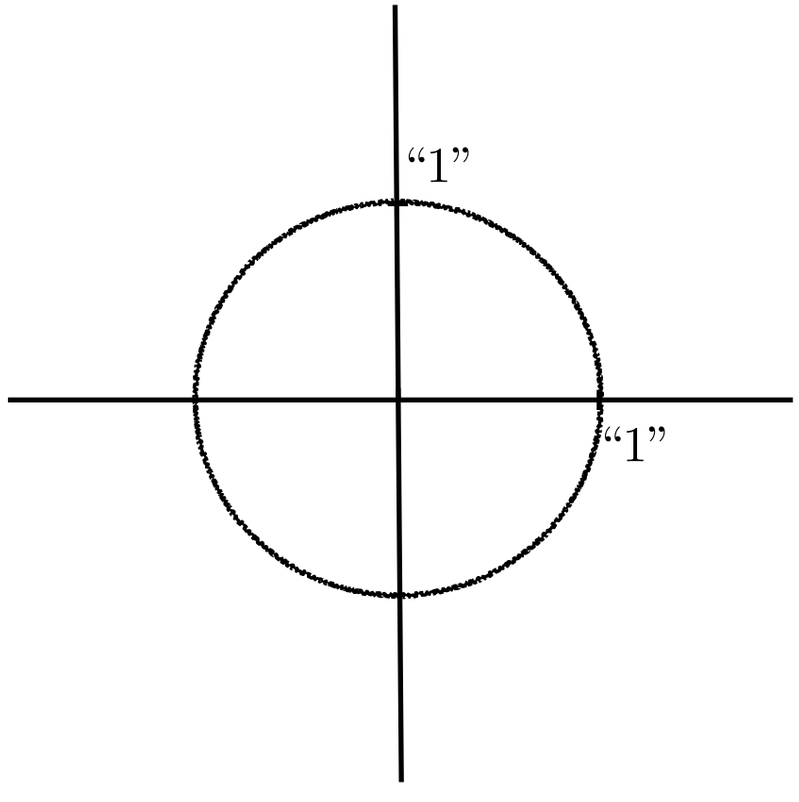
Multiple axions/U(1)s

Cheung, Remmen '14
Rudelius '15

- The WGC translates into a "convex hull" condition on the spectrum of instantons/particles:

$$\vec{z}_i \equiv \frac{M_P}{M_i}\begin{pmatrix} Q_i^1 & Q_i^2 \end{pmatrix} \qquad \left(= \frac{M_P}{\sqrt{2}\, m_i}\begin{pmatrix} 1/f_i^1 & 1/f_i^2 \end{pmatrix}\right)$$

$|\vec{z}_{EBH}| \equiv$ "1"



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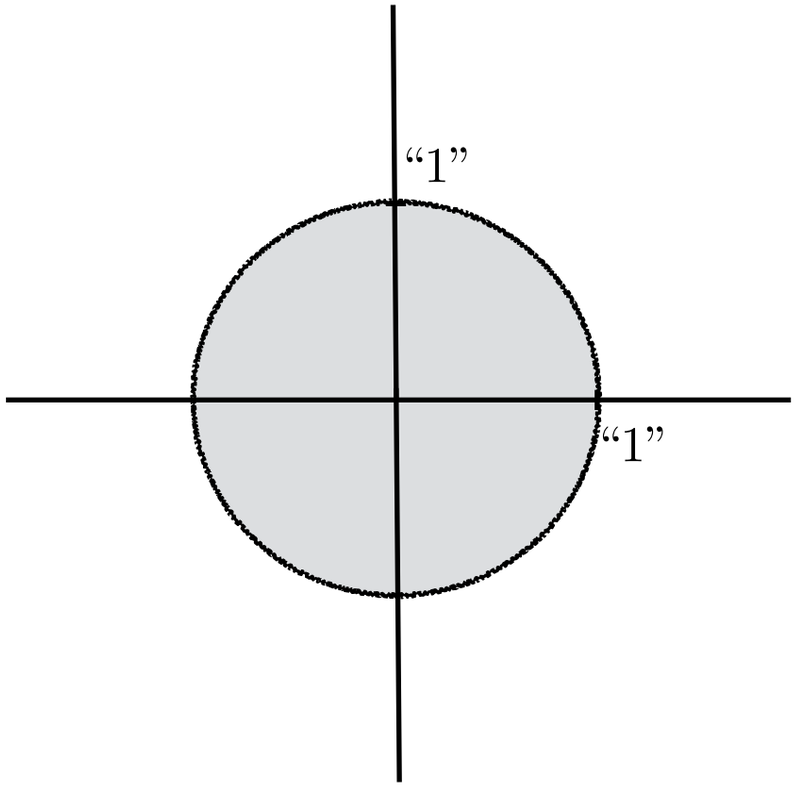

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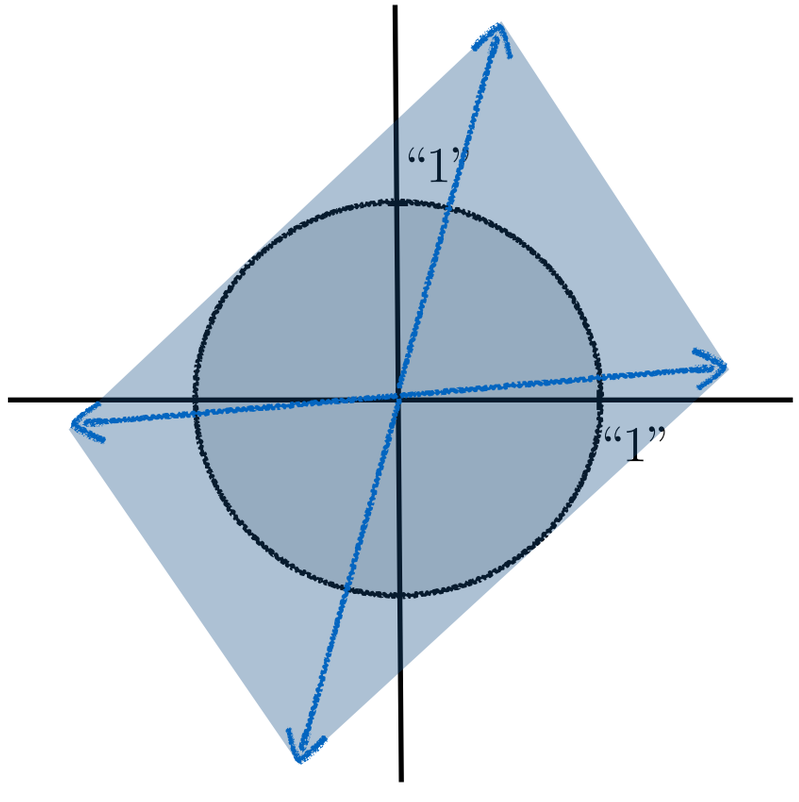

WGC

$$|\vec{z}| = \text{“1”}$$

$$\cap$$

$$\text{Convex Hull}\left\{\vec{z}_{p1}, \vec{z}_{p2}\right\}$$

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- **A set of instantons which satisfies the convex hull condition does not yield trans-Planckian field ranges (in controlled regimes).**

$$(\Delta\phi)_{max} \leq \frac{2\pi}{m_{min}}$$

Brown et al. '15

How to evade this “no-go”?

1. Use “spectator” instantons

A possible loophole

The WGC requires f·m<1 for ONE instanton, but not for ALL

$$V = e^{-m}\left[1-\cos\left(\frac{\Phi}{F}\right)\right] + e^{-M}\left[1-\cos\left(\frac{\Phi}{f}\right)\right]$$

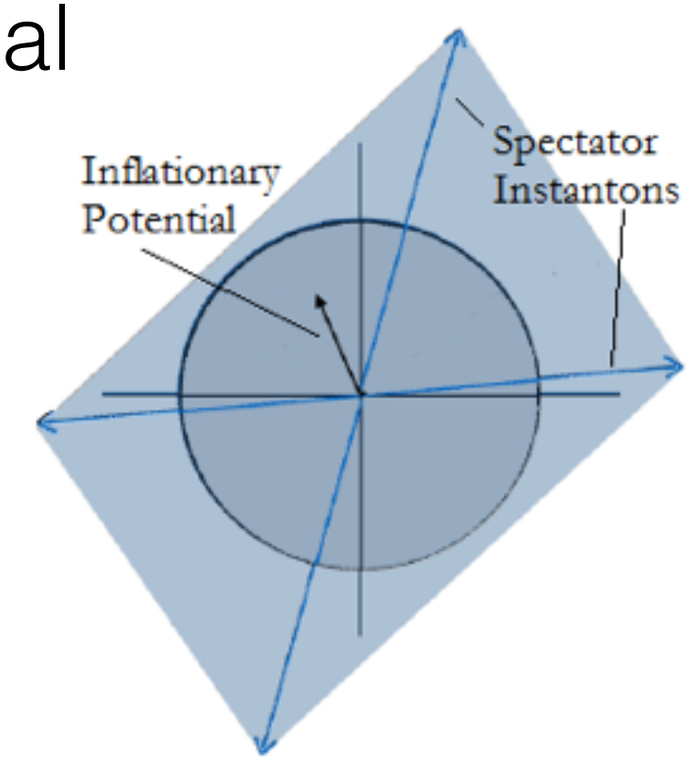
With $1 < m \ll M, \qquad F \gg M_P > f, \qquad M \times f \ll 1$

- The second instanton fulfills the WGC, but is negligible, an “spectator”. Inflation is governed by the first term.

Possible implementation in
Hebecker et al. ’15
…needs further exploration

A possible loophole

- In the presence of “spectator” (negligible) instantons that fulfill the WGC, dominant instantons can generate an inflationary potential



Montero et al. ‘15
Brown et al. ’15
Bachlechner et al. ‘15

- What is the origin of the spectator instantons?
  - Could have different origin than the inflationary ones
  - Could be bound states of inflationary ones?
- In tension with “strong” versions of the WGC.

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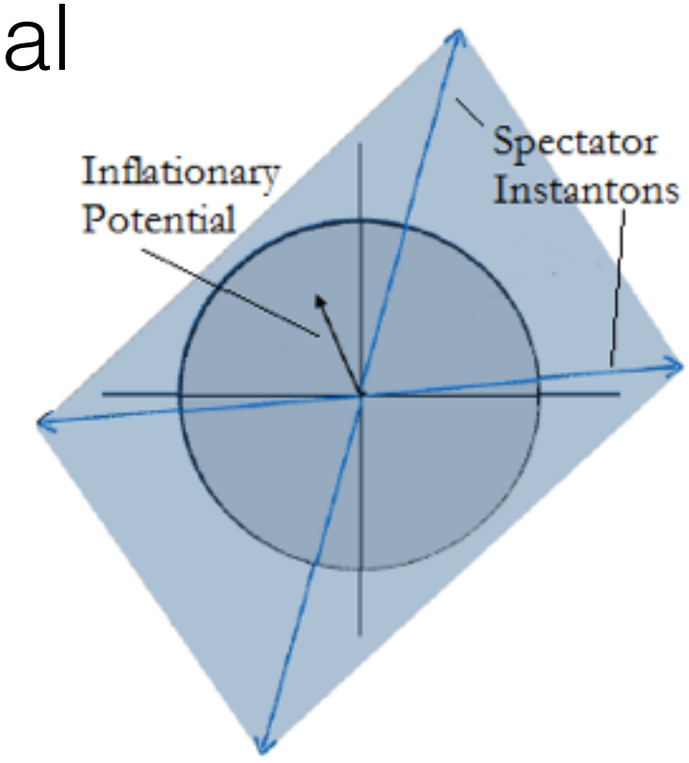

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Need explicit examples!

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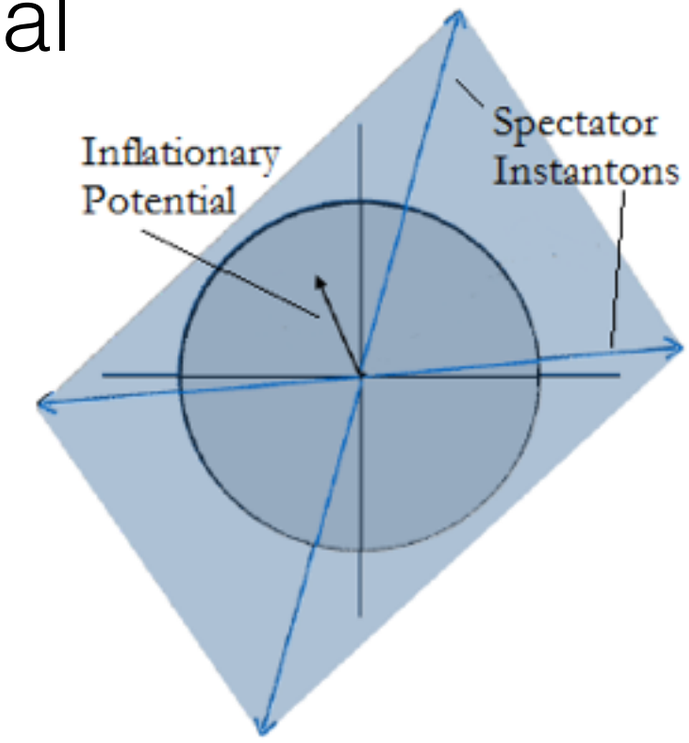

- What is the origin of the spectator instantons?
  - In SUSY setups, spectators could have extra zero-modes that prevent them from entering the super-potential
  - They enter non-holomorphic couplings and become specially relevant after ~~SUSY~~

How to evade this “no-go”?

2. Give up parametric control: extra-natural inflation

Extra-Natural Inflation

Arkani-Hamed, Cheng, Creminelli, Randall

- 5D U(1) gauge theory compactified on a circle:

$$\delta V(\phi) = \frac{3(-1)^S}{4\pi^2}\frac{1}{(2\pi R)^4}\sum_{n\in\mathbb{Z}} c_n e^{-2\pi n R m_5}\,\mathrm{Re}\,e^{in\phi/f}$$

$$c_n(2\pi R m_5) = \frac{(2\pi R m_5)^2}{3n^3} + \frac{2\pi R m_5}{n^4} + \frac{1}{n^5},$$

  - KK one-loop potential
  - WGC $\implies$ $2\pi R\, m_5 \cdot f \leq 1$

- Give up parametric control, rely on n^{-5} suppressions

$$f = \frac{1}{2\pi R\, g_5} \gg 1\ ; \qquad R \cdot m_5 \ll 1$$

Extra-Natural Inflation

- The limit $f \sim 1/Rg \gg 1$ clashes with the "magnetic" WGC:

$$\begin{array}{c} M_{mag} \sim \Lambda/g^2 \\ Q_{mag} \sim 1/g \end{array} \implies \frac{M_{mag}}{Q_{mag}} \leq M_p \Leftrightarrow \Lambda \leq g M_p \ll 1/R$$

- Can one use (KNP)-alignment to avoid this clash?
  - Use two U(1)s in 5d, with particles of charges (1,0) & (N,1)

$$V = V_0 \left[1 - \cos \frac{A}{f_A}\right] + \tilde{V}_0 \left[1 - \cos\left(\frac{NA}{f_A} + \frac{B}{f_B}\right)\right]$$

  - If $N \gg 1$, then $f_{eff} \sim N f_B$. No need to take $f_{A/B} \gg M_p$

de la Fuente, Saraswat, Sundrum

- Problem: $N \gg 1$ can lead to strong coupling $gN \sim 1$
- Problem: Unknown magnetic spectrum. Does it satisfy WGC?

Conclusions

Conclusions

- Weak Gravity Conjecture applies to (a large class of) axions
- WGC not addressed by most attempts to generate natural inflation (N-flation, KNP, kinetic alignment)
- A set of instantons which satisfies the WGC does not yield trans-Planckian (parametrically controlled) field ranges
- Evading the WGC would require some new insight:
  - Sub-dominant “spectator” instantons
  - Give up parametric control: extra-natural inflation
- These possibilities require further study

Thanks