Anomaly-free Chiral Fermion Sets and Gauge Coupling Unification

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Based on:

Minimal anomaly-free chiral fermion sets and gauge coupling unification
LMC, D. Emmanuel-Costa, R. González Felipe, C. Simões
Phys. Rev. D 90, 125037 (2014) [arXiv:1409.0805]

IV Postgradute Meeting on Theoretical Physics

Outline

- Introduction and Motivation
- Minimal Anomaly-free Chiral Fermion Sets
- SU(5)-inspired Anomaly-free Chiral Fermion Sets
- Concluding Remarks

Introduction and Motivation

Standard Model (SM)

- Particle content is chiral (not vectorlike) $(d_3(R), d_2(R))_{y_R}$

$$(\overline{\mathbf{3}}, \mathbf{2})_{-1/6} \oplus (\mathbf{3}, \mathbf{1})_{2/3} \oplus (\mathbf{3}, \mathbf{1})_{-1/3} \oplus (\mathbf{1}, \mathbf{2})_{1/2} \oplus (\mathbf{1}, \mathbf{1})_{-1}$$

➔ No gauge-invariant mass term $m\overline{\psi}\psi$

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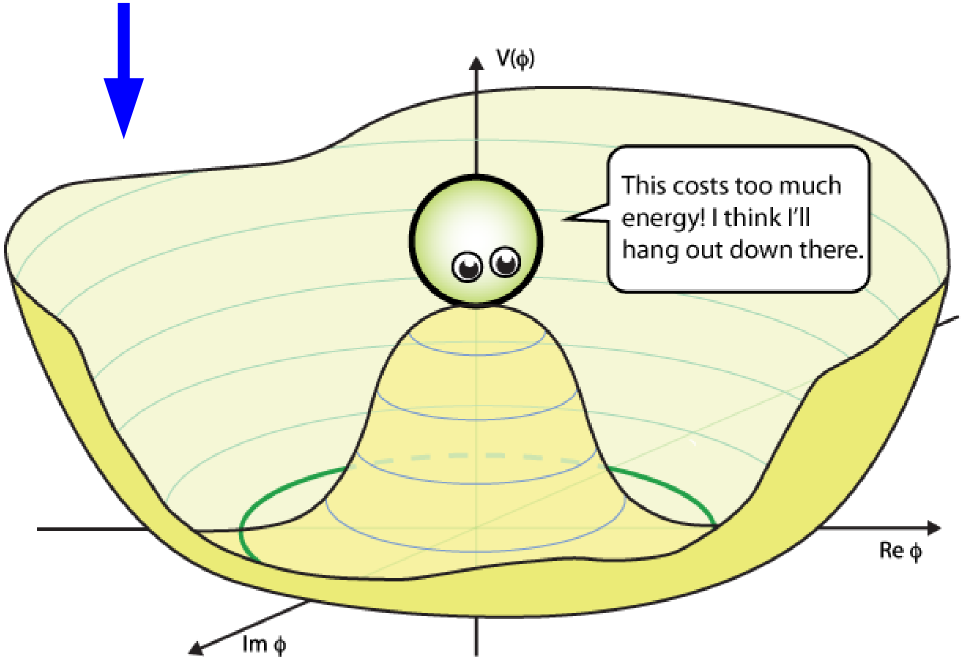
➔ Higgs mechanism



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- SM has shortcomings: Dark matter, Neutrino mass, Hierarchy problem...

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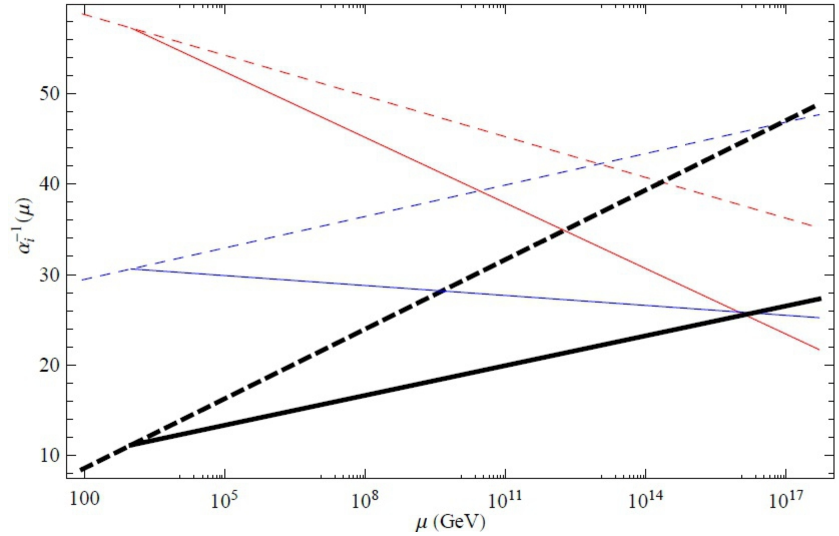
- SM has shortcomings: Dark matter, Neutrino mass, Hierarchy problem...

  ➔ No gauge coupling unification in the SM



Gauge coupling unification (GCU)

- Renormalization group equations at one loop level
  - in SM: $\alpha_i^{-1}(\lambda) = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i} b_i^{SM} \ln\left(\frac{\lambda}{M_Z}\right)$
  - SM with N extra particles:

$$\alpha_U^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i}\left(b_i^{SM} + \sum_{j=1}^{N} b_i^j r_j\right) \ln\left(\frac{\Lambda}{M_Z}\right)$$

$$r_j = \frac{\ln(\Lambda/M_j)}{\ln(\Lambda/M_Z)} \qquad b_i = \frac{2}{3} t_i(R) \prod_{j \neq i} d_j(R)$$

$d_i(R)$ is the dimension and $t_i(R)$ the Dynkin index

Gauge coupling unification (GCU)

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- Connection between the strength of different forces
- In Grand Unified Theories (GUTs) there is also a connection between Yukawa couplings

⟶ GCU decreases some arbitrariness in SM parameters

Gauge coupling unification (GCU)

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GCU $\xrightarrow{\text{this framework}}$ $\sum_{j=1}^{N} b_i^j r_j \neq 0$ $\longrightarrow$ **Particles beyond the SM ones**

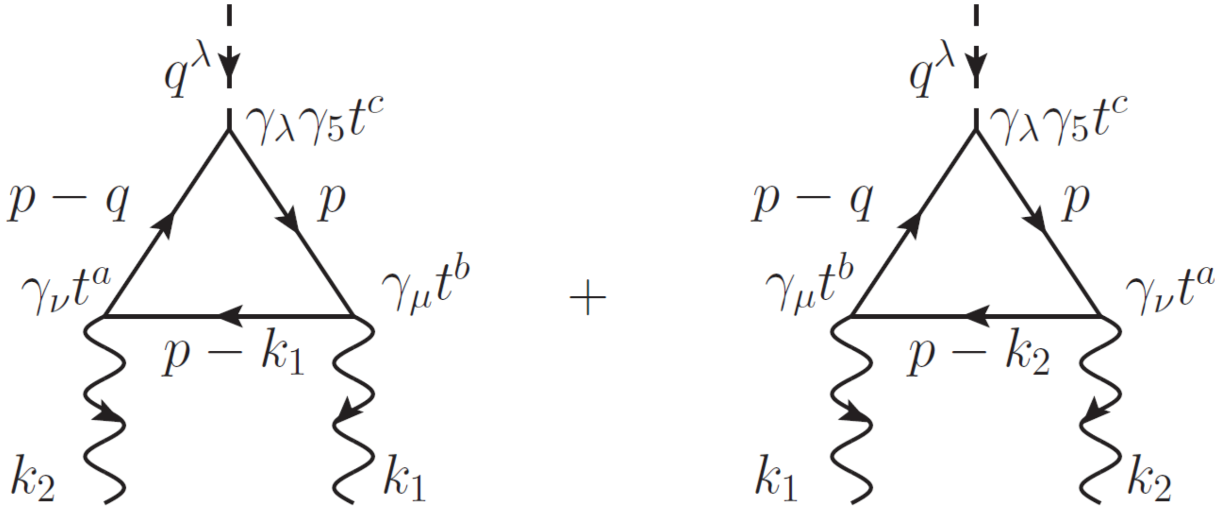
Anomalies

- Renormalizability implies Ward identities

$$\mathcal{M}(k) = \epsilon_\mu(k)\mathcal{M}^\mu(k) \longrightarrow k_\mu \mathcal{M}^\mu(k) = 0$$

➔ Renormalizable theory must be free of triangular anomalies

$\mathcal{M}^{abc}_{\mu\nu\lambda}$:



$$q^\lambda \mathcal{M}^{abc}_{\mu\nu\lambda} = \mathcal{A}^{abc}_{\mu\nu} = \frac{1}{4\pi^2}\varepsilon_{\mu\nu\alpha\beta}k_1^\alpha k_2^\beta \mathrm{tr}_R\left[\left\{t^a, t^b\right\} t^c\right]$$

Anomalies

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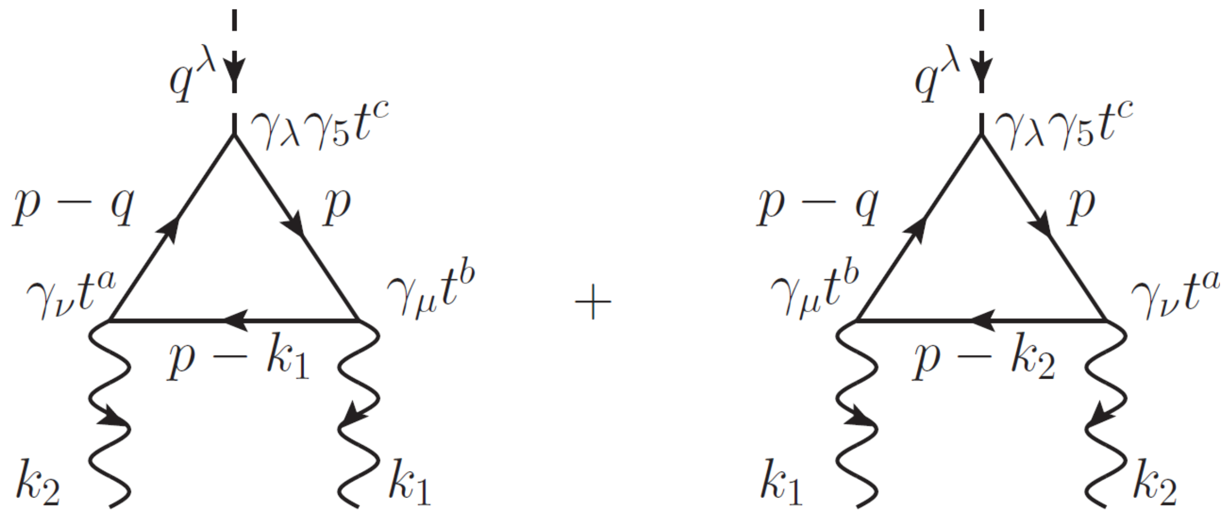


$\longrightarrow$ Anomaly-free condition: $\sum_f \mathrm{tr}_R \left[\{t^a, t^b\} t^c \right] = 0$

Anomalies

- Renormalizability implies Ward identities

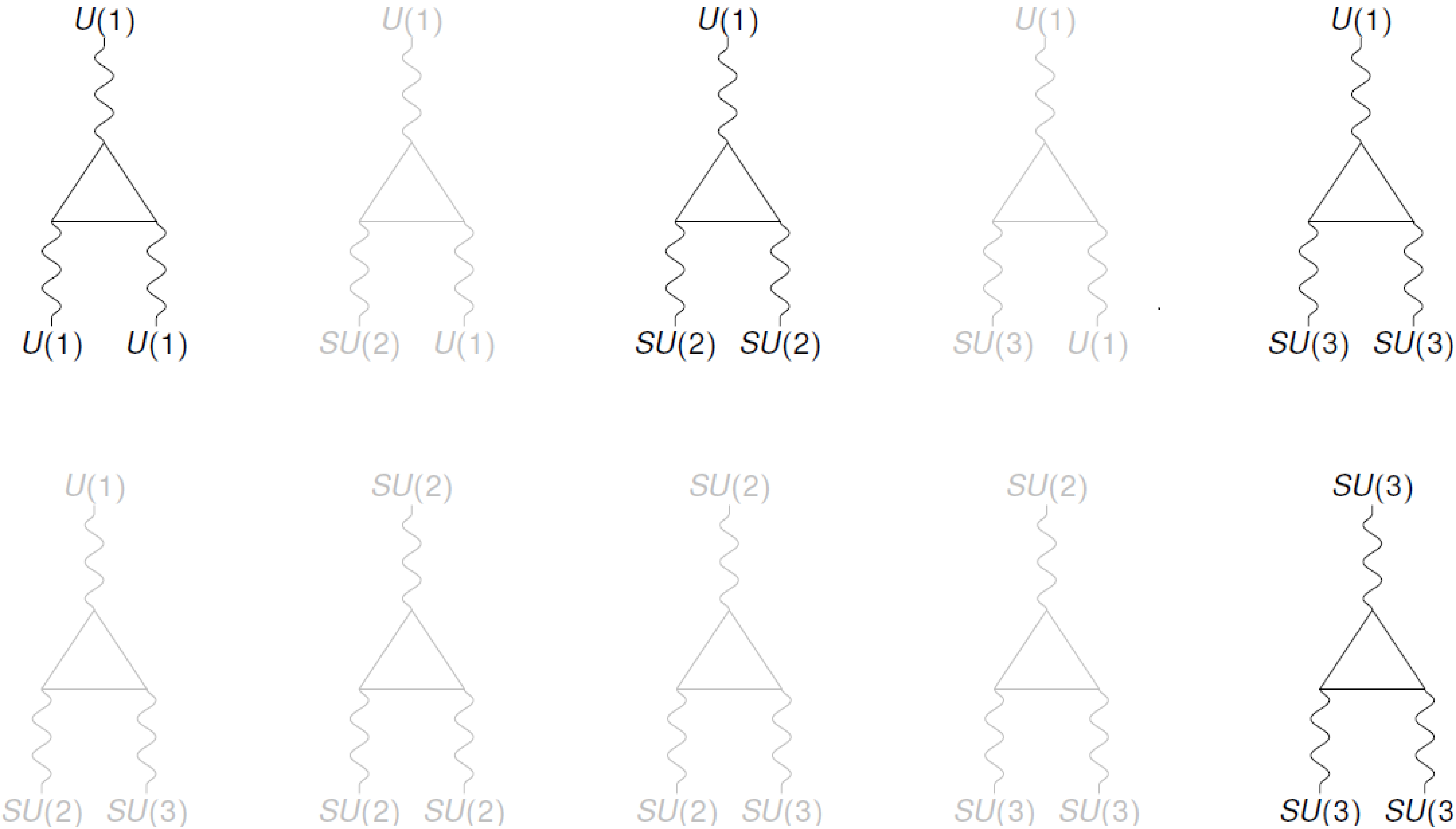
$$\mathcal{M}(k) = \epsilon_\mu(k)\mathcal{M}^\mu(k) \longrightarrow k_\mu \mathcal{M}^\mu(k) = 0$$

→ Consistent theory must be free of

- Gauge anomalies
- Mixed gauge-gravitational anomaly
- SU(2) Witten's anomaly

→ The number of chiral SU(2) doublets must be even

Anomalies


U(1)
U(1) U(1)
U(1)
SU(2) U(1)
U(1)
SU(2) SU(2)
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Anomalies

- 4 non-vanhising gauge anomalies and 1 mixed gauge-gravitational anomaly

$$[\mathsf{SU}(3)\,\text{-}\,\mathsf{SU}(3)\,\text{-}\,\mathsf{SU}(3)]:\quad \sum_R A_3(R)\,d_2(R) \;=\; 0$$

$$[\mathsf{SU}(3)\,\text{-}\,\mathsf{SU}(3)\,\text{-}\,\mathsf{U}(1)]:\quad \sum_R y_R\,t_3(R)\,d_2(R) \;=\; 0$$

$$[\mathsf{SU}(2)\,\text{-}\,\mathsf{SU}(2)\,\text{-}\,\mathsf{U}(1)]:\quad \sum_R y_R\,t_2(R)\,d_3(R) \;=\; 0$$

$$[\mathsf{U}(1)\,\text{-}\,\mathsf{U}(1)\,\text{-}\,\mathsf{U}(1)]:\quad \sum_R y_R^3\,d_2(R)\,d_3(R) \;=\; 0$$

$$[\text{gravity}\,\text{-}\,\text{gravity}\,\text{-}\,\mathsf{U}(1)]:\quad \sum_R y_R\,d_2(R)\,d_3(R) \;=\; 0$$

Anomalies

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$$\sum_R y_R^3\, d_2(R)\, d_3(R) \;=\; 0$$

$$\sum_R y_R\, d_2(R)\, d_3(R) \;=\; 0$$

$$\mathrm{tr}_R\left(\left\{t^a, t^b\right\} t^c\right) \;=\; A(R)\,\mathrm{tr}\left(\left\{t^a, t^b\right\} t^c\right)$$

Anomaly index: $A(R)$

$$t(R)\delta^{ab} = \mathrm{tr}_R\left[t^a t^b\right]$$

Dynkin index: $t(R)$

Anomalies

- 4 non-vanhising gauge anomalies and 1 mixed gauge-gravitational anomaly

U(1): $t_1(R) = y_R^2$

SU(2): $t_2(R) = \dfrac{d_2(R)\,[d_2^2(R) - 1]}{12}$

SU(3):

SU(3)-irrep	$\mathbf{3}$	$\mathbf{6}$	$\mathbf{8}$	$\mathbf{10}$	$\mathbf{15}$	$\mathbf{15'}$
t_3	$\frac{1}{2}$	$\frac{5}{2}$	3	$\frac{15}{2}$	10	$\frac{35}{2}$
A_3	1	7	0	27	14	77

Main goal

- Requirements:
  - Chiral fermion set beyond the SM
  - CGU at intermediate scale
  - Anomaly-free conditions verified
  - Vectorlike with respect to color SU(3) and electromagnetic U(1)

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$$(\mathbf{1},\mathbf{3})_1\,;(\mathbf{1},\mathbf{2})_{-3/2} \xrightarrow{\text{EWSB}} \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_2\ ;\ \mathbf{1}_{-1} \oplus \mathbf{1}_{-2}$$

- $\mathbf{d}_Q$ denotes states with SU(3) dimension $\mathbf{d}$ and eletric charge Q

Main goal

- Requirements:

SM ✓ → Chiral set

SM ✗ → CGU at intermediate scale

SM ✓ → Anomaly-free conditions verified

SM ✓ → Vectorlike with respect to color SU(3) and electromagnetic U(1)

Main goal

- Requirements:
  - Chiral fermion set beyond the SM
  - CGU at intermediate scale
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↓

Find the minimal chiral sets of fermions that are not only anomaly-free but also vectorlike with respect to color SU(3) and electromagnetic U(1), leading to GCU.

Minimal Anomaly-free Chiral Fermion Sets

Extra fermion content

- Adding 1 fermion:

$$
\begin{aligned}
[\mathsf{SU}(3)\text{ - }\mathsf{SU}(3)\text{ - }\mathsf{SU}(3)]: &\quad A_3(R_1)\, d_2(R_1) \;=\; 0\\
[\mathsf{SU}(3)\text{ - }\mathsf{SU}(3)\text{ - }\mathsf{U}(1)]: &\quad y_{R_1} t_3(R_1)\, d_2(R_1) \;=\; 0\\
[\mathsf{SU}(2)\text{ - }\mathsf{SU}(2)\text{ - }\mathsf{U}(1)]: &\quad y_{R_1} t_2(R_1)\, d_3(R_1) \;=\; 0\\
[\mathsf{U}(1)\text{ - }\mathsf{U}(1)\text{ - }\mathsf{U}(1)]: &\quad y^3_{R_1} d_2(R_1)\, d_3(R_1) \;=\; 0\\
[\text{gravity - gravity - }\mathsf{U}(1)]: &\quad y_{R_1} d_2(R_1)\, d_3(R_1) \;=\; 0
\end{aligned}
$$

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[\text{gravity}\text{-}\text{gravity}\text{-}\mathsf{U}(1)]: &\quad y_{R_1} d_2(R_1)\,d_3(R_1) \;=\; 0
\end{aligned}
$$

↓

One type of solution: $(\mathbf{8},\mathbf{d}')_0$ or $(\mathbf{1},\mathbf{d}')_0$

Singlet or adjoint representation with zero hypercharge

Extra fermion content

- Adding 2 fermions:

$$\begin{aligned} A_3(R_1)\, d_2(R_1) \;+\; A_3(R_2)\, d_2(R_2) \;&=\; 0 \\ y_{R_1} t_3(R_1)\, d_2(R_1) \;+\; y_{R_2} t_3(R_2)\, d_2(R_2) \;&=\; 0 \\ y_{R_1} t_2(R_1)\, d_3(R_1) \;+\; y_{R_2} t_2(R_2)\, d_3(R_2) \;&=\; 0 \\ y^3_{R_1} d_2(R_1)\, d_3(R_1) \;+\; y^3_{R_2} d_2(R_2)\, d_3(R_2) \;&=\; 0 \\ y_{R_1} d_2(R_1)\, d_3(R_1) \;+\; y_{R_2} d_2(R_2)\, d_3(R_2) \;&=\; 0 \end{aligned} \quad \longrightarrow \quad \begin{aligned} y_R &= z\, y'_R \\ y^2_{R_1} &= y^2_{R_2} \end{aligned}$$

Extra fermion content

- Adding 2 fermions:

$$
\begin{aligned}
A_3(R_1)\,d_2(R_1) \;+\; A_3(R_2)\,d_2(R_2) \;&=\; 0 \\
y_{R_1}t_3(R_1)\,d_2(R_1) \;+\; y_{R_2}t_3(R_2)\,d_2(R_2) \;&=\; 0 \\
y_{R_1}t_2(R_1)\,d_3(R_1) \;+\; y_{R_2}t_2(R_2)\,d_3(R_2) \;&=\; 0 \\
y^3_{R_1}d_2(R_1)\,d_3(R_1) \;+\; y^3_{R_2}d_2(R_2)\,d_3(R_2) \;&=\; 0 \\
y_{R_1}d_2(R_1)\,d_3(R_1) \;+\; y_{R_2}d_2(R_2)\,d_3(R_2) \;&=\; 0
\end{aligned}
\quad \longrightarrow \quad
\begin{aligned}
y_R &= z\, y'_R \\
y^2_{R_1} &= y^2_{R_2}
\end{aligned}
$$

- $y_{R_1} = y_{R_2} = y_R$
  - ➔ Cancellation for each multiplet (as before)
  - ➔ Cancellation between multiplets ⟶ $(\mathbf{6},\mathbf{1})_0 \oplus (\overline{\mathbf{3}},\mathbf{7})_0$
    - ⟶ Not vectorlike with respect to SU(3) nor U(1)

Extra fermion content

- Adding 2 fermions:

$$A_3(R_1)\,d_2(R_1) + A_3(R_2)\,d_2(R_2) = 0$$
$$y_{R_1}t_3(R_1)\,d_2(R_1) + y_{R_2}t_3(R_2)\,d_2(R_2) = 0$$
$$y_{R_1}t_2(R_1)\,d_3(R_1) + y_{R_2}t_2(R_2)\,d_3(R_2) = 0$$
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$$y_{R_1}d_2(R_1)\,d_3(R_1) + y_{R_2}d_2(R_2)\,d_3(R_2) = 0$$

→

$$y_R = z\,y'_R$$
$$y^2_{R_1} = y^2_{R_2}$$

- $y_{R_1} = -y_{R_2} \neq 0$

➔ Non-trivial equation: $\dfrac{d_3(R_1)}{d_3(R_2)} = \dfrac{t_3(R_1)}{t_3(R_2)} = -\dfrac{A_3(R_1)}{A_3(R_2)} = 1$

→ Imposing vectorlike condition: $(\mathbf{d}, \mathbf{d}')_y \oplus (\overline{\mathbf{d}}, \mathbf{d}')_{-y}$

Extra fermion content

- Adding 2 fermions:

$$A_3(R_1)\,d_2(R_1) + A_3(R_2)\,d_2(R_2) = 0$$
$$y_{R_1}t_3(R_1)\,d_2(R_1) + y_{R_2}t_3(R_2)\,d_2(R_2) = 0$$
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→

$$y_R = z\,y'_R$$
$$y^2_{R_1} = y^2_{R_2}$$

$(\mathbf{d},\mathbf{d}')_y \oplus (\overline{\mathbf{d}},\mathbf{d}')_{-y}$ is a vectorlike particle

↓

3 or more chiral fermion are required to fulfil the constraints

Minimal chiral fermion sets

- Adding 3 fermions: ($\mathbf{d} \equiv d_3(R) \leq 10$ and $d_2(R) \leq 5$)

Set	Particle content				
P1	$(\mathbf{d},\mathbf{1})_{5z/6}$	$\oplus$	$(\mathbf{d},\mathbf{2})_{-2z/3}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{3})_{z/6}$
P2	$(\mathbf{d},\mathbf{1})_{7z/6}$	$\oplus$	$(\mathbf{d},\mathbf{3})_{-5z/6}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{4})_{z/3}$
P3	$(\mathbf{d},\mathbf{1})_{3z/2}$	$\oplus$	$(\mathbf{d},\mathbf{4})_{-z}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{5})_{z/2}$
P4	$(\mathbf{d},\mathbf{2})_{4z/3}$	$\oplus$	$(\mathbf{d},\mathbf{3})_{-7z/6}$	$\oplus$	$(\overline{\mathbf{d}},\mathbf{5})_{z/6}$

- P1 and P4 have even $\mathbf{d}$ due to Witten's anomaly
  - ➔ With $d_3(R) > 10$ ⟶ $(\mathbf{15},\mathbf{1})_{z/6} \oplus (\overline{\mathbf{6}},\mathbf{2})_{-z/3} \oplus (\mathbf{1},\mathbf{3})_{z/2}$

Minimal chiral fermion sets

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P2	$(\mathbf{d}, \mathbf{1})_{7z/6}$	$\oplus$	$(\mathbf{d}, \mathbf{3})_{-5z/6}$	$\oplus$	$(\overline{\mathbf{d}}, \mathbf{4})_{z/3}$
P3	$(\mathbf{d}, \mathbf{1})_{3z/2}$	$\oplus$	$(\mathbf{d}, \mathbf{4})_{-z}$	$\oplus$	$(\overline{\mathbf{d}}, \mathbf{5})_{z/2}$
P4	$(\mathbf{d}, \mathbf{2})_{4z/3}$	$\oplus$	$(\mathbf{d}, \mathbf{3})_{-7z/6}$	$\oplus$	$(\overline{\mathbf{d}}, \mathbf{5})_{z/6}$

- P1 and P4 have even $\mathbf{d}$ due to Witten's anomaly

→ Imposing vectorlike condition: $|z| = 0, 1, 3, \ \ \mathbf{d} = 1, 8$

$|z| = 1, \ \ \mathbf{d} = 3, 6, 10$

↓

Chiral sets to test GCU

Minimal sets and unification

- Recall the evolution of gauge couplings

$$\alpha_U^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{1}{2\pi\kappa_i}\left(b_i^{SM} + \sum_{j=1}^{N} b_i^j r_j\right)\ln\left(\frac{\Lambda}{M_Z}\right)$$

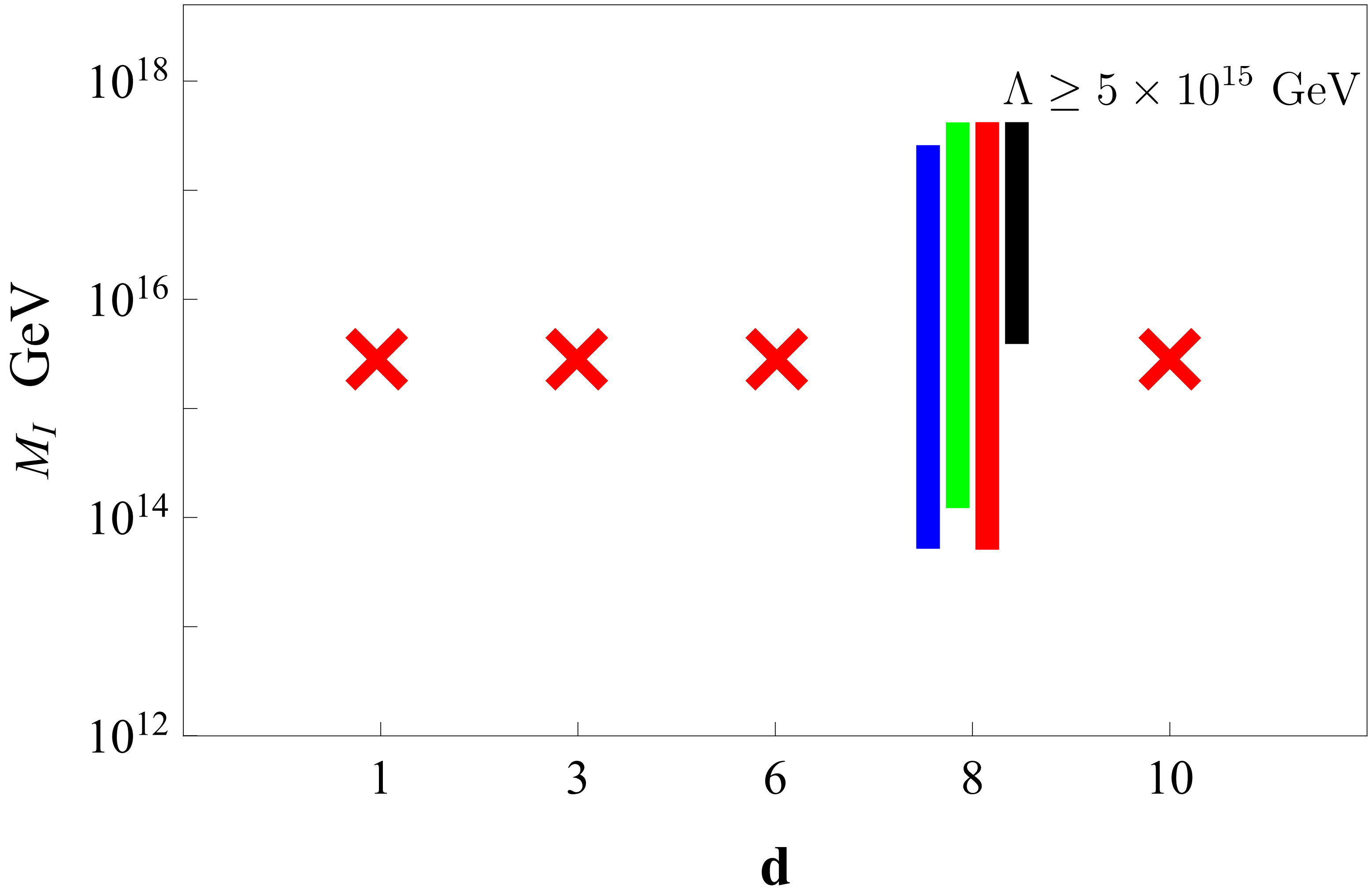
$$r_j = \frac{\ln(\Lambda/M_j)}{\ln(\Lambda/M_Z)} \qquad b_i = \frac{2}{3}t_i(R)\prod_{j\neq i} d_j(R)$$

➔ $z = 0 \longrightarrow \sum_{j}^{N} b_1^j r_j = 0$

No solutions with zero hypercharge

Minimal sets and unification

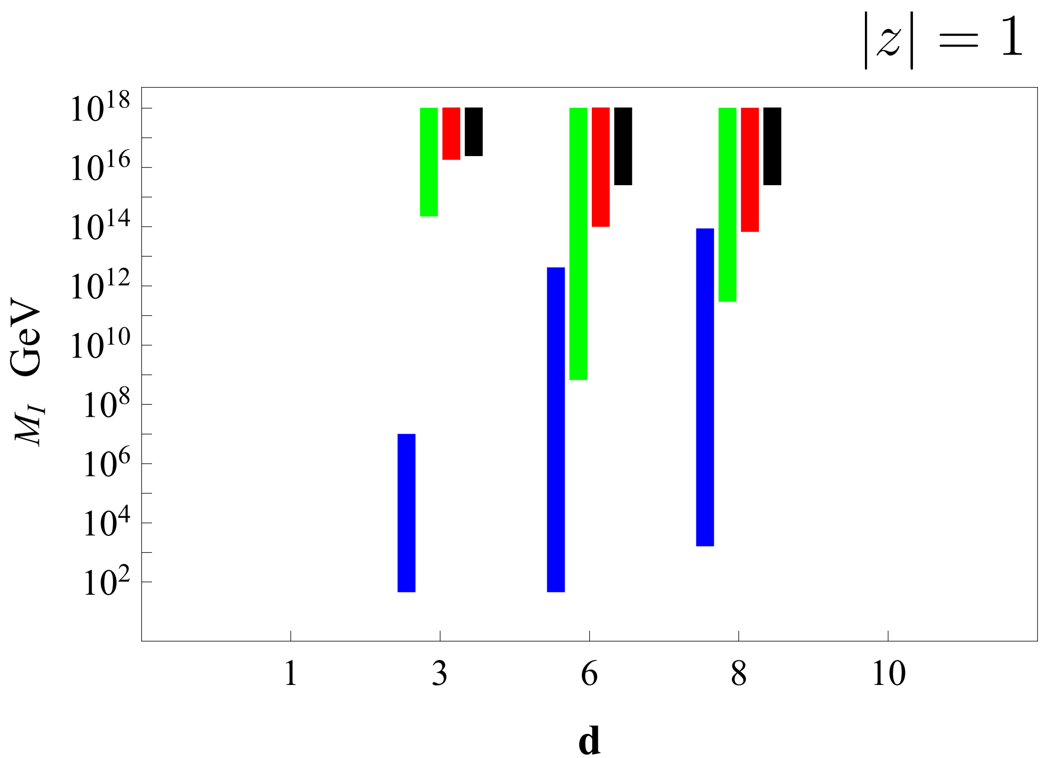
· P1: $(\mathbf{d},\mathbf{1})_{5z/6} \oplus (\mathbf{d},\mathbf{2})_{-2z/3} \oplus (\overline{\mathbf{d}},\mathbf{3})_{z/6}$ $|z| = 3$

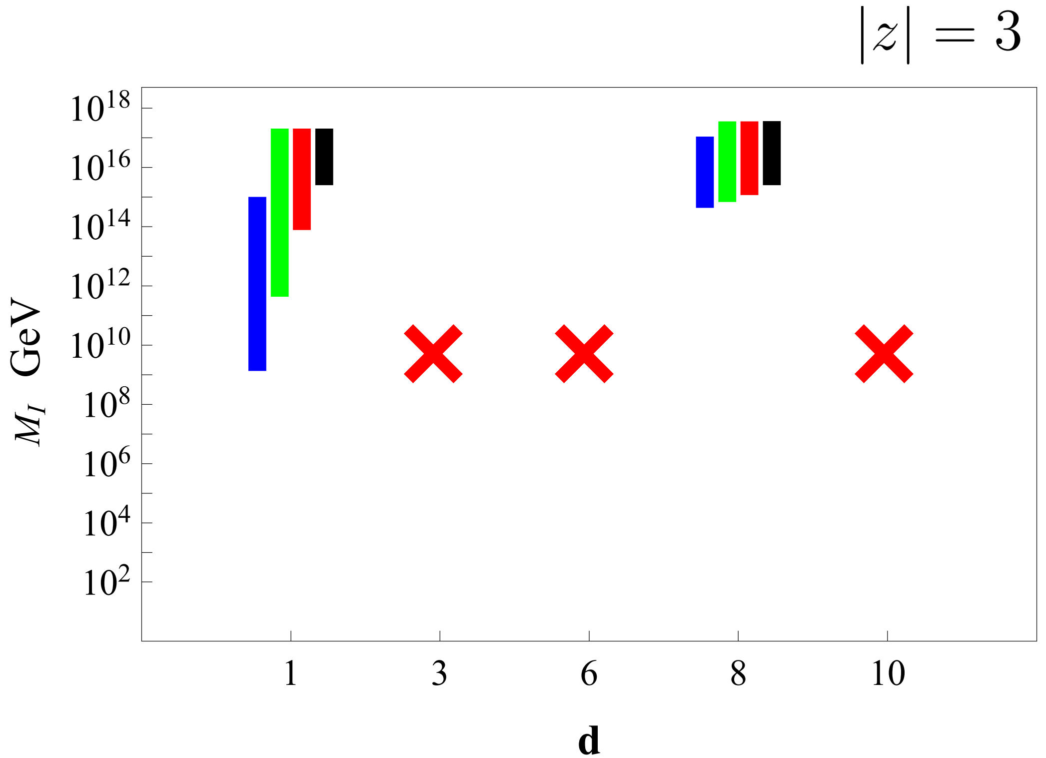


$\mathbf{d}$ is even

Minimal sets and unification

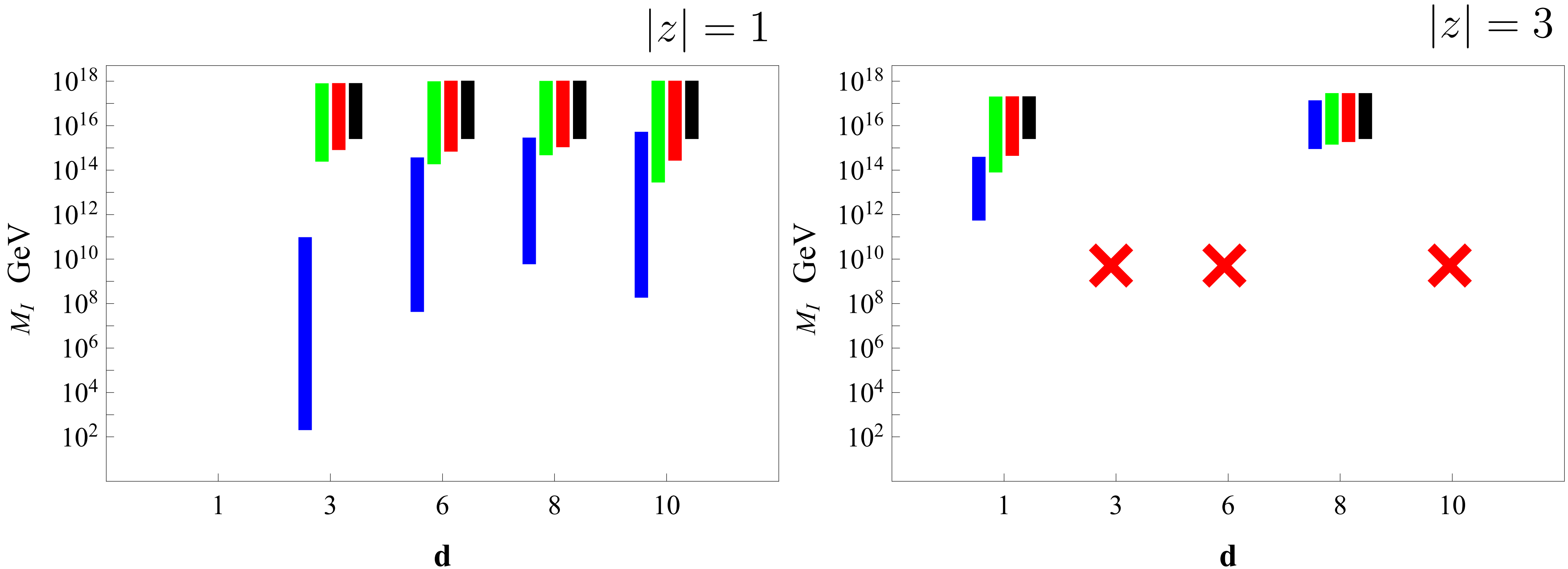
- P2 $(\mathbf{d},\mathbf{1})_{7z/6} \oplus (\mathbf{d},\mathbf{3})_{-5z/6} \oplus (\overline{\mathbf{d}},\mathbf{4})_{z/3}$





Minimal sets and unification

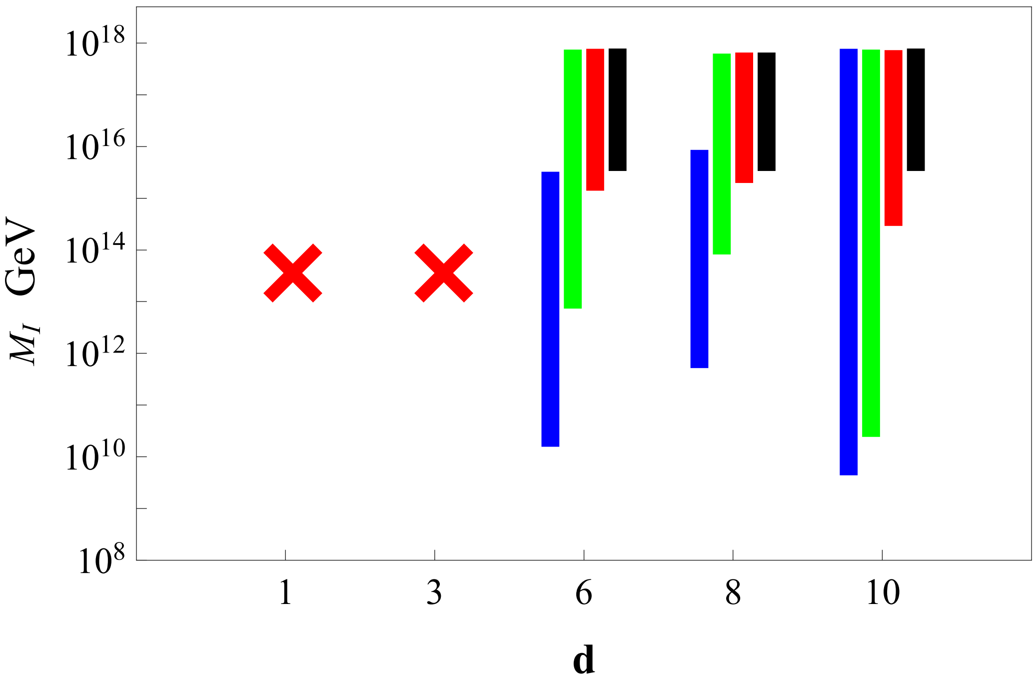
- P3 $(\mathbf{d},\mathbf{1})_{3z/2} \oplus (\mathbf{d},\mathbf{4})_{-z} \oplus (\overline{\mathbf{d}},\mathbf{5})_{z/2}$



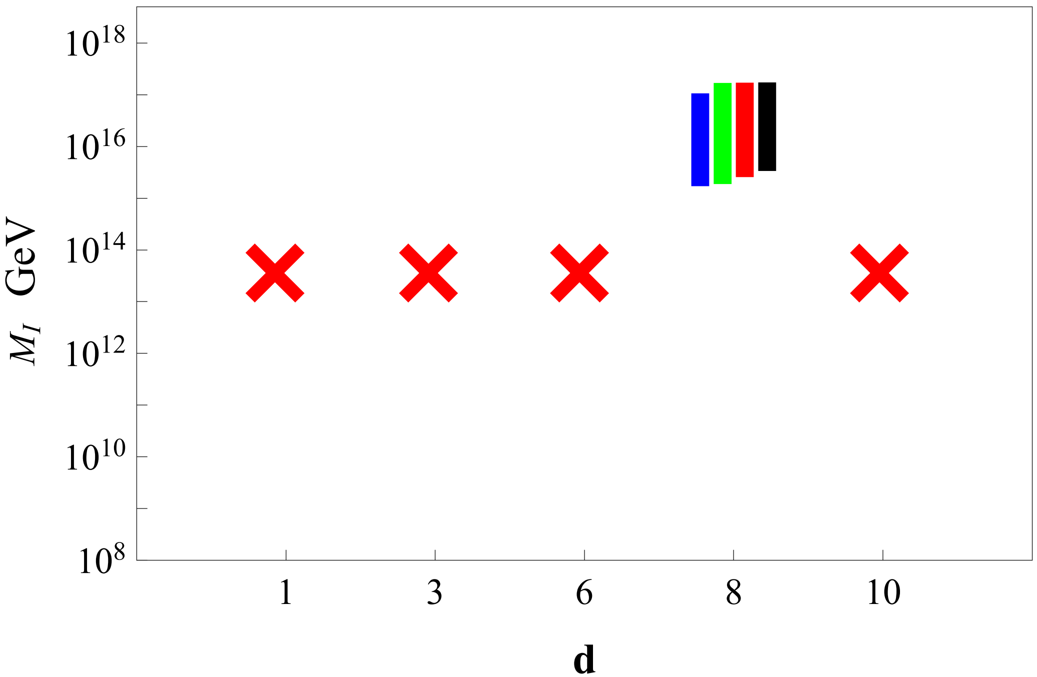
Minimal sets and unification

- P4 $(\mathbf{d},\mathbf{2})_{4z/3} \oplus (\mathbf{d},\mathbf{3})_{-7z/6} \oplus (\overline{\mathbf{d}},\mathbf{5})_{z/6}$ $\mathbf{d}$ is even

$|z| = 1$



$|z| = 3$



Minimal sets and unification

- 16 solutions (11 are consistent with string-scale unification)
- Possible TeV or even lower mass states, although at least one fermion has high intermediate energy scale
- The usual mass mechanism for these new chiral fermions through Yukawa interactions with scalars is tightly constrained by electroweak precision data
- Quantum numbers are different than those present in SM GUT embeddings into SU(5) or SO(10) groups

SU(5)-inspired Anomaly-free Chiral Fermion Sets

Setup

- Requirements:
  - Chiral fermion set beyond the SM
  - CGU at intermediate scale
  - Anomaly-free conditions verified
  - Vectorlike with respect to color SU(3) and electromagnetic U(1)
    - SU(5) representations with $d_5(R) \leq 50$
    - Canonical groups normalization $\kappa_1 = 5/3$, $\kappa_2 = \kappa_3 = 1$

Particle content

Label	Multiplet	SU(5)-rep	(b_1, b_2, b_3)	Label	Multiplet	SU(5)-rep	(b_1, b_2, b_3)
1	$(\mathbf{1},\mathbf{2})_{1/2}$	5, 45	$(1/3, 1/3, 0)$	12	$(\mathbf{1},\mathbf{2})_{-3/2}$	40	$(3, 1/3, 0)$
2	$(\mathbf{3},\mathbf{1})_{-1/3}$	5, 45, 50	$(2/9, 0, 1/3)$	13	$(\mathbf{8},\mathbf{1})_{1}$	40	$(16/3, 0, 2)$
3	$(\mathbf{1},\mathbf{1})_{1}$	10	$(2/3, 0, 0)$	14	$(\mathbf{3},\mathbf{3})_{-1/3}$	45	$(2/3, 4, 1)$
4	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3}$	10, 40	$(8/9, 0, 1/3)$	15	$(\overline{\mathbf{3}},\mathbf{1})_{4/3}$	45	$(32/9, 0, 1/3)$
5	$(\mathbf{3},\mathbf{2})_{1/6}$	10, 15, 40	$(1/9, 1, 2/3)$	16	$(\overline{\mathbf{3}},\mathbf{2})_{-7/6}$	45, 50	$(49/9, 1, 2/3)$
6	$(\mathbf{1},\mathbf{3})_{1}$	15	$(2, 4/3, 0)$	17	$(\overline{\mathbf{6}},\mathbf{1})_{-1/3}$	45	$(4/9, 0, 5/3)$
7	$(\mathbf{6},\mathbf{1})_{-2/3}$	15	$(16/9, 0, 5/3)$	18	$(\mathbf{8},\mathbf{2})_{1/2}$	45, 50	$(8/3, 8/3, 4)$
8	$(\mathbf{1},\mathbf{4})_{-3/2}$	35	$(6, 10/3, 0)$	19	$(\mathbf{1},\mathbf{1})_{-2}$	50	$(8/3, 0, 0)$
9	$(\overline{\mathbf{3}},\mathbf{3})_{-2/3}$	35, 40	$(8/3, 4, 1)$	20	$(\overline{\mathbf{6}},\mathbf{3})_{-1/3}$	50	$(4/3, 8, 5)$
10	$(\overline{\mathbf{6}},\mathbf{2})_{1/6}$	35, 40	$(2/9, 2, 10/3)$	21	$(\mathbf{6},\mathbf{1})_{4/3}$	50	$(64/9, 0, 5/3)$
11	$(\overline{\mathbf{10}},\mathbf{1})_{1}$	35	$(20/3, 0, 5)$				

Anomalies and vectorlike constraints

- $\mathbf{d}_Q$ denotes states with SU(3) dimension $\mathbf{d}$ and eletric charge Q

$$\mathbf{1}_1 : \quad n_1 + n_3 + n_6 - n_8 - n_{12} = 0$$
$$\mathbf{3}_{-1/3} : \quad n_2 + n_5 - n_9 + n_{14} = 0$$
$$\overline{\mathbf{3}}_{-2/3} : \quad n_4 - n_5 + n_9 - n_{14} + n_{16} = 0$$
$$\mathbf{1}_2 : \quad n_6 - n_8 - n_{12} - n_{19} = 0$$
$$\mathbf{6}_{-2/3} : \quad n_7 - n_{10} - n_{20} = 0$$
$$\mathbf{1}_{-3} : \quad n_8 = 0$$
$$\overline{\mathbf{3}}_{-5/3} : \quad n_9 + n_{16} = 0$$
$$\overline{\mathbf{6}}_{-1/3} : \quad n_{10} + n_{17} + n_{20} = 0$$
$$\overline{\mathbf{10}}_1 : \quad n_{11} = 0$$
$$\mathbf{8}_1 : \quad n_{13} + n_{18} = 0$$
$$\mathbf{3}_{-4/3} : \quad n_{14} - n_{15} = 0$$
$$\overline{\mathbf{6}}_{-4/3} : \quad n_{20} - n_{21} = 0$$

- Two independent anomaly equations

[SU(2) - SU(2) - U(1)] :

$$2n_1 + 9n_2 + 3n_3 + 17n_4 - 9n_5 - 5n_6 + 16n_7 - 18n_{10} + 8n_{13} = 0$$

[U(1) - U(1) - U(1)] :

$$54n_1 + 243n_2 + 81n_3 + 459n_4 - 243n_5 - 135n_6 + 432n_7 - 486n_{10} + 216n_{13} = 0$$

Scalar content

- Self-contained unification include scalars to obtain the proper symmetry breaking

$$\mathrm{SU}(5) \xrightarrow{\mathbf{24}} \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \xrightarrow[H \sim (\mathbf{1},\mathbf{2})_{1/2}]{\mathbf{5}} \mathrm{SU}(3) \times \mathrm{U}(1)_Q$$

- Extra content:

$$\Sigma_3 \sim (\mathbf{1},\mathbf{3})_0,\ \Sigma_8 \sim (\mathbf{8},\mathbf{1})_0,\ (X,\overline{Y}) \sim (\mathbf{3},\mathbf{2})_{-5/6},\ T \sim (\mathbf{3},\mathbf{1})_{-1/3}$$

- X, Y and T can mediate proton decay so we assume that their masses are of the order of the unification scale

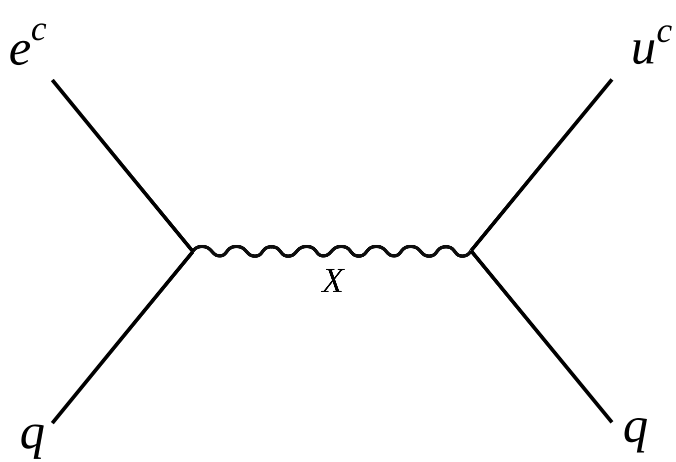
Scalar content

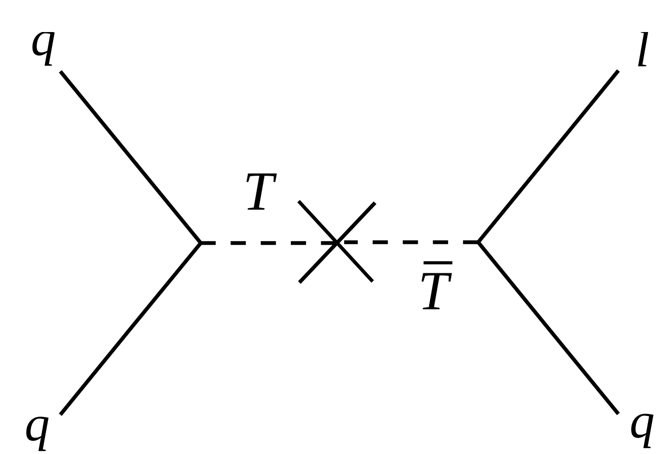
- Self-contained unification include scalars to obtain the proper symmetry breaking

$$\mathrm{SU}(5) \xrightarrow{\mathbf{24}} \mathrm{SU}(3) \times \mathsf{SU}(2) \times \mathsf{U}(1) \xrightarrow[H \sim (\mathbf{1},\mathbf{2})_{1/2}]{\mathbf{5}} \mathrm{SU}(3) \times \mathsf{U}(1)_Q$$

- Extra content:

$$\Sigma_3 \sim (\mathbf{1},\mathbf{3})_0,\ \Sigma_8 \sim (\mathbf{8},\mathbf{1})_0,\ (X,\overline{Y}) \sim (\mathbf{3},\mathbf{2})_{-5/6},\ T \sim (\mathbf{3},\mathbf{1})_{-1/3}$$





Minimal sets and unification

Set	n_s	Particle content	G
S1	4	$3\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus (1,2)_{-3/2} \oplus (1,1)_{2}$	–
S2	5	$(1,2)_{1/2} \oplus (3,1)_{-1/3} \oplus (1,1)_{-1} \oplus (3,1)_{2/3} \oplus (\overline{3},2)_{-1/6}$	–
S3	5	$(1,1)_{-1} \oplus (1,3)_{1} \oplus (8,1)_{1} \oplus (8,2)_{-1/2} \oplus (1,1)_{-2}$	–
S4	5	$2\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus (\overline{6},1)_{2/3} \oplus (6,2)_{-1/6} \oplus (\overline{6},1)_{-1/3}$	–
S5	5	$(1,2)_{1/2} \oplus (1,1)_{1} \oplus (1,3)_{1} \oplus 3\,(1,2)_{-3/2} \oplus 2\,(1,1)_{2}$	–
S6	5	$2\,(1,2)_{1/2} \oplus 3\,(1,1)_{-1} \oplus (1,3)_{-1} \oplus 2\,(1,2)_{3/2} \oplus (1,1)_{-2}$	–
S7	5	$2\,(1,2)_{1/2} \oplus 2\,(3,1)_{-1/3} \oplus 2\,(1,1)_{-1} \oplus 2\,(3,1)_{2/3} \oplus 2\,(\overline{3},2)_{-1/6}$	✓
S8	5	$2\,(1,1)_{-1} \oplus 2\,(1,3)_{1} \oplus 2\,(8,1)_{1} \oplus 2\,(8,2)_{-1/2} \oplus 2\,(1,1)_{-2}$	✓
S9	6	$(\overline{3},1)_{1/3} \oplus (\overline{3},2)_{-1/6} \oplus (3,3)_{2/3} \oplus (3,3)_{-1/3} \oplus (\overline{3},1)_{4/3} \oplus (\overline{3},2)_{-7/6}$	*
S10	6	$(\overline{3},1)_{1/3} \oplus (\overline{3},1)_{-2/3} \oplus (8,1)_{-1} \oplus (3,3)_{-1/3} \oplus (\overline{3},1)_{4/3} \oplus (8,2)_{1/2}$	✓
S11	6	$(3,1)_{2/3} \oplus (\overline{3},2)_{-1/6} \oplus (3,3)_{2/3} \oplus (8,1)_{1} \oplus (\overline{3},2)_{-7/6} \oplus (8,2)_{-1/2}$	✓
S12	6	$(1,2)_{1/2} \oplus (6,1)_{-2/3} \oplus (\overline{6},2)_{1/6} \oplus (1,2)_{-3/2} \oplus (6,1)_{1/3} \oplus (1,1)_{2}$	–
S13	6	$(\overline{6},1)_{2/3} \oplus 2\,(8,1)_{1} \oplus (\overline{6},1)_{-1/3} \oplus 2\,(8,2)_{-1/2} \oplus (6,3)_{1/3} \oplus (\overline{6},1)_{-4/3}$	✓
S14	6	$2\,(\overline{3},1)_{1/3} \oplus 2\,(3,3)_{2/3} \oplus (6,2)_{-1/6} \oplus 2\,(\overline{3},2)_{-7/6} \oplus (\overline{6},3)_{-1/3} \oplus (6,1)_{4/3}$	✓
S15	6	$2\,(\overline{3},1)_{1/3} \oplus 2\,(\overline{3},1)_{-2/3} \oplus 2\,(3,2)_{1/6} \oplus (\overline{6},1)_{2/3} \oplus (6,2)_{-1/6} \oplus (\overline{6},1)_{-1/3}$	✓
S16	6	$2\,(\overline{3},2)_{-1/6} \oplus (\overline{6},2)_{1/6} \oplus 2\,(3,3)_{-1/3} \oplus 2\,(\overline{3},1)_{4/3} \oplus (6,3)_{1/3} \oplus (\overline{6},1)_{-4/3}$	✓
S17	6	$(1,2)_{1/2} \oplus 2\,(\overline{3},1)_{1/3} \oplus 2\,(\overline{3},1)_{-2/3} \oplus 2\,(3,2)_{1/6} \oplus (1,2)_{-3/2} \oplus (1,1)_{2}$	✓
S18	6	$(1,2)_{1/2} \oplus 2\,(1,3)_{1} \oplus 3\,(1,2)_{-3/2} \oplus (8,1)_{1} \oplus (\overline{8},2)_{-1/2} \oplus (1,1)_{2}$	–
S19	6	$3\,(1,2)_{1/2} \oplus 3\,(1,1)_{-1} \oplus (1,3)_{1} \oplus (1,2)_{-3/2} \oplus (8,1)_{1} \oplus (\overline{8},2)_{-1/2}$	*
S20	6	$2\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus 2\,(1,3)_{-1} \oplus 2\,(1,2)_{3/2} \oplus (\overline{8},1)_{-1} \oplus (8,2)_{1/2}$	*

Minimal sets and unification

Set	n_s	Particle content	G
S1	4	$3\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus (1,2)_{-3/2} \oplus (1,1)_{2}$	–
S2	5	$(1,2)_{1/2} \oplus (3,1)_{-1/3} \oplus (1,1)_{-1} \oplus (3,1)_{2/3} \oplus (\overline{3},2)_{-1/6}$	–
S3	5	$(1,1)_{-1} \oplus (1,3)_{1} \oplus (8,1)_{1} \oplus (8,2)_{-1/2} \oplus (1,1)_{-2}$	–
S4	5	$2\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus (\overline{6},1)_{2/3} \oplus (6,2)_{-1/6} \oplus (\overline{6},1)_{-1/3}$	–
S5	5	$(1,2)_{1/2} \oplus (1,1)_{1} \oplus (1,3)_{1} \oplus 3\,(1,2)_{-3/2} \oplus 2\,(1,1)_{2}$	–
S6	5	$2\,(1,2)_{1/2} \oplus 3\,(1,1)_{-1} \oplus (1,3)_{-1} \oplus 2\,(1,2)_{3/2} \oplus (1,1)_{-2}$	–
S7	5	$2\,(1,2)_{1/2} \oplus 2\,(3,1)_{-1/3} \oplus 2\,(1,1)_{-1} \oplus 2\,(3,1)_{2/3} \oplus 2\,(\overline{3},2)_{-1/6}$	✓
S8	5	$2\,(1,1)_{-1} \oplus 2\,(1,3)_{1} \oplus 2\,(8,1)_{1} \oplus 2\,(8,2)_{-1/2} \oplus 2\,(1,1)_{-2}$	✓
S9	6	$(\overline{3},1)_{1/3} \oplus (\overline{3},2)_{-1/6} \oplus (3,3)_{2/3} \oplus (3,3)_{-1/3} \oplus (\overline{3},1)_{4/3} \oplus (\overline{3},2)_{-7/6}$	*
S10	6	$(\overline{3},1)_{1/3} \oplus (\overline{3},1)_{-2/3} \oplus (8,1)_{-1} \oplus (3,3)_{-1/3} \oplus (\overline{3},1)_{4/3} \oplus (8,2)_{1/2}$	✓
S11	6	$(3,1)_{2/3} \oplus (\overline{3},2)_{-1/6} \oplus (3,3)_{2/3} \oplus (8,1)_{1} \oplus (\overline{3},2)_{-7/6} \oplus (8,2)_{-1/2}$	✓
S12	6	$(1,2)_{1/2} \oplus (6,1)_{-2/3} \oplus (\overline{6},2)_{1/6} \oplus (1,2)_{-3/2} \oplus (6,1)_{1/3} \oplus (1,1)_{2}$	–
S13	6	$(\overline{6},1)_{2/3} \oplus 2\,(8,1)_{1} \oplus (\overline{6},1)_{-1/3} \oplus 2\,(8,2)_{-1/2} \oplus (6,3)_{1/3} \oplus (\overline{6},1)_{-4/3}$	✓
S14	6	$2\,(\overline{3},1)_{1/3} \oplus 2\,(3,3)_{2/3} \oplus (6,2)_{-1/6} \oplus 2\,(\overline{3},2)_{-7/6} \oplus (\overline{6},3)_{-1/3} \oplus (6,1)_{4/3}$	✓
S15	6	$2\,(\overline{3},1)_{1/3} \oplus 2\,(\overline{3},1)_{-2/3} \oplus 2\,(3,2)_{1/6} \oplus (\overline{6},1)_{2/3} \oplus (6,2)_{-1/6} \oplus (\overline{6},1)_{-1/3}$	✓
S16	6	$2\,(\overline{3},2)_{-1/6} \oplus (\overline{6},2)_{1/6} \oplus 2\,(3,3)_{-1/3} \oplus 2\,(\overline{3},1)_{4/3} \oplus (6,3)_{1/3} \oplus (\overline{6},1)_{-4/3}$	✓
S17	6	$(1,2)_{1/2} \oplus 2\,(\overline{3},1)_{1/3} \oplus 2\,(\overline{3},1)_{-2/3} \oplus 2\,(3,2)_{1/6} \oplus (1,2)_{-3/2} \oplus (1,1)_{2}$	✓
S18	6	$(1,2)_{1/2} \oplus 2\,(1,3)_{1} \oplus 3\,(1,2)_{-3/2} \oplus (8,1)_{1} \oplus (\overline{8},2)_{-1/2} \oplus (1,1)_{2}$	–
S19	6	$3\,(1,2)_{1/2} \oplus 3\,(1,1)_{-1} \oplus (1,3)_{1} \oplus (1,2)_{-3/2} \oplus (8,1)_{1} \oplus (\overline{8},2)_{-1/2}$	*
S20	6	$2\,(1,2)_{1/2} \oplus 2\,(1,1)_{-1} \oplus 2\,(1,3)_{-1} \oplus 2\,(1,2)_{3/2} \oplus (\overline{8},1)_{-1} \oplus (8,2)_{1/2}$	*

Minimal sets and unification

			Intermediate mass scales [GeV]					
Set	α_U^{-1}	$\Lambda_{\max}$[GeV]	rep	min	max	rep	min	max
S7	$[30.5, 37.6]$	1.0×10^{17}	$(\mathbf{1}, \mathbf{2})_{1/2}$	M_Z	7.6×10^{16}	$(\mathbf{3}, \mathbf{1})_{-1/3}$	M_Z	1.3×10^{16}
			$(\mathbf{1}, \mathbf{1})_{-1}$	8.0×10^{3}	7.8×10^{16}	$(\mathbf{3}, \mathbf{1})_{2/3}$	2.6×10^{4}	9.4×10^{16}
			$(\overline{\mathbf{3}}, \mathbf{2})_{-1/6}$	M_Z	5.8×10^{7}			
S8	$[1.4, 18.8]$	5.0×10^{17}	$(\mathbf{1}, \mathbf{1})_{-1}$	2.2×10^{3}	2.8×10^{17}	$(\mathbf{1}, \mathbf{3})_{1}$	M_Z	4.0×10^{5}
			$(\mathbf{8}, \mathbf{1})_{1}$	8.9×10^{14}	4.7×10^{17}	$(\mathbf{8}, \mathbf{2})_{-1/2}$	M_Z	6.2×10^{7}
			$(\mathbf{1}, \mathbf{1})_{-2}$	9.4×10^{12}	3.4×10^{17}			
S10	$[8.0, 35.0]$	5.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	M_Z	4.2×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$	M_Z	4.9×10^{17}
			$(\mathbf{8}, \mathbf{1})_{-1}$	6.7×10^{3}	5.0×10^{17}	$(\mathbf{3}, \mathbf{3})_{-1/3}$	M_Z	1.1×10^{12}
			$(\overline{\mathbf{3}}, \mathbf{1})_{4/3}$	M_Z	5.0×10^{17}	$(\mathbf{8}, \mathbf{2})_{1/2}$	M_Z	4.7×10^{13}
S11	$[4.0, 34.5]$	5.3×10^{17}	$(\mathbf{3}, \mathbf{1})_{2/3}$	M_Z	4.9×10^{17}	$(\overline{\mathbf{3}}, \mathbf{2})_{-1/6}$	M_Z	2.7×10^{17}
			$(\mathbf{3}, \mathbf{3})_{2/3}$	M_Z	9.7×10^{14}	$(\mathbf{8}, \mathbf{1})_{1}$	8.4×10^{3}	4.6×10^{17}
			$(\overline{\mathbf{3}}, \mathbf{2})_{-7/6}$	4.0×10^{5}	5.1×10^{17}	$(\mathbf{8}, \mathbf{2})_{-1/2}$	M_Z	4.3×10^{13}

- The intermediate mass scales of Σ_3 and Σ_8 can take any value up to the unification scale

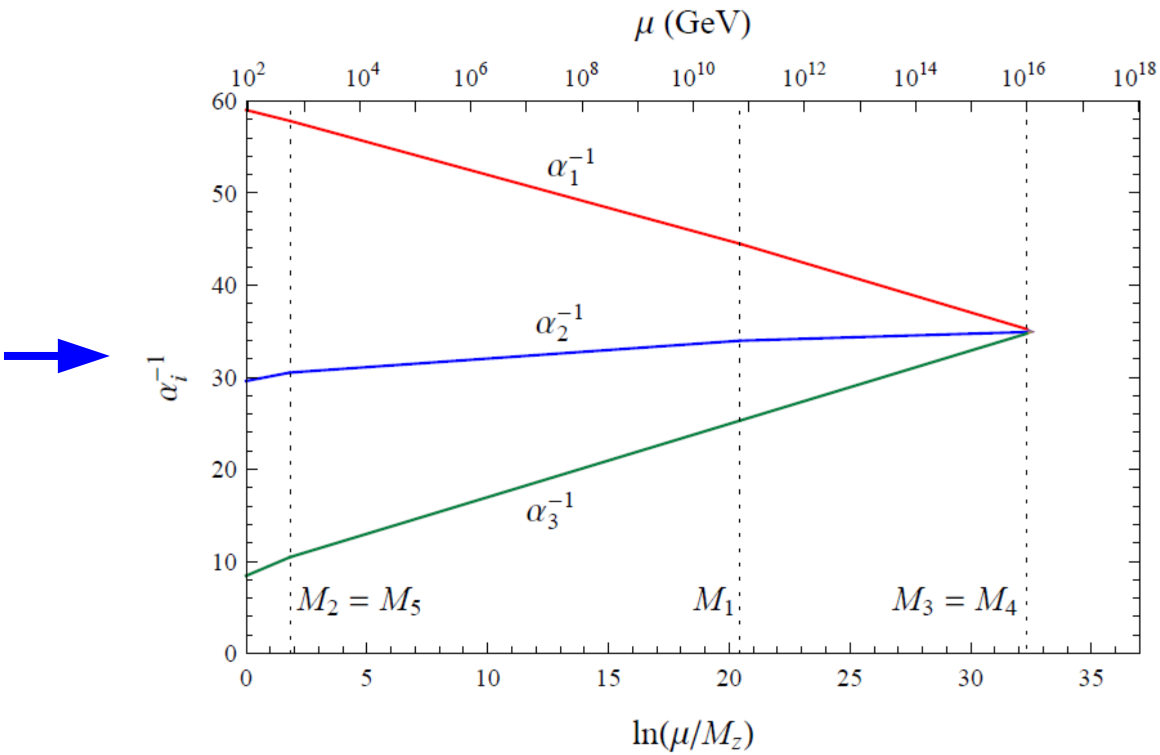
Minimal sets and unification

			Intermediate mass scales [GeV]					
Set	α_U^{-1}	$\Lambda_{\max}$[GeV]	rep	min	max	rep	min	max
S13	$[1.0, 35.5]$	5.3×10^{17}	$(\overline{\mathbf{6}}, \mathbf{1})_{2/3}$	7.4×10^4	5.1×10^{17}	$(\mathbf{8}, \mathbf{1})_1$	1.9×10^7	5.0×10^{17}
			$(\overline{\mathbf{6}}, \mathbf{1})_{-1/3}$	5.5×10^7	5.0×10^{17}	$(\mathbf{8}, \mathbf{2})_{-1/2}$	1.4×10^{13}	5.2×10^{17}
			$(\mathbf{6}, \mathbf{3})_{1/3}$	M_Z	7.1×10^{13}	$(\overline{\mathbf{6}}, \mathbf{1})_{-4/3}$	M_Z	5.0×10^{17}
S14	$[1.0, 35.9]$	5.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	M_Z	4.7×10^{17}	$(\mathbf{3}, \mathbf{3})_{2/3}$	6.4×10^7	4.9×10^{17}
			$(\mathbf{6}, \mathbf{2})_{-1/6}$	M_Z	2.8×10^{16}	$(\overline{\mathbf{3}}, \mathbf{2})_{-7/6}$	2.9×10^3	5.0×10^{17}
			$(\overline{\mathbf{6}}, \mathbf{3})_{-1/3}$	2.5×10^3	4.4×10^{17}	$(\mathbf{6}, \mathbf{1})_{4/3}$	M_Z	4.9×10^{17}
S15	$[32.3, 37.3]$	5.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	9.0×10^6	4.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$	1.5×10^{11}	4.3×10^{17}
			$(\mathbf{3}, \mathbf{2})_{1/6}$	M_Z	9.8×10^5	$(\overline{\mathbf{6}}, \mathbf{1})_{2/3}$	2.1×10^{13}	5.0×10^{17}
			$(\mathbf{6}, \mathbf{2})_{-1/6}$	8.1×10^{10}	8.5×10^{16}	$(\overline{\mathbf{6}}, \mathbf{1})_{-1/3}$	5.4×10^{12}	4.8×10^{17}
S16	$[1.0, 37.1]$	5.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{2})_{-1/6}$	M_Z	5.1×10^{17}	$(\overline{\mathbf{6}}, \mathbf{2})_{1/6}$	M_Z	6.0×10^{15}
			$(\mathbf{3}, \mathbf{3})_{-1/3}$	9.5×10^7	5.3×10^{17}	$(\overline{\mathbf{3}}, \mathbf{1})_{4/3}$	M_Z	5.0×10^{17}
			$(\mathbf{6}, \mathbf{3})_{1/3}$	880	4.9×10^{17}	$(\overline{\mathbf{6}}, \mathbf{1})_{-4/3}$	M_Z	5.0×10^{17}
S17	$[31.9, 37.1]$	6.0×10^{16}	$(\mathbf{1}, \mathbf{2})_{1/2}$	M_Z	2.0×10^{16}	$(\overline{\mathbf{3}}, \mathbf{1})_{1/3}$	M_Z	8.3×10^{15}
			$(\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$	2.9×10^6	3.3×10^{16}	$(\mathbf{3}, \mathbf{2})_{1/6}$	M_Z	2.0×10^7
			$(\mathbf{1}, \mathbf{2})_{-3/2}$	6.5×10^{10}	5.0×10^{16}	$(\mathbf{1}, \mathbf{1})_2$	6.7×10^{10}	5.9×10^{16}

Minimal sets and unification

- Out of 20 sets, 9 unify safely with proton decay bounds (6 are consistent with string-scale unification)
- More TeV or even lower mass states possibilities, but also more fermions at high intermediate energy scales

Two extra SM generations



Complete SU(5) multiplets

- None of the previous sets corresponds to a complete representation
- Anomaly-free solutions with SU(5) multiplets

$$[\mathsf{SU}(5)\text{-}\mathsf{SU}(5)\text{-}\mathsf{SU}(5)]: \quad \sum_R A_5(R) = 0$$

SU(5)-irrep	5	10	15	24	35	40	45	50
A_5	1	1	9	0	-44	-16	-6	-15

↓

$$\mathsf{n}_5 + \mathsf{n}_{10} + 9\mathsf{n}_{15} - 44\mathsf{n}_{35} - 16\mathsf{n}_{40} - 6\mathsf{n}_{45} - 15\mathsf{n}_{50} = 0$$

Complete SU(5) multiplets

- Low-energy vectorlike conditions

$$\mathsf{n}_5 + \mathsf{n}_{10} - \mathsf{n}_{40} = 0\,,$$
$$\mathsf{n}_{15} - \mathsf{n}_{40} - \mathsf{n}_{50} = 0\,,$$
$$\mathsf{n}_{15} + \mathsf{n}_{45} = 0\,,$$
$$\mathsf{n}_{35} = 0$$

- Anomaly-free condition

$$\mathsf{n}_5 + \mathsf{n}_{10} + 9\mathsf{n}_{15} - 44\mathsf{n}_{35} - 16\mathsf{n}_{40} - 6\mathsf{n}_{45} - 15\mathsf{n}_{50} = 0 \checkmark$$

Complete SU(5) multiplets

- Minimal sets

$$\overline{5} \oplus 10\,, \quad \longrightarrow \quad \text{One SM generation (trivial)}$$

$$\overline{5} \oplus \overline{40} \oplus 50\,,$$

$$10 \oplus 40 \oplus \overline{50}\,,$$

$$15 \oplus \overline{45} \oplus 50\,,$$

$$\overline{5} \oplus \overline{15} \oplus \overline{40} \oplus 45\,,$$

$$10 \oplus 15 \oplus 40 \oplus \overline{45} \quad \longrightarrow \quad \text{One SM generation}$$

- Complete SM fermion content in a non-standard way

$$3 \times 10 \oplus 4 \times 15 \oplus 3 \times 40 \oplus 4 \times \overline{45} \oplus 50$$

→ No GCU without mass splittings within each SU(5) multiplet

Complete SU(5) multiplets

- Minimal sets

$\overline{5} \oplus 10\,,$ → One SM generation (trivial)

$\overline{5} \oplus \overline{40} \oplus 50\,,$

$10 \oplus 40 \oplus \overline{50}\,,$

$15 \oplus \overline{45} \oplus 50\,,$

$\overline{5} \oplus \overline{15} \oplus \overline{40} \oplus 45\,,$

$10 \oplus 15 \oplus 40 \oplus \overline{45}$ → One SM generation

- Complete SM fermion content in a non-standard way

$$3 \times 10 \oplus 4 \times 15 \oplus 3 \times 40 \oplus 4 \times \overline{45} \oplus 50$$

→ **Future works require a detailed analysis of mass mechanisms for the different types of minimal sets**

Conclusions

- We searched for minimal chiral sets of fermions beyond the SM that are vectorlike particles with respect to color and electromagnetism, form an anomaly-free set and allow for the unification of gauge couplings
- Adding only a minimal chiral content to SM and requiring gauge unification enforces that some of the new particles should decouple from the theory at intermediate scales much larger than the electroweak scale
- So far, supersymmetry has not been observed, thus nonsupersymmetric extensions of the SM remain plausible alternatives that are worth being investigated