

The Landscape of the Free Fermionic String Vacua

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Outline

- Heterotic String Phenomenology
- Free Fermionic Construction
- $SO(10)$ GUTs
- Flipped $SU(5)$ Models
- Classification
- Recent and Future Work

Free Fermionic Construction

- 4D Theory
- $N = 1$ Supersymmetry
- 3 Generation Standard Model Fermions
- $SO(10)$ GUTs
- Absence of exotic states

Properties

- Conformally invariance
- Decoupling left and right moving modes
- $D = 4$ theory

Result

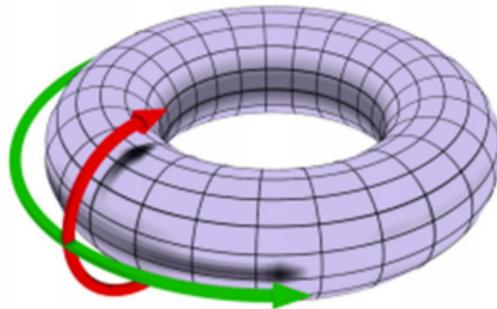
- $C_L = -26 + 11 + D + \frac{D}{2} + \frac{N_{f_L}}{2} = 0$
 \implies 18 left-moving real fermions
- $C_R = 0$
 \implies 44 right-moving real fermions

Free Fermionic Construction

- Partition function is used to include all physical states

$$Z = \sum_{\alpha, \beta} C \binom{\alpha}{\beta} Z [\alpha, \beta]$$

- Taking the one-loop partition function transforms the worldsheet into a torus.



Free Fermionic Construction

$$\alpha = \left\{ \psi^{1,2}, \chi^i, y^i, \omega^i | \bar{y}^i, \bar{\omega}^i, \bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}, \bar{\phi}^{1,\dots,8} \right\}$$

Where $i = 1, \dots, 6$

- Left-movers

- X_L^μ , $\mu = 1, 2$ 2 transverse coordinates
- ψ_L^μ , $\mu = 1, 2$ The fermionic partners
- Ω^j , $j = 1, \dots, 18$ 18 internal real fermions

- Right-movers

- X_R^μ , $\mu = 1, 2$ 2 transverse coordinates
- $\bar{\Omega}^j$, $j = 1, \dots, 44$ 44 internal real fermions

Free Fermionic Construction

- ABK Rules
 - $\sum_i m_i b_i = 0$
 - $N_{ij} \cdot b_i \cdot b_j = 0 \bmod 4$
 - $N_i \cdot b_i \cdot b_i = 0 \bmod 8$
 - $1 \in \Xi$
 - Even number of fermions
- One-Loop Phases
 - $C \binom{b_i}{b_j} = \pm 1 \text{ or } \pm i$
- GSO Projection
 - $e^{i\pi b_i \cdot F_\alpha} |s\rangle_\alpha = \delta_\alpha C \binom{\alpha}{b_i}^* |s\rangle_\alpha$
- Virasoro Condition
 - $M_L^2 = -\frac{1}{2} + \frac{\alpha_L^2}{8} + \sum v_L = -1 + \frac{\alpha_R^2}{8} + \sum v_R = M_R^2$

[I.Antoniadis et al, Nuclear Physics B (1987) 78-108]

$SO(10)$ Models

Basis Vectors

- $v_1 = 1 = \{\psi^\mu, \chi^{1,\dots,6}, y^{1,\dots,6}, \omega^{1,\dots,6} | \bar{y}^{1,\dots,6}, \bar{\omega}^{1,\dots,6}, \bar{\eta}^{1,2,3}, \bar{\psi}^{1,\dots,5}, \bar{\phi}^{1,\dots,8}\}$
- $v_2 = S = \{\psi^\mu, \chi^{1,\dots,6}\}$
- $v_{2+i} = e_i = \{y^i, \omega^i | \bar{y}^i, \bar{\omega}^i\} \quad i = 1, \dots, 6$
- $v_9 = b_1 = \{\chi^{34}, \chi^{56}, y^{34}, y^{56} | \bar{y}^{34}, \bar{y}^{56}, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}$
- $v_{10} = b_2 = \{\chi^{12}, \chi^{56}, y^{12}, y^{56} | \bar{y}^{12}, \bar{y}^{56}, \bar{\eta}^2, \bar{\psi}^{1,\dots,5}\}$
- $v_{11} = z_1 = \{\bar{\phi}^{1,\dots,4}\}$
- $v_{12} = z_2 = \{\bar{\phi}^{5,\dots,8}\}$

Gauge Group

$$SO(10) \times U(1)_1 \times U(1)_2 \times U(1)_3 \times SO(8)_1 \times SO(8)_2$$

$SO(10)$ Breaking Basis Vectors

Pati-Salam Models

- $v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$

Flipped $SU(5)$ Models

- $v_{12} = \alpha = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,4} = \frac{1}{2}, \bar{\phi}^5 = 1\}$

$SU(4) \times SU(2) \times U(1)$ Models

- $v_{13} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$
- $v_{14} = \beta = \{\bar{\psi}^{4,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,6} = \frac{1}{2}\}$

Standard-Like Models

- $v_{12} = \alpha = \{\bar{\psi}^{4,5}, \bar{\phi}^{1,2}\}$
- $v_{13} = \beta = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,4} = \frac{1}{2}, \bar{\phi}^5 = 1\}$

Flipped $SU(5)$ Models

There are 3 basis vectors that can break the $SO(10)$ gauge group giving rise to the flipped $SU(5)$ models. The possible choices of α are then given by:

- $\alpha_1 = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,4} = \frac{1}{2}, \bar{\phi}^5 = 1\}$
- $\alpha_2 = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,\dots,8} = \frac{1}{2}\}$
- $\alpha_3 = \{\bar{\eta}^{1,2,3} = \frac{1}{2}, \bar{\psi}^{1,\dots,5} = \frac{1}{2}, \bar{\phi}^{1,2} = \frac{1}{2}, \bar{\phi}^{5,6} = \frac{1}{2}, \bar{\phi}^8 = 1\}$

The matter content are encoded in the 16 and $\overline{16}$ representations of $SO(10)$ decomposed under $SU(5) \times U(1)$

$$\begin{aligned}\mathbf{16} &= (\bar{\mathbf{5}}, -\frac{3}{2}) + (\mathbf{10}, +\frac{1}{2}) + (\mathbf{1}, +\frac{5}{2}), \\ \mathbf{\overline{16}} &= (\mathbf{5}, +\frac{3}{2}) + (\overline{\mathbf{10}}, -\frac{1}{2}) + (\mathbf{1}, -\frac{5}{2}).\end{aligned}$$

The Standard Model Particles

We can decompose the flipped $SU(5)$ representation under $SU(3) \times SU(2) \times U(1)$ as

$$(\bar{\mathbf{5}}, -\frac{3}{2}) = (\bar{3}, 1, -\frac{2}{3})_{u^c} + (1, 2, -\frac{1}{2})_L,$$

$$(\mathbf{10}, +\frac{1}{2}) = (3, 2, +\frac{1}{6})_Q + (\bar{3}, 1, +\frac{1}{3})_{d^c} + (1, 1, 0)_{\nu^c},$$

$$(\mathbf{1}, +\frac{5}{2}) = (1, 1, +1)_{e^c},$$

GGSO Coefficients

$$(v_i | v_j) = \begin{pmatrix} S & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & b_1 & b_2 & z_1 & z_2 & \alpha \\ S & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ e_1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ e_2 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ e_3 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ e_4 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ e_5 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ e_6 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ b_1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 1/2 \\ b_2 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & -1/2 \\ z_1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ z_2 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1/2 \\ \alpha & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

Where $C(v_i | v_j) = e^{i\pi(v_i | v_j)}$ $(b_i | b_j) \in \{-\frac{1}{2}, 0, \frac{1}{2}, 1\}$

GGSO Coefficients

	S	e_1	e_2	e_3	e_4	e_5	e_6	b_1	b_2	z_1	z_2	α
S												
e_1		0 ₁	1 ₂	1 ₃	1 ₄	0 ₅	0 ₆		1 ₇		1 ₈	
e_2			1 ₉	1 ₁₀	1 ₁₁	0 ₁₂	0 ₁₃		1 ₁₄		0 ₁₅	
e_3				0 ₁₆	1 ₁₇	0 ₁₈		0 ₁₉	1 ₂₀		0 ₂₁	
e_4					0 ₂₂	0 ₂₃		1 ₂₄	0 ₂₅		1 ₂₆	
e_5						0 ₂₇	0 ₂₈	1 ₂₉	0 ₃₀		0 ₃₁	
e_6							1 ₃₂	1 ₃₃	0 ₃₄		1 ₃₅	
b_1								0 ₃₆		1 ₃₇	1/2 ₃₈	
b_2									0 ₃₉	1 ₄₀	-1/2 ₄₁	
z_1										0 ₄₂	0 ₄₃	
z_2											1/2 ₄₄	
α												

Enhancements

Depending on the choices of the projection coefficients, extra gauge bosons may arise. These arise with any linear combination of z_1 , z_2 and α , which are massless. Such as

$$\mathbf{G} = \left\{ \begin{array}{l} z_1, \quad z_2, \quad z_1 + z_2, \quad \alpha, \\ \alpha + z_1, \quad \alpha + z_2, \quad \alpha + z_1 + z_2, \quad x \end{array} \right\}$$

Where

$$x = 2\alpha + z_1 = \{\bar{\psi}^{1,\dots,5}, \bar{\eta}^{1,2,3}\}$$

Observable Gauge Group Enhancements

x is a sector which can enlarge the observable gauge group.
Enhancement takes place when the following conditions are satisfied

Sector Condition	
$(z_1 + 2\alpha e_i) = (z_1 + 2\alpha z_k) = 0$	
Enhancement Condition	Resulting Enhancement
$(z_1 + 2\alpha \alpha) = (z_1 + 2\alpha b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3$ $\longrightarrow SU(6) \times SU(2) \times U(1)^2$
$(z_1 + 2\alpha \alpha) \neq (z_1 + 2\alpha b_2)$	$SU(5)_{obs} \times U(1)_5 \times U(1)_1 \times U(1)_2 \times U(1)_3$ $\longrightarrow SO(10) \times U(1)^3$

The pre-stated conditions hold for all $i = 1, \dots, 6$.

Chiral Matter

The chiral matter spectrum arises from the twisted sectors. The chiral spinorial representations of the observable $SU(5) \times U(1)$ arise from the sectors

$$\begin{aligned} B_{pqrs}^{(1)} &= S + b_1 + pe_3 + qe_4 + re_5 + se_6, \\ &= \{\psi^\mu, \chi^{12}, (1-p)y^3\bar{y}^3, p\omega^3\bar{\omega}^3, (1-q)y^4\bar{y}^4, q\omega^4\bar{\omega}^4, \\ &\quad (1-r)y^5\bar{y}^5, r\omega^5\bar{\omega}^5, (1-s)y^6\bar{y}^6, s\omega^6\bar{\omega}^6, \bar{\eta}^1, \bar{\psi}^{1,\dots,5}\}, \end{aligned}$$

$$B_{pqrs}^{(2)} = S + b_2 + pe_1 + qe_2 + re_5 + se_6,$$

$$B_{pqrs}^{(3)} = S + b_3 + pe_1 + qe_2 + re_3 + se_4,$$

Where $p, q, r, s = 0, 1$ and $b_3 = b_1 + b_2 + 2\alpha + z_1$.

Projectors

The states in the sector $B_{pqrs}^{(1)}$ can be projected out of the spectrum by the GGSO projection of the vectors e_1, e_2, z_1 and z_2 . Similarly for all sectors, we can define a projector P such that the states survive when $P = 1$ and are projected out when $P = 0$:

$$P_{pqrs}^{(1)} = \frac{1}{16} \left(1 - C\left(\frac{e_1}{B_{pqrs}^{(1)}}\right) \right) \cdot \left(1 - C\left(\frac{e_2}{B_{pqrs}^{(1)}}\right) \right) \cdot \left(1 - C\left(\frac{z_1}{B_{pqrs}^{(1)}}\right) \right) \cdot \left(1 - C\left(\frac{z_2}{B_{pqrs}^{(1)}}\right) \right)$$

$$P_{pqrs}^{(2)} = \frac{1}{16} \left(1 - C\left(\frac{e_3}{B_{pqrs}^{(2)}}\right) \right) \cdot \left(1 - C\left(\frac{e_4}{B_{pqrs}^{(2)}}\right) \right) \cdot \left(1 - C\left(\frac{z_1}{B_{pqrs}^{(2)}}\right) \right) \cdot \left(1 - C\left(\frac{z_2}{B_{pqrs}^{(2)}}\right) \right)$$

$$P_{pqrs}^{(3)} = \frac{1}{16} \left(1 - C\left(\frac{e_5}{B_{pqrs}^{(3)}}\right) \right) \cdot \left(1 - C\left(\frac{e_6}{B_{pqrs}^{(3)}}\right) \right) \cdot \left(1 - C\left(\frac{z_1}{B_{pqrs}^{(3)}}\right) \right) \cdot \left(1 - C\left(\frac{z_2}{B_{pqrs}^{(3)}}\right) \right)$$

Projectors

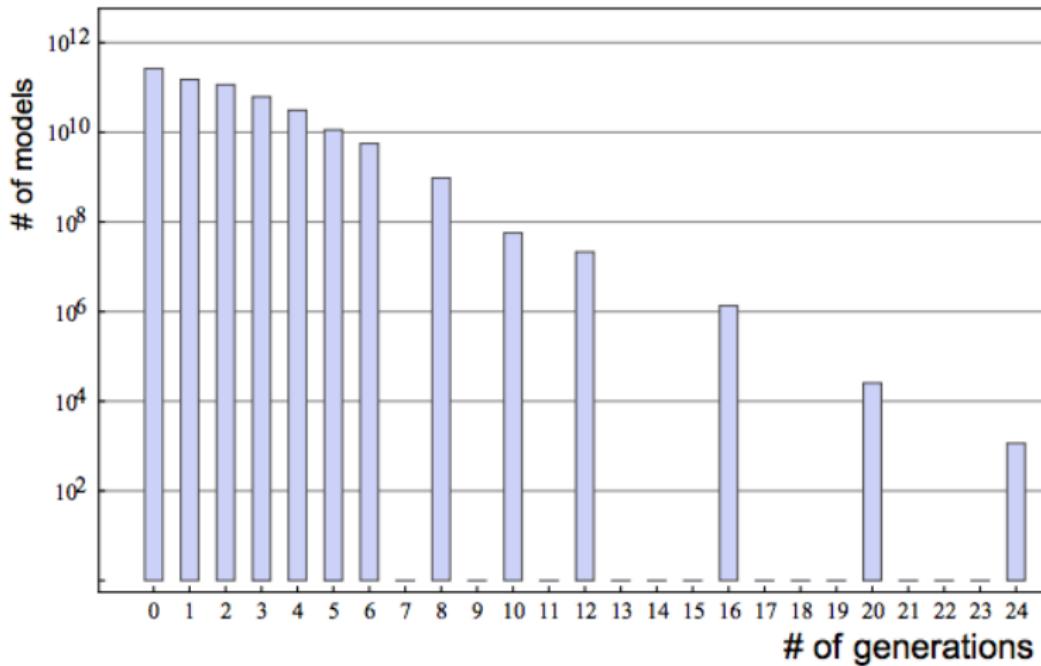
These projectors can be expressed as a system of linear equations with p , q , r and s as unknowns. The solutions of a specific system of equations yield the different combinations of p , q , r and s for which sectors survive the GSO projections.

$$\begin{pmatrix} (e_1|e_3) & (e_1|e_4) & (e_1|e_5) & (e_1|e_6) \\ (e_2|e_3) & (e_2|e_4) & (e_2|e_5) & (e_2|e_6) \\ (z_1|e_3) & (z_1|e_4) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_3) & (z_2|e_4) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_1|b_1) \\ (e_2|b_1) \\ (z_1|b_1) \\ (z_2|b_1) \end{pmatrix},$$

$$\begin{pmatrix} (e_3|e_1) & (e_3|e_2) & (e_3|e_5) & (e_3|e_6) \\ (e_4|e_1) & (e_4|e_2) & (e_4|e_5) & (e_4|e_6) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_5) & (z_1|e_6) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_5) & (z_2|e_6) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_3|b_2) \\ (e_4|b_2) \\ (z_1|b_2) \\ (z_2|b_2) \end{pmatrix},$$

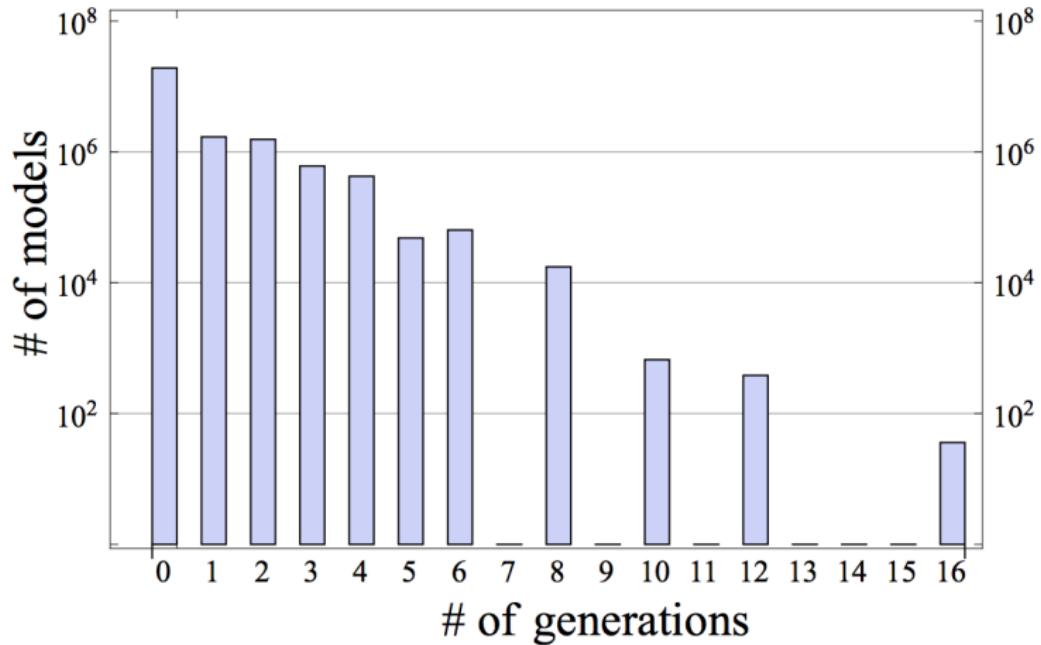
$$\begin{pmatrix} (e_5|e_1) & (e_5|e_2) & (e_5|e_3) & (e_5|e_4) \\ (e_6|e_1) & (e_6|e_2) & (e_6|e_3) & (e_6|e_4) \\ (z_1|e_1) & (z_1|e_2) & (z_1|e_3) & (z_1|e_4) \\ (z_2|e_1) & (z_2|e_2) & (z_2|e_3) & (z_2|e_4) \end{pmatrix} \begin{pmatrix} p \\ q \\ r \\ s \end{pmatrix} = \begin{pmatrix} (e_5|b_3) \\ (e_6|b_3) \\ (z_1|b_3) \\ (z_2|b_3) \end{pmatrix}.$$

SO(10) Classification



[Figure 1: John Rizos, arXiv:1105.1243]

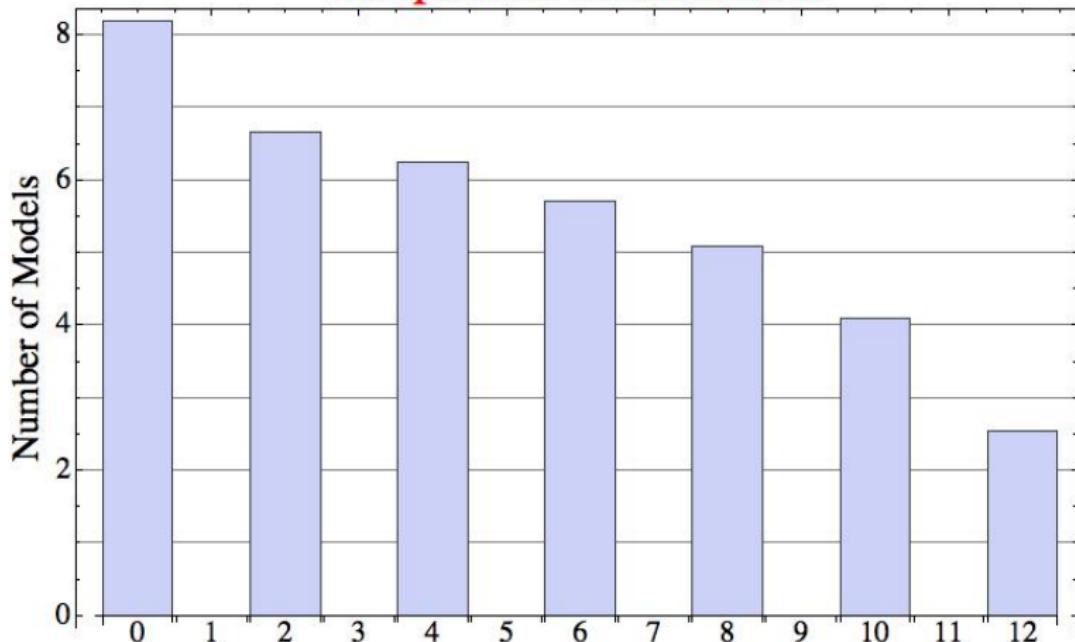
Pati-Salam Classification



[Figure 3: B.Assel et al, arXiv:1007.2268]

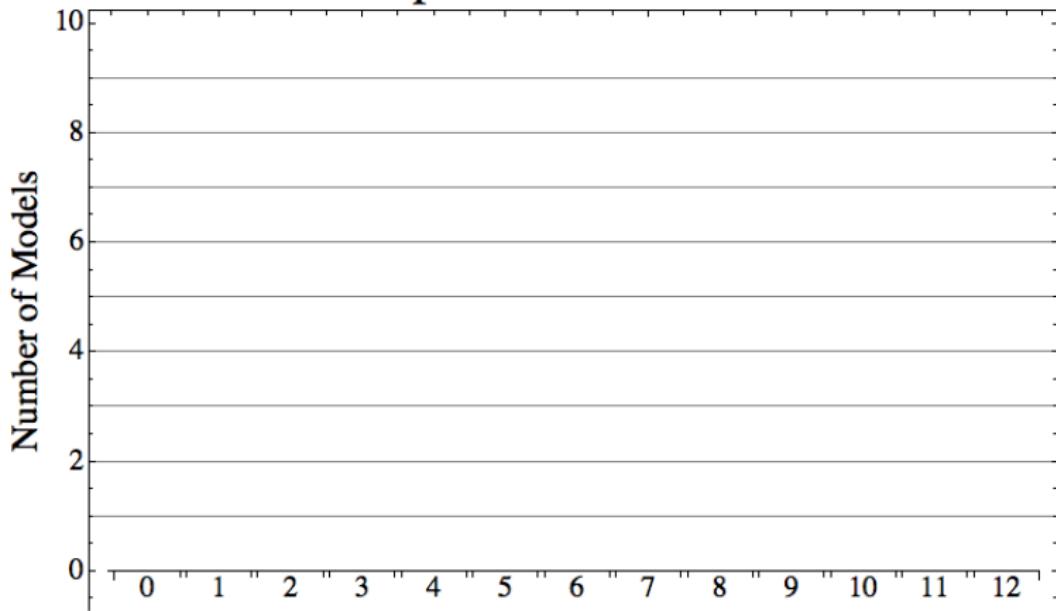
Flipped $SU(5)$ Classification

Exophobic Generations



[Figure 2: H. Sonmez et al, arXiv:1403.4107]

Exophobic Generations



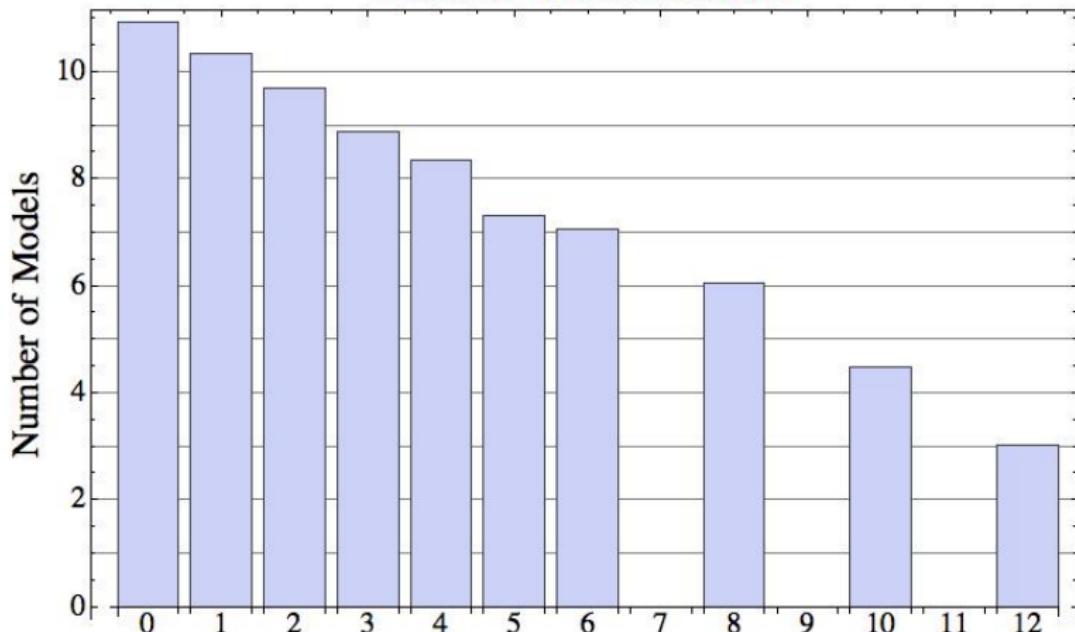
[H. Sonmez et al, arXiv:1412.2839]

SLM Classification



Flipped $SU(5)$ Classification

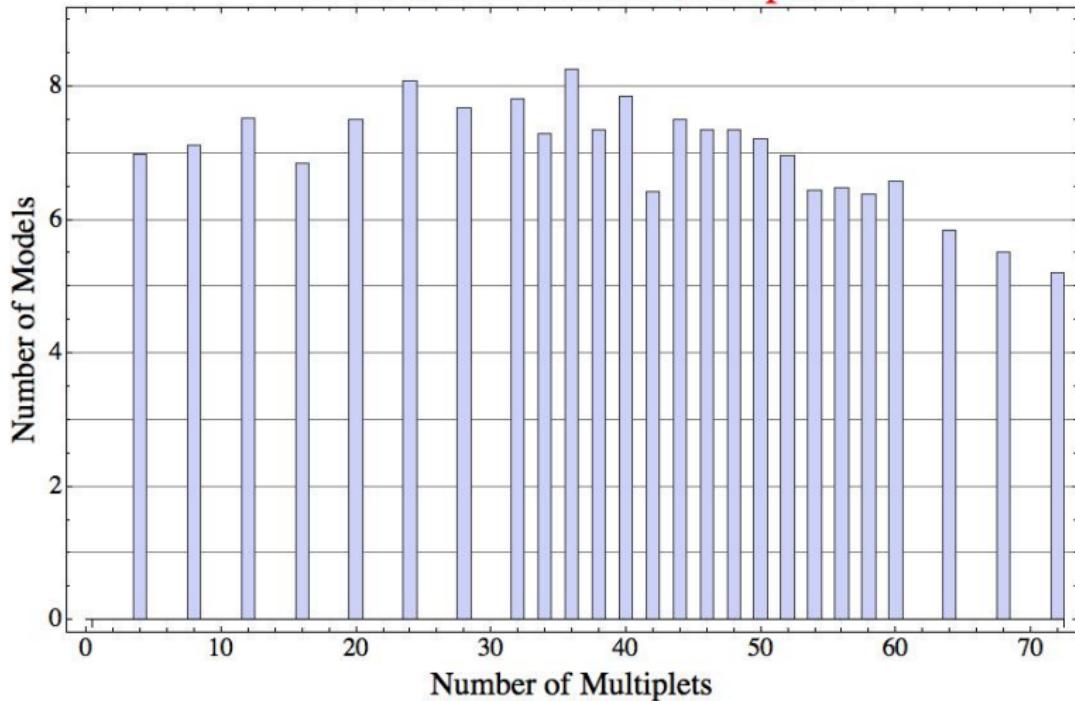
Exotic Generations



[Figure 1: H. Sonmez et al, arXiv:1403.4107]

Flipped $SU(5)$ Classification

3 Generation Exotic Multiplets



[Figure 3: H. Sonmez et al, arXiv:1403.4107]

Flipped $SU(5)$ Classification

	Constraints	Total models in sample	Probability	Estimated number of models in class
	No Constraints	10000000000000	1	1.76×10^{13}
(1)	+ No Enhancements	762269298719	7.62×10^{-1}	1.34×10^{13}
(2)	+ Anomaly Free Flipped $SU(5)$	139544182312	1.40×10^{-1}	2.45×10^{12}
(3)	+ 3 Generations	738045321	7.38×10^{-4}	1.30×10^{10}
(4a)	+ SM Light Higgs	706396035	7.06×10^{-4}	1.24×10^{10}
(4b)	+ Flipped $SU(5)$ Heavy Higgs	46470138	4.65×10^{-5}	8.18×10^8
(5)	+ SM Light Higgs + & Heavy Higgs	43624911	4.36×10^{-5}	7.67×10^8
(6a)	+ Minimal Flipped $SU(5)$ Heavy Higgs	42310396	4.23×10^{-5}	7.44×10^8
(6b)	+ Minimal SM Light Higgs	25333216	2.53×10^{-5}	4.46×10^8
(7)	+ Minimal Flipped $SU(5)$ Heavy Higgs + & Minimal SM Light Higgs	24636896	2.46×10^{-5}	4.33×10^8
(8)	+ Minimal Exotic States	1218684	1.22×10^{-6}	2.14×10^7

[Figure 3: H. Sonmez et al, arXiv:1403.4107]

Recent and Future Work

Recent and Future Work:

- ① $SU(4) \times SU(2)_L \times U(1)_L$
- ② $SU(3)_C \times U(1)_C \times SU(2)_L \times U(1)_L$
- ③ $SU(3)_C \times U(1)_C \times SU(2)_L \times SU(2)_R$
- ④ $SU(5) \times U(1)$
- ⑤ Non-Supersymmetric Heterotic String Vacua

THANK YOU