A new Look at the Axion Window through Mixing Axions

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based on 1503.01015, 1503.02965 [hep-th] with G. Shiu & F. Ye
1409.1236, in progress with J. Ecker & G. Honecker

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Axions in a Nutshell

Peccei-Quinn, Wilczek, Weinberg (1977), KSVZ (1979), DFSZ (1979) , . . .

axions: CP-odd $\mathbb{R}$ scalars with $a \to a + \varepsilon$

shift symm. broken by nonpert. effects $\to$ cosine-potential:

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a \partial^\mu a - \Lambda^4 \left[1 - \cos\frac{a}{f_a}\right] \quad \Rightarrow \quad m_a^2 \sim \frac{1}{f_a^2}$$

$\rightsquigarrow$ suitable building blocks for particle physics + cosmology:

axion window

- Strong CP problem
- Dark Matter

$\Big\} \Rightarrow$ 10^9 GeV $< f_a < 10^{12}$ GeV

- Large Field Inflation $\Rightarrow$ $f_a > M_{Pl}$ for slow roll Freese-Frieman-Olinto (1990)

String Theory: plethora of axions from KK compact. of p-forms

see e.g. Witten (1984), Banks-Dine-Fox-Gorbatov (2003), Svrček-Witten (2006)

☞ BUT generically $f_a \sim M_{string}$ ↓ does not fit within either range

1 closed string axion is unable to solve either question

note: LVS scenario with $M_{string} \sim 10^{12}$ GeV Conlon (2006), Cicoli-Goodsell-Ringwald (2012)

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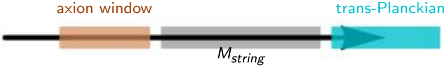

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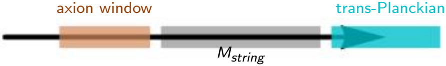

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Cracking a hard Nut

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  - ⋆ within chiral multiplet @ intersection point of two D-branes
  - ⋆ construct KSVZ or DFSZ models Honecker-WS (2013), Ecker-Honecker-WS (2014, soon) see e.g. Buchbinder-Constantin-Lukas (2014) for HE
  - ⋆ Decoupled from inflaton axion $\subset$ closed string sector

☞ $\exists$ no-go theorems and arguments forbidding $f > M_{Pl}$ (1 axion)
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☞ Question: $\exists$ alternative scenarios to obtain $f_a > M_{Pl}$?

Axions & String Theory

reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014)

- Closed string axions a^i from dim. red. of p-forms $C_{(p)}$ on $\mathcal{M}_{1,3} \times \mathcal{X}_6$ ($C_{(p)} \in$ RR-forms + NS 2-form in Type II)

$$a^i \equiv (2\pi)^{-1} \int_{\Sigma^i} C_{(p)}, \qquad p\text{−cycle } \Sigma^i \subset \mathcal{X}_6, \qquad i \in \left\{1, \ldots, \begin{array}{c} h_{11} \\ h_{21}+1 \end{array} \right\}$$

  Kinetic terms for p-forms $C_{(p)} \rightsquigarrow$ kinetic terms for a^i

- Including D-branes wrapping p-cycle Σ^i:
  - ⋆ D-brane Chern-Simons terms $\rightsquigarrow$ anomal. coupling "$a^i \, Tr(G \wedge G)$"
  - ⋆ Eucl. D-branes $\rightsquigarrow$ additional non-pert. effects (D-brane instantons)

- For a single D-brane wrapping p-cycle with $\Omega\mathcal{R}(\Sigma^i) \neq \Sigma^i$ $\rightsquigarrow$ a^i acquires Stückelberg charges under $U(1)$

reviews: Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12)

e.g. Type IIA D6-branes on $CY_3/\Omega\mathcal{R} \rightsquigarrow \Sigma^i = \Sigma^i_+ + \Sigma^i_-$

$$\int_{\Sigma^i_-} C_{(5)} \wedge F \neq 0 \qquad \rightsquigarrow \qquad \text{Stückelberg term with } a^i$$

An Effective Action...

w/ Shiu & Ye 1503.01015, 1503.02965 [hep-th]

- String Theory compactifications $\rightsquigarrow$ effective action with mixing axions

$$\mathcal{S}^{\rm eff}_{axion} = \int \left[\frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (\mathrm{d}a^i - k^i A) \wedge \star_4 (\mathrm{d}a^j - k^j A) - \frac{1}{8\pi^2} \left(\sum_{i=1}^{N} r_i a^i \right) \mathrm{Tr}(G \wedge G) + \mathcal{L}_{gauge} \right]$$

- 2 types of kinetic mixing
  - (1) metric mixing: $\mathcal{G}_{ij}$ is not diagonal see Bachlechner-Long-McAllister (2014/15)
  - (2) $U(1)$ mixing: $k^i \neq 0$ for some $i \in \{1, \ldots, N\}$
    gauged axions: $a^i \rightarrow a^i + k^i \chi, \quad A \rightarrow A + d\chi$
- $\mathrm{Tr}(G \wedge G)$-term associated to non-Abelian gauge group
  $\rightsquigarrow$ collective periodicity: $\sum_{i=1}^N r_i a^i \simeq \sum_{i=1}^N r_i a^i + 2\pi$
- axions couple to D-brane instantons
  $\rightsquigarrow$ individual periodicity: $a^i \rightarrow a^i + 2\pi\nu^i$, $\nu^i \in \mathbb{Z}$
  note: effective contributions of D-brane instantons to $\mathcal{S}^{\rm eff}_{axion}$ is model-dependent
  see e.g. Ibáñez-Uranga (2007, 2012), Blumenhagen-Cvetič-Kachru-Weigand (2009)

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To determine axion decay constants
To figure out axions eaten by $A_{U(1)}$ } $\Longrightarrow$ Diagonalise kinetic terms

eigenbasis for kinetic terms $\neq$ eigenbasis for potentials

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axionic directions with large f_a?

Note: different from N-flation Dimopoulos-Kachru-McGreevy-Wacker (2005),
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- 1 axion eaten by $U(1)$ gauge boson, $\perp$ axion ξ with decay constant:

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}$$

with $\lambda_\pm$ eigenvalues of $\mathcal{G}_{ij}$ and $M_{st}\equiv\sqrt{\lambda_+(k^+)^2+\lambda_-(k^-)^2}$

$$\cos\theta=\frac{\mathcal{G}_{11}-\mathcal{G}_{22}}{\lambda_+-\lambda_-},\quad \sin\theta=\frac{2\mathcal{G}_{12}}{\lambda_+-\lambda_-},\qquad \begin{pmatrix}k^+\\k^-\end{pmatrix}=\begin{pmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}\\ \sin\frac{\theta}{2} & -\cos\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}k^1\\k^2\end{pmatrix}$$

- Contour plot representation of f_ξ (in units $\sqrt{\mathcal{G}_{11}}$)

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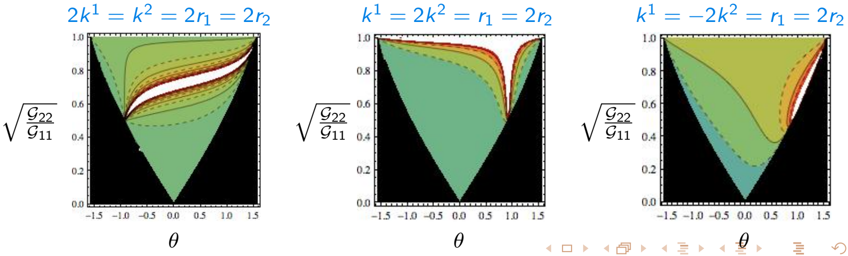

Conclusions

- Isotropy relations among moduli λ_- & λ_+ (white regions) $\rightsquigarrow f_\xi > M_{Pl}$ for mixing axions

- Explicit examples in String Theory: $\left\{ \begin{array}{l} \text{IIA w/ inters. D6-branes} \\ \text{IIB w/ inters. D7-branes} \end{array} \right.$
  $\longrightarrow$ more suitable examples are desired

Open issues

☞ No-go theorems for multiple axions??
Brown-Cottrell-Shiu-Soler (2015), Montero-Uranga-Valenzuela (2015), Junghans (2015),
Rudelius (2014/15), Hebecker-Mangat-Rompineve-Witkowski (2015), Bachlechner-Long-McAllister (2015)

☞ Validity of eff. description requires moduli stabilisation??
Conlon (2006), Choi-Jeong (2006), Hristov (2008), Higaki-Kobayashi (2011),
Cicoli-Dutta-Maharana (2014)

Conclusions

- Isotropy relations among moduli λ_- & λ_+ (white regions) $\rightsquigarrow f_\xi > M_{Pl}$ for mixing axions

- Explicit examples in String Theory: $\begin{cases} \text{IIA w/ inters. D6-branes} \\ \text{IIB w/ inters. D7-branes} \end{cases}$

  $\longrightarrow$ more suitable examples are desired

Open issues

☞ No-go theorems for multiple axions??
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Eskerrik asko

Moitas grazas

Moltes gràcies

Muchas gracias

WGC &The Madison Swampland

- WGC (weak form) states that $\exists$ state with $\left(\frac{M}{Q}\right) \leq M_{Pl}$
  $\rightsquigarrow$ conjectured generalisation for 0-forms with $\mathcal{S}_{inst} \leq \frac{M_{Pl}}{f}$
  Arkhani-Hamed-Motl-Nicolis-Vafa (2006)
- Consistent compactifications for

$$\begin{array}{ccc} \text{M-theory} & \overset{S,T}{\longleftrightarrow} & \text{Type IIB} \\ \mathcal{M}_{1,2} \times S^1_M \times \tilde{S}^1 \times X_6 & & \mathcal{M}_{1,2} \times S^1 \times X_6 \end{array}$$

  $\rightsquigarrow$ Constraints on effective axion decay constant by applying WGC on 5dim BH in M-theory
  Brown-Cottrell-Shiu-Soler (2015)
- Resolving ambiguity in defining effective axion decay constant
  $\rightsquigarrow$ possible axionic directions with $f > M_{Pl}$
  Bachlechner-Long-McAllister (2014/15), Junghans (2015)
- **BUT** in our simple model: 2 axions + 1 $U(1)$ + 1 instanton
  - apply WGC at appropriate scale $\Lambda > M_{\mathrm{gauge}}$
    $\Rightarrow \exists$ axionic direction with $f_2 = \frac{M_{\mathrm{gauge}}}{\tilde{k}^2} < M_{Pl}$ ✓WGC
  - Higher harmonics are still troublesome for $\perp$ axion

  Montero-Uranga-Valenzuela (2015)

Inflation and Axions

- Single field models discriminated by Lyth-bound: Lyth (1996)

$$\frac{\Delta\phi}{M_{Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2} \qquad \begin{array}{ll} r < 0.01 & \text{small field inflation } (\Delta\phi < M_{Pl}) \\ r > 0.01 & \text{large field inflation } (\Delta\phi > M_{Pl}) \end{array}$$

- measurable tensor perb. $\Delta_T^2(k)$ for $r \equiv \frac{\Delta_T^2(k)}{\Delta_S^2(k)} > 0.01$

  $\rightsquigarrow$ problematic sensitivity to dim 6 operators $\Rightarrow |\Delta\eta| \gg \mathcal{O}(1)$ ↯ slow roll

- Shift symmetry of axions forbids such corrections, while inflaton potential follows from non-perturb. effects

$$V(a) \sim \Lambda^4 \left[1 - \cos\frac{a}{f}\right]$$

  $\Rightarrow$ natural inflation Freese-Frieman-Olinto (1990) with $f > M_{Pl}$ for slow roll

- String Theory: reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), ...
  - ⋆ UV complete theory with plethora of axions
  - ⋆ local origin of shift symmetries Banks-Dixon (1988), Kallosh-Linde-Linde-Susskind (1995), Banks-Seiberg (2010), ...

But ∃ no-go theorems and arguments forbidding $f > M_{Pl}$ (1 axion)

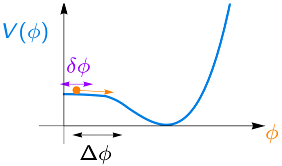
Banks-Dine-Fox-Gorbatov (2003), Svrček-Witten (2006), Arkhani-Hamed-Motl-Nicolis-Vafa (2006)

Some aspects of Inflation

reviews: Baumann (2009); Baumann-McAllister (2009,2014); Westphal (2014); ...

- Inflation = cure for horizon problem and flatness problem
- Inflation = explanation for fluctuations in nearly scale-invariant, nearly Gaussian CMB
- typical model: slow-roll single scalar field with potential V satisfying

$$\epsilon \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \qquad |\eta| \equiv \left|M_{Pl}^2 \frac{V''}{V}\right| \ll 1 \qquad \text{during inflation}$$



$\Delta\phi$: distance traveled during inflation

- QM fluctuations $\delta\phi$ during inflation

$$\Rightarrow \begin{cases} \text{scalar pert. } \Delta_S^2(k) & \sim 10^{-9} \text{ (WMAP+PLANCK)} \\ \text{tensor pert. } \Delta_T^2(k) & \end{cases}$$

Some aspects of Inflation

- spectral index n_s: deviation from scale-invariance

$$\Delta_S(k) = A_S \left(\frac{k}{k_\star}\right)^{n_s - 1} \qquad k_\star : \text{reference scale}$$

- tensor-to-scalar ratio r: $r \equiv \frac{\Delta_T^2(k)}{\Delta_S^2(k)}$ sets the inflation scale

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\text{GeV}$$

- Lyth bound: relates field displacements $\Delta\phi$ to r during inflation

$$\frac{\Delta\phi}{M_{Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2}$$

$r < 0.01$ small field inflation ($\Delta\phi < M_{Pl}$)
$r > 0.01$ large field inflation ($\Delta\phi > M_{Pl}$)

- (n_s, r) are related to slow-roll parameters (ϵ, η):

$$n_s - 1 = 2\eta - 6\epsilon \qquad r = 16\epsilon$$

$\Rightarrow (n_s, r)$ measurements give direct info about potential V

Some Considerations about Axions

- axions = CP-odd real scalars with a continuous shift symmetry:

$$a \to a + \varepsilon, \qquad \varepsilon \in \mathbb{R}$$

- shift symmetry is broken to a discrete symmetry by non-perturbative effects (gauge instantons, D-brane instantons, etc.):

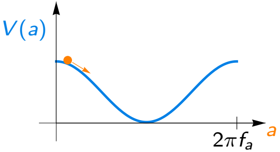
$$a \to a + 2\pi n, \qquad n \in \mathbb{Z}$$

☞ symmetry constrains axionic couplings:

$$\mathcal{S} = \int \frac{f_a^2}{2} \mathrm{d}a \wedge \star_4 \mathrm{d}a - \Lambda^4 \left[1 \pm \cos(a)\right] \star_4 \mathbf{1}$$

f_a: axion decay constants (coupling strength of axion to other matter)

- natural inflation: axion as inflaton candidate Freese-Frieman-Olinto (1990)



$$\epsilon = \frac{M_{Pl}^2}{2 f_a^2} \left(\frac{\sin(a/f_a)}{1 \pm \cos(a/f_a)} \right)^2 \ll 1$$

$$\eta = \frac{M_{Pl}^2}{f_a^2} \left| \frac{\cos(a/f_a)}{1 \pm \cos(a/f_a)} \right| \ll 1$$

"Lifting" the flat direction

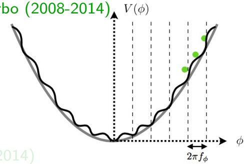
(1) Monodromy effects: shift symmetry is softly broken

- F-term (torsional cycles, fluxes) $\longrightarrow V(\xi) \sim \xi^{p\geq 2}$

  Marchesano-Shiu-Uranga (2014), McAllister-Silverstein-Westphal-Wrase (2014), Blumenhagen-Herschmann-Plauschinn (2014), Kaloper-Lawrence-Sorbo (2008-2014)

- D-term (D-branes) $\longrightarrow V(\xi) = \sqrt{L^4 + \xi^2} \sim \xi$

  Silverstein-Westphal (2008), McAllister-Silverstein-Westphal (2008)



(2) Alignment effects: 2 non-Abelian gauge groups

Kim-Nilles-Peloso (2004), Kappl-Krippendorf-Nilles (2014); Choi-Kim-Yun (2014)

$$V_{axion}^{\mathrm{eff}} = \Lambda_1^4\left[1-\cos\left(\frac{a^-}{f_1}+\frac{a^+}{g_1}\right)\right] + \Lambda_2^4\left[1-\cos\left(\frac{a^-}{f_2}+\frac{a^+}{g_2}\right)\right]$$

see also BenDayan-Pedro-Westphal (2014), Long-McAllister-McGuirk (2014)

(3) $U(1)$ gauge symmetry:

- 1 axionic direction eaten by $U(1)$ boson (Stückelberg mechanism)
- orthogonal direction acquires mass due to non-perturbative effect

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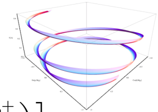

$$V_{axion}^{\text{eff}} = \Lambda_1^4 \left[1 - \cos\left(\frac{a^-}{f_1} + \frac{a^+}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{a^-}{f_2} + \frac{a^+}{g_2}\right)\right]$$

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- orthogonal direction acquires mass due to non-perturbative effect

Aligned natural inflation

- Consider 2 strongly coupled non-Abelian gauge groups:

$$V_{axion}^{\text{eff}} = \Lambda_1^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_1} + \frac{\hat{a}^+}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_2} + \frac{\hat{a}^+}{g_2}\right)\right]$$

  with decay constants

$$f_1 = \frac{\sqrt{\lambda_-}}{|r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}|} \quad g_1 = \frac{\sqrt{\lambda_+}}{|r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}|} \quad f_2 = \frac{\sqrt{\lambda_-}}{|s_1 \sin\frac{\theta}{2} - s_2 \cos\frac{\theta}{2}|} \quad g_2 = \frac{\sqrt{\lambda_+}}{|s_1 \cos\frac{\theta}{2} + s_2 \sin\frac{\theta}{2}|}$$

- Perfect alignment:

$$\frac{f_1}{g_1} = \frac{f_2}{g_2} \qquad \Rightarrow \qquad \left|\frac{r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}}{r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}}\right| = \left|\frac{s_1 \cos\frac{\theta}{2} + s_2 \sin\frac{\theta}{2}}{s_1 \sin\frac{\theta}{2} - s_2 \cos\frac{\theta}{2}}\right|$$

- Deviation from perfect alignment

$$\alpha_{dev} \equiv g_2 - \frac{f_2}{f_1} g_1 = \frac{\sqrt{\lambda_+}\,(s_1 r_2 - r_1 s_2)}{\left(\frac{s_1^2 - s_2^2}{2} \sin\theta - s_1 s_2 \cos\theta\right)\left(r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}\right)}$$

- Continuous parameter θ allows for $\alpha_{\text{dev}} \approx 0.009\sqrt{\lambda_+}$ with $r_i, s_i \sim O(1-10)$

$U(1)$ gauge invariance

- $U(1)$ gauge invariance: $A \to A + \mathrm{d}\chi$, $a'^2 \to a'^2 + \tilde{k}\chi$

$$\mathcal{S}_{sub} = \int \left[\frac{f_2^2}{2} \left(\mathrm{d}a'^2 - \tilde{k}A\right) \wedge \star_4 \left(\mathrm{d}a'^2 - \tilde{k}A\right) - \frac{1}{g_1^2} F \wedge \star_4 F - \underbrace{\frac{1}{8\pi^2} a'^2 \,\mathrm{Tr}(\, G \wedge G)}_{\text{not } U(1) \text{ invariant}} \right]$$

$\rightsquigarrow$ requires presence of chiral fermions

"reversed" GS mechanism $\Rightarrow \delta\mathcal{S}_{anom}^{mix} = -\int \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{mix}} \chi \,\mathrm{Tr}(\, G \wedge G)$

- If anomaly also contains non-symmetric contributions

$\rightsquigarrow$ Generalised Chern-Simons terms Aldazabel-Ibáñez-Uranga (2003), Anastasopoulos et al (2006) De Rydt-Rosseel-Schmidt-Van Proeyen-Zagerman (2007)

$$\mathcal{S}_{sub}^{\mathrm{GCS}} = \int \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{GCS}} A \wedge \Omega,$$

- Full $U(1)$ gauge invariance: $\boxed{\tilde{k} + \mathcal{A}^{mix} + \mathcal{A}^{\mathrm{GCS}} = 0}$

- $+$ 3 additional constraints from non-Abelian and mixed Abelian/non-Abelian anomaly cancelation

Anomalies

- Spectrum of chiral fermions

	$SU(N)$	$U(1)$
ψ_L^i	R_1^i	q_L^i
ψ_R^i	R_2^i	q_R^i

- mixed anomaly coefficient:
$\mathcal{A}^{\rm mix} = \sum_i \left[{\rm Tr}(q_L^i \{T_a^{R_1^i}, T_b^{R_1^i}\}) - {\rm Tr}(q_R^i \{T_a^{\overline{R}_2^i}, T_b^{\overline{R}_2^i}\}) \right]$

- mixed anomaly: $\mathcal{A}^{\rm GCS} - \mathcal{A}^{\rm mix} = 0$

- cubic $U(1)$ anomaly: $\mathcal{A}^{U(1)^3} = \sum_i \left[(q_L^i)^3 - (q_R^i)^3 \right] = 0$

- cubic $SU(N)$ anomaly:
$\mathcal{A}^{SU(N)^3} = \sum_i \left[{\rm Tr}(T_a^{R_1^i}\{T_b^{R_1^i}, T_c^{R_1^i}\}) - {\rm Tr}(T_a^{\overline{R}_2^i}\{T_b^{\overline{R}_2^i}, T_c^{\overline{R}_2^i}\}) \right] = 0$

Integrating out

- Potential for a'^1? $\to$ integrating out A ($+\ a'^2$)

Step 1: e.o.m for massive A in unitary gauge

$$-\frac{1}{g_1^2}d(\star_4 dA)+(f_{\tilde{a}_2}\tilde{k}^2)^2\star_4 A=-\frac{\mathcal{A}^{\rm GCS}}{8\pi^2}\Omega-\star_4\mathcal{J}_\psi$$

Step 2: Deduce Lorenz-gauge condition

$$(f_{\tilde{a}^2}\tilde{k}^2)^2 d(\star_4 A)=-\frac{\mathcal{A}^{\rm GCS}+\mathcal{A}^{\rm mix}}{\mathcal{A}^{\rm mix}}d(\star_4\mathcal{J}_\psi)$$

Step 3: Re-insert relation between A and $\mathcal{J}_\psi$

$$\mathcal{S}=\int\frac{f_1^2}{2}\mathrm{d}a'^1\wedge\star_4\mathrm{d}a'^1-\frac{1}{8\pi^2}a'^1\mathrm{Tr}(G\wedge G)-\frac{\mathcal{C}}{f_2^2}\underbrace{\mathcal{J}_\psi\wedge\star_4\mathcal{J}_\psi}_{4-fermion}$$

- Potential for a'^1? $\to$ integrating out non-Abelian gauge group + chiral fermions

$$\mathcal{S}=\int\frac{f_1^2}{2}\mathrm{d}a'^1\wedge\star_4\mathrm{d}a'^1-\Lambda^4\left[1-\cos\left(a'^1\right)\right]\star_4\mathbf{1}$$

Integrating out

- Potential for a'^1? $\rightarrow$ integrating out A $(+\ a'^2)$

Step 1: e.o.m for massive A in unitary gauge

$$-\frac{1}{g_1^2} d(\star_4 dA) + (f_{\tilde{a}_2} \tilde{k}^2)^2 \star_4 A = -\frac{\mathcal{A}^{\mathrm{GCS}}}{8\pi^2}\Omega - \star_4 \mathcal{J}_\psi$$

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$$(f_{\tilde{a}2} \tilde{k}^2)^2 d(\star_4 A) = -\frac{\mathcal{A}^{\mathrm{GCS}} + \mathcal{A}^{\mathrm{mix}}}{\mathcal{A}^{\mathrm{mix}}} d(\star_4 \mathcal{J}_\psi)$$

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$$\mathcal{S} = \int \frac{f_1^2}{2} \mathrm{d}a'^1 \wedge \star_4 \mathrm{d}a'^1 - \frac{1}{8\pi^2} a'^1 \mathrm{Tr}(G \wedge G) - \frac{\mathcal{C}}{f_2^2} \underbrace{\mathcal{J}_\psi \wedge \star_4 \mathcal{J}_\psi}_{4-fermion}$$

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Axions from String Theory

Dimensional reduction of String Theory on $\mathcal{M}_{1,3} \times \mathcal{X}_6$

$\Rightarrow$ plethora of axions see e.g. Witten (1984), Banks-Dine-Fox-Gorbatov (2003), Svrček-Witten (2006)

(1) Closed String Axions

- 1 Model-independent: NS 2-form B_2 along $\mathcal{M}_{1,3}$
- $h_{11} + h_{21}$ Model-dependent: $\left\{ \begin{array}{l} \text{NS 2-form } B_2 \\ \text{RR-forms } C_p \end{array} \right.$ along $\mathcal{X}_6$

(2) Open String Axions

- Wilson-line: reduction of (D-brane) gauge field

  see e.g. ArkaniHamed-Cheng-Creminelli-Randal, Marchesano-Shiu-Uranga (2014)
- Field-type: phase of $\mathbb{C}$ scalar field in $\mathcal{N} = 1$ chiral multiplet @ intersection of 2 D-branes

String Theory embedding I

Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011)

- Type IIA on $\mathcal{M}_{1,3} \times \mathcal{X}_6$ with D6-branes
  $\mathcal{M}_{1,3}$ maximally symmetric 4-dim spacetime
  $\mathcal{X}_6$ admits a symplectic basis (α_i, β^j) for $H^3(\mathcal{X}_6)$: $\int_{\mathcal{X}_6} \alpha_i \wedge \beta^j = \ell_s^6 \delta_i{}^j$
  e.g. $\mathcal{X}_6 = CY_3/\Omega\mathcal{R}$
- axions emerge from reduction of RR-form C_3
  & charges under (D-brane) $U(1)$ from reduction of RR-form C_5:

$$C_3 = \frac{1}{2\pi} \sum_{i=1}^{b_3} \xi^i(x)\, \alpha_i(y) + \dots \qquad C_5 = \frac{1}{2\pi} \sum_{i=1}^{b_3} D_{(2)i} \wedge \beta^i + \dots$$

- Reduction of bulk action $\longrightarrow$ kinetic terms

$$\mathcal{S}_R^{\text{bulk}} \ni -\frac{\pi}{2\ell_s^8} \int \mathrm{d}C_i \wedge \star_4 \mathrm{d}C_{i=3,5} \longrightarrow \mathcal{S}_{kin} = -\frac{1}{4\ell_s^2} \int d\xi^i \wedge \star_4 d\xi^j \mathcal{K}_{ij} + dD_{(2)i} \wedge \star_4 dD_{(2)_j} \mathcal{K}^{ij}$$

with $\mathcal{K}_{ij} = \frac{1}{2\pi\ell_s^6} \int_{\mathcal{X}_6} \alpha_i \wedge \star_6 \alpha_j$ and $\mathcal{K}^{ij} = \mathcal{K}_{ij}^{-1}$

String Theory embedding II

- D6-brane wraps $\mathcal{M}_{1,3} \times \Delta_3$, w.r.t. de Rahm-dual basis (γ_i, δ^j)

$$\Delta_3 = \sum_{i=1}^{b_3/2} (r^i \gamma_i + p_i \delta^i) \qquad \int_{\gamma_j} \alpha_i = \ell_s^3 \delta_i{}^j = \int_{\delta^i} \beta^j$$

- Reduction of D-brane action $\longrightarrow$ anomalous coupling + $U(1)$ charges

$$\mathcal{S}_{CS}^{D6} \ni \int_{D6} \frac{1}{4\pi \ell_s^3} C_3 \wedge F^2 + \frac{1}{\ell_s^5} C_5 \wedge F \longrightarrow \frac{1}{8\pi^2} \sum_{i=1}^{b_3/2} r^i \int_{\mathcal{M}_{1,3}} \xi^i F \wedge F + \frac{1}{2\pi \ell_s^2} \sum_{j=1}^{b_3/2} p_j \int_{\mathcal{M}_{1,3}} D_{(2)j} \wedge \mathrm{d}A$$

- $D_{(2)i}$ is Hodge-dual to $\xi^i \rightsquigarrow$ dualisation in favour of ξ^i:

$$\boxed{\mathcal{S}_{axion} = -\frac{1}{2\ell_s^2} \int_{\mathcal{M}_{1,3}} \left[\frac{1}{2} \left(d\xi^i - \frac{p_i}{\pi} A \right) \wedge \star_4 \left(d\xi^j - \frac{p_j}{\pi} A \right) \mathcal{K}_{ij} \right] + \frac{1}{8\pi^2} \sum_i r^i \int_{\mathcal{M}_{1,3}} \xi^i F \wedge F}$$

- similar reasoning for C_4-axions in type IIB with D7-branes

Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)

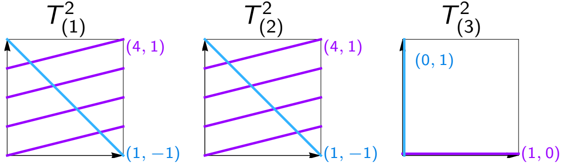
Factorisable D6-branes on $T^6/\Omega\mathcal{R}$

- Three-cycle Π_x decomposed with respect to (γ_i, δ^j):

$$\Pi_x = r_x^i \, \gamma_i + s_x^i \, \delta^i \qquad r_x^i, s_x^i \in \mathbb{Z}$$

factorisable three-cycles: r_x^i and s_x^i are given in terms of 1-cycle wrapping numbers (n_x^i, m_x^i) on $T^2_{(i)}$

- local 2-stack model $U(1)_a \times U(N_b)$



with axions ξ^1 and ξ^2 charged under $U(1)_a$:

$$\mathcal{S} \ni \int -\frac{1}{2\ell_s^2} \sum_{I=1,2} \mathcal{K}_{II} (d\xi^I - s_a^I A_a) \wedge \star_4 (d\xi^I - s_a^I A_a) + \frac{1}{8\pi^2} \left(r_b^1 \xi^1 + r_b^2 \xi^2 \right) \mathrm{Tr}(G_b \wedge G_b)$$