Stability of Supersymmetric Moduli in Kähler Uplifted Vacua

in collaboration with A. Achúcarro and P. Ortiz

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Outline

Stability of the complex structure/dilaton sector

Type-IIB flux compactifications

KKLT framework: *three-step approach*

1. The complex structure and the dilaton are stabilised by the background fluxes at a large energy scale compared to the SSB scale.
2. The Kähler moduli are stabilised at a lower energies by the non-perturbative effects at an AdS supersymmetric critical point.
3. Supersymmetry is broken at low energies by including $\overline{D3}$, $D-$terms, matter fields ($F-$term breaking)... , to *uplift* the vacuum to dS.

(*Giddings et al. '02, Kachru et al. '03*)

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Metastability of the **complex structure/dilaton sector**

- In the KKLT framework it is guaranteed by the large hierarchy between the c.s. masses and the supersymmetry breaking scale.
- This large hierarchy requires the fine tunning of $W_0 \equiv W_{flux}|_{Z_0} \ll 1$ (*Gallego et al. '10, Baumann et al. '14*)

$$m_{3/2}^2 = e^K |W_0|^2 \ll M_{flux}^2 \sim \mathcal{O}(1)$$

Stability of the complex structure/dilaton sector

Type-IIB flux compactifications

Large Volume Scenarios

Balasubramanian et al. '05, Conlon et al. '05

- The complex structure and the dilaton are stabilised by the background fluxes at a supersymmetric configuration.
- The Kähler moduli are stabilised by the combined effect of α' and non-perturbative corrections.
    - The volume of the internal space $\mathcal{V}$ is exponentially large.
    - The configuration is a non-supersymmetric AdS critical point.
- Uplifting is still required to obtain a positive cosmological constant.

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Metastability of the **complex structure/dilaton sector**

- The v.e.v. of the flux superpotential is not fine-tuned $W_0 \sim \mathcal{O}(1)$.
- The volume modulus has a small mass $m_{\mathcal{V}} \sim \mathcal{V}^{3/2}$,
- but there is no large mass hierarchy the c.s. sector and the rest of the Kähler moduli, $m_k \sim m_{3/2} \sim 1/\mathcal{V}$.

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Metastability of the **complex structure/dilaton sector**

- The scalar potential is an exponentially small deformation, $\mathcal{O}(\xi/\mathcal{V}^3)$, of the tree-level potential.
  - The zero-order potential is positive definite, $V_{tree} = e^K |D_\alpha W_{flux}|^2 \geq 0$.
  - SUSY configurations of the c.s. sector are stable at zero-order.
- The stability is *almost* guaranteed after adding the corrections,
- but massless fields at tree-level could become tachyonic.

Stability of the complex structure/dilaton sector

Type-IIB flux compactifications

Kähler uplifted dS vacua

Balasubramanian et al. '05, Westphal '07

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- There is no need to include additional sources of SUSY breaking.

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Metastability of the **complex structure/dilaton sector**

- The corrections of the tree-level potential are no longer exponentially suppressed,
- they could destabilise c.s. fields with small masses at tree-level.
- we need to characterise the tree-level mass spectrum of the c.s. moduli.

Type-IIB flux compactifications

Mass spectrum of the no-scale potential

Tree-level mass spectrum at a supersymmetric configuration

- The hessian in the the c.s and dilaton sector takes the simple form

$$V_{tree} = \mathrm{e}^K |D_I W_{flux}|^2 \quad \Longrightarrow \quad \mathcal{H}_{tree} = \mathrm{e}^K \Big(|W_0| \cdot \mathbb{1} + \mathcal{M} \Big)^2$$

- $\mathcal{M}$ is the $2(h_{2,1}+1) \times 2(h_{2,1}+1)$ fermion mass matrix

$$\mathcal{M} = \begin{pmatrix} 0 & D_I D_J W \\ D_{\bar{I}} D_{\bar{J}} \overline{W} & 0 \end{pmatrix}.$$

- We can express the scalar masses in terms of the fermionic spectrum. After normalising all masses by $m_{3/2}$:

$$\mathcal{M} \to \mathrm{diag}(\pm m_\lambda), \qquad \mu^2_{\pm\lambda} = (1 \pm m_\lambda)^2$$

$$K = -2 \log \mathcal{V} - \log[-\mathrm{i}(\tau - \bar{\tau})] - \log \mathrm{i} \int_{CY} \Omega \wedge \bar{\Omega},$$

$$W_{flux} = \int_{CY} G_3 \wedge \Omega$$

Statistical analysis

Random Matrix Theory

$$\mathcal{M} = \begin{pmatrix} 0 & D_I D_J W \\ D_{\bar{I}} D_{\bar{J}} \overline{W} & 0 \end{pmatrix}.$$

(*Denef et al. '04, '05, Marsh et al. '12, Bachlechner et al. '12*)

Joint probability distribution of the entries:

- Components of the fermion mass matrix

$$\mathbb{E}[D_I D_J W] = 0 \qquad \mathbb{E}[D_I D_J W \,.\, D_{\bar{K}} D_{\bar{L}} \overline{W}] = \frac{m_h^2 m_{3/2}^2}{4N} \left(\delta_{I\bar{K}}\delta_{J\bar{L}} + \delta_{I\bar{L}}\delta_{J\bar{K}}\right)$$

- m_h measures the ratio between the typical fermion masses and $m_{3/2}$.
- No higher moments are needed (universality of RMT).

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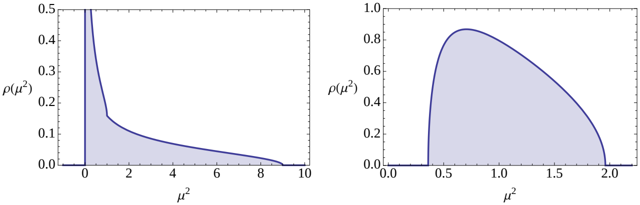
- With this choices $\mathcal{M}$ can be identified as an element of the Altland-Zirnbauer CI ensemble *(Altland '97)*.
- To leading order in $1/h_{2,1}$ the density function of fermion masses (in units of $m_{3/2}$) reads

$$\rho(m_\lambda) dm_\lambda = \frac{2}{\pi m_h^2} \sqrt{m_h^2 - m_\lambda^2}\, dm_\lambda.$$

Statistical analysis

Mass spectrum of complex structure moduli

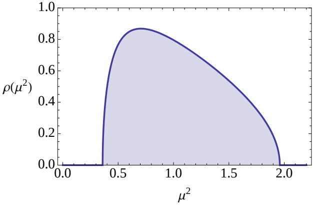


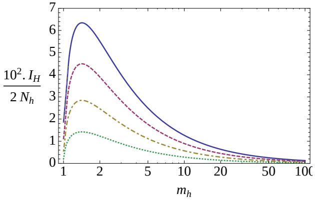
- The plots represent the typical scalar mass spectrum of the complex structure sector at tree-level, with $m_h = 2$ (left), $m_h = 0.4$ (right).
- m_h sets the ratio between the mass scale of the truncated sector and $m_{3/2}$ (e.g.: $m_h \sim \mathcal{O}(1)$ in LVS)

$$m_h \sim \frac{|D_I D_J W_{flux}|}{|W_{flux}|} \sim \mathcal{O}(1 - 10)$$

- The spectrum contains no tachyons regardless of the value of m_h.
- If $m_h > 1$ typical configurations might contain arbitrarily light scalar fields.
- If $m_h < 1$ all fermions are lighter than the gravitino and the scalar mass spectrum develops a mass gap.

(P. Ortiz and KS '14)





- We need to characterise the effect α' and non-perturbative corrections on the mass spectrum of the c.s. sector.

$$\mu^2_{\pm\lambda} = \mu^2_{\pm\lambda}|_{tree} + \mathcal{O}(\xi/\mathcal{V})$$

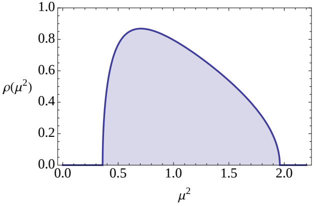
- the squared masses, (normalised by $m^2_{3/2}$), receive $\mathcal{O}(\xi/\mathcal{V})$ corrections.
- Light moduli, $\mu^2 \lesssim \xi/\mathcal{V}$, may be destabilised.

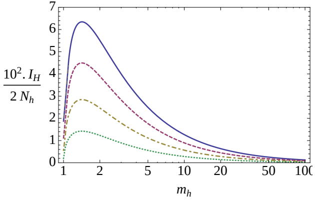
$$K = -2\log(\mathcal{V} + \tfrac{\xi}{2}) - \log[-\mathrm{i}(\tau - \bar{\tau})] - \log \mathrm{i} \int_{CY} \Omega \wedge \bar{\Omega},$$

$$W = W_{flux} + \sum_{\alpha} A_\alpha \, \mathrm{e}^{-a_\alpha T_\alpha}$$

Statistical analysis

New Branch of Vacua





- We have calculated the percentage of light fields on the c.s. sector, i.e. $\mu^2 \leq \xi/\mathcal{V} \sim 10^{-2} - 10^{-4}$, (right).
- There are three stability regimes:
  1. For $m_h \gg 1$ a large mass hierarchy protects stability (KKLT case).
  2. For $\xi/\mathcal{V} \ll 1$ we approach the no-scale limit (in LVS $\xi/\mathcal{V} \sim 10^{-10}$).
  3. **NEW REGIME:** For $m_h < 1$ the mass gap protects stability.

$$\mu^2_{\pm\lambda} = (1 \pm m_\lambda)^2$$

Engineering the fermion mass spectrum

"Type-IIB Attractor Equations"

Type-IIB Attractor Equations: *Denef et al. '04, Kallosh '05, Bellucci et al. '07*

At any supersymmetric critical point (τ_0, z_0^i) the quantised fluxes f_a, h_a can be decomposed in terms of the period vector $\Pi_a = (Z^I, F_I)$:

$$f - \tau_0\, h = \mathrm{i}\, \mathrm{e}^{K_{cs}} \left[a_0 \cdot \overline{\Pi}(z_0) - \bar{a}_i \cdot D_i \Pi(z_0) \right]$$

(Hodge decomposition of the fluxes)

$$f^I_{A,B} = \ell_s^{-2} \int_{A^I, B_I} F_3 \in \mathbb{Z}, \qquad Z^I = \int_{A^I} \Omega, \qquad F_I = \int_{B_I} \Omega$$

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- Given a particular compactification, (i.e. a period vector Π_a),
- the fluxes obtained by the above formula ensure that (τ_0, z_0^i) is supersymmetric with:

$$a_0 = W|_{\tau_0, z_0}, \qquad a_i = D_\tau D_i W|_{\tau_0, z_0} \qquad D_i D_j W|_{\tau_0, z_0} = f_{ijk}\; a_k$$

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We have some control over the properties of the fermion mass spectrum

- in order to have a large mass hierarchy between the c.s. sector and the SUSY breaking scale ($m_\lambda \gg 1$) we choose $|a_0|^2 \ll |a_i|^2$.
- Conversely, if $|a_0|^2 \gg |a_i|^2$ the fermion masses will be small compared to the gravitino mass ($m_\lambda \ll 1$).

Engineering the fermion mass spectrum

Given a CY compactification:

1. Choose a tentative field configuration $(\tilde{\tau}_0, \tilde{z}_0)$
2. and the expectation value of the superpotential $a_0 = W_0$.
3. Solve for the fluxes

$$\tilde{f} - \tilde{\tau}_0 \, \tilde{h} = \mathrm{i} \, \mathrm{e}^{K_{cs}} \, a_0 \cdot \overline{\Pi}(\tilde{z}_0)$$

- with this fluxes $(\tilde{\tau}_0, \tilde{z}_0)$ is a supersymmetric critical point,
- and all fermions in the c.s. secto have zero mass $\mathcal{M} = 0$.
- However $(\tilde{f}_a, \tilde{h}_a)$ will not be integers in general.

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   - However $(\tilde{f}_a, \tilde{h}_a)$ will not be integers in general.

- Find the closest quantised flux configuration $(\tilde{f}_a, \tilde{h}_a) \to (f_a, h_a)$.
- With these fluxes solve the SUSY critical point equations for the moduli (τ, z^i)

$$D_\tau W|_{\tau_0, z_0} = 0, \qquad D_i W|_{\tau_0, z_0} = 0$$

   - The resulting supersymmetric configuration (τ_0, z_0^i) has a light fermion mass spectrum $m_\lambda \lesssim 1$,
   - and thus, the tree-level mass spectrum of the c.s. sector will have mass gap of order $\mathcal{O}(m_{3/2})$.

Example: dS vacuum in the $P^4_{[1,1,1,6,9]}$ model

Candelas et al. '94, Denef Douglas and Florea '04

- The moduli space contains $h_{1,1} = 2$ Kähler and $h_{2,1} = 276$ c.s. moduli.

$$\mathcal{V} = \frac{1}{36}\left((T_1 + T_1)^{3/2} - (T_2 + T_2)^{3/2}\right)$$

- We consider the dynamics of the only two c.s. moduli $\{z^1, z^2\}$ invariant under the symmetry $\Gamma = \mathbb{Z}_6 \times \mathbb{Z}_{18}$.
  - The c.s. and flux configurations were constructed as described before.
  - the stabilisation of the Kähler sector following the method in *Rummel et al. '11, Louis et al. '12* for "swiss-cheese" Calabi-Yau's.

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$C_0 + ig_s^{-1}$	$< z^1 >$	$< z^2 >$	$< \mathcal{V} >$	$W_0 = m_{3/2}\mathcal{V}$	$< V >$
$-0.057 + i1.7$	$0.35 + i1.9$	$0.31 + i1.5$	376.1	6.7	8.7×10^{-9}

m_λ	$\mu^2_{+\lambda}$	$\mu^2_{-\lambda}$	$\mu^2_{t_1}$	$\mu^2_{t_2}$	$m^2_{3/2}$	$\xi/(2\mathcal{V})$
0.33	1.8	0.43	0.013	47.5	3.2×10^{-4}	1.9×10^{-3}
0.21	1.9	0.60	0.0028	42.4	$--$	$--$
0.071	1.1	0.89	$--$	$--$	$--$	$--$

- The masses are in units of $m_{3/2}$ and $\mathcal{V}$ in units of ℓ_s.

Conclusions

- We have discussed the perturbative stability of *complex structure moduli* at Kähler uplifted dS vacua.
- Using tools from RMT we have characterised the tree-level mass spectrum of the complex structure moduli sector.
  - when $W_0 \sim 1$ the tree-level mass spectrum typically contains a large fraction of light fields,
  - which can be destabilised by the α' and non-pertubative corrections.

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  - which can be destabilised by the α' and non-pertubative corrections.

- We have presented a **new class of metastable configurations** where
  - the chiral fermions in the c.s. sector are lighter than the gravitino.
  - The mass spectrum of the c.s. sector presents a mass gap,
  - which ensures metastability after adding corrections to the tree-level potential.
  - It does not require a large mass hierarchy
  - or the exponential suppression of the non-separable corrections.
- We have provided a systematic way to construct these vacua,
- and presented an example in the Calabi-Yau $P_{[11169]}$.