Heterotic Superpotentials and Moduli

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based on work in collaboration with Edward Hardy and Xenia de la Ossa,

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Introduction

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- String Compactifications.
- Gukov-Vafa-Witten superpotential and supersymmetry conditions.
- First order deformations, holomorphic structures and moduli.
- Conclusions and outlook..

General String Compactifications

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First need to find massless spectrum, i.e. infinitesimal moduli!

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Need a nicer description to deal with moduli [Anderson et al 10, Anderson et al 14, de la Ossa EES 14, Garcia-Fernandez et al 15].

Superpotential and Supersymmetry

The Superpotential

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and where

$$\omega^A_{CS} = \mathrm{tr}\left(A \wedge \mathrm{d}A + \frac{2}{3} A \wedge A \wedge A\right) \, .$$

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Ignore D-terms and conformally balanced condition for this talk, and assume stable bundles.

The Infinitesimal Moduli Space

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At the supersymmetric locus, the four-dimensional mass-matrix reads

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$$\begin{aligned}\delta_{12}W|_{\delta W=0} &= \int_X \frac{\alpha'}{2}\Big(\mathrm{tr}\,\delta_1 A\wedge\delta_2(F\wedge\Omega) - \mathrm{tr}\,\delta_1\Theta\wedge\delta_2(R\wedge\Omega)\Big)\\ &\quad+\int_X \mathrm{d}\tau_1\wedge\delta_2\Omega + \int_X \delta_2(H+i\mathrm{d}\omega)\wedge\delta_1\Omega\\ &\quad+\int_X (H+i\mathrm{d}\omega)\wedge\delta_2\delta_1\Omega\ .\end{aligned}$$

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where $\Delta_2 \in H^{(0,1)}(TX)$, $\alpha_2 \in \Omega^{(0,1)}(\mathrm{End}(V))$.

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Note: Deformations $\delta_2\nabla = \kappa_2$ non-physical. Can be thought of as infinitesimal field redefinitions preserving Strominger system [de la Ossa EES 14].

Kernels and the Atiyah Algebroid

It follows that Δ_2 is in the kernel of [Anderson et al 10]

$$\begin{aligned} \mathcal{F} \;:\quad & H^{(0,1)}(TX) \to H^{(0,1)}(\text{End}(V)) \\ \mathcal{R} \;:\quad & H^{(0,1)}(TX) \to H^{(0,1)}(\text{End}(TX)) \;. \end{aligned}$$

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Conditions from the Anomaly

We also have the terms

$$\int_X \delta_2(H + i\mathsf{d}\omega) \wedge \delta_1\Omega + \int_X (H + i\mathsf{d}\omega) \wedge \delta_2\delta_1\Omega \in \delta_{12}W|_{\delta W=0}$$

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Algebra: $\Rightarrow$ arrive at the following conditions

$$\overline{\partial}\tau_2^{(0,2)} = 0$$

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It follows that $x = (\Delta,\alpha,\kappa)\in H^{(0,1)}(\mathcal{Q}_1)$ is in the kernel of

$$\mathcal{H}\ :\quad H^{(0,1)}(\mathcal{Q}_1)\to H^{(0,2)}(T^*X)\ .$$

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$$0 \to T^*X \to \mathcal{Q}_2 \to \mathcal{Q}_1 \to 0 \, ,$$

with corresponding holomorphic structure

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Get same kernel structure.

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- Heterotic string is a nice playground for phenomenology, but the moduli problem is hard.
- From the heterotic superpotential, we derived the massless moduli space, and saw that it agrees with the 10d computation of [Anderson et al 14, de la Ossa EES 14] for the infinitesimal moduli space of solutions to the Strominger system.
- We note that the heterotic anomaly condition may lead to lifting extra moduli, even in Calabi-Yau compactifications.

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$\Rightarrow$ Spoils holomorphic structure $\overline{\partial}_2$.

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Thank you for your attention!