Entanglement Entropy in Shock Wave Collisions

Based on work with Daniel Grumiller, Stefan Stricker 1506.02658 (JHEP); & Wilke van der Schee (15XX.XXXXX)

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Motivation

<u>Central question:</u>

How does a **strongly coupled** quantum system **evolve** from a **far-from equilibrium** state to **equilibrium?**

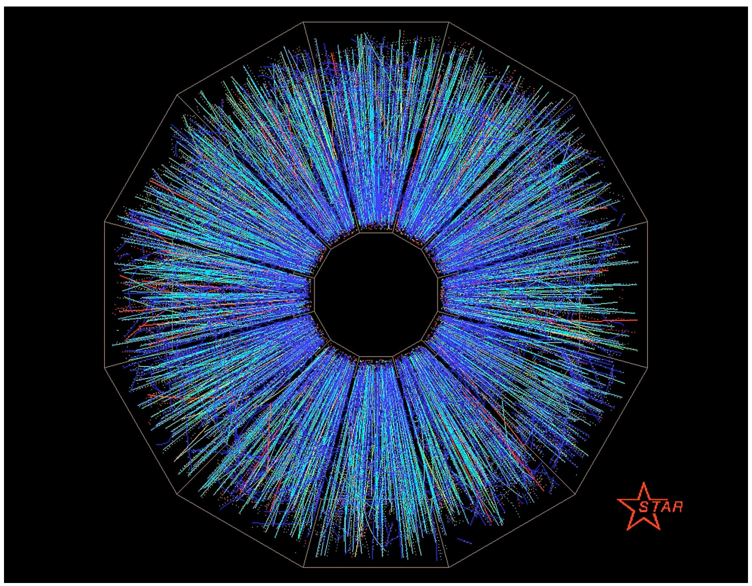
[picture: S. Schlichting]

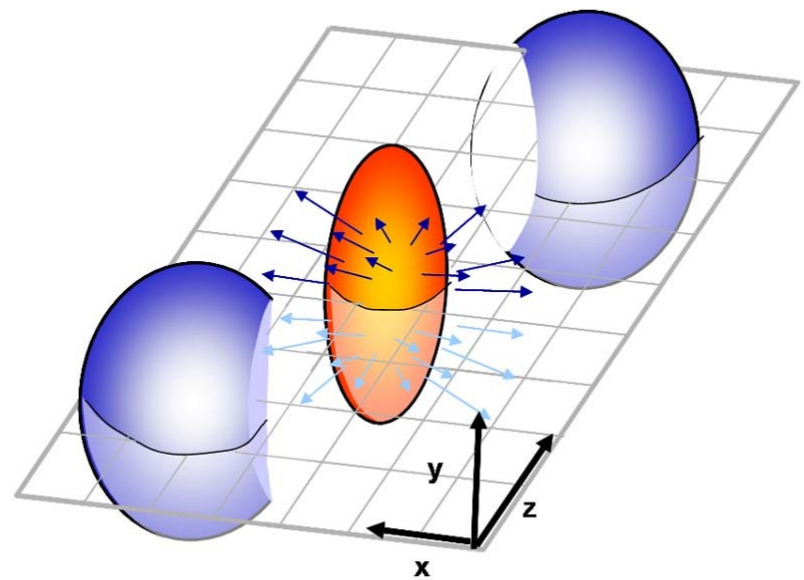
<u>Challenges:</u>

- Due to **strong coupling perturbative** methods are **not applicable**.
- **Far-from equilibrium** we are **outside the regime** of **linear response theory**.

Quark-gluon plasma in heavy ion collisions [see talk by K. Landsteiner]

Quark-gluon plasma (QGP) is a **deconfined phase of quarks and gluons.**
It is produced in **heavy ion collision** (HIC) experiments at **RHIC** and **LHC**.





Why AdS/CFT?

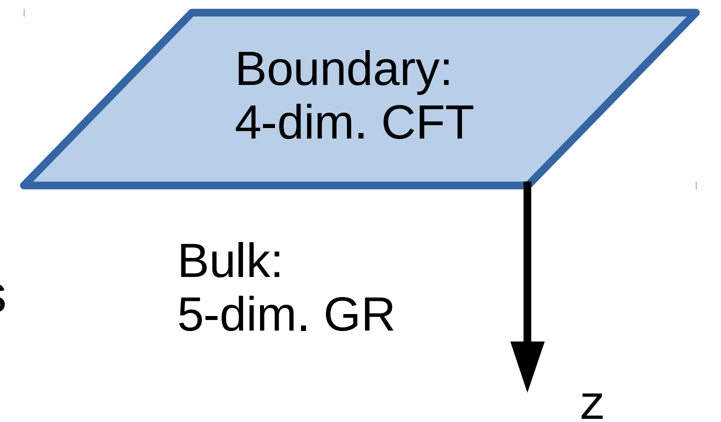
- The QGP produced in HIC's behaves like a **strongly coupled liquid** rather than a **weakly coupled gas**.
- Due to strong coupling we **can't use perturbation theory**.
- To study real time dynamics we **can't use lattice QCD**.

AdS/CFT correspondence

[see talks by K. Landsteiner, D.R. Fernandez]

AdS/CFT correspondence: [Maldacena 97]

Type IIB string theory on $AdS_5 \times S^5$ is equivalent to $\mathcal{N}=4$ super symmetric $SU(N_c)$ **Yang-Mills theory** in 4D.



Supergravity limit:

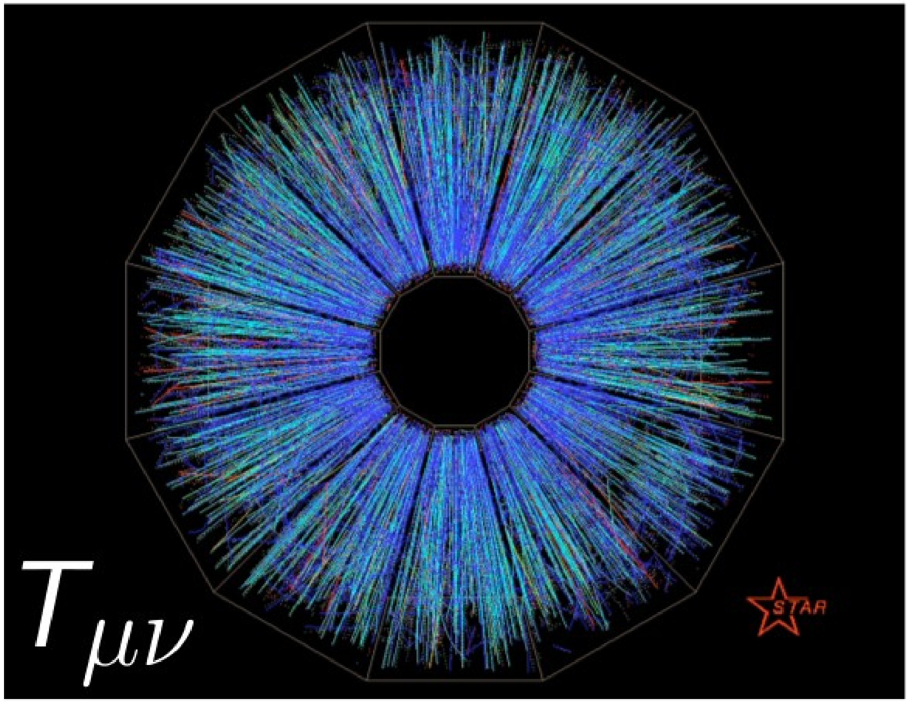
Strongly coupled large N_c $\mathcal{N}=4$ $SU(N_c)$ SYM theory is equivalent to **classical (super)gravity** on AdS_5.

Strategy:

- Use $\mathcal{N}=4$ SYM as **toymodel** for **QCD**.
- Build a **gravity model** dual to HICs, like colliding **gravitational shock waves**.
- Switch on the computer and solve the 5-dim. gravity problem **numerically**.
- Use the **holographic dictionary** to compute **observables in the 4 dim. field theory** form the gravity result. [Skenderis 01]

Holographic thermalization

Thermalization = Black hole formation



- **Holographic dictionary** translates thermodynamic **properties (S,T,M)** of **black holes** to the corresponding **properties of the gauge theory**.
- Computing **black hole formation on AdS** in general requires methods from **numerical relativity**.
- The **observable** we use to **study thermalization** is **entanglment entropy**.

Entanglement entropy

Divide the system into **two parts** A,B.
The total Hilbert space factorizes:

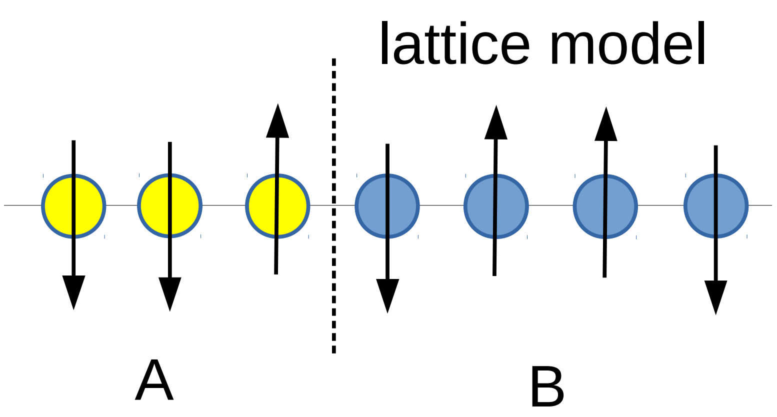
$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

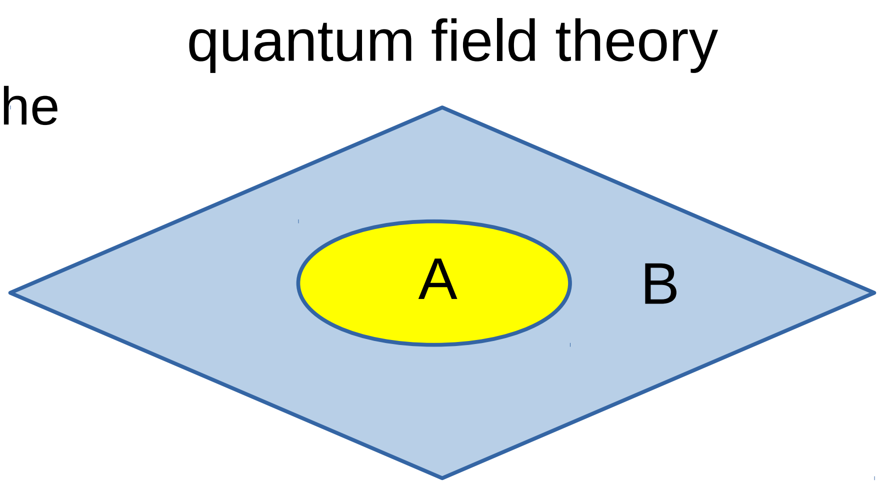
The **reduced density matrix** of A is obtained by the trace over $\mathcal{H}_\mathcal{B}$

$$\rho_A = \mathrm{Tr}_B \rho$$

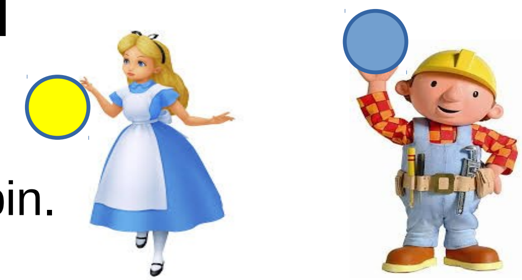
Entanglement entropy is defined as the **von Neumann entropy** of ρ_A:

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A$$





Entanglement entropy in a two quantum bit system

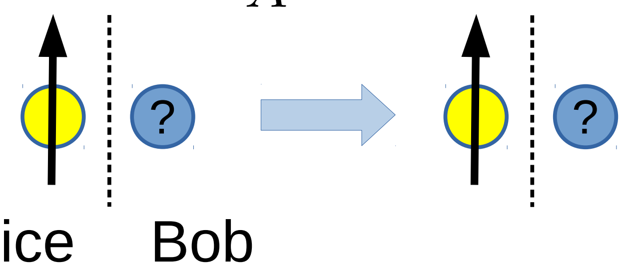


Consider a quantum system of two spin 1/2 dof's.
Observer Alice has only access to one spin and Bob to the other spin.

A **product state (not entangled)** in a two spin 1/2 system:

$$S_A = 0$$

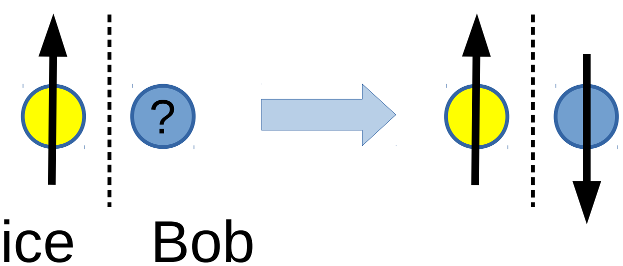
$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle + |\downarrow_A\rangle) \otimes (|\uparrow_B\rangle + |\downarrow_B\rangle)$$



A (maximally) **entangled state** in a two spin 1/2 system:

$$S_A = \log 2$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\rangle \otimes |\downarrow_B\rangle - |\downarrow_A\rangle \otimes |\uparrow_B\rangle)$$



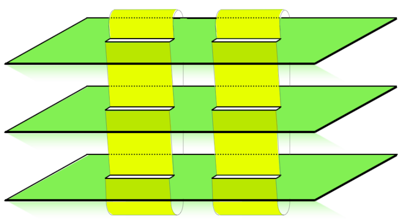
Entanglement entropy is a **measure** for how much a given quantum state is **entangled**.

Entanglement entropy in quantum field theories

The Basic Method to compute entanglement entropy in quantum field theories is the **replica method.**

Involves path integrals over n-sheeted Riemann surfaces ~ it's **complicated!**

With the **replica method** one gets **analytic results** for **1+1 dim. CFTs**. [Holzhey-Larsen-Wilczek 94]



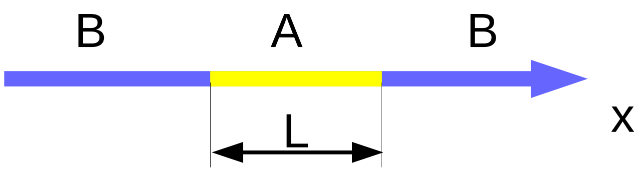
3-sheeted Riemann surface

One finds **universal scaling** with interval size:

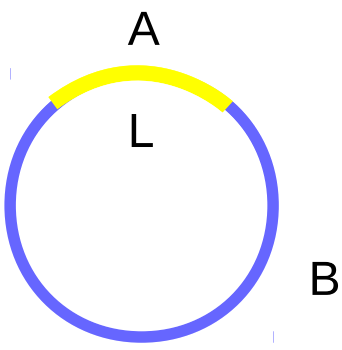
central charge of the CFT

$$S_A = \frac{c}{3} \log \frac{L}{a} + finite$$

UV cut off



1+1 dim. CFTs



Message: Computing entanglement entropy in interacting QFTs is complicated and analytically only possible in 1+1 dim. CFTs.

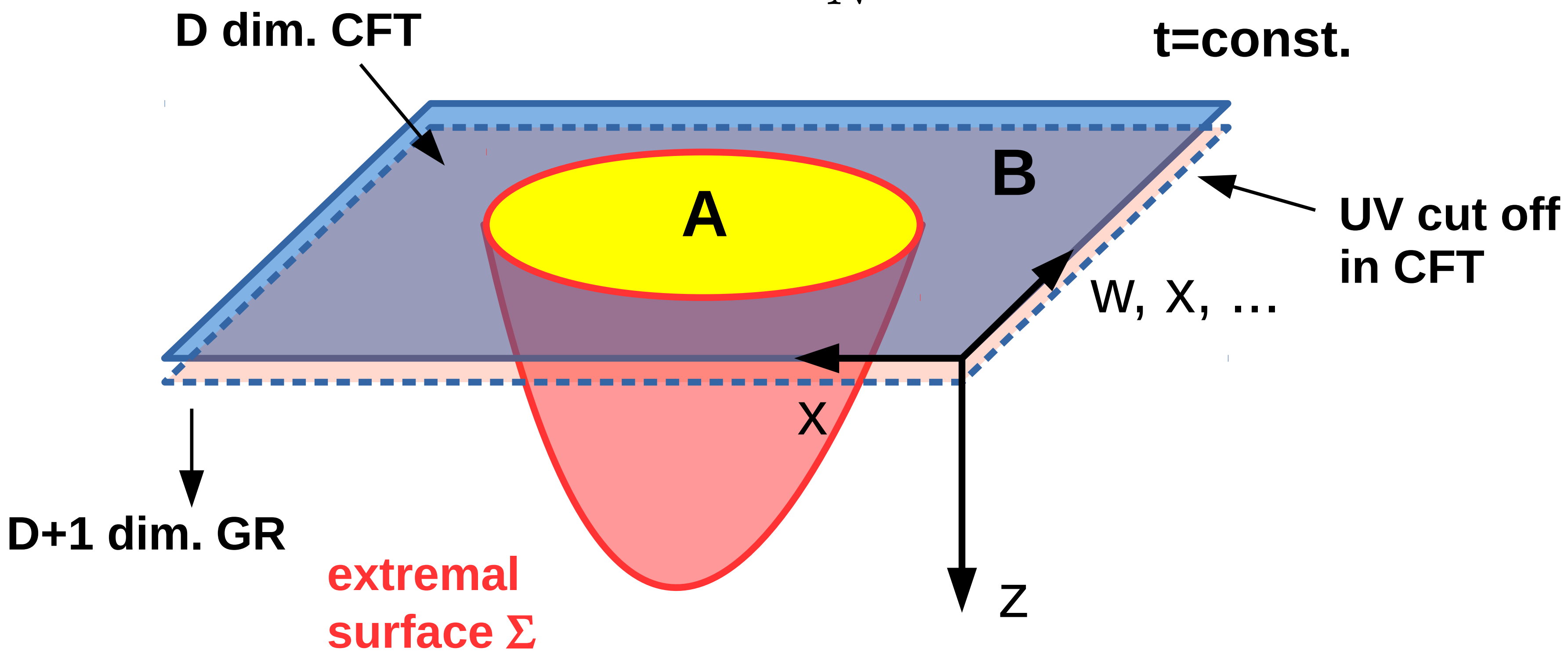
AdS/CFT provides a **simpler method** that works also in **higher dimensions**.

Holographic entanglement entropy

Within **AdS/CFT** entanglement entropy can be computed form the **area of minimal (extremal) surfaces** in the gravity theory.

$$S_A = \frac{\text{Area}(\Sigma)}{4G_N}$$

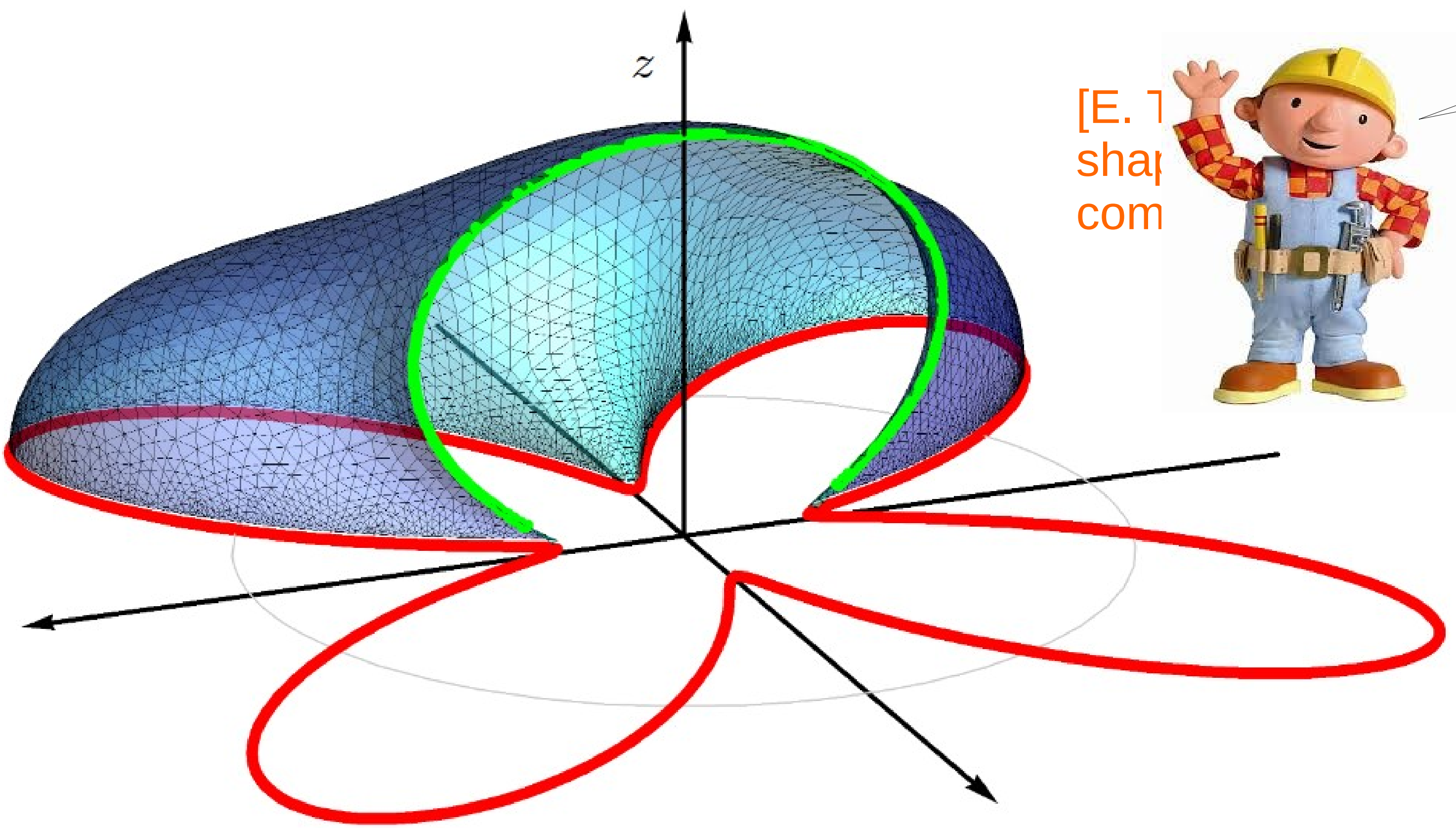
[Ryu-Takayanagi 06, Hubeny-Rangamani-Takayanagi 07]



Holographic entanglement entropy

- In practice computing extremal co-dim. 2 hyper-surfaces is numerically involved.
- Can we somehow simplify our lives?

Yes we can!

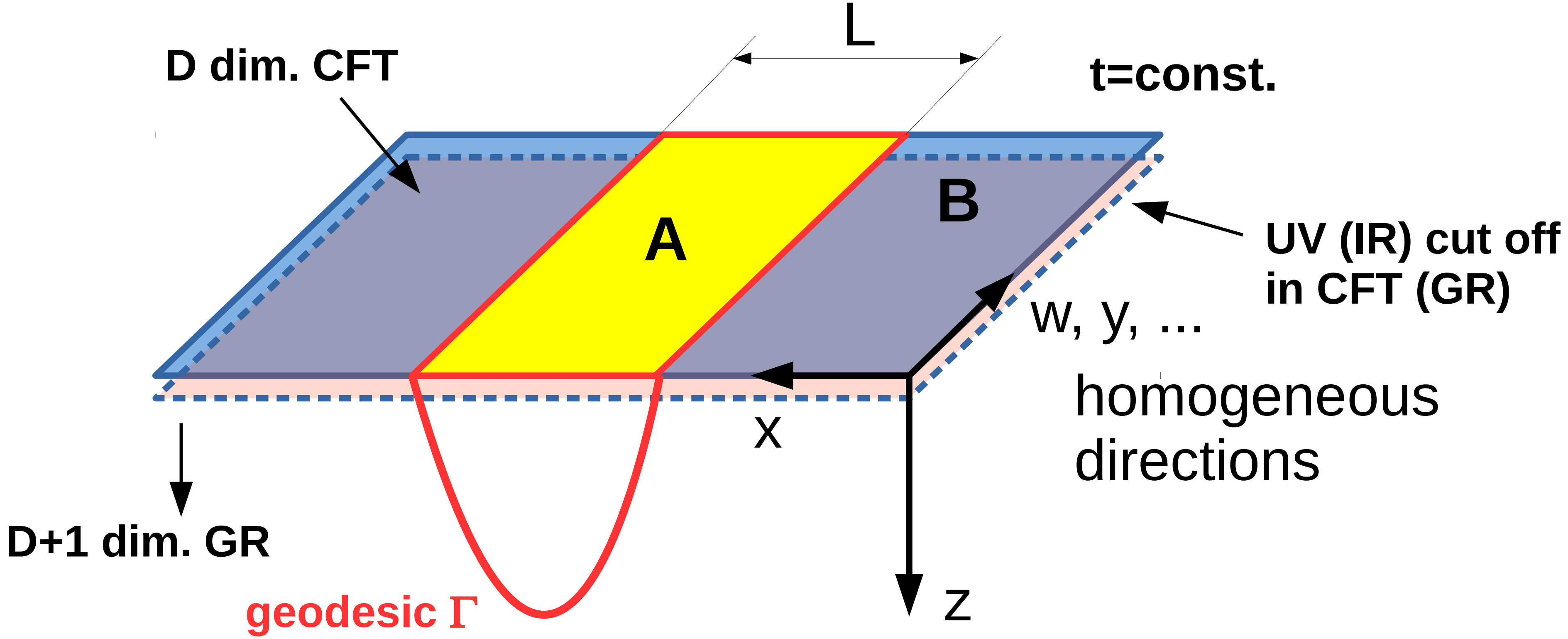


[E. T nal surface for a star
shap region (red) in AdS4
com rface Evolver]

Entanglement entropy from geodesics

Consider a **stripe region** of **infinite extend** in **homogeneous directions** of the geometry.
The **entanglement entropy** is prop. to the **geodesics length** in an **auxiliary spacetime**.

$$S_A = const.\frac{\mathrm{Length}(\Gamma)}{4G_N} \qquad \tilde{g}_{\mu\nu} = \Omega(z,t,x)^2 g_{\mu\nu}$$

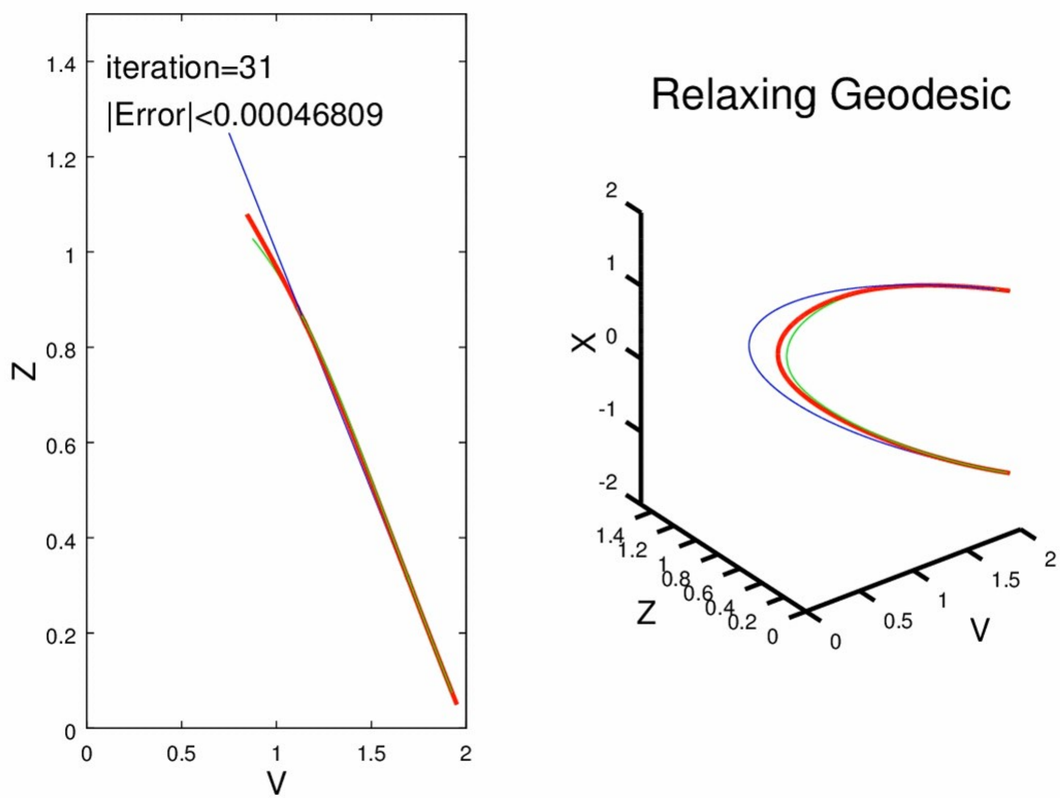


Numerics: relax, don't shoot!

Geodesic equation as **two point boundary value problem**.

$$\ddot{X}^\mu(\tau) + \Gamma^\mu_{\alpha\beta}\dot{X}^\alpha(\tau)\dot{X}^\beta(\tau) = 0$$

$$BCs: \quad (V(\pm1), Z(\pm1), X(\pm1)) = (t_0, 0, L/2)$$



- There are two **standard numerical methods** for solving two point boundary value problems.

  [see Numerical Recipes]

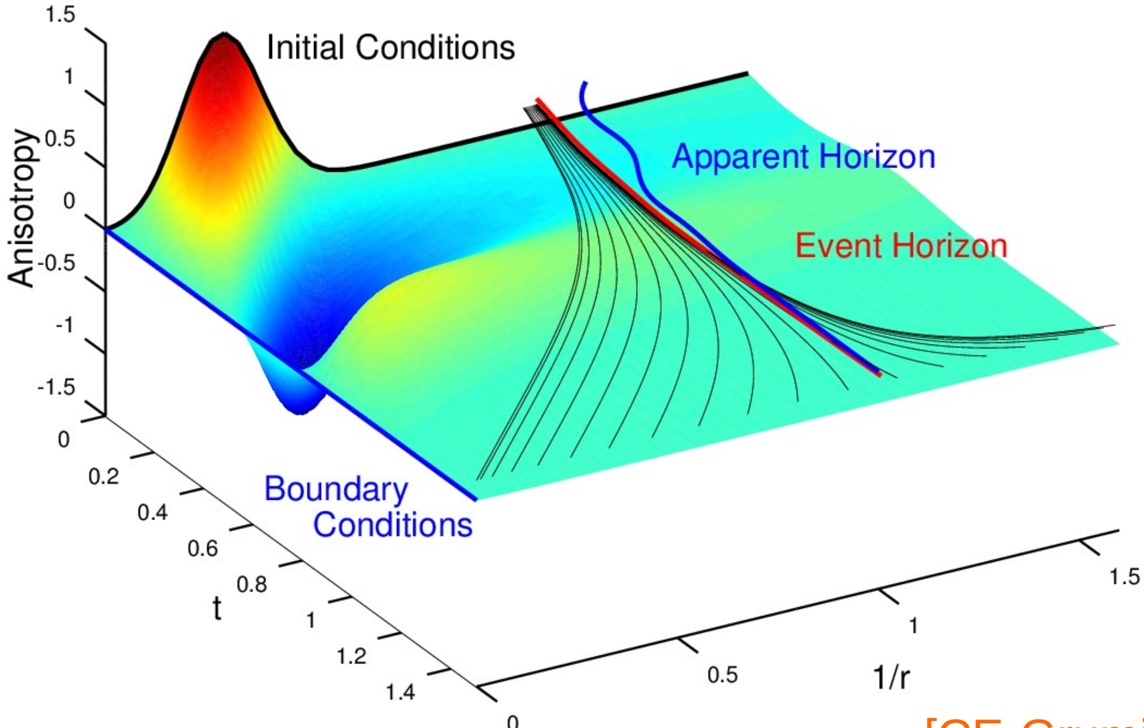
Shooting:
Very **sensitive to initialization** on **asymptotic AdS** spacetimes.

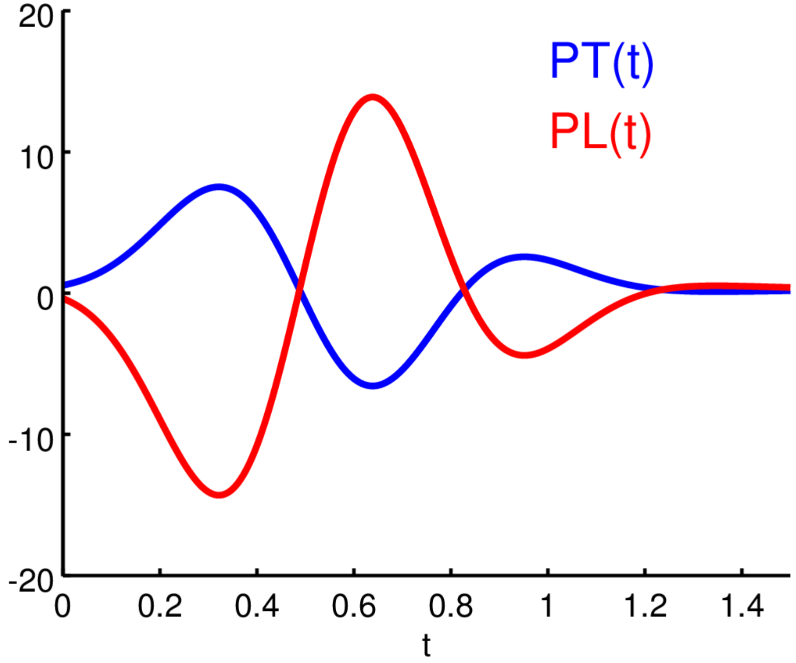
Relaxation:
Converges very **fast** if **good initial geodesic** is provided.

Isotropization of homogeneous plasma

An **initially highly anisotropic** (N=4 SYM) plasma **relaxates** to its **isotropic equilibrium state**. [Chesler-Yaffe 09]

The **dual gravity model** describes the **formation of a black brane** in an **anisotropic AdS_5** geometry.

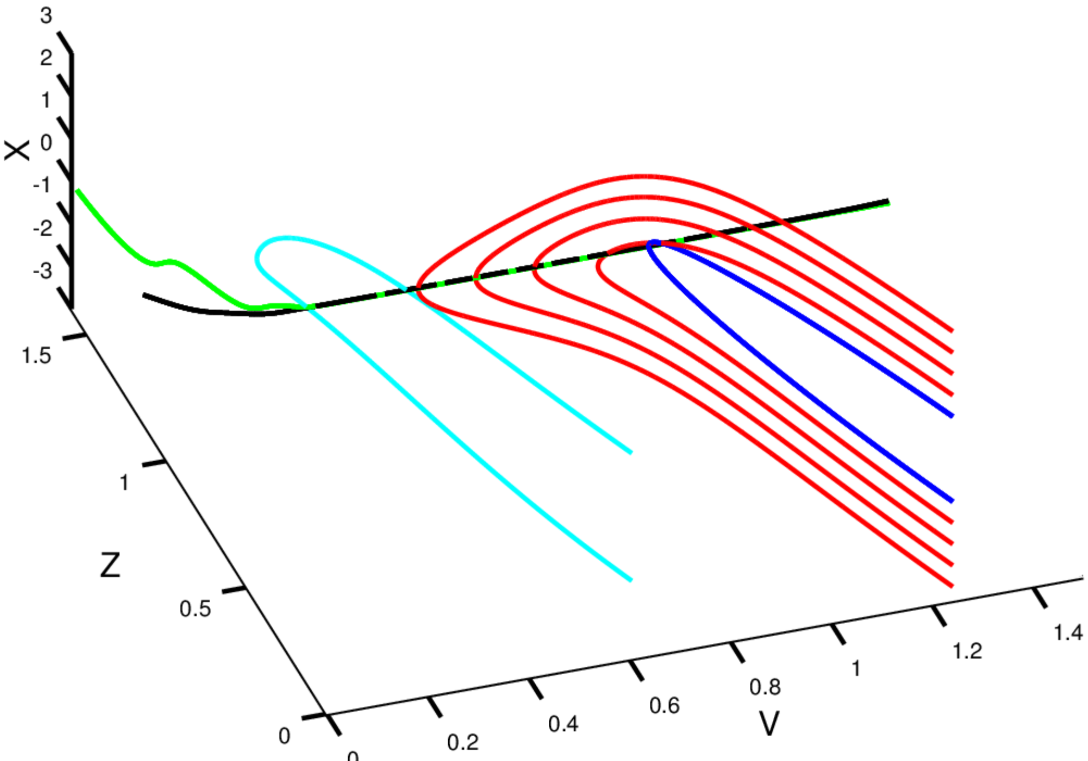


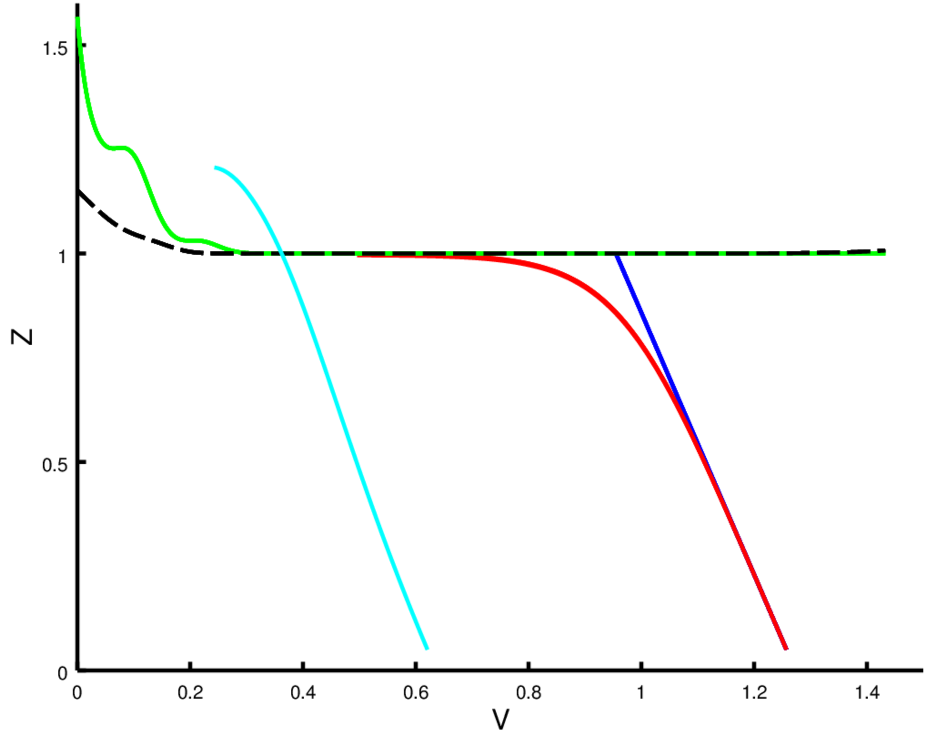


[CE-Grumiller-Stricker 15]

Geodesics in anisotropic AdS_5 black brane background

- Far-from equilibrium geodesics can go beyond the horizon.
- Near equilibrium geodesics stay outside the horizon.





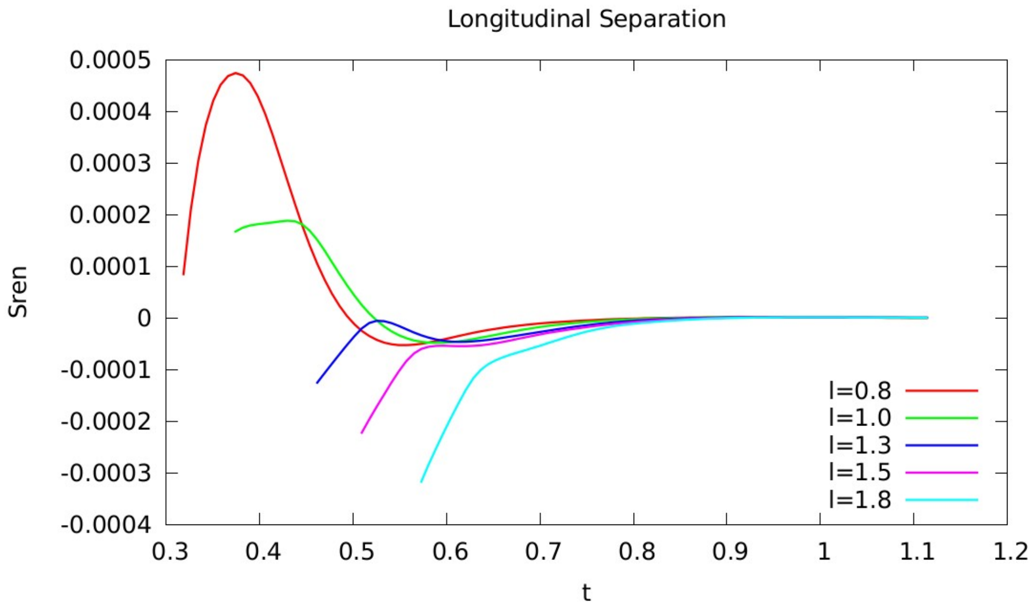
[CE-Grumiller-Stricker 15]

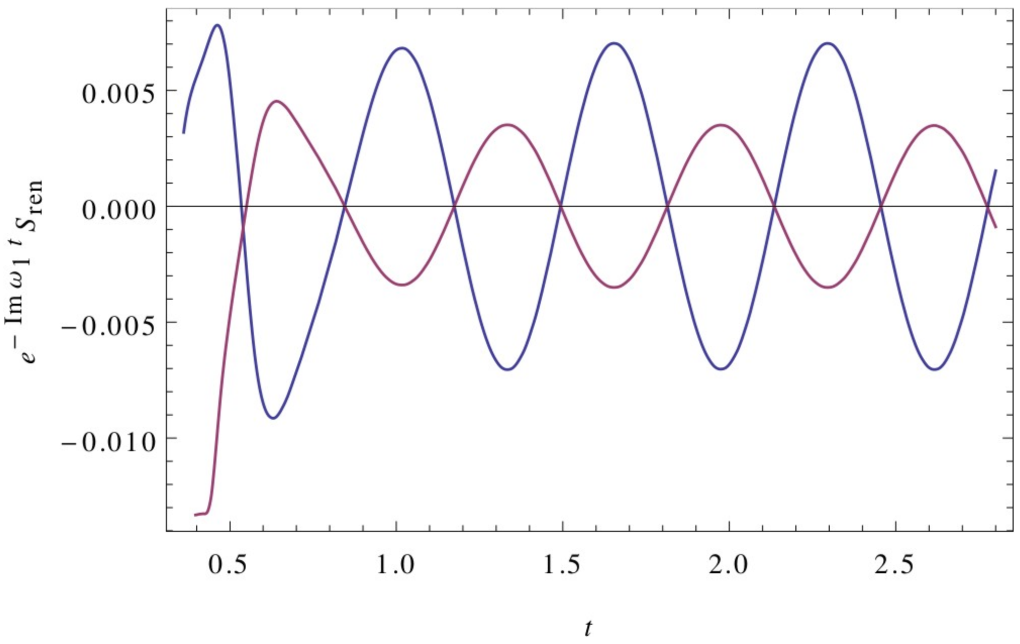
Quasinormal ringdown of entanglement entropy

[see also talk by W. Sybesma]

The **late time dynamics** of EE is captured by a **single (complex) number**:

$$\frac{\omega_1}{\pi T} = \pm 3.1119452 - 2.746676i$$

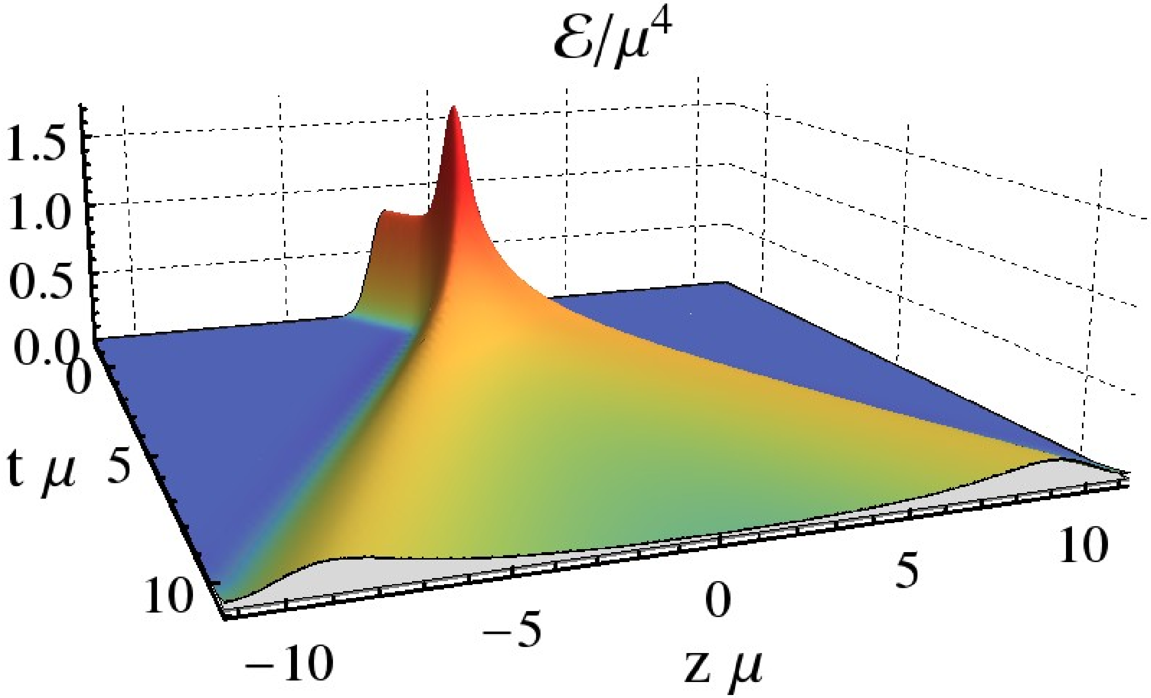


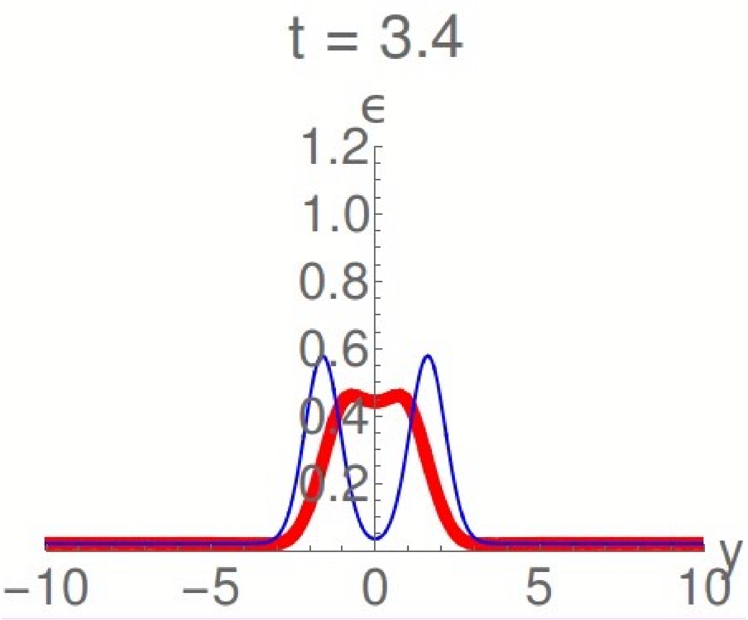


[CE-Grumiller-Stricker 15]

Holographic shock wave collisions

HIC is modeled by **two colliding sheets of energy** with **infinite extend in transverse direction** and **Gaussian profile** in **beam direction**. [Chesler-Yaffe 10]





Solving the numerical GR problem is tricky: [Chesler-Yaffe 13]

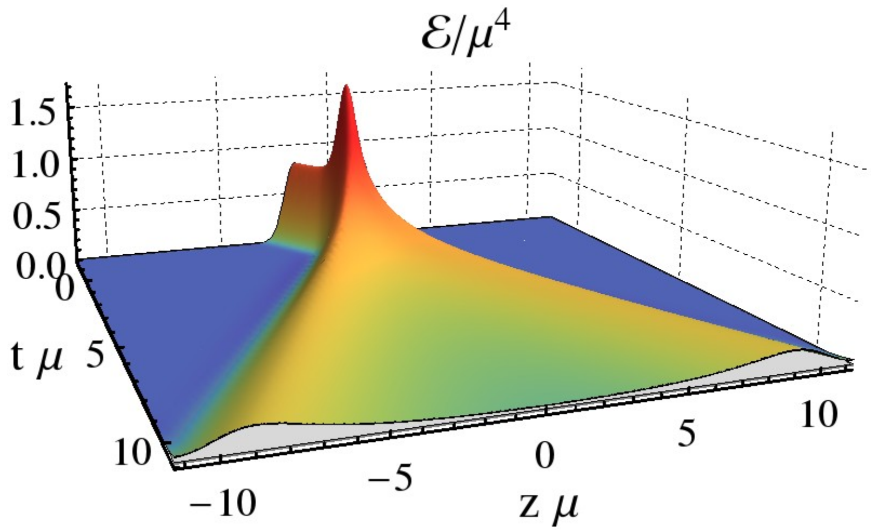
- **Characteristic formulation** decouples EE to **nested system of ODEs**
- ODEs solved by **spectral method** [see textbooks by Boyd, Trefethen]
- **Time evolution** with simple **Adams-Bashforth** algorithm

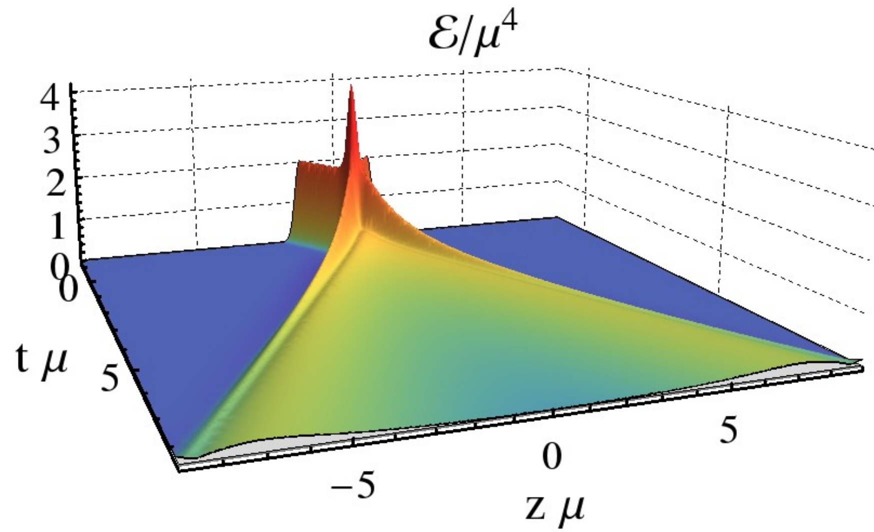
Wide vs. narrow shocks

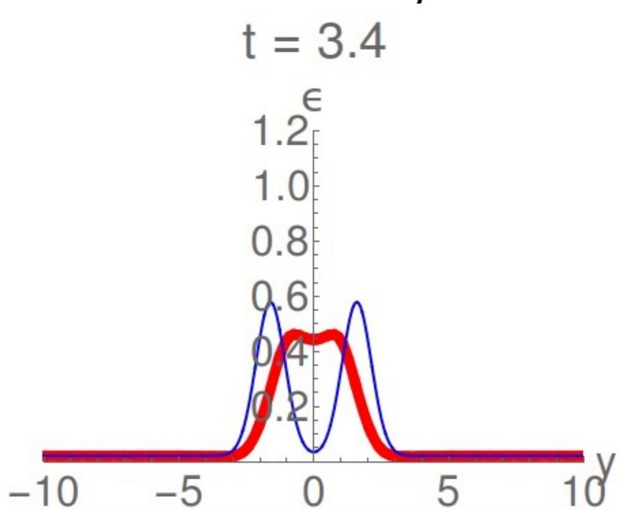
Two qualitatively different dynamical regimes

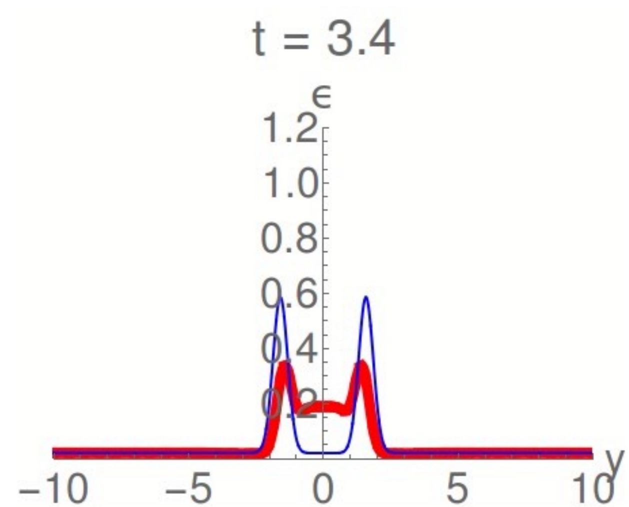
[Solana-Heller-Mateos-van der Schee 12]

- **Wide** shocks (~RHIC): **full stopping**
- **Narrow** shocks (~LHC): **transparency**

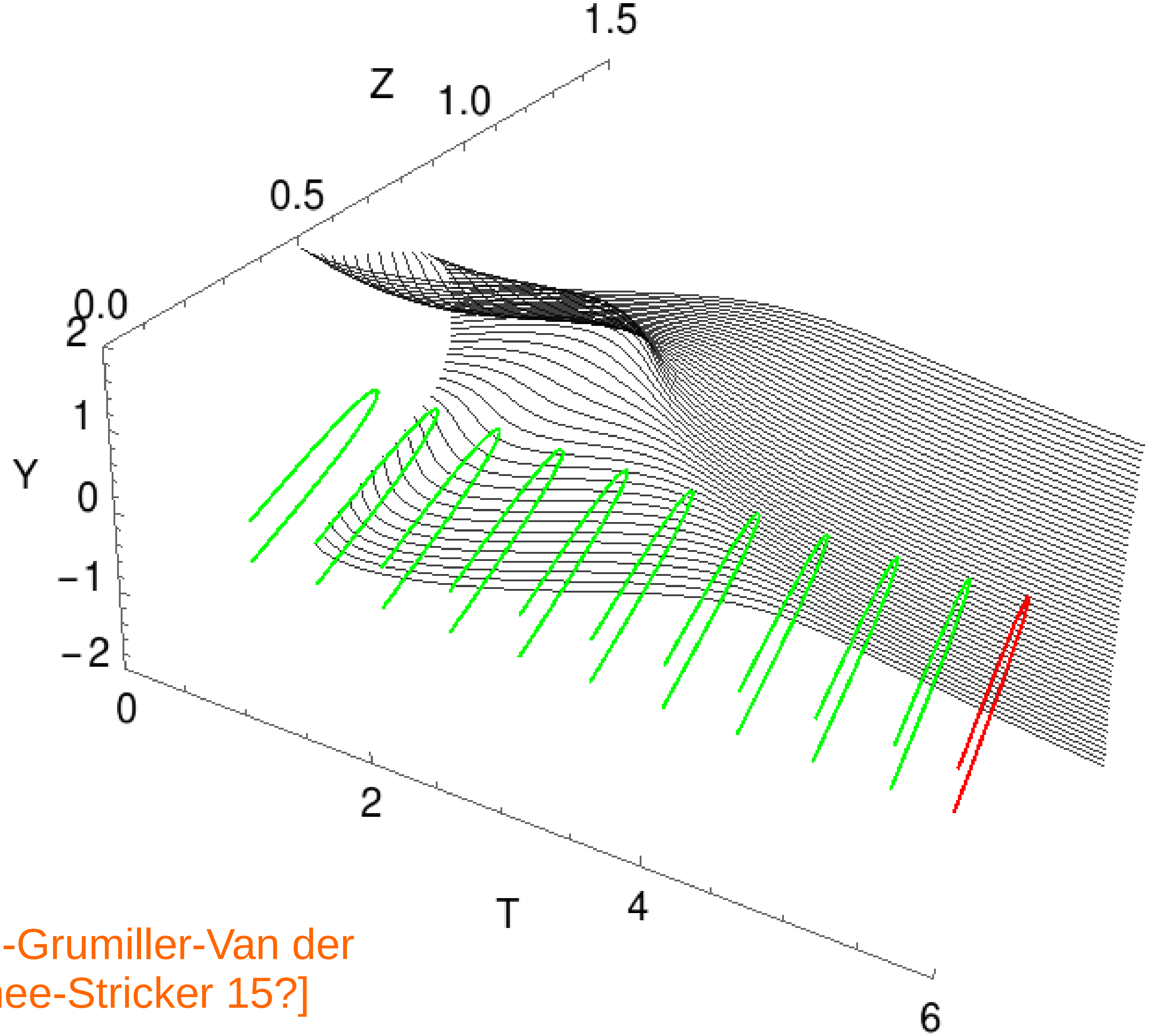








Geodesics and apparent horizon

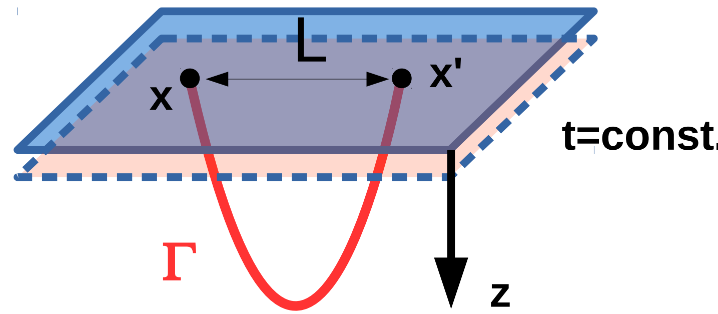


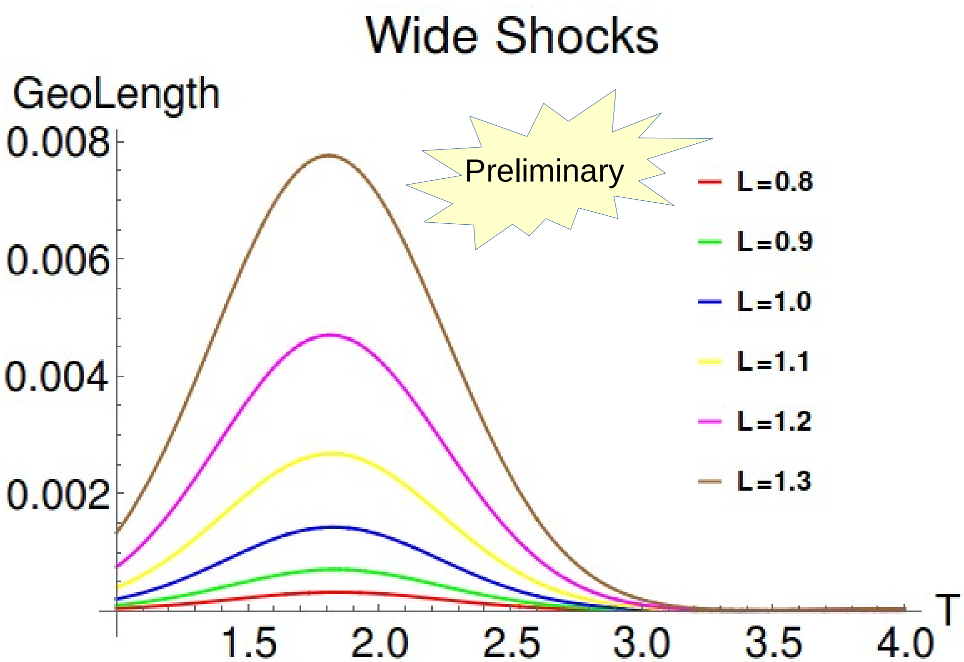
[CE-Grumiller-Van der Schee-Stricker 15?]

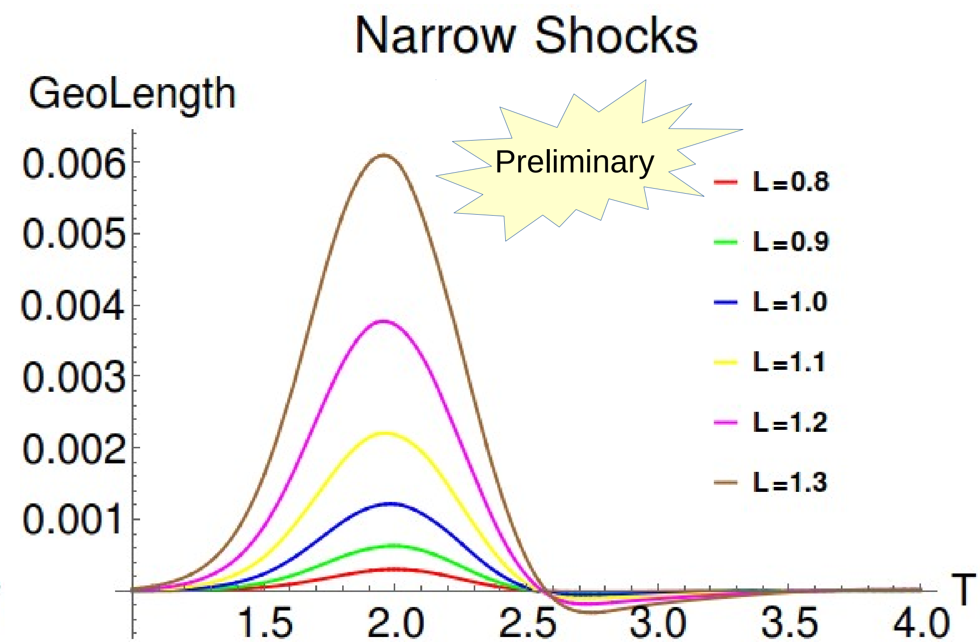
Two point functions

Within AdS/CFT **two point functions** for operators **O(t,x)** of **large conformal weight Δ** can be computed form the **length of geodesics**. [Balasubramanian-Ross 00]

$$\langle \mathcal{O}(t,x)\mathcal{O}(t,x')\rangle \propto e^{-\Delta \mathrm{Length}(\Gamma)}$$



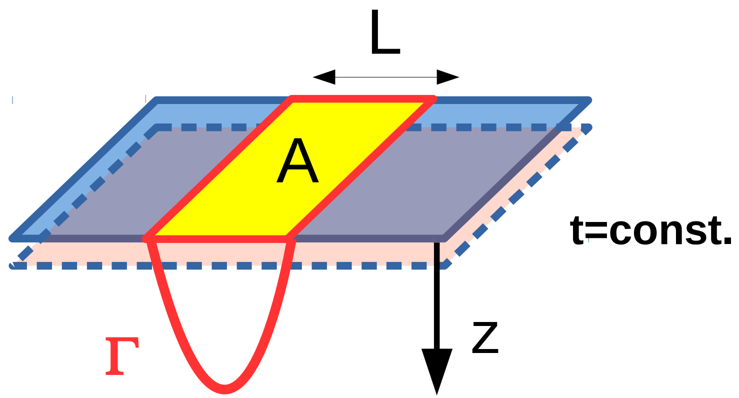


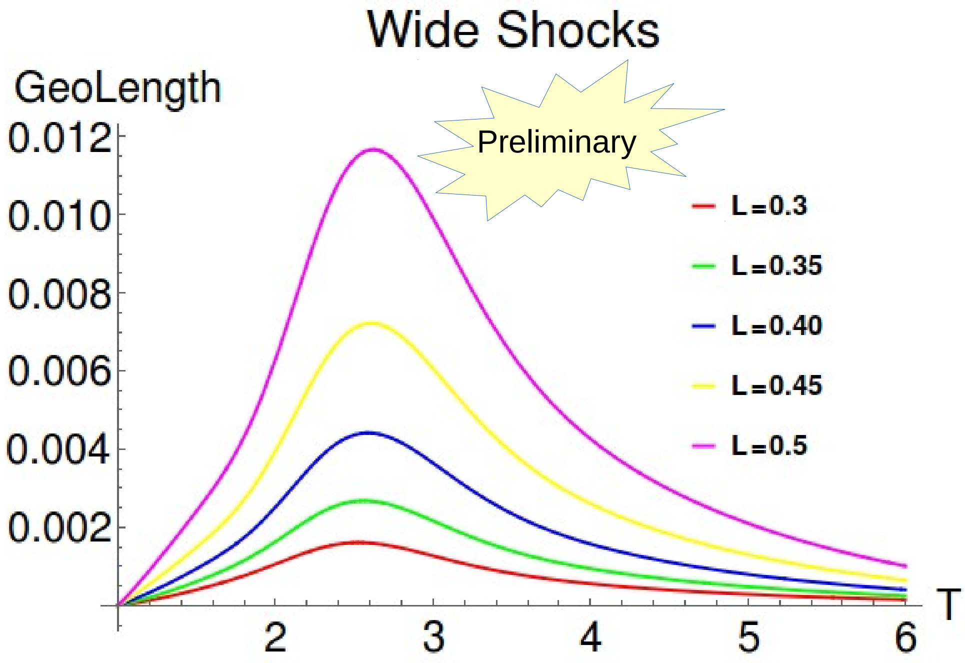


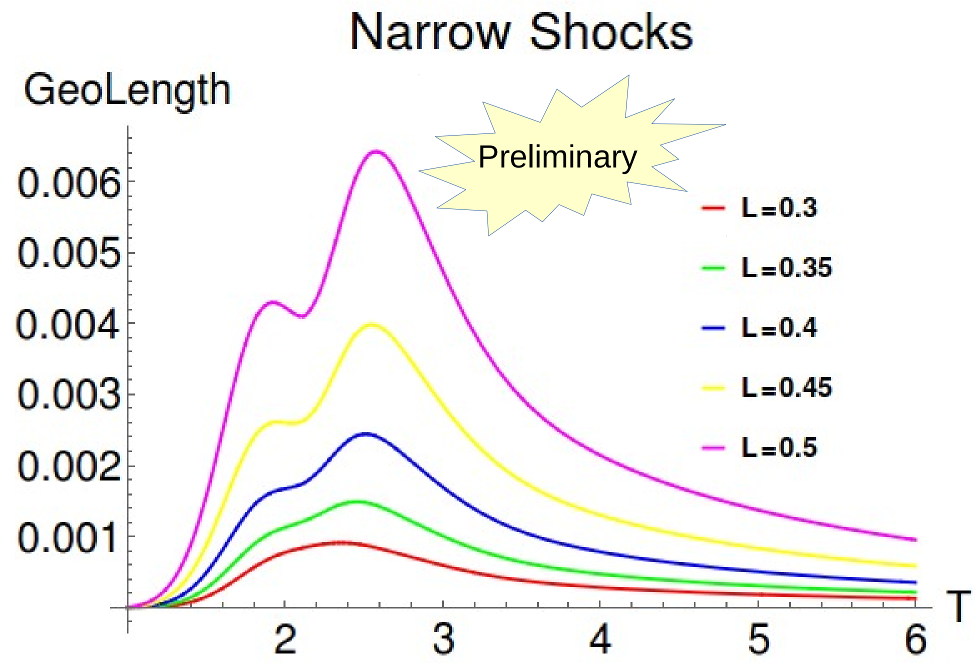
[CE-Grumiller-Van der Schee-Stricker 15?]

Entanglement entropy

$$S_A = const.\frac{\text{Length}(\Gamma)}{4G_N}$$





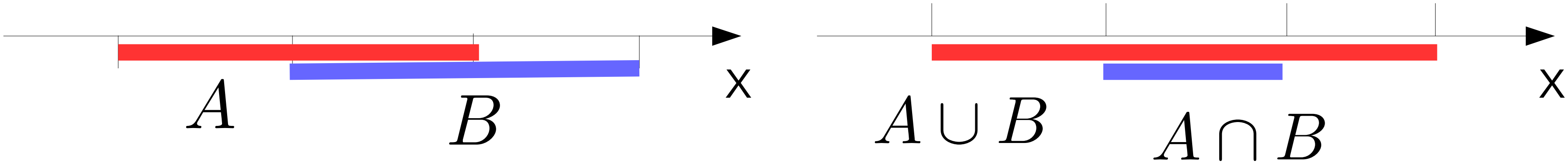


[CE-Grumiller-Van der Schee-Stricker 15?]

Strong subadditivity

- A **fundamental property** of entanglement entropy is strong subadditivity.
- Hard to prove within QFT, very **intuitive** in the **dual gravity picture**.

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$



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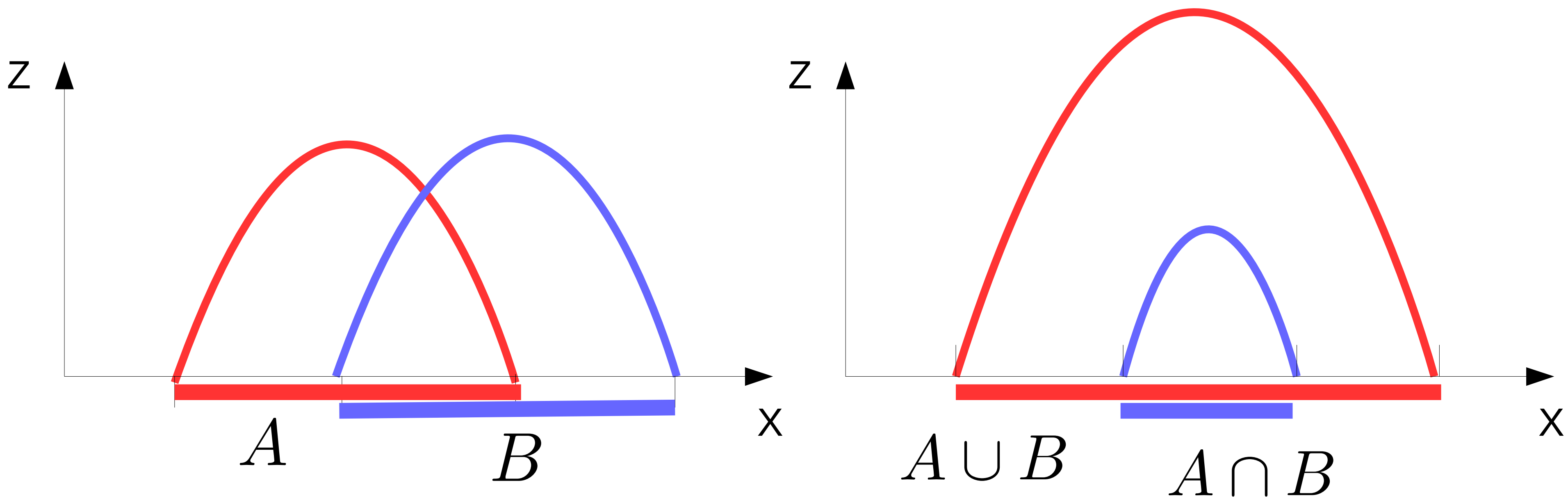

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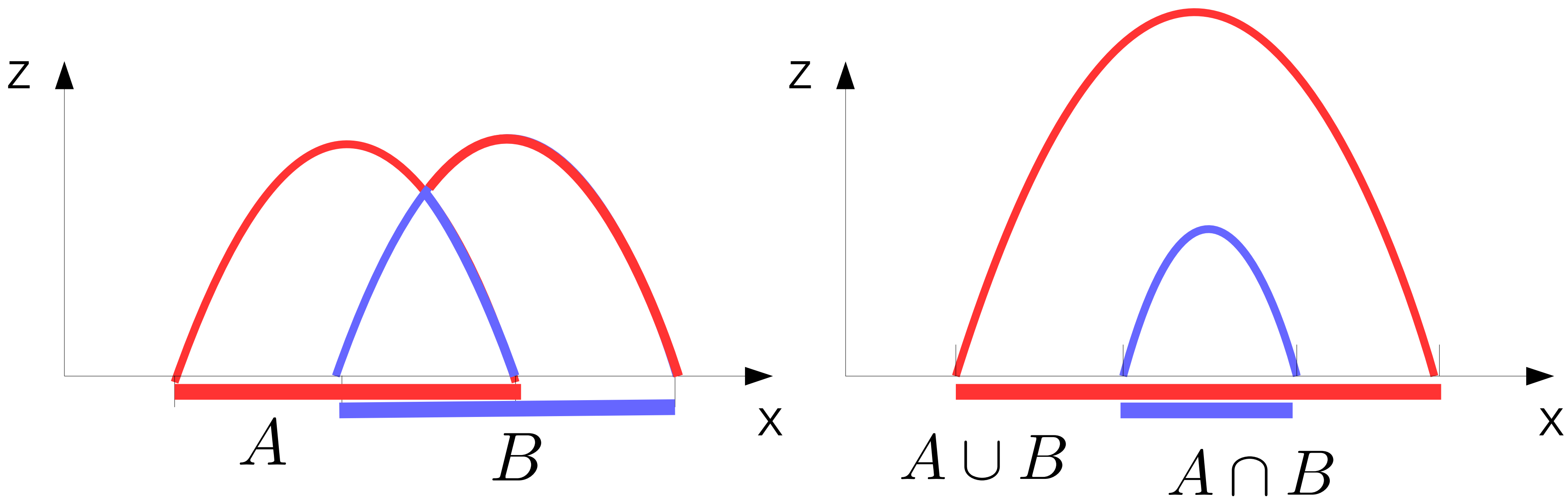
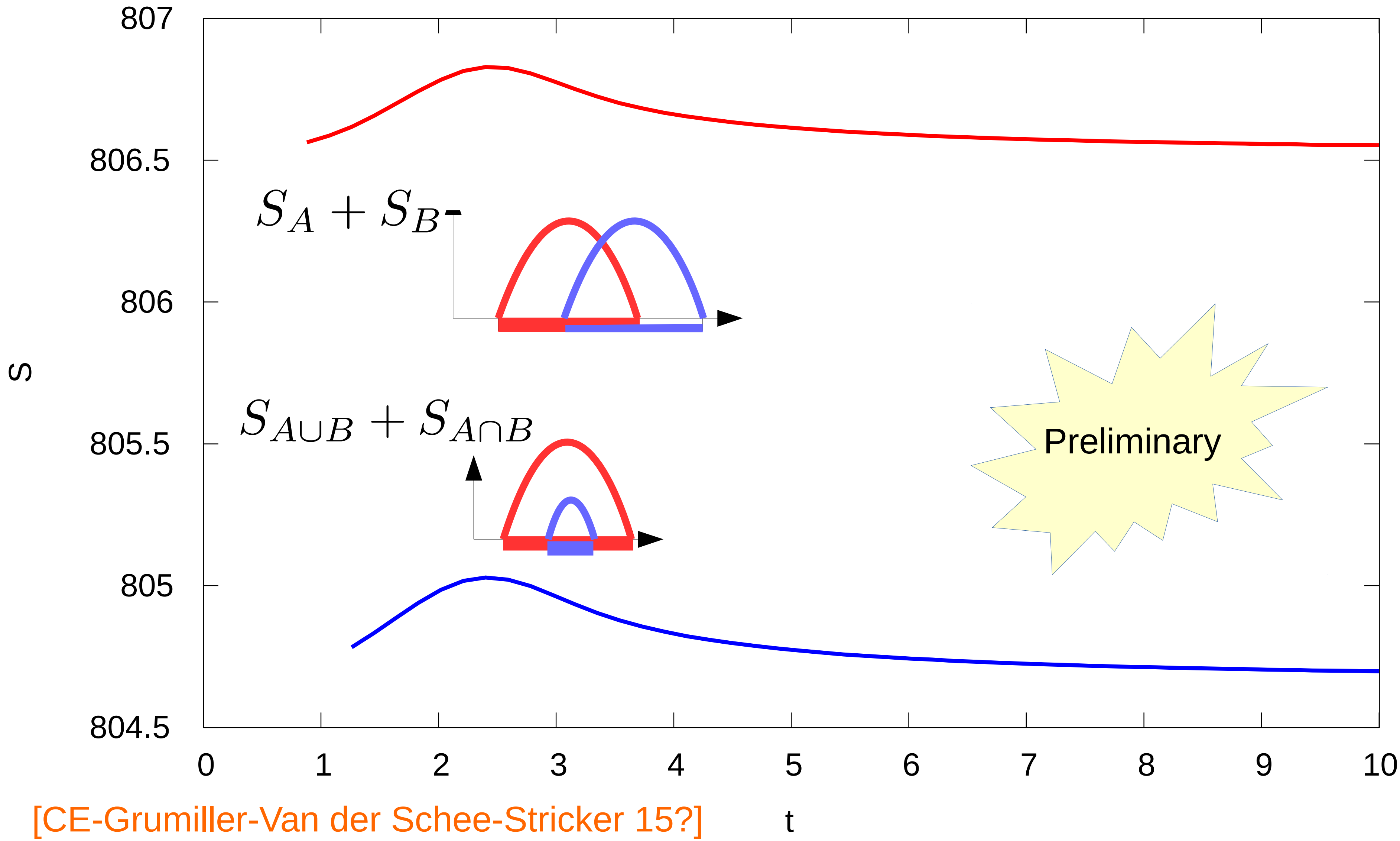


Numerical check of strong subadditivity



[CE-Grumiller-Van der Schee-Stricker 15?]

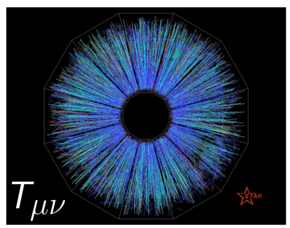
Summary

- The **near equilibrium dynamics** of holographic entanglement shows **quasinormal mode** behaviour.
- In holographic **shock wave collisions** the **entanglement entropy** and the **two point function** may serve as **order parameter** for the **full stopping–transparency transition**.
- We **numerically checked** the **strong subadditivity** condition.

Take home message

Within AdS/CFT complicated stuff in the CFT often has a very intuitive geometric interpretation on the AdS side.

thermalization = **black hole formation**



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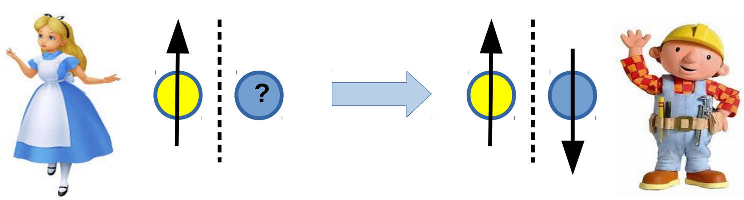
entanglement entropy = **area of extremal surface**



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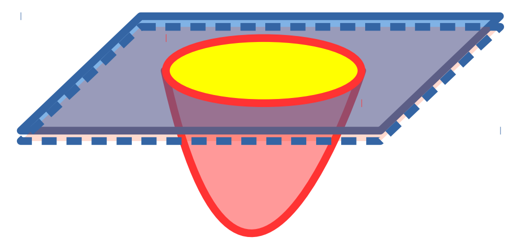

two point function = **length of geodesic**

$$\langle \mathcal{O}(t,x)\mathcal{O}(t,x')\rangle$$

=