

The Higgs VEV with gravity

Franco D. Albareti

*PhD student
under the supervision of*

Prof. Antonio L. Maroto and Prof. Francisco Prada



Instituto de
Física
Teórica
UAM-CSIC

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Acknowledgements



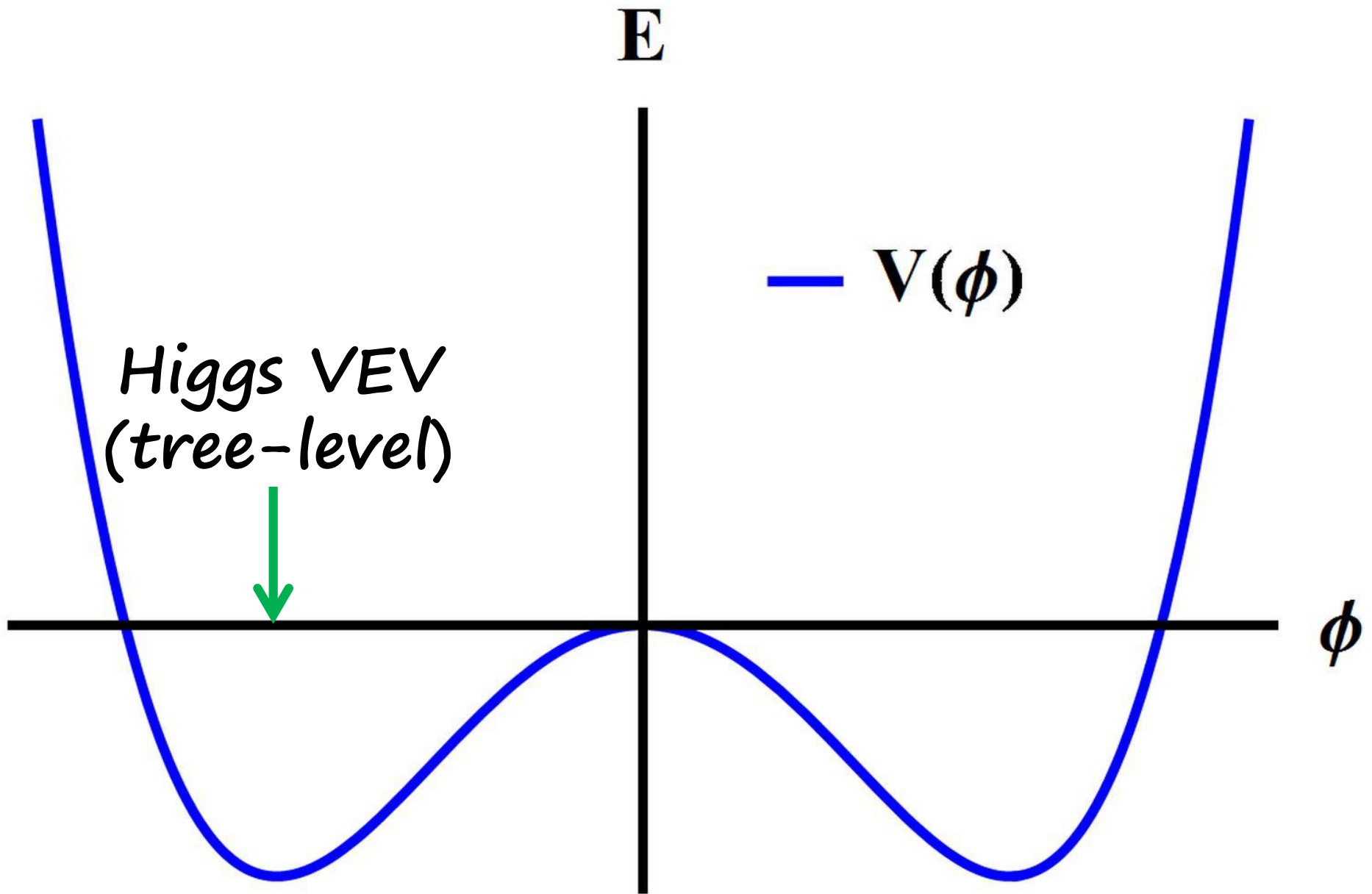
Obra Social
Fundación "la Caixa"

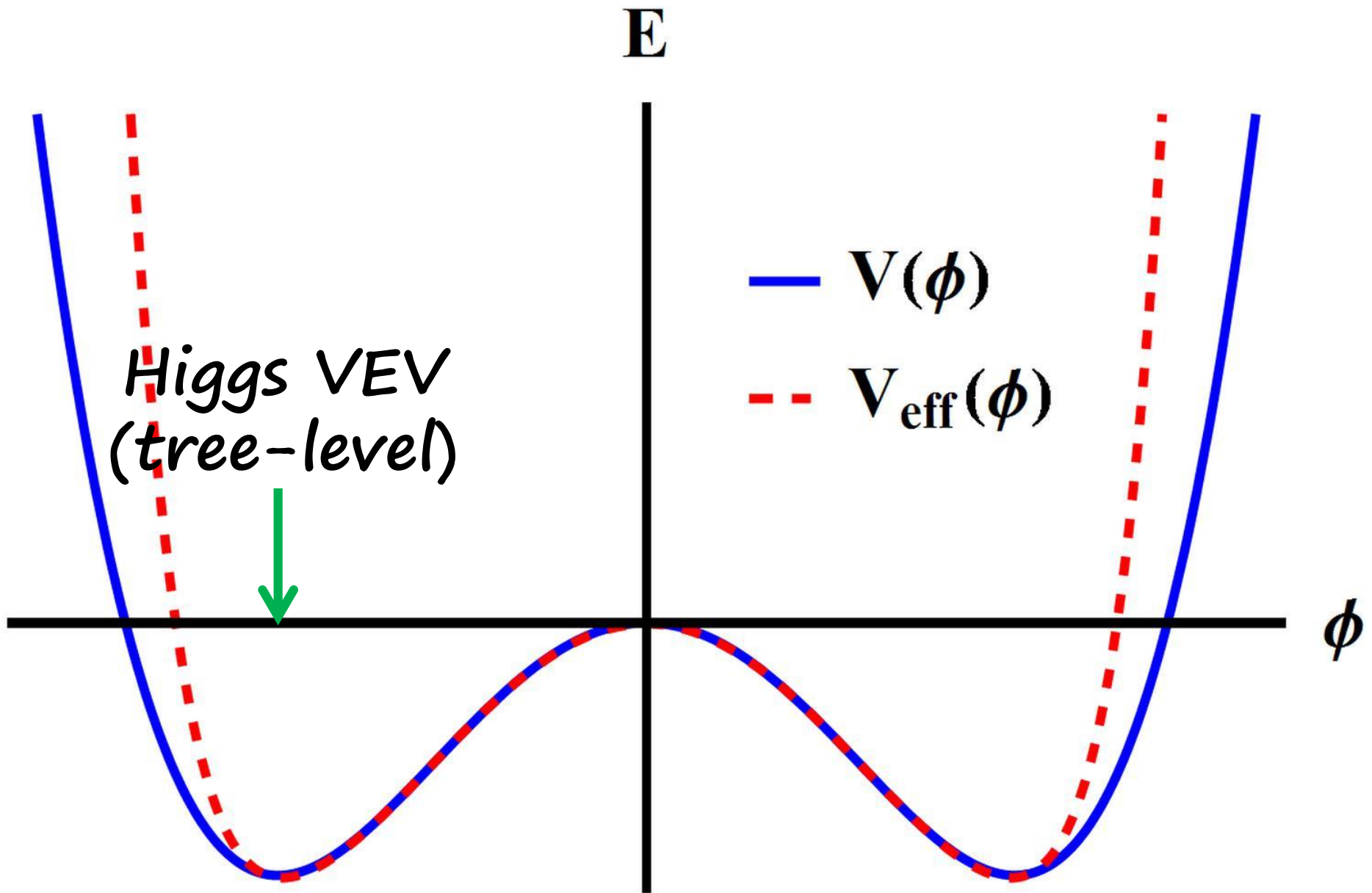
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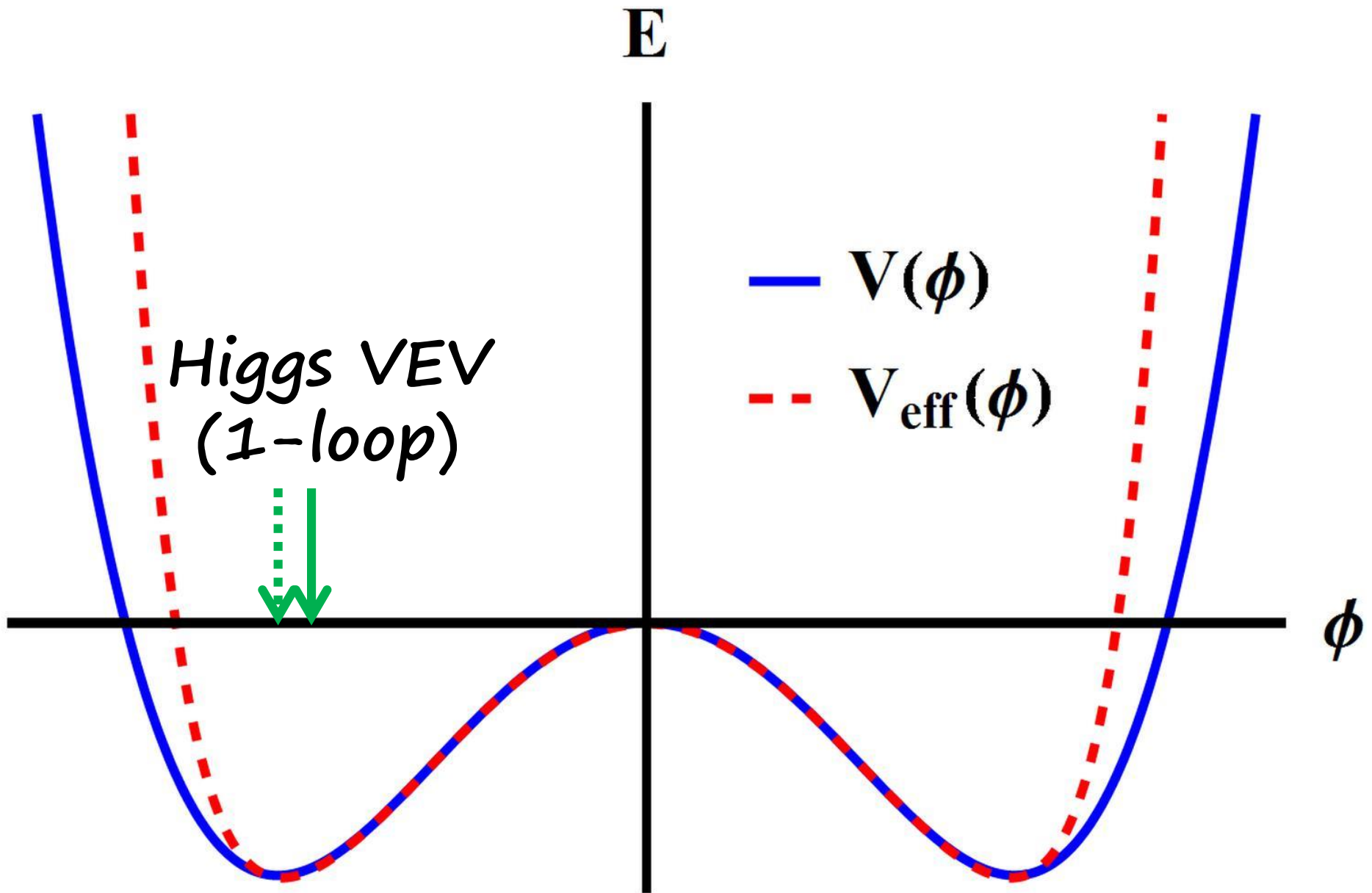
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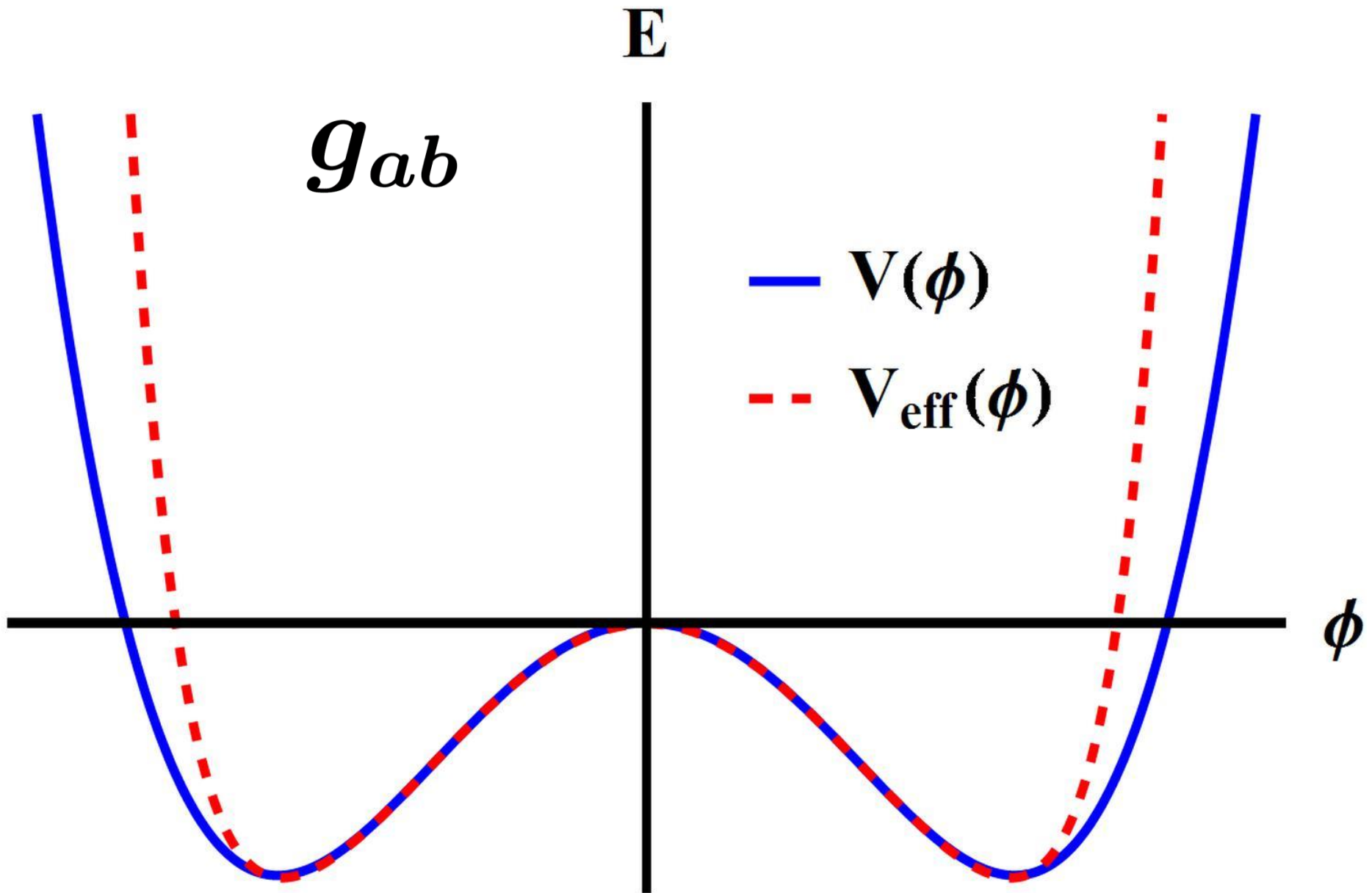


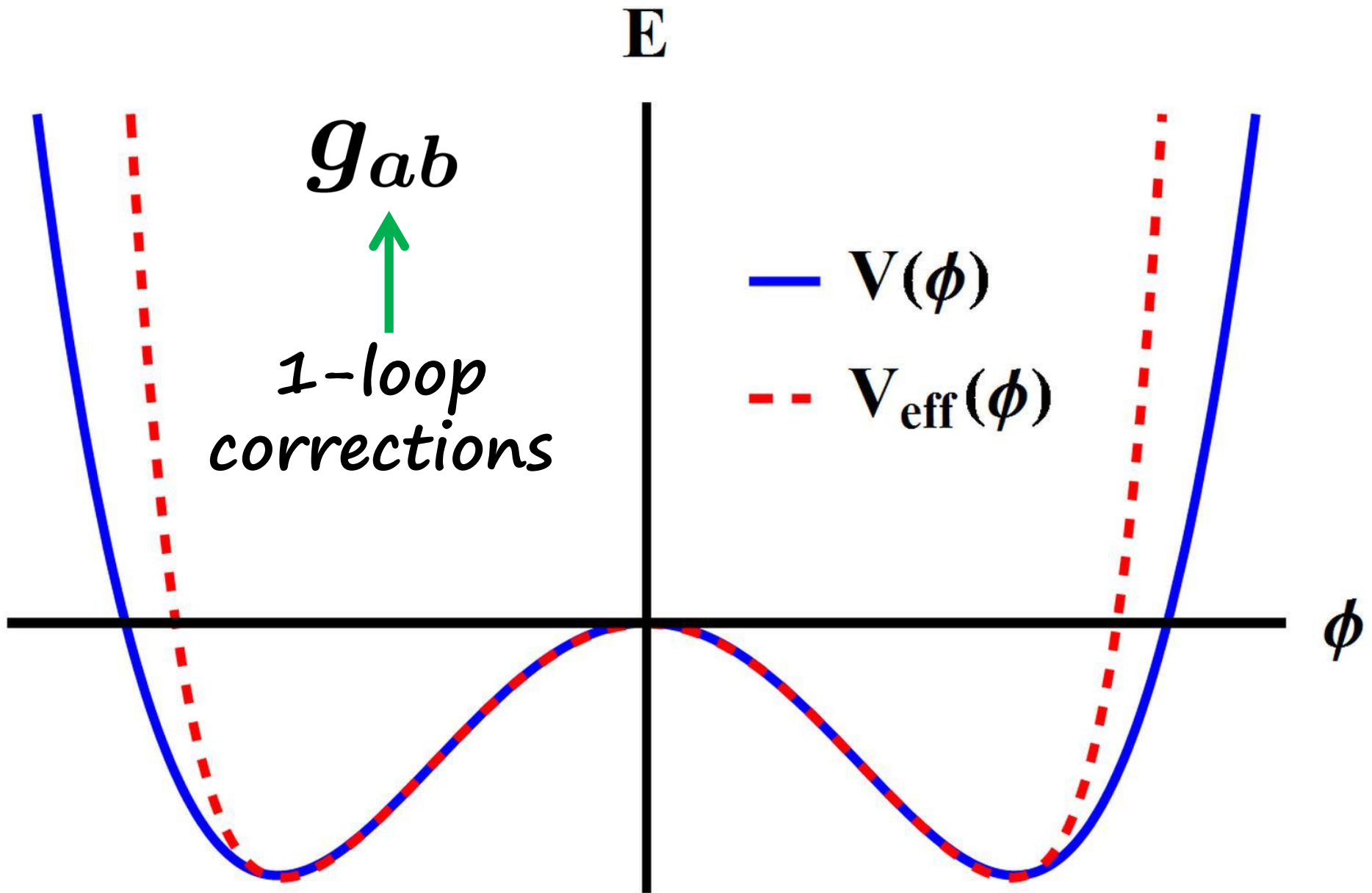
*Does gravity affect the
Higgs VEV?*











Outline

- **Introduction**
- Modes
- Results
- Observational effects
- Conclusions

Introduction

KG eq.

$$\square \phi + V'(\phi) = 0$$

Introduction

KG eq.

$$\square \phi + V'(\phi) = 0$$

Quantum operator



Introduction

KG eq.

$$\square \phi + V'(\phi) = 0$$

Quantum operator

$$\hat{\phi} + \delta\phi$$

Background
field

Quantum
perturbation

Introduction

KG eq.

$$\square \phi + V'(\phi) = 0$$

Quantum operator

where...

$$\langle 0 | \delta\phi | 0 \rangle = 0$$

$$\hat{\phi} + \delta\phi$$

$$\hat{\phi} = \langle 0 | \hat{\phi} + \delta\phi | 0 \rangle$$

Background
field

Quantum
perturbation

Introduction

$$\square \phi + V'(\phi) = 0$$

$$V'(\hat{\phi}) + V''(\hat{\phi}) \delta\phi + \frac{1}{2} V'''(\hat{\phi}) \delta\phi^2 + \dots$$

Introduction

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$$V'(\hat{\phi}) + V''(\hat{\phi}) \delta\phi + \frac{1}{2} V'''(\hat{\phi}) \delta\phi^2 + \dots$$

Take $\langle \mathbf{0} | \dots | \mathbf{0} \rangle$

Introduction

$$\square \hat{\phi} + V'_{\text{eff}}(\hat{\phi}) = 0$$

$$V'(\hat{\phi}) + V''(\hat{\phi}) \langle \delta\phi \rangle + \frac{1}{2} V'''(\hat{\phi}) \langle \delta\phi^2 \rangle + \dots$$

Take $\langle 0 | \dots | 0 \rangle$

Introduction

$$\square \hat{\phi} + V'_{\text{eff}}(\hat{\phi}) = 0$$

$$V'(\hat{\phi}) + \cancel{V''(\hat{\phi}) \langle \delta\phi \rangle} + \frac{1}{2} V''''(\hat{\phi}) \langle \delta\phi^2 \rangle + \dots$$

○ Quantum fluctuations
(1-loop)

Take $\langle 0 | \dots | 0 \rangle$

Introduction

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Introduction

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$$V'^{\circ}_{\text{eff}}(\hat{\phi})$$

Introduction

1-loop effective potential for $\hat{\phi}$

$$V_{\text{eff}}^{\circ}(\hat{\phi}) = V(\hat{\phi}) + V^{\circ}(\hat{\phi})$$

$$V^{\circ}(\hat{\phi}) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 \langle \mathbf{0} | \delta\phi^2 | \mathbf{0} \rangle$$

Introduction

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Quadratic operator 

Introduction

Flat space-time

$$V_{\text{eff}}^{\circ} = V + \underbrace{\frac{\hbar}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + m^2}}_{\substack{\text{Vacuum energy of} \\ \text{quantum fluctuations}}}$$

$\frac{1}{2} \hbar \omega$

Introduction

Flat space-time

$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{2} \int \frac{d^3 k}{(2\pi)^3} \sqrt{k^2 + m^2}$$

Vacuum energy of
quantum fluctuations $\frac{1}{2} \hbar \omega$

$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{64\pi^2} m^4 \log \left(\frac{m^2}{\mu^2} \right)$$

Coleman & Weinberg '73
Including W, Z and top contributions

Introduction

Flat FRW

$$V_{\text{eff}}^{\circ} = V + \underbrace{\frac{\hbar}{2} \int \frac{d^3(\mathbf{k}/a)}{(2\pi)^3} \sqrt{(\mathbf{k}/a)^2 + m^2}}_{\text{Vacuum energy of quantum fluctuations}}$$

Vacuum energy of quantum fluctuations $\frac{1}{2} \hbar \omega$

$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{64\pi^2} m^4 \log \left(\frac{m^2}{\mu^2} \right)$$

Introduction

Flat FRW

$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{2} \int \frac{d^3(\mathbf{k}/a)}{(2\pi)^3} \sqrt{(k/a)^2 + m^2}$$

Comoving scales get redshifted



$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{64\pi^2} m^4 \log \left(\frac{m^2}{\mu^2} \right)$$

Physical renormalization scale



Introduction

Flat space-time

Flat FRW

} *Homogeneous effect*

Non-homogeneous effect?

Introduction

Perturbations

$$ds^2 = a^2(\eta) \{ [1 + 2\Phi(\eta, x)] d\eta^2 - [1 - 2\Psi(\eta, x)] dx^2 \}$$

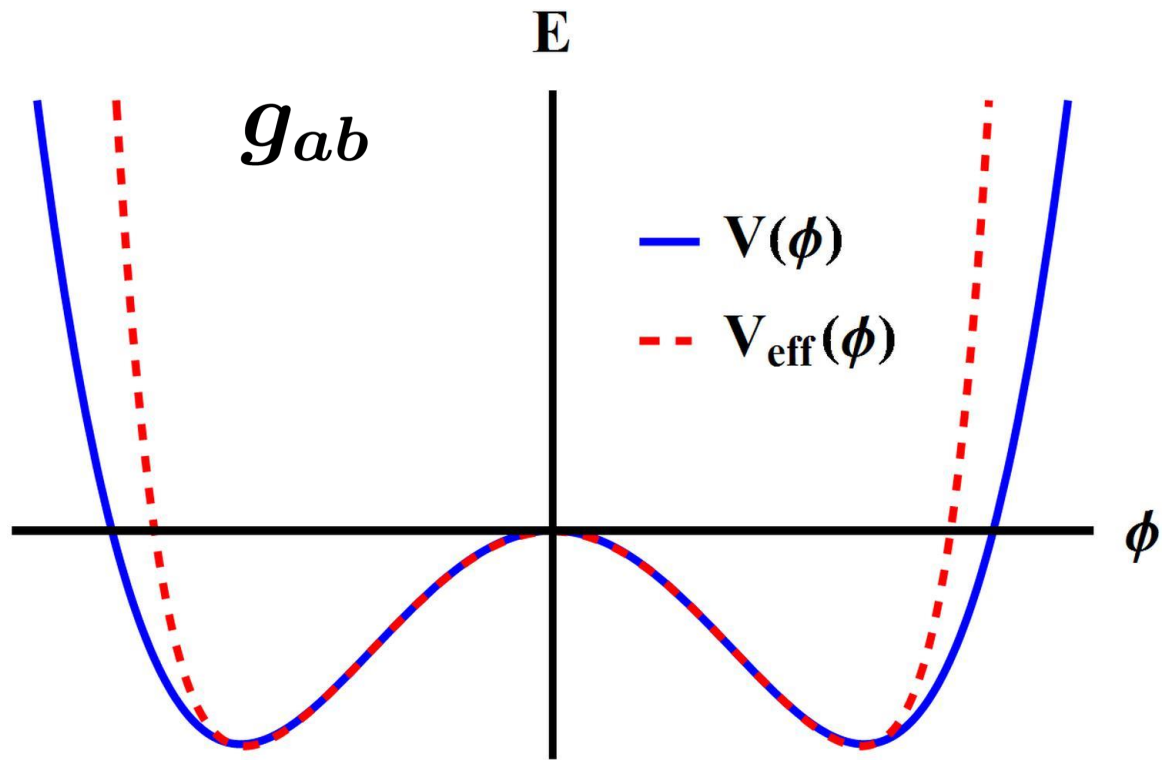
Φ, Ψ

*Static weak
gravitational field
(Solar System...)*

*Cosmological effects
due to the LSS*

So...

1-loop
corrections



due to the interactions
with SM particles

sensitive to the space-
time geometry

Φ, Ψ

In this work,
Higgs self-interactions

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Modes

1-loop effective potential for $\hat{\phi}$

$$V_{\text{eff}}^{\circ}(\hat{\phi}) = V(\hat{\phi}) + V^{\circ}(\hat{\phi})$$

$$V^{\circ}(\hat{\phi}) = \frac{1}{2} \int_0^{m^2(\hat{\phi})} dm^2 \langle \mathbf{0} | \delta\phi^2 | \mathbf{0} \rangle$$

Quadratic operator 

Modes

Quadratic operator $\langle 0 | \delta\phi^2 | 0 \rangle$

• Divergent part

- Local
- Independent of $|0\rangle$
- Covariant (curvature tensors)
- Schwinger-de Witt

• Finite part



Renormalized
physical quantities

- Non-local
- Depends on $|0\rangle$
- Not manifestly covariant
- “Brute force” (mode summation)

Modes

this is all about classical gravity

with quantum fields

QFT in curved spacetimes

(Birrel & Davies '82)

Modes

1) Quantize the fluctuations canonically

$$\delta\phi = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \left(a_{\mathbf{k}} \delta\phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \delta\phi_{\mathbf{k}}^* \right)$$

2) Find the modes

- Solve KG to first order (in metric perturbations...)
- Using a WKB ansatz

$$\delta\phi_{\mathbf{k}} = f_{\mathbf{k}} e^{i\theta_{\mathbf{k}}}$$

$$\underline{\omega^2 \gg \mathcal{H}^2, \nabla^2(\Phi, \Psi)}$$

Modes

1) Quantize the fluctuations canonically

$$\delta\phi = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \left(a_{\mathbf{k}} \delta\phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \delta\phi_{\mathbf{k}}^* \right)$$

2) Find the modes

- Solve KG to first order (in metric perturbations...)
- Using a WKB ansatz
- Boundary conditions

Match the perturbed modes to the unperturbed ones at $\eta = 0$

(adiabatic vacuum)

Modes

1) Quantize the fluctuations canonically

$$\delta\phi = \int \frac{d\mathbf{k}^3}{(2\pi)^3} \left(a_{\mathbf{k}} \delta\phi_{\mathbf{k}} + a_{\mathbf{k}}^\dagger \delta\phi_{\mathbf{k}}^* \right)$$

2) Find the modes

$$\delta\phi_{\mathbf{k}} = f_{\mathbf{k}} e^{i\theta_{\mathbf{k}}}$$

3) Compute

$$\langle \mathbf{0} | \delta\phi^2 | \mathbf{0} \rangle$$

4) Regularize & Renormalize

Modes

4) Regularize & Renormalize

$$\langle \mathbf{0} | \delta\phi^2 | \mathbf{0} \rangle$$

- Fourier space
- Expand in powers of p
- Dimensional regularization for the integration over quantum modes k

Renormalization?

- The same **UV** behaviour than in flat space-time
- Contributions from Φ , Ψ to the V_{eff}° are **finite**.

Outline

- **Introduction** ✓
- **Modes** ✓
- **Results**
- **Observational effects**
- **Conclusions**

Outline

- **Introduction** ✓
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Results

$$V_{\text{eff}}^{\circ} = V + \frac{\hbar}{64\pi^2} m^4 \log\left(\frac{m^2}{\mu^2}\right) + \frac{\hbar}{16\pi^2} m^4 (H_{\Phi} + H_{\Psi})$$

Results

$$V_{\text{eff}}^{\circ} = V + \underbrace{\frac{\hbar}{64\pi^2} m^4 \log\left(\frac{m^2}{\mu^2}\right)}_{\text{Homogeneous contribution}} + \frac{\hbar}{16\pi^2} m^4 (H_{\Phi} + H_{\Psi})$$

Homogeneous
contribution

- *Divergent*
- *Does not depend on geometry to this order*

Results

$$V_{\text{eff}}^{\circ} = V + \underbrace{\frac{\hbar}{64\pi^2} m^4 \log\left(\frac{m^2}{\mu^2}\right)}_{\text{Homogeneous contribution}} + \underbrace{\frac{\hbar}{16\pi^2} m^4 (H_{\Phi} + H_{\Psi})}_{\text{Non-homogeneous contribution}}$$

*Homogeneous
contribution*

*Non-homogeneous
contribution*

- *Divergent*
- *Does not depend on geometry to this order*

- *Finite*
- *Non-local*

Results

Why not effective action?

$$\Gamma_Q = \int d^4x (-V_{\text{eff}}) + \dots (\partial\phi)^2 + \dots$$

$$\omega^2 \gg \mathcal{H}^2, \nabla^2(\Phi, \Psi)$$

Quantum
frequencies

Higgs boson ~ 125 GeV
mass

Gravitational
frequencies

Solar System $\sim 10^{-25}$ GeV
Cosmology $\sim 10^{-39}$ GeV

Results

Higgs VEV $\hat{\phi}(t, x) = v + \Delta v(t, x)$

Homogeneous Space-time dependent

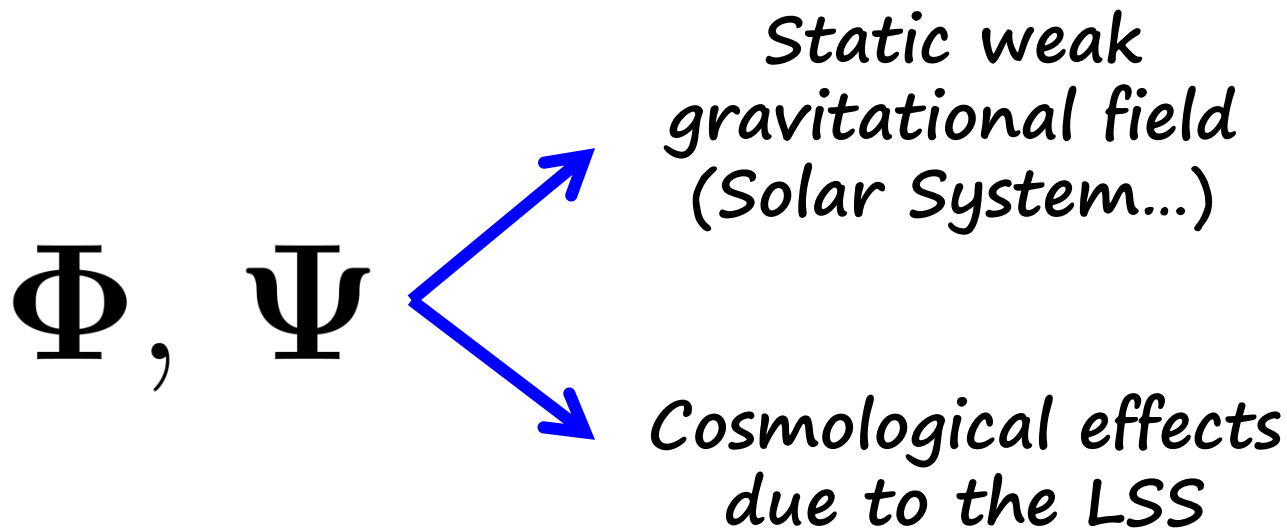
$$\Delta_{\text{Higgs}} = \frac{\Delta v}{v} = -\frac{3\lambda}{4\pi^2} (H_{\Phi} + H_{\Psi})$$

- Non-homogeneous effect, the field value which minimizes the potential is different for each point of spacetime.

Results

Perturbations

$$ds^2 = a^2(\eta) \{ [1 + 2\Phi(\eta, x)] d\eta^2 - [1 - 2\Psi(\eta, x)] dx^2 \}$$



Results

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$$ds^2 = a^2(\eta) \{ [1 + 2\Phi(\eta, x)] d\eta^2 - [1 - 2\Psi(\eta, x)] dx^2 \}$$

Φ, Ψ

Static weak
gravitational field
(Solar System..)

Cosmological effects
due to the LSS

Results

Solar System (momentum space)

$$H_{\Phi+\Psi} = \left(\frac{\sin(pt)}{pt} - \cos(pt) \right) \left(\frac{\Phi(p) + \Psi(p)}{2} \right)$$

$$H_{\Phi-\Psi} = \left(\frac{\sin(pt)}{pt} - 1 \right) \left(\frac{\Phi(p) - \Psi(p)}{2} \right)$$

Results

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$$H_{\Phi+\Psi}, H_{\Phi-\Psi} \xrightarrow{p \rightarrow 0} 0$$

There is no shift in the spacetime mean
value of Higgs VEV

Results

Newtonian
potential

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There is no shift in the spacetime mean
value of Higgs VEV

Results

Solar System (real space, momentum space)

- Newtonian potential

$$\Phi_N(r) = -\frac{1}{r} G M \quad \longleftrightarrow \quad \Phi_N(p) = -\frac{4\pi}{p^2} G M$$

Results

Solar System (real space)

$$\left. \begin{aligned} H_{\Phi+\Psi}^N &= \begin{pmatrix} r \\ - \\ t \end{pmatrix} \Phi_N \\ H_{\Phi-\Psi}^N &= \frac{1}{2} \begin{pmatrix} r \\ - \\ t \end{pmatrix} - \mathbf{1} \Phi_N (1 - \gamma) \end{aligned} \right\} \times \theta(t^2 - r^2)$$

Results

Solar System (real space)

$$H_{\Phi+\Psi}^N = \left(\frac{r}{t} \right) \Phi_N$$

$$H_{\Phi-\Psi}^N = \frac{1}{2} \left(\frac{r}{t} - 1 \right) \Phi_N (1 - \gamma)$$

$$\left. \begin{array}{l} H_{\Phi+\Psi}^N \\ H_{\Phi-\Psi}^N \end{array} \right\} \times \frac{\theta(t^2 - r^2)}{} \downarrow \text{Causality}$$

Results

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Remarks

- $\frac{r}{t} \rightarrow$ Boundary effects from the bc's of the modes

Results

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Remarks

- $\frac{r}{t} \rightarrow$ Boundary effects from the bc's of the modes
 $\underline{t} \rightarrow \infty$

Results

Solar System (real space)

$$H_{\Phi+\Psi}^N = 0$$

$$H_{\Phi-\Psi}^N = -\frac{1}{2}\Phi_N (1 - \underbrace{\gamma}_{\text{Eddington parameter}})$$

Remarks

Eddington
parameter

- $\frac{r}{t} \rightarrow$ Boundary effects from the bc's of the modes

$$\underline{t} \rightarrow \infty$$

Results

Solar System (real space)

$$H_{\Phi+\Psi}^N = 0$$

$$H_{\Phi-\Psi}^N = -\frac{1}{2}\Phi_N (1 - \underbrace{\gamma}_{\text{Eddington parameter}})$$

Remarks

- $\gamma = \frac{\Psi}{\Phi} \rightarrow$ In GR $\gamma = 1$
no effect in GR

Results

Solar System (real space, momentum space)

- Newtonian potential

$$\Phi_N(r) = -\frac{1}{r} G M \quad \longleftrightarrow \quad \Phi_N(p) = -\frac{4\pi}{p^2} G M$$

Results

Solar System (real space, momentum space)

- Newtonian potential

$$\Phi_N(r) = -\frac{1}{r} G M \longleftrightarrow \Phi_N(p) = -\frac{4\pi}{p^2} G M$$

- General potential

$$\Phi(r) = -\frac{1}{r} \sum_{l=0}^{\infty} \sum_{m=l}^l \frac{Q_{lm}}{r^l} \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta, \phi)$$



$$\Phi(p) = -\frac{4\pi}{p^2} \sum_{l=0}^{\infty} i^l \sum_{m=l}^l \frac{Q_{lm}}{(2p)^l} \sqrt{\frac{4\pi}{2l+1}} Y_{lm}(\theta_p, \phi_p)$$

Results

Solar System (real space)

Newtonian results

$$H_{\Phi+\Psi}^N = 0$$

$$H_{\Phi-\Psi}^N = -\frac{1}{2}\Phi_N (1 - \underbrace{\gamma}_{\text{Eddington parameter}})$$

Remarks

Eddington
parameter

- $\gamma = \frac{\Psi}{\Phi} \rightarrow$ In GR $\gamma = 1$
no effect in GR

Results

Solar System (real space)

General results

$$H_{\Phi+\Psi}^{\times} = 0$$

$$H_{\Phi-\Psi}^{\times} = -\frac{1}{2}\Phi^{\times} (1 - \underbrace{\gamma}_{\text{Eddington parameter}})$$

Remarks

Eddington
parameter

- $\gamma = \frac{\Psi}{\Phi} \rightarrow$ In GR $\gamma = 1$
no effect in GR

Outline

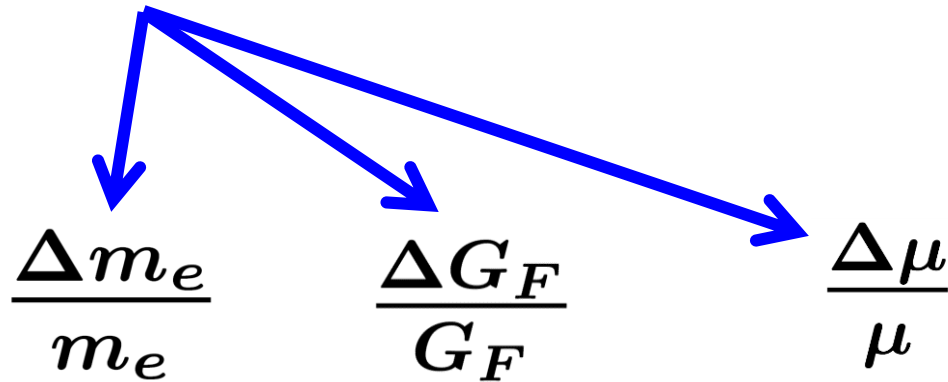
- **Introduction** ✓
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Observational effects

Δ Higgs



Observational effects

Δ Higgs

$$\frac{\Delta m_e}{m_e}$$

$$\frac{\Delta G_F}{G_F}$$

$$\frac{\Delta \mu}{\mu}$$

Observational effects

Solar System

$$\Delta_{\text{Higgs}} = \frac{3\lambda}{8\pi^2} \underbrace{\Delta\Phi}_{\text{Gravitational potential}} (1 - \gamma)$$

Eddington parameter

$\lambda \simeq 1/8$

Observational effects

Solar System

$$\Delta_{\text{Higgs}} = \frac{3\lambda}{8\pi^2} \Delta\Phi (1 - \gamma)$$

$$\frac{\Delta\mu}{\mu} = -\Delta_{\text{Higgs}}$$

Proton-to-electron mass ratio

$$\frac{\Delta\mu}{\mu} < 10^{-16} \quad \text{Atomic clocks
on Earth}$$

Huntemann, et al.
2014

Observational effects

Solar System

$$|\gamma - 1| < 10^{-5}$$

*Cassini bound,
Bertotti, et al. 2003*

$$\Delta_{\text{Higgs}} = \frac{3\lambda}{8\pi^2} \Delta\Phi (1 - \gamma)$$

$$\frac{\Delta\mu}{\mu} = -\Delta_{\text{Higgs}}$$

$$|\gamma - 1| < 10^{-4} \quad \text{on Earth}$$

$$\Delta\Phi_{\oplus} \approx 10^{-10}$$

Proton-to-electron mass ratio

$$|\gamma - 1| < 10^{-8} \quad \text{around the Sun}$$

$$\Delta\Phi_{\odot} \approx 10^{-6}$$

$$\frac{\Delta\mu}{\mu} < 10^{-16} \quad \text{Atomic clocks
on Earth}$$

*Huntemann, et al.
2014*

Franco D. Albareti
19th November 2015
IV Post. Meet. Theor. Phys.

Observational effects

Solar System

$$\Delta^i_{\text{Higgs}} \approx \frac{3\lambda}{8\pi^2} \Delta\Phi (1 - \gamma)$$

- *Higgs self-interactions*
- *Vector bosons*
- *Top quark*

Observational effects

Solar System

$$\Delta^i_{\text{Higgs}} \approx \frac{3\lambda}{8\pi^2} \Delta\Phi (1 - \gamma) \times n_{\text{eff}} \times$$




Bosons,
Fermions

- Higgs self-interactions
- Vector bosons
- Top quark

Observational effects

Solar System

$$\Delta^i_{\text{Higgs}} \approx \frac{3\lambda}{8\pi^2} \Delta\Phi (1 - \gamma) \times n_{\text{eff}} \times \left(\frac{g_i}{\lambda}\right) \times \left(\frac{m_i}{m_{\text{Higgs}}}\right)^4$$



Bosons,
Fermions

Mass/coupling
factors

- Higgs self-interactions
- Vector bosons
- Top quark

Work in progress...

Outline

- **Introduction** ✓
- **Modes** ✓
- **Results** ✓
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Outline

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Conclusions

- *Metric perturbations contribute to the finite part of the Higgs 1-loop effective potential*
- *This leads to a space-time dependent Higgs VEV which translates into variations on the masses of all the elementary particles.*
- *Within the Solar System, constraints on the Eddington parameter can be obtained from measurements of the proton-to-electron mass ratio.*