Gauge groups and matter in generic F-theory models

String Pheno 2015
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Washington (Wati) Taylor, MIT

Based on work with L. Anderson, A. Grassi, T. Grimm, J. Halverson, S. Johnson, D. Morrison, J. Shaneson, and Y. Wang, particularly:

arXiv: 1201.1943, 1204.0283, 1412.6112	D. Morrison, WT
arXiv: 1409.8295	A. Grassi, J. Halverson, J. Shaneson, WT
arXiv: 1506.nnnnn	J. Halverson, WT
arXiv: 150m.nnnnn	WT, Y. Wang

How do we go from string theory/F-theory/M-theory → 4D phenomenology?

Most Work:

– Consider a specific class of string constructions

– Figure out what kinds of gadgets give gauge fields, matter, etc.

– Try to "engineer" string vacua with desirable features.

– Tuning is expensive (cf. Watari talk)

This talk: top-down approach, "Follow the theory"

– Consider the full space of F-theory constructions
(cf Klevers, Schäfer-Nameki, . . . talks)

– Identify "generic" features

– See how SM, dark matter can arise in this framework

Result: perhaps surprising how much we might get "for free,"
in generic/typical F-theory vacua. Structure interesting in any case.

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F-theory: nonperturbative IIB string theory [Vafa, Morrison/Vafa]

Requires an *elliptically fibered* Calabi-Yau X

$\pi : X \to B$ (B curved), fiber $\pi^{-1}(p) \cong T^2$

Geometry of $B_2, B_3 \longrightarrow$ 6D, 4D supergravity.

To construct an F-theory model:

1. Choose base B

2. Choose point in CS moduli space
(Weierstrass model $y^2 = x^3 + fx + g$ encodes IIB $\tau = \chi + ie^{-\phi}$ as fun. on B.

A theme of this talk & much research over last five years:
Focusing on geometry of $B \to$ simplifications and insight

Primary goals/results:

A) Classifying and enumerating elliptic Calabi-Yau threefolds and fourfolds

B) Generic features of $\mathcal{N} = 1$ F-theory vacua in 6D and in 4D

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What does "generic" mean?

Two distinct aspects:

I. Generic features of vacua with fixed B
i.e., features for *arbitrary* Weierstrass moduli ("non-Higgsable structure")

II. Features present for "most" B's

6D: Finite number of B's, fairly clear global picture
$\rightarrow$ almost all B's have "non-Higgsable clusters," certain typical G's

4D: Picture emerging, *surprisingly parallel to 6D picture*
but more complicated technically.
Many open questions for theory/pheno

First: I and II in 6D, where story is clear. Next: emerging related 4D story

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6D F-theory vacua and elliptic Calabi-Yau threefolds

I. Non-Higgsable clusters in 6D

Gauge groups in F-theory:

7-branes on curve (divisor) S_i
$\rightarrow$ gauge factor G_i
Singularity type: $\text{ord}_{S_i} f, g$ in Weierstrass

For certain B_2, G nontrivial $\forall$ moduli

Example:

F-theory on $\mathbb{F}_3 \leftrightarrow$ heterotic $E_8 \times E_8$ on K3 w/ (15, 9) instantons
$\Rightarrow SU(3)$ with no matter (non-Higgsable)

- In other cases, non-Higgsable matter (e.g. E_7 w/ $\frac{1}{2}$ **56**)
  Can't Higgs: can't satisfy D-term constraints
- When no smooth heterotic dual, $\exists$ non-Higgsable product groups w/ matter

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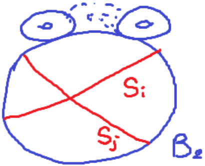

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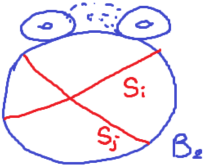

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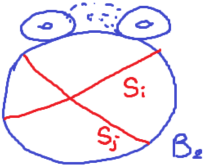

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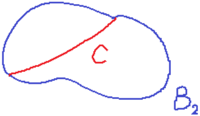
Geometry of non-Higgsable groups

The base B_2 is a complex surface.

Contains complex curves C_i



For $C \cong \mathbb{P}^1 \cong S^2$, local geometry encoded by *normal bundle* $\mathcal{O}(m)$

m encodes self-intersection of curve C

If $N_C \cong \mathcal{O}(-n), n > 0$, C is *rigid* (no deformations)

For $\mathcal{O}(-n), n > 2$, base space is so curved that 7-branes must pile up to preserve Calabi-Yau structure on total space $\Rightarrow$ non-Higgsable gauge group

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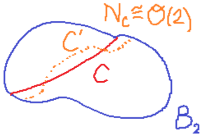

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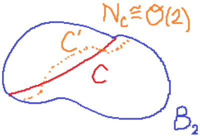

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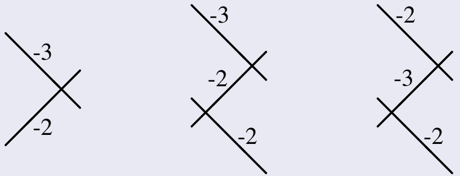
Classification of 6D "Non-Higgsable Clusters" (NHC's) [Morrison/WT]

Clusters of curves imposing generic nontrivial codimension one singularities:

$-m$
$(m = 3, 4, 5, 6, 7, 8, 12)$



$\mathfrak{su}(3), \mathfrak{so}(8), \mathfrak{f}_4$
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$\mathfrak{g}_2 \oplus \mathfrak{su}(2)$ $\mathfrak{g}_2 \oplus \mathfrak{su}(2)$ $\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$

- Identified using *Zariski decomposition* $-nK = \sum_i q_i C_i + X$
- Any other combination including -3 or below $\Rightarrow$ (4, 6) at point/curve

NHC's useful in classification of compact (SUGRA) F-theory models (next) and 6D SCFTs [Heckman/Morrison/Vafa,+ del Zotto, Park, Rudelius,. . .]

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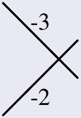

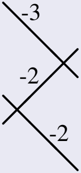

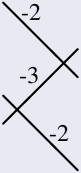

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6D II. Classification/enumeration of elliptic Calabi-Yau threefolds

Gross: $\exists$ finite number of topologically distinct elliptic CY threefolds
(up to birational equivalence)

Mathematical minimal model program to classify surfaces:

1. Given surface S, find -1 curve C $(C \cdot C = -1)$, blow down $\rightarrow S'$
2. Repeat until done $\Rightarrow$ minimal surface

Grassi: minimal surfaces for B_2 bases:
$\mathbb{P}^2, \mathbb{F}_m (m \leq 12)$, Enriques ($-K \sim$ trivial)

Program: start with $\mathbb{P}^2, \mathbb{F}_m$, blow up to get all bases B_2, constrained by NHC's

Given bases, tune Weierstrass $\rightarrow$ all elliptic CY3's, all 6D F-theory models

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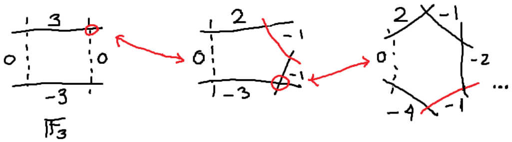

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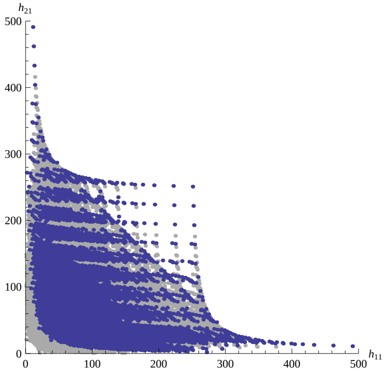
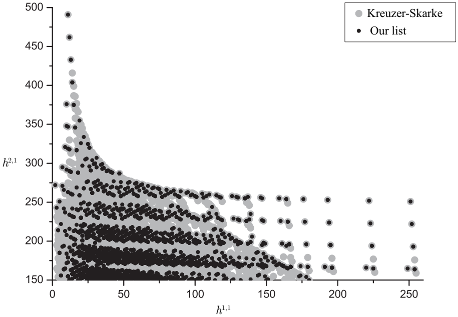
Found all toric B_2 Generic EF Hodge #'s [Morrison/WT, WT]



- 61,539 toric bases (some not strictly toric: -9, -10, -11 curves)
- Reproduces large subset of Kreuzer-Skarke database of CY3 Hodge #'s
  Boundary of "shield" from generic elliptic fibrations over blowups of $\mathbb{F}_{12}$.
- In principle: start with $\mathbb{F}_m, \mathbb{P}^2$, blow up/tune $\rightarrow$ *all* EF CY3's

Beyond toric: approach allows construction of general (non-toric) bases

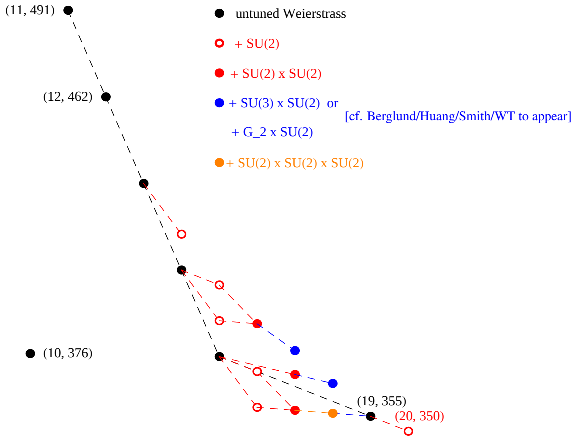
- Computed all 162, 404 "semi-toric" bases w/ 1 $\mathbb{C}^*$-structure [Martini/WT]
- All bases for EF CY threefolds w/ $h^{2,1}(X) \geq 150$ [WT/Wang]



Technical issues at large $h^{1,1}(X)$, small $h^{2,1}(X)$:
Infinite generators for cone, Multiply intersecting -1 curves

Upshot: modest expansion of possibilities beyond toric, semi-toric

EF CY3's with $h^{2,1} \geq 350$, $\mathbb{F}_m+$ tuning $\rightarrow$ complete classification [Johnson/WT]



- Matches KS; non-toric + toric at (19, 355); new non-toric below 350
- Empirical data on Calabi-Yau's suggests: "most" (known) CY's are elliptic, particularly at large Hodge numbers (cf. [Gray/Haupt/Lukas])

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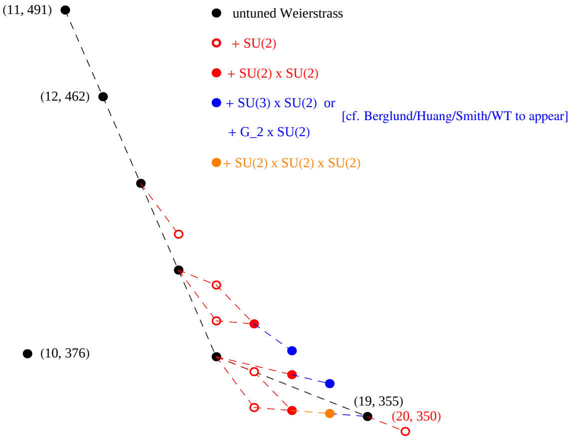

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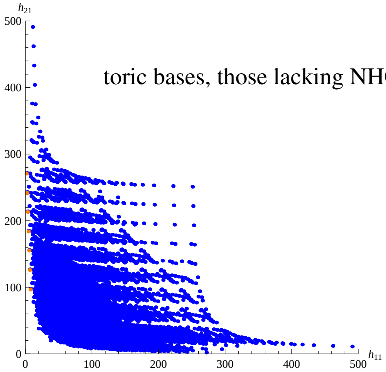
"Most" bases B_2 have NHC's



The only bases B_2 that lack NHC's are weak Fano = (generalized) del Pezzo

Ten corresponding Hodge pairs $(2 + T, 272 - 29T), T = 0, \ldots, 9$

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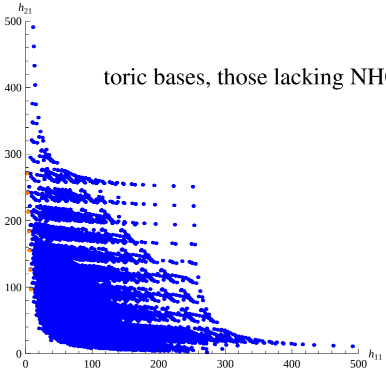

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Upshot for a 6D "phenomenologist"

Physics of a "typical" 6D SUSY F-theory compactification

$T \sim 25$, rank $G \sim 35$,

$$G \sim (G_2 \times SU(2))^3 \times E_8^2 \times F_4 \times SU(3) \times SO(8)$$

Non-Higgsable gauge groups are "generic" in two senses:

– G + matter are non-Higgsable $\Rightarrow$ persist throughout CS moduli space

– Almost all B's have non-Higgsable G, matter.
No tuning necessary

Now how about 4D? Remarkably, story is closely parallel *at level of geometry*

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Now how about 4D? Remarkably, story is closely parallel *at level of geometry*

4D F-theory compactifications

Story parallel in many ways:

– Compactify on Calabi-Yau fourfold, base B_3 = complex threefold

– Underlying moduli space of EF CY fourfolds on rational $B_3 \sim \{\text{EFCY3}'\text{s}\}$

But:

– G-flux $\Rightarrow$ superpotential, lifts flat directions

– Brane worldvolume DOF nontrivial $\Rightarrow$ can e.g. break gauge group

– F-theory only covers part of $\mathcal{N} = 1$ theory space

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4D non-Higgsable clusters [Morrison/WT]
(see also: Anderson/WT, Grassi/Halverson/Shaneson/WT)

At level of geometry, similar to 6D but more complicated

Toric bases: straightforward to compute using dual monomials to rays v_i

$$f, g \in \text{span}\,\{m : \langle m, v_i \rangle \geq -4, -6\}$$

Generally: 7-branes wrap divisors = surfaces S. Orders of vanishing of f, g depend on normal bundle N_S.

Expanding in coordinate z, $S = \{z = 0\}$,

$$f = f_0 + f_1 z + f_2 z^2 + \cdots$$

Up to leading non-vanishing term,

$$\begin{aligned} f_k &\in \mathcal{O}(-4K_S + (4-k)N_S) \\ g_k &\in \mathcal{O}(-6K_S + (6-k)N_S) \end{aligned}$$

Can do computations using geometry of *surfaces*

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Example: non-Higgsable $SU(2)$

Take $S = \mathbb{P}^2$, $-K_S = 3H, N = -4H \Rightarrow SU(2)$ on S

For global model, consider $\mathbb{P}^1$ bundle over $S = \mathbb{P}^2$, "twist" $= -4H$
Threefold $\tilde{\mathbb{F}}_4$, 3D analogue of Hirzebruch surfaces

Single group clusters: $\boxed{SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8}$

In particular, cannot have: non-Higgsable $SU(5), SO(10)$

Two-factor non-Higgsable group products

Either as an isolated 2-group cluster, *or within larger clusters*,
the only 2-factor products that can appear are:

$$\boxed{\begin{array}{lll} G_2 \times SU(2), & SO(7) \times SU(2), & SU(2) \times SU(2), \\ SU(3) \times SU(2), & SU(3) \times SU(3) & \end{array}}$$

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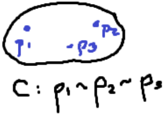
4D: clusters can have complicated structure

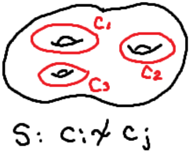
Basic issue:

On curve, all points homologous

On surface, many distinct curves

Can support independent matter





Branching

Unlike in 6D, cluster structure can have branching in "quiver diagram"

e.g. (higher branchings also possible)

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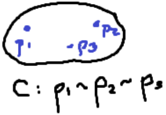

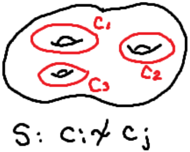

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e.g.

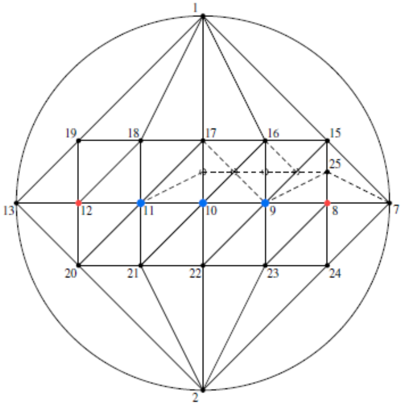
$$\begin{tikzcd} \mathfrak{g}_2 & & \mathfrak{g}_2 \\ & \mathfrak{su}_2 \arrow[ul, leftrightarrow] \arrow[ur, leftrightarrow] \arrow[d, leftrightarrow] & \\ & \mathfrak{su}_2 & \end{tikzcd}$$

(higher branchings also possible)

Clusters can also have long chains

$$\mathfrak{su}_2 \longleftrightarrow \mathfrak{su}_3 \longleftrightarrow \mathfrak{su}_3 \longleftrightarrow \mathfrak{su}_3 \longleftrightarrow \mathfrak{su}_2$$

(A)

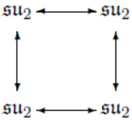


(B)

Similar for $\mathbb{F}_8$: $SU(2) \times SU(3)^{11} \times SU(2)$

And closed loops

Multiple blowup of $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \Rightarrow$



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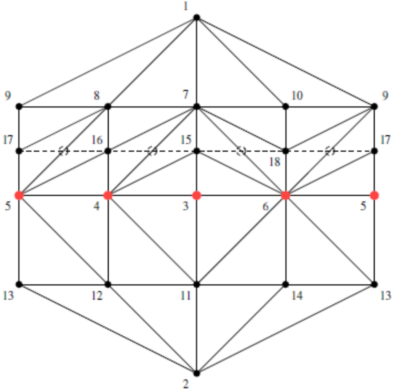

(B)

Upshot: local structure restricted (9 groups, 5 products),
but global structure can be very complex

And closed loops

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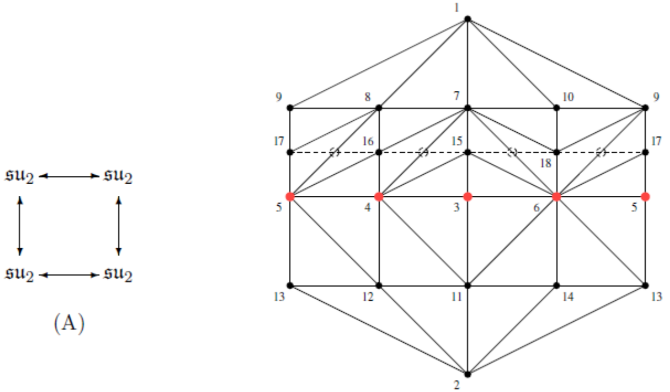
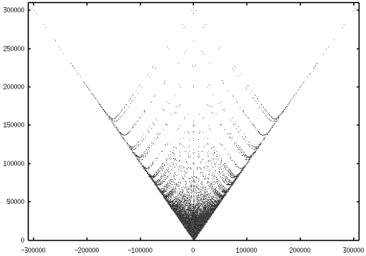


(A) (B)

Upshot: local structure restricted (9 groups, 5 products), but global structure can be very complex

Classification of elliptic Calabi-Yau fourfolds

Harder than threefolds → Mori theory. No proofs, some exploration:
w/Halverson: $\mathbb{P}^1$ bundles over toric bases B_2
w/Wang: Monte Carlo exploration by blowing up/down toric bases
Structure seems closely analogous to EF CY threefolds



[Lynker, Schimmrigk, Wisskirchen]
Hypersurfaces in weighted $\mathbb{P}^5$ (1998)

"minimal models" $\sim \mathbb{F}_m$
but more complex

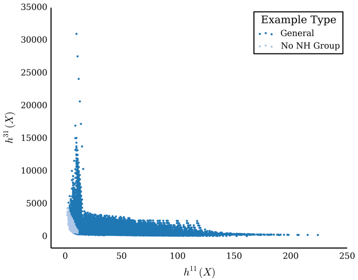
Blow up curves, points:
$h^{3,1} \downarrow, h^{1,1} \uparrow$

Generic EF CY 4:
Lots of G, matter
at level of geometry

Exploring threefold bases B_3: $\mathbb{P}^1$ bundles [Halverson/WT, to appear]

Consider $B_3 = \mathbb{P}^1$ bundle over a surface S.

Characterized by "twist" T = normal bundle of section $\cong S$.



- Enumerated $\sim 100,000$ distinct B_3 for toric B_2
- Each represents distinct divisor $S + N$ geometry
  $\rightarrow$ Exploration of generic fourfolds and of local geometry

Smooth heterotic dual when S = generalized del Pezzo [Anderson/WT]

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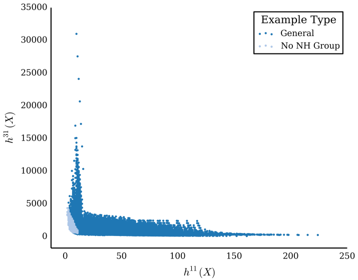

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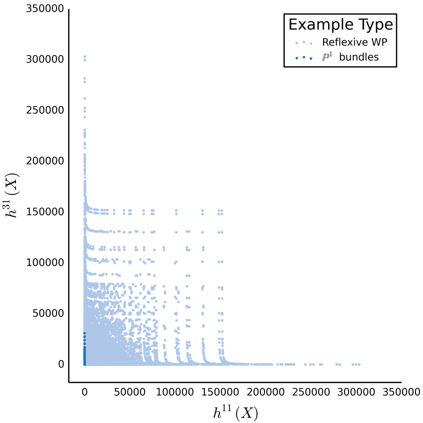
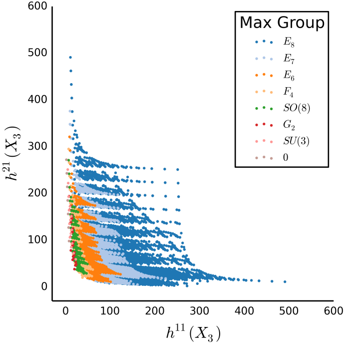
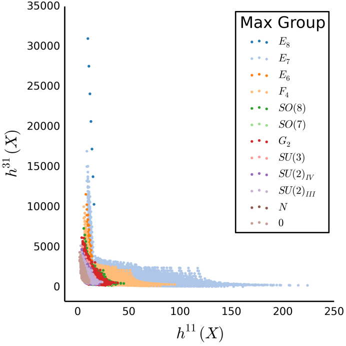
Fits into set of known fourfolds



Similar pattern of non-Higgsable groups to 6D



6D



4D

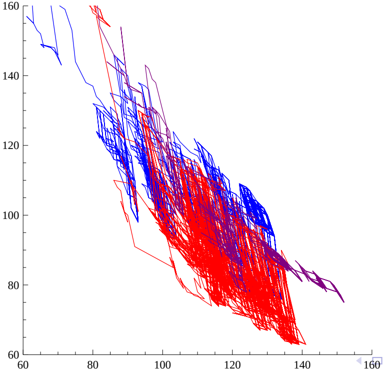
Note: dropped bundles in 4D with E_8 and bad curves, kept in 6D

Exploring threefold bases B_3: Monte Carlo [Wang/WT, to appear]

Start with e.g. $B_3 = \mathbb{P}^3$, blow up and down randomly
(Random walk on graph: samples vertices proportional to incident edges)

Note: does not connect to all bases, need singular intermediaries
but expect as in 6D, may give decent sampling of bulk

Average rank of NHC $\sim$ 50



Some statistics

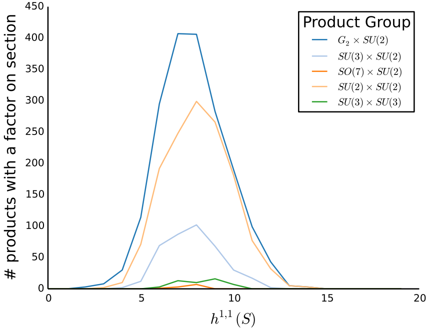
	$\mathbb{P}^1$ bundles	$\mathbb{P}^1$ bdl. $S_\pm$	Monte Carlo
$SU(2)$	0.44	0.35	0.47
G_2	0.23	0.18	0.31
F_4	0.16	0.17	0.09
$SU(3)$	0.005	0.04	0.11
$SO(8)$	0.008	0.04	0.01
E_7	0.10	0.11	0.01

Frequency of single non-Higgsable factors

Statistics are very rough. Systematic issues with each, only covering subsets

Statistics on two-group product factors

Statistics of products in $\mathbb{P}^1$-bundles, using section



Monte Carlo frequency of two-group product factors:

$$G_2 \times SU(2) : 54\% \qquad SU(2) \times SU(2) : 30\%$$
$$SU(3) \times SU(2) : 8\% \qquad SU(3) \times SU(3) : 7\%$$

Realizing the standard model in F-theory

Ignoring the outstanding issues of G-flux and seven-brane DOF,
what are the options for realizing $G = SU(3) \times SU(2) \times U(1)$?

1. Tune the whole thing — but not on divisors with NHC's
   (F-theory GUT e.g. [Donagi/Wijnholt, Beasley/Heckman/Vafa])
2. Tune part of G, get part from NHC; *e.g.* NH $SU(3)$, tune $SU(2) \times U(1)$
3. Get all of G from non-Higgsable structure

$NHC \Rightarrow SU(3) \times SU(2)$ reasonably common, $\sim 8\%$ of $G_1 \times G_2$ in MC study

$NHC \Rightarrow U(1)$ open question (some NH $U(1)$'s in 6D [Martini/WT])

$SU(3) \times SU(2)$ spectrum matches SM well, given $U(1)$, no anomalies
[U(1): cf. Schäfer-Nameki, Klevers talks]

Unification

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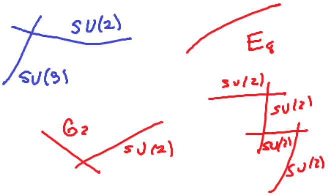
Dark matter candidates

Two possibilities:

I) "hidden sector" dark matter, e.g. from a disconnected cluster



II) WIMP dark matter (from $SU(2) \times G$, $G = SU(2), SU(3), SO(7), SO(8)$)

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If $SU(2)$ is in an NHC, non-Higgsable in SUSY theory

Just means stabilized by some mechanism in UV SUSY vacuum

Negative quadratic term can be induced by radiative corrections

Example of how this may occur

$SU(2)$ on divisor S

Matter curve C could support ϕ, H_u, H_d

Superpotential term $W = \phi \epsilon_{ij} H_u^i H_d^j$

F-terms combine with D-terms to stabilize H_u, H_d at quartic order

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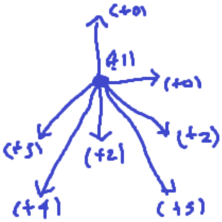

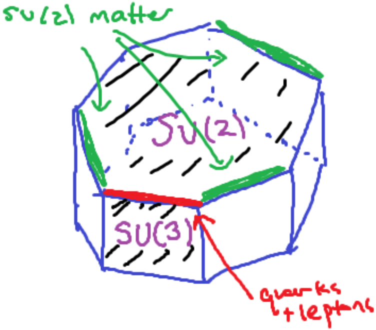

Conclusions

– 6D: good control of elliptic CY threefolds, NHC's ubiquitous

– 4D: distribution of NHC's seems similar: bigger groups at larger $h^{1,1}$

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less frequent than $G_2 \times SU(2)$, but reasonably prevalent

– Scenarios for dark matter, Higgs sector

Generic appearance of gauge groups and matter leads to new physics questions

Can we realize natural phenomenological scenarios with generic gauge/matter?

Natural mechanisms to produce $U(1)$?

Signals of non-Higgsable dark matter?

How does G-flux and the superpotential interact with non-Higgsable clusters?

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Muchas
gracias!!