

Stringpheno 2015 at IFT Madrid: June 10 (Wed)

# Statistics of Effective Theories in F-theory Flux Compactifications

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- review
- 1401.5908, 1408.6156, 1408.6167 w/ A.Braun and Kimura
- 1506.xxxxxx in progress

# Flux Compactification

$$H^{(3)}, F^{(3)} \in H^3(M_3; \mathbb{Z})$$

$$W \propto \int_{M_3} (F - \tau H) \wedge \Omega_M$$

$$(F^{(3)} - \tau H^{(3)})_{(1,2)} = 0.$$

$$\int_{M_3} H^{(3)} \wedge F^{(3)} \leq Q_{O3}.$$

- choose a topological **flux**
- potential generated
- cpx. str. vev is determined so that
- generate an **ensemble of fluxes** by requiring

$$G^{(4)} \in H^4(X_4; \mathbb{Z})$$

$$W \propto \int_{X_4} G \wedge \Omega_X$$

$$(G^{(4)})_{(1,3)} = 0.$$

$$\frac{1}{2} \int_{X_4} G \wedge G \leq \frac{\chi(X_4)}{24}.$$

## Ashok-Douglas-Denef's treatment

- scatter plot on the moduli space  $\mathcal{M}$  → distribution  $(m,m)$ -form on  $\mathcal{M}$  by contin. approx.

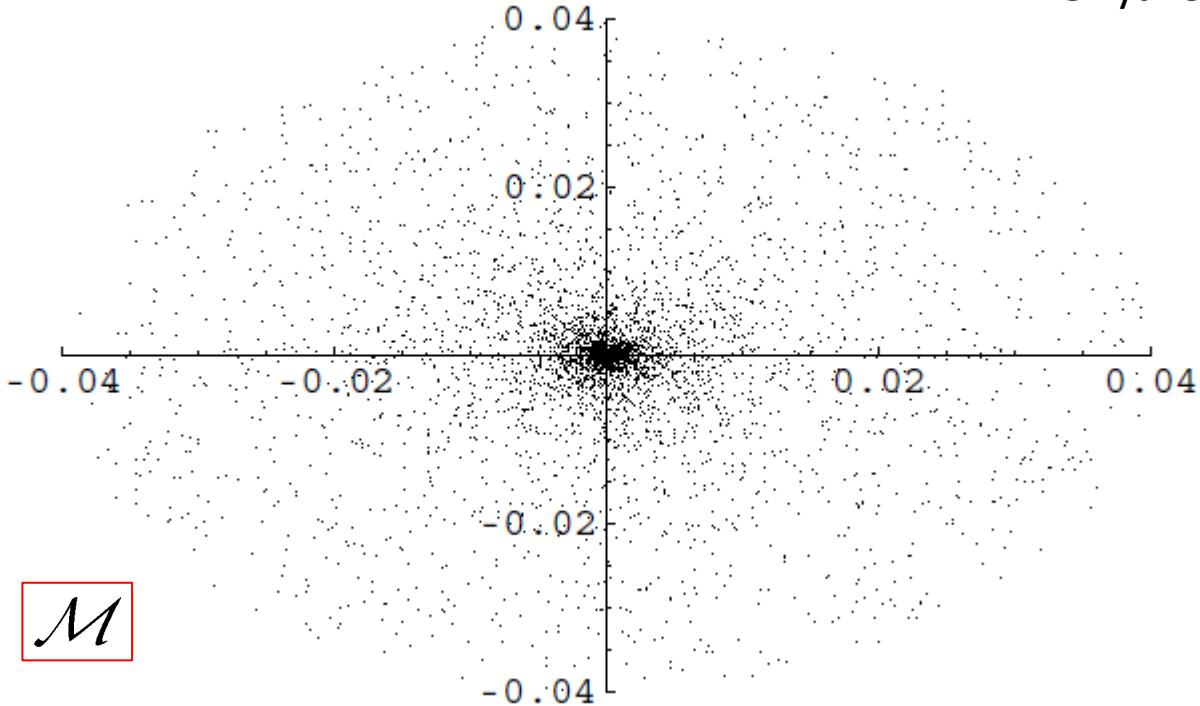
$$d\mu_I \simeq \frac{(2\pi L_*)^{K/2}}{(K/2)!} \det \left( -\frac{R}{2\pi i} + \frac{\omega}{2\pi} \mathbf{1}_{m \times m} \right) \quad \text{if } K \ll L_*.$$

Ashok Douglas '03  
Denef Douglas '04

- typical approach in IIB :

$$m = h^{2,1}(M_3) + 1, \quad K = 2b_3(M_3), \quad L_* = Q_{O3}.$$

- prefactor (like Bousso Polchinski '00) → popular  $10^{O(500)}$
- distribution: agree with numerical studies



distribution near  
the conifold point  
in a model with

$$h^{2,1} = 1.$$

- **conifold limit**  $\{a, a \ln(a), \dots\} \rightarrow \infty \frac{da \wedge d\bar{a}}{|a|^2 [\ln(1/|a|^2)]^2} \sim \frac{d[\ln(1/|a|^2)]}{[\ln(1/|a|^2)]^2}$ .
  - **large cpx str limit**  $\{1 + \ln(\zeta) + \dots + \ln^3(\zeta) + \zeta + \zeta^2 + \dots\}$
- $\rightarrow \infty \frac{d \ln(\zeta) \wedge d \ln(\bar{\zeta})}{[\ln(1/|\zeta|^2)]^2} \sim \frac{d[\ln(1/|\zeta|^2)]}{[\ln(1/|\zeta|^2)]^2}$ .

# plan of this talk

- Introduction
- F-theory application
- Gauge group and  $N_{\text{gen}}$
- (comments [incl.  $h^{3,1}(X_4) - h^{1,1}(X_4)$  ] )
- cost of an extra U(1) symmetry
- using Ashok-Douglas distribution

# F-theory application

- fix a topology of  $(B_3, [S])$  and a symmetry R on S
  - $\mathcal{M}$ : moduli space of ell. fibr.  $\pi: X_4 \rightarrow B_3$ .
  - $\mathcal{M}^{R=A_4} \subset \mathcal{M}$ : SU(5) gauge group is on S.
- Ashok-Douglas-Denef works !

$$d\mu_I \simeq \frac{(2\pi L_*)^{K/2}}{(K/2)!} \det \left( -\frac{R}{2\pi i} + \frac{\omega}{2\pi} \mathbf{1}_{m \times m} \right)$$

if  $K \ll L_*$ .

prefactor replaced by  $\frac{K^{L_*}}{(L_*)!}$

if  $K \gg L_*$ .

- now, with

$$m = h^{3,1}, \quad K \geq 2(h^{3,1} + 1) + h_H^{2,2},$$

$$H^{2,2}(X) = H_H^{2,2} \oplus H_V^{2,2} \oplus H_{RM}^{2,2}.$$

$$L_* = \frac{\chi(X_4)}{24} - \frac{1}{2} (G_{gen} + G_{sym.br.})^2.$$

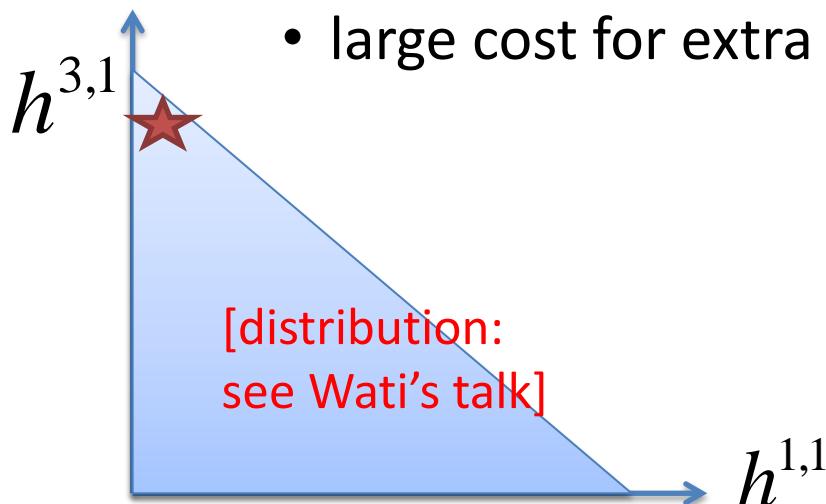
# $N_{\text{gen}}$ and gauge group

- fix  $(B_3, [S], R)$  and ask distrib. on  $N_{\text{gen}}$ .
  - prefactor  $\frac{K^{L_*}}{L_*!} \approx \exp[L_* \ln(K / L_*)]$
  - $L_* = \frac{\chi(X_4)}{24} - \frac{1}{2}(G_{\text{gen}} + G_{\text{sym.br.}})^2.$
  - $N_{\text{gen}} = \int G_{\text{gen}}^{(4)}.$
  - Gaussian distribution in  $N_{\text{gen}}$ ; variance  $O(1)$ .
- fix  $(B_3, [S])$ ; how the #(flux vacua) depends on R

$B_3$ $(S = \mathbb{P}^2)$	$P^3$	$n=-3$	$n=0$	$n=3$	$\mathbb{P}[O \oplus O(n)]$
$\Delta L_* : \text{none} \rightarrow SU(5)$	418.	776.	281.	45.	<b>rk=4 gauge group</b>
$\Delta L_* : SU(5) \rightarrow SO(10)$	3.5	7.	2.25	0	<b>fraction</b> $10^{-O(500)}$ .

# comments

- not considered:
  - non-geometric CFT target, fraction D3-branes, Ricci-flat but not Kahler (non-SUSY)
- $B_3 = \mathbb{P}^3, \mathbb{P}^1 \times \mathbb{P}^2, \dots$  are not typical examples.
  - they are in the corner with large  $h^{3,1}, \Delta h^{3,1}, \Delta K$ .
    - large cost for extra rank, large #(flux vacua).



- Kahler moduli need to be stabilized;
- inflation ?
- $\langle W \rangle \ll (\text{Planck})^3$  ??

# cost of an extra U(1) symmetry

TW '15

- R-parity v.s. spontaneous R-parity violation
  - statistical cost of the extra U(1) shouldn't be ignored.
- fix  $B_3 = \mathbb{P}^1 \times \mathbb{P}^2$ ;  $S = \mathbb{P}^2$ .  $\Delta L_* \sim (\Delta h^{3,1})/4$

$$WP_{[1:2:3]}\text{-fibred, no gauge group} \quad h^{3,1} = 3277,$$

$$WP_{[1:2:3]}\text{-fibred, SU}(5) \text{ gauge group} \quad h^{3,1} = 2148,$$

$$WP_{[1:2:3]}\text{-fibred, SO}(10) \text{ gauge group} \quad h^{3,1} = 2138$$

$$WP_{[1:2:3]}\text{-fibred, SU}(6) \text{ gauge group} \quad h^{3,1} = 1905 \sim 1918,$$

$$\text{Bl}_{[1:0:0]} WP_{[1:2:3]}^2\text{-fibred} \quad h^{3,1} = 932,$$

$$F_1\text{-fibred, no.2} \quad h^{3,1} = 7(b_2)^2 - 12b_2 + 372, \quad (0 < b_2 < 3).$$

- an extra U(1) via MW costs as much as SU(5) itself.

# using the AD distribution

$$\det\left(-\frac{R}{2\pi i} + \frac{\omega}{2\pi} \mathbf{1}_{m \times m}\right)$$

TW '15

- many applications
- an example: approximate U(1)

$$\pi_X : X_4 \rightarrow B_3. \quad y(y + A_1 x + A_3) = x(x^2 + A_2 x + A_4) + A_6; \quad A_n \in \Gamma(B_3; \mathcal{O}(-nK_B)).$$

- a U(1) sym. in the  $A_6 \rightarrow 0$  limit;  $\mathcal{M}^{U(1)} \subset \mathcal{M}$ .
- around  $\mathcal{M}^{U(1)}$ ,  $X_4$  is like deformed conifold along the curve  $\Sigma$ :  $x = y = A_3 = A_4 = 0$ . (e.g., Grimm Weigand '10)
- $\Delta h^{3,1} = g(\Sigma)$ . The period integrals may be  $\{a_i, a_i \ln(a_i)\}$

(preliminary)

$i = 1, \dots, g(\Sigma)$ .

$$AD \propto \prod_{i=1}^g \frac{da_i \wedge d\bar{a}_i}{|a_i|^2 [\ln |a_i|^2]^2} \quad \longrightarrow \quad P(|a_i|^2 < \varepsilon, \forall i = 1, \dots, g) \approx \frac{1}{[\ln(1/\varepsilon)]^{g(\Sigma)}}.$$

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