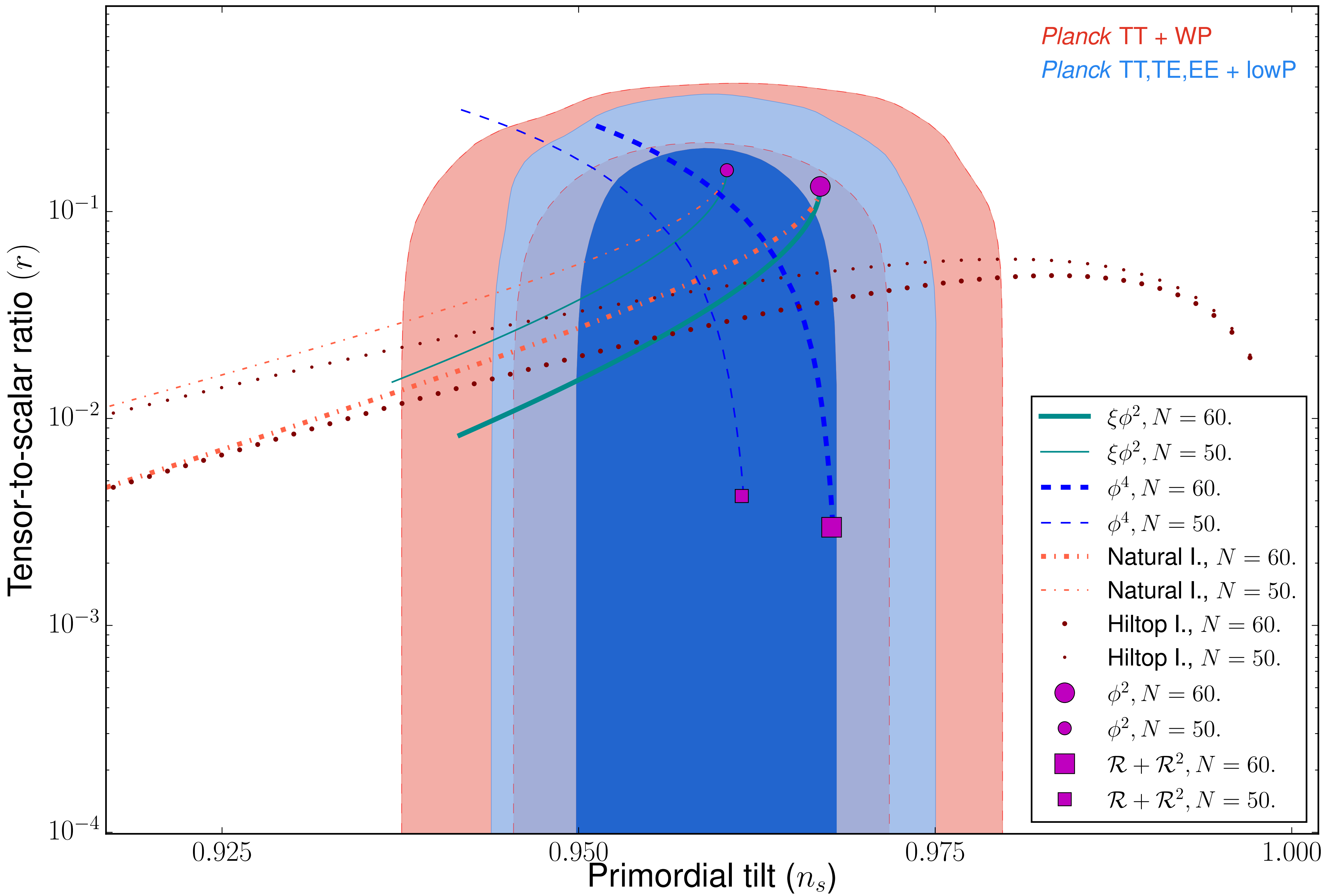
The present and future of the most favoured Inflationary Models

Miguel Escudero Abenza

Based on arXiv:1509.05419

In collaboration with Héctor Ramirez, Olga Mena, Lotfi Boubekeur and Elena Giusarma

Motivation



Planck TT + WP
Planck TT,TE,EE + lowP
$\xi\phi^2, N = 60.$
$\xi\phi^2, N = 50.$
$\phi^4, N = 60.$
$\phi^4, N = 50.$
Natural I., $N = 60.$
Natural I., $N = 50.$
Hiltop I., $N = 60.$
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$\phi^2, N = 60.$
$\phi^2, N = 50.$
$\mathcal{R} + \mathcal{R}^2, N = 60.$
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Tensor-to-scalar ratio (r)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
0.925
0.950
0.975
1.000
Primordial tilt (n_s)

Contents

FLRW Universe

Background FLRW:

$$ds^2 = a^2(\tau)\left\{-d\tau^2 + dx^i dx_i\right\}$$

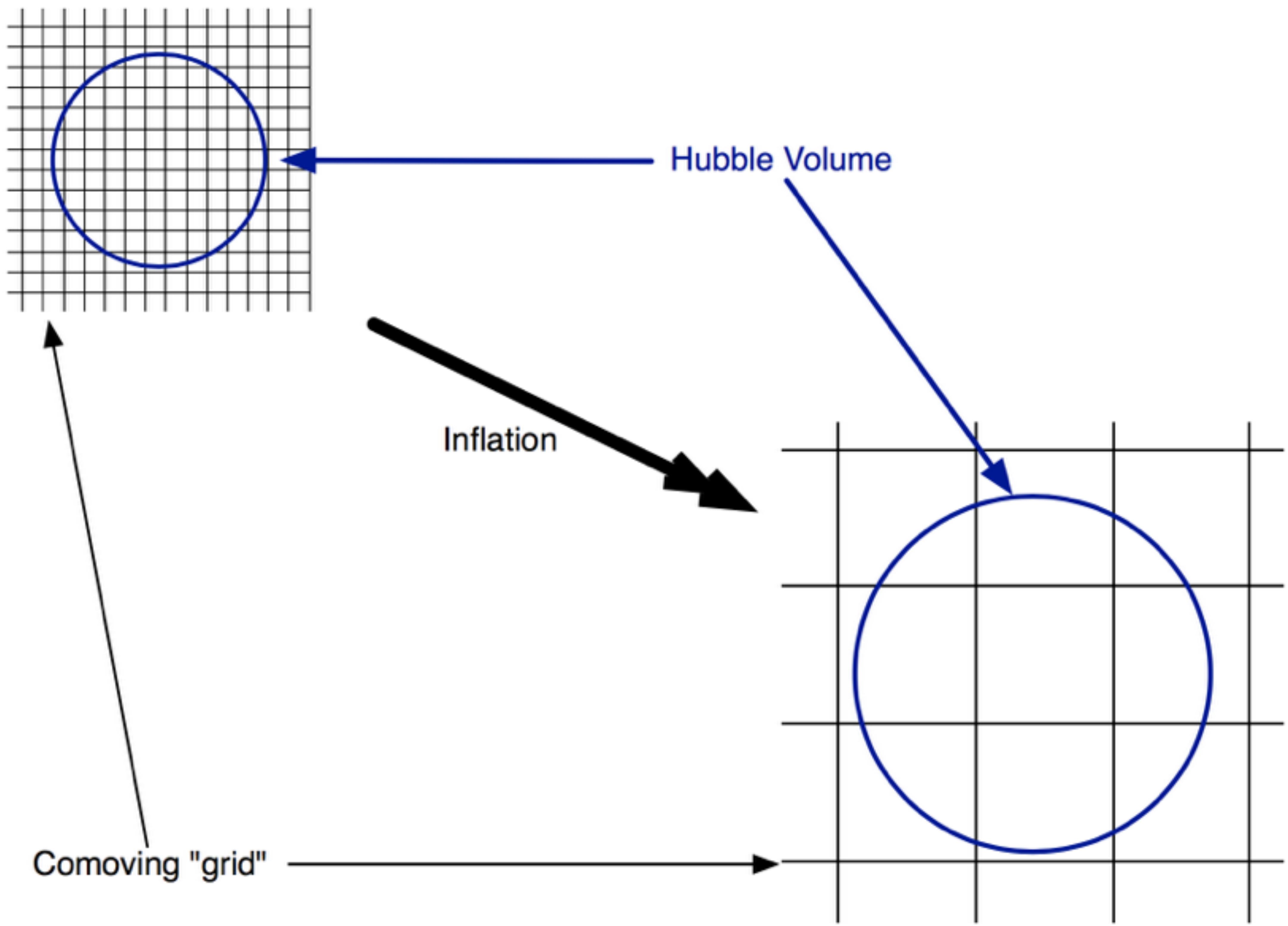
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Perturbed FLRW metric in Newtonian gauge:

$$ds^2 = a^2(\tau)\left\{-(1+2\psi)d\tau^2 + (1-2\phi)dx^i dx_i - h_{ij} dx^i dx^j\right\}$$

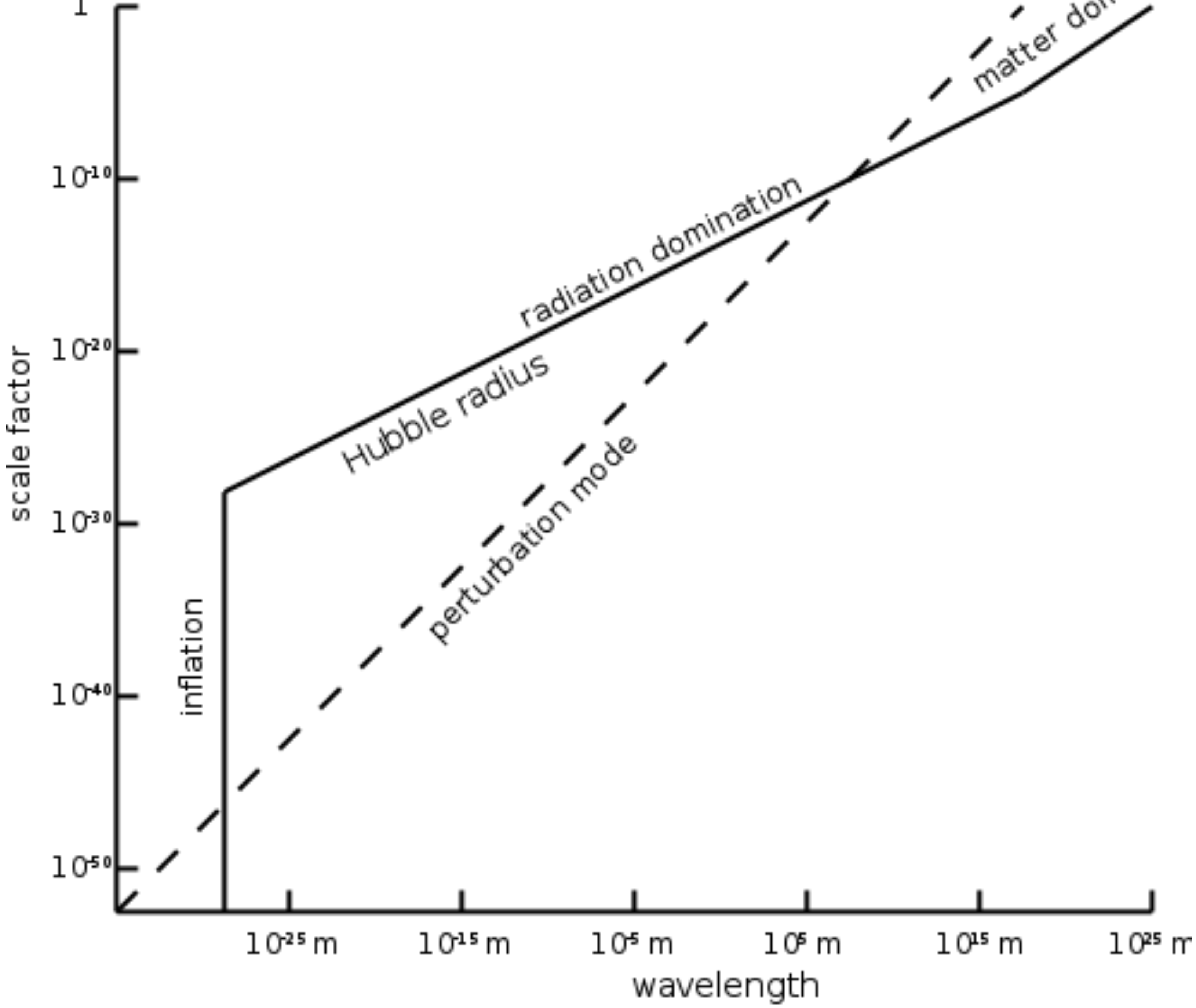
$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4} \delta T_{\mu\nu}$$

Exponential Expansion

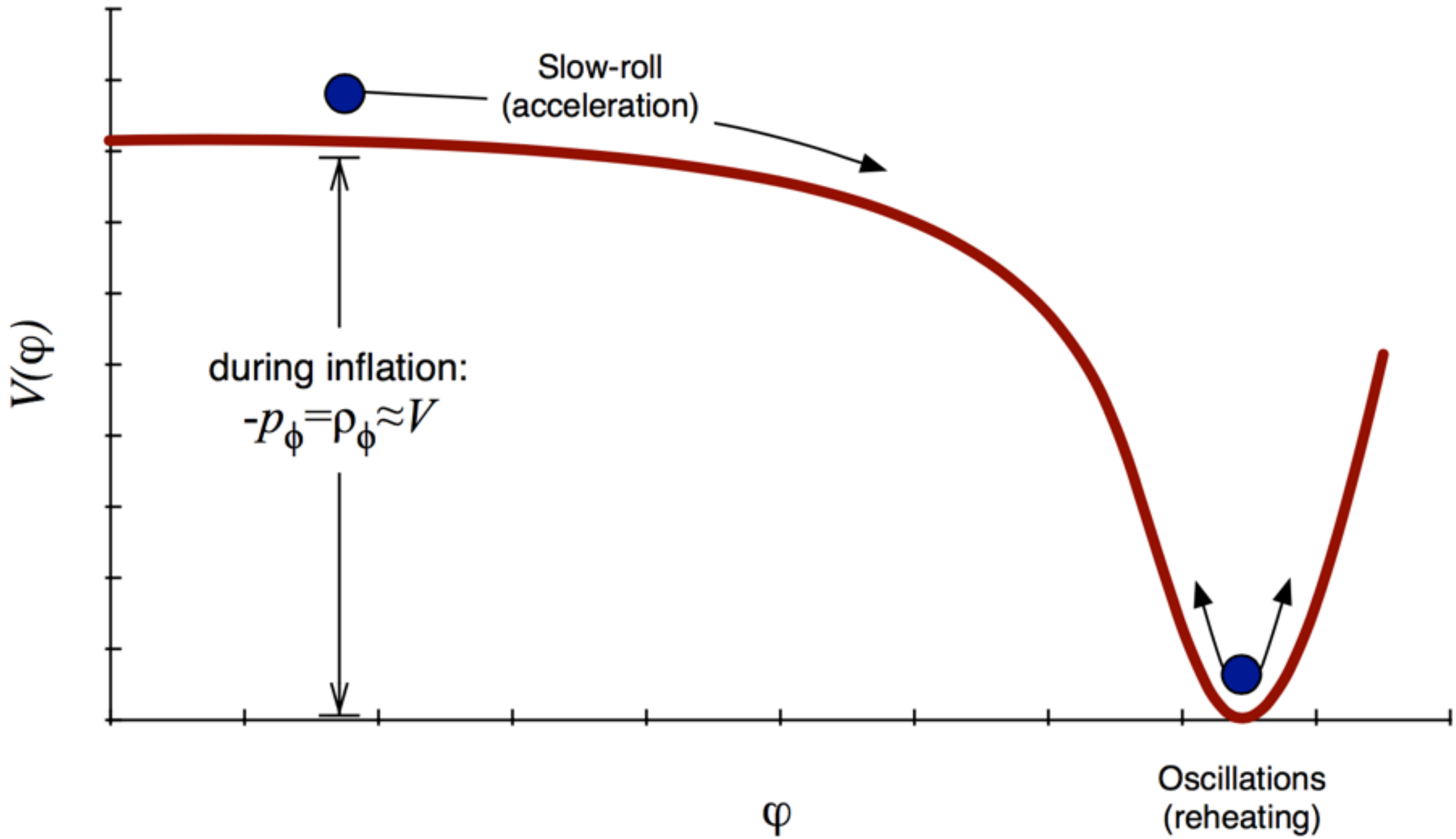


Scales

Frozen outside the Horizon!



Slow Roll


Slow-roll
(acceleration)
V(φ)
during inflation:
$-p_\phi = \rho_\phi \approx V$
φ
Oscillations
(reheating)

Inflationary Models

Chaotic scenario

$$V(\phi) \propto \phi^2$$

Natural Inflation

$$V(\phi) = V_0 \left[1 - \cos(\phi/f)\right]$$

Hilltop scenarios

$$V(\phi) = V_0 \left[1 - (\phi/\mu)^p\right]$$

Non-minimally coupled Higgs-like scenario

$$V(\phi) = \frac{\lambda \left(\phi^2 - v^2\right)^2}{4\left(1 + \xi\phi^2/M_{\rm pl}^2\right)^2} \quad v = 0$$

$\xi \to \infty$ $\quad R^2$ Starobinsky

$\xi \to 0$ $\quad V \propto \phi^4$

Fundamentals of Inflation

Attractor solution

Final state independent of the initial conditions

Exponential acceleration

Solves the Horizon, Flatness and Monopole problems

Predictions

Perturbations seed (adiabatic/isocurvature)

Nearly scale invariant power spectrum

Stochastic background of gravitational waves

Implications of Inflation

Curvature perturbation in Fourier space: $\mathcal{R}_{\vec{k}}$

$$\langle \mathcal{R}_{\vec{k}} \rangle = 0 \qquad \langle \mathcal{R}_{\vec{k}} \mathcal{R}_{\vec{p}} \rangle = (2\pi)^3 \delta^{(3)}(\vec{p} + \vec{k}) \frac{2\pi^2}{k^3} \mathcal{P}_{\mathcal{R}}(k)$$

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Nearly scale invariant power spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathrm{s}} \left(\frac{k}{k_*} \right)^{n_{\mathrm{s}} - 1 + \frac{1}{2!} \alpha_{\mathrm{s}} \ln(k/k_*) + \frac{1}{3!} \beta_{\mathrm{s}} (\ln(k/k_*))^2 + \ldots}$$

$$k_* = 0.05\,\mathrm{Mpc}^{-1}$$

$$\mathcal{P}_{\mathrm{t}}(k) = A_{\mathrm{t}} \left(\frac{k}{k_*} \right)^{n_{\mathrm{t}} + \ldots} \qquad r = \frac{\mathcal{P}_{\mathrm{t}}(k_*)}{\mathcal{P}_{\mathcal{R}}(k_*)}$$

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Predictions for single field slow-roll Inflation:

$$r : \{10^{-4} \to 0.1\}$$

$$n_t : \{-0.02 \to 0\}$$

$$n_s : \{0.94 \to 0.98\}$$

$$\alpha_s : \{-10^{-3} \to -10^{-4}\}$$

$$\beta_s : \{-10^{-4} \to -10^{-5}\}$$

CMB Anisotropies

Intensity, $I \Rightarrow T$

Polarization, Q, U

$\mathcal{A}, \mathcal{B} = T, E, B \qquad \theta = 180^\circ/\ell$

$$\Delta\mathcal{A}(\theta,\phi) = \sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m}^{\mathcal{A}} Y_{\ell m}(\theta,\phi)$$

$$C_\ell^{\mathcal{A},\mathcal{B}} = \langle \Delta\mathcal{A}\Delta\mathcal{B}\rangle = \frac{\ell(\ell+1)}{2\pi}\sum_{m=-\ell}^{\ell}\frac{a_{\ell m}^{\mathcal{A}\,*} a_{\ell m}^{\mathcal{B}}}{2\ell+1}$$

CMB Anisotropies

$$C_\ell^{\mathcal{AB},\mathrm{s}} = \int_0^\infty \frac{\mathrm{d}k}{k}\ \Delta^{\mathrm{s}}_{\ell,\mathcal{A}}(k)\ \Delta^{\mathrm{s}}_{\ell,\mathcal{B}}(k)\ \mathcal{P}_{\mathcal{R}}(k)$$

$$C_\ell^{\mathcal{AB},\mathrm{t}} = \int_0^\infty \frac{\mathrm{d}k}{k}\ \Delta^{\mathrm{t}}_{\ell,\mathcal{A}}(k)\ \Delta^{\mathrm{t}}_{\ell,\mathcal{B}}(k)\ \mathcal{P}_{\mathrm{t}}(k)$$

$$C_\ell^{\mathcal{AB},\mathrm{tot}} = C_\ell^{\mathcal{AB},\mathrm{s}} + C_\ell^{\mathcal{AB},\mathrm{t}}$$

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4}\delta T_{\mu\nu}$$

$$\mathcal{A},\mathcal{B} = T, E, B$$

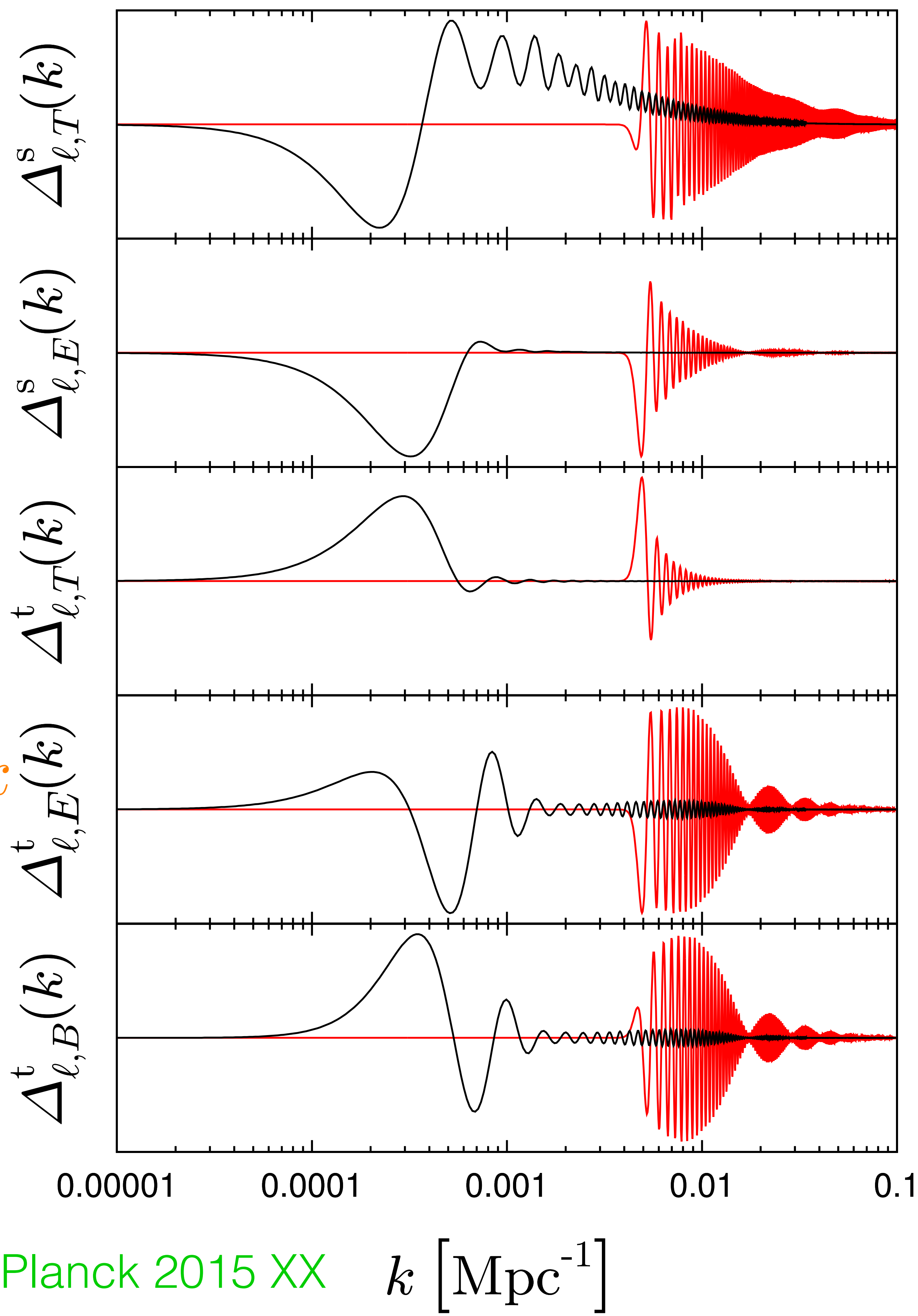
Transfer Functions:

Red $\ell = 65$

Black $\ell = 2$

Small Scales: large ℓ, large k

Large Scales: small ℓ, small k

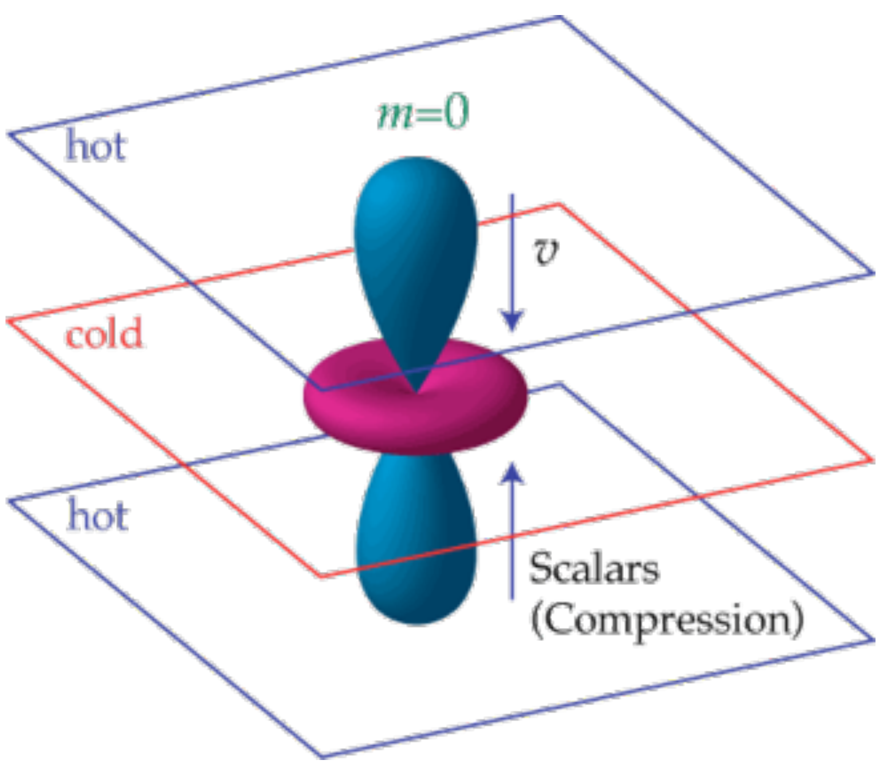


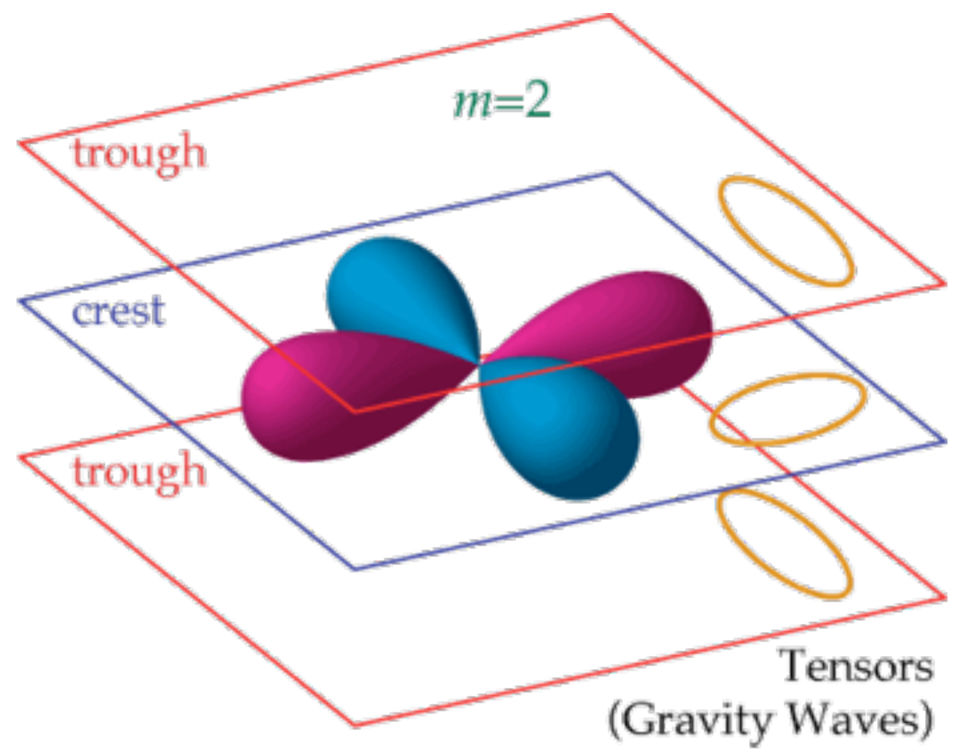
Planck 2015 XX

CMB anisotropies

Perturbations and Thompson Scattering: Kosowski 96'

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi}(\epsilon \cdot \epsilon') = \frac{3\sigma_T}{8\pi}\frac{1+\cos(\theta)^2}{2}$$





Hu (background.uchicago.edu/)

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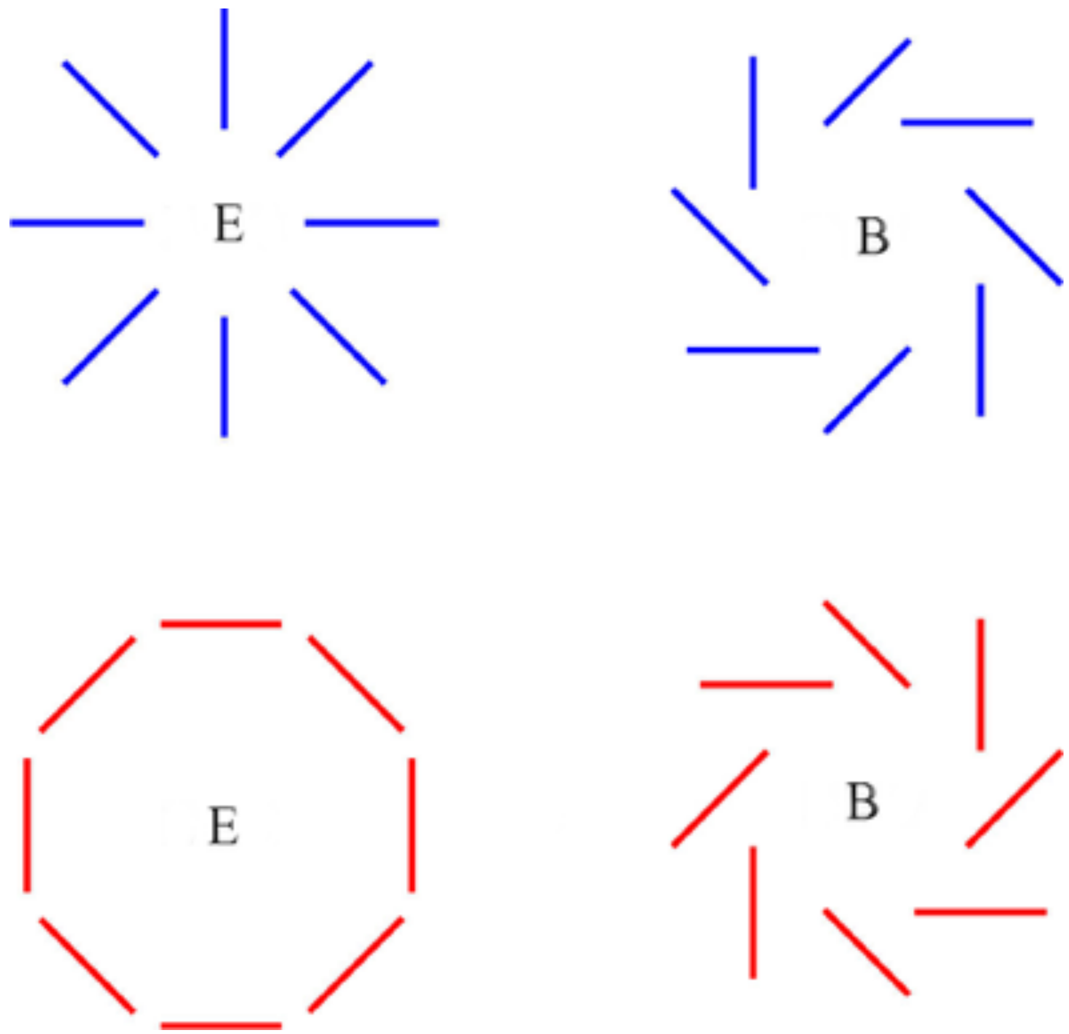
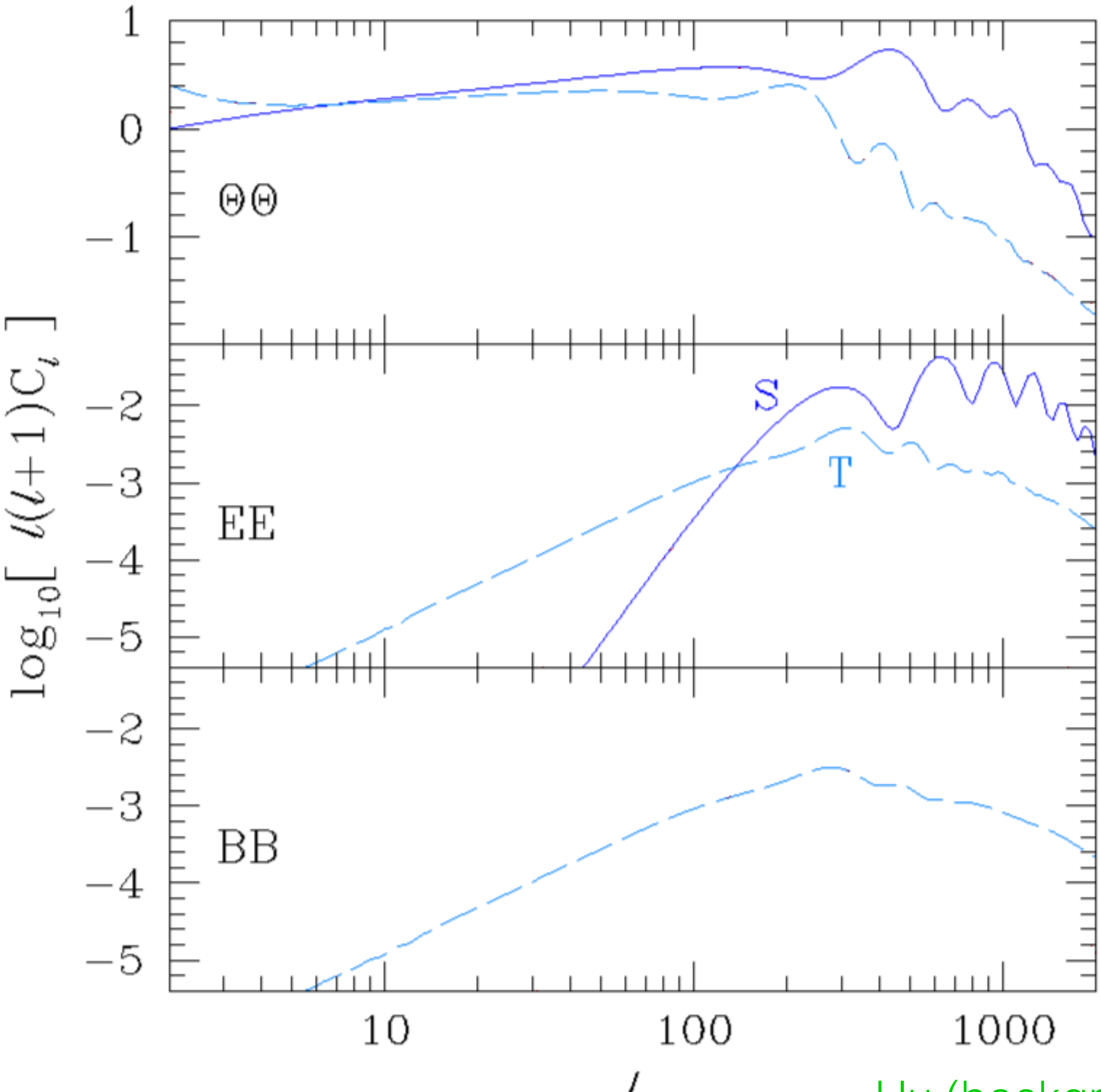


Hu (background.uchicago.edu/)

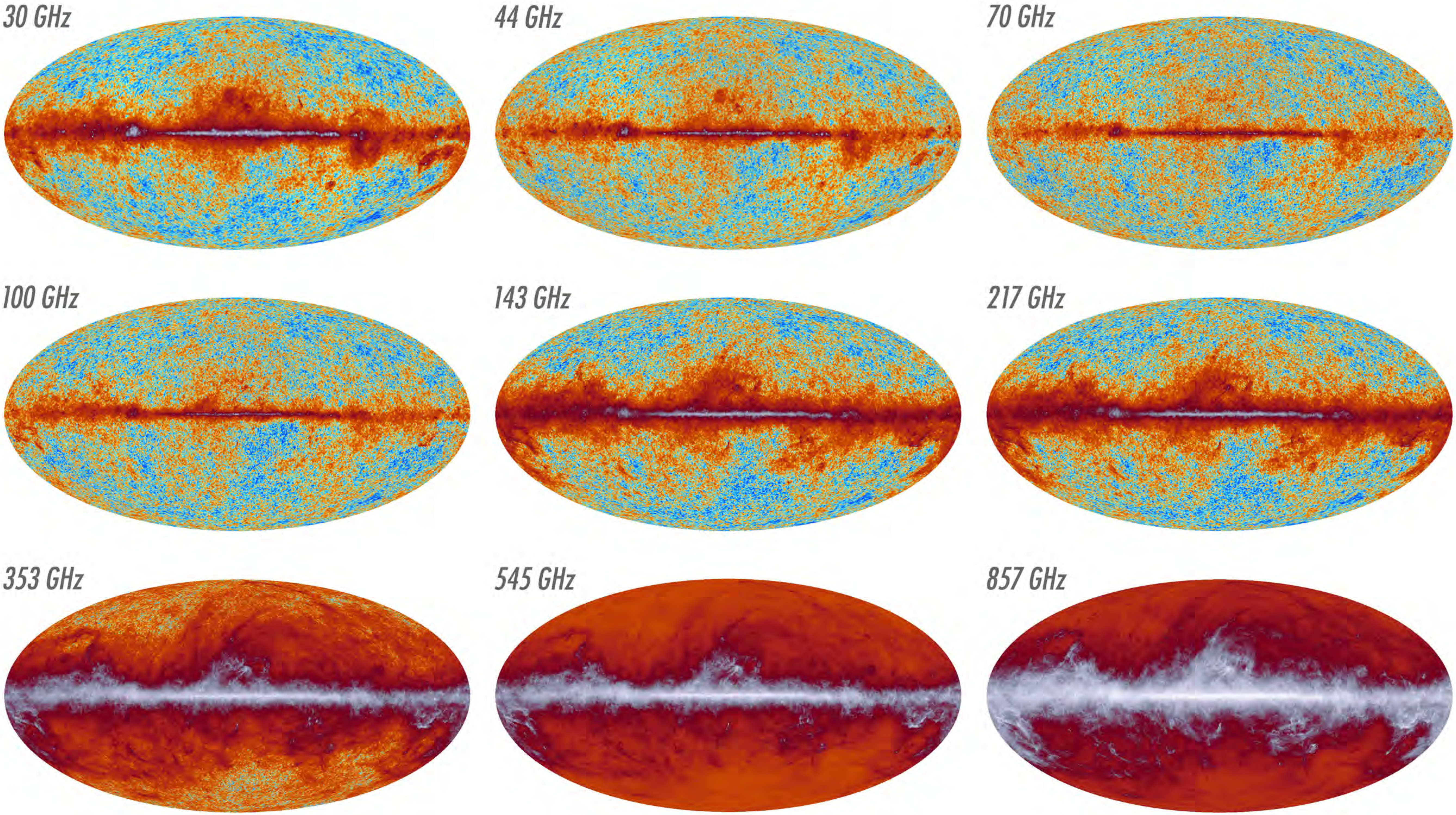
CMB Spectra



$$\delta G^s_{\mu\nu} \not\Rightarrow \frac{8\pi G}{c^4}\delta T^{\gamma^B}_{\mu\nu}$$

Hu (background.uchicago.edu/)

The sky in radio



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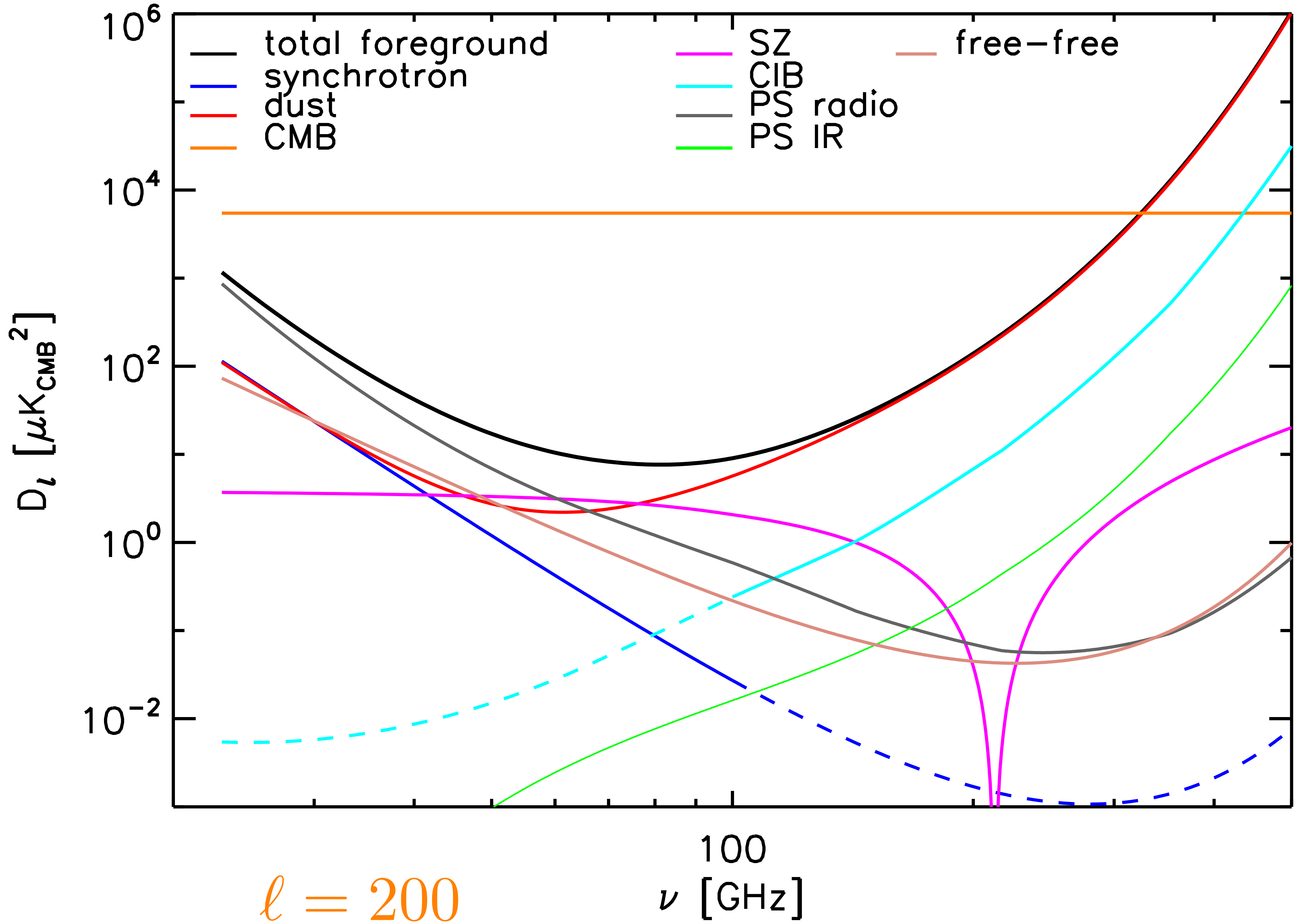
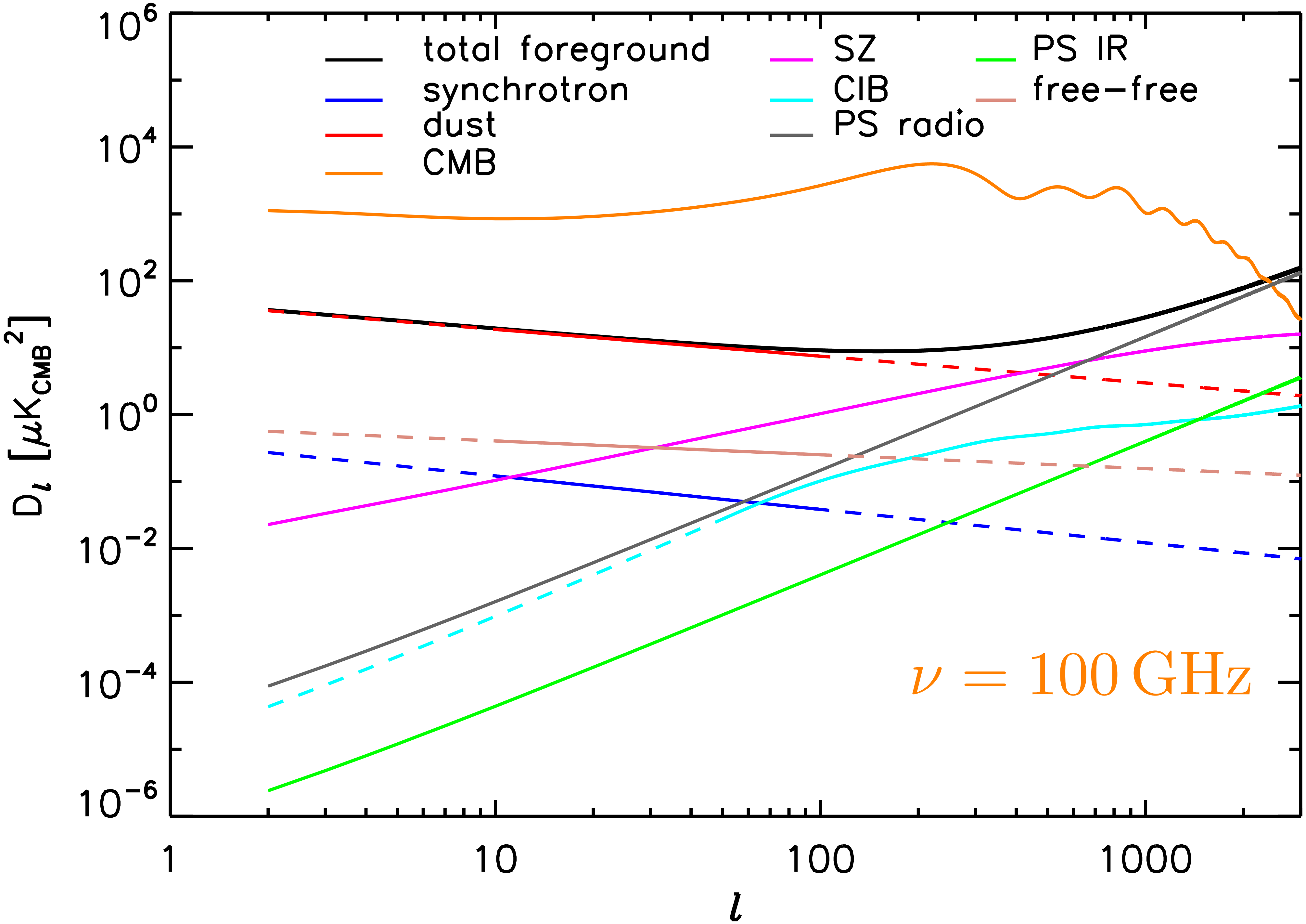
Foreground Contamination



$\ell = 200$

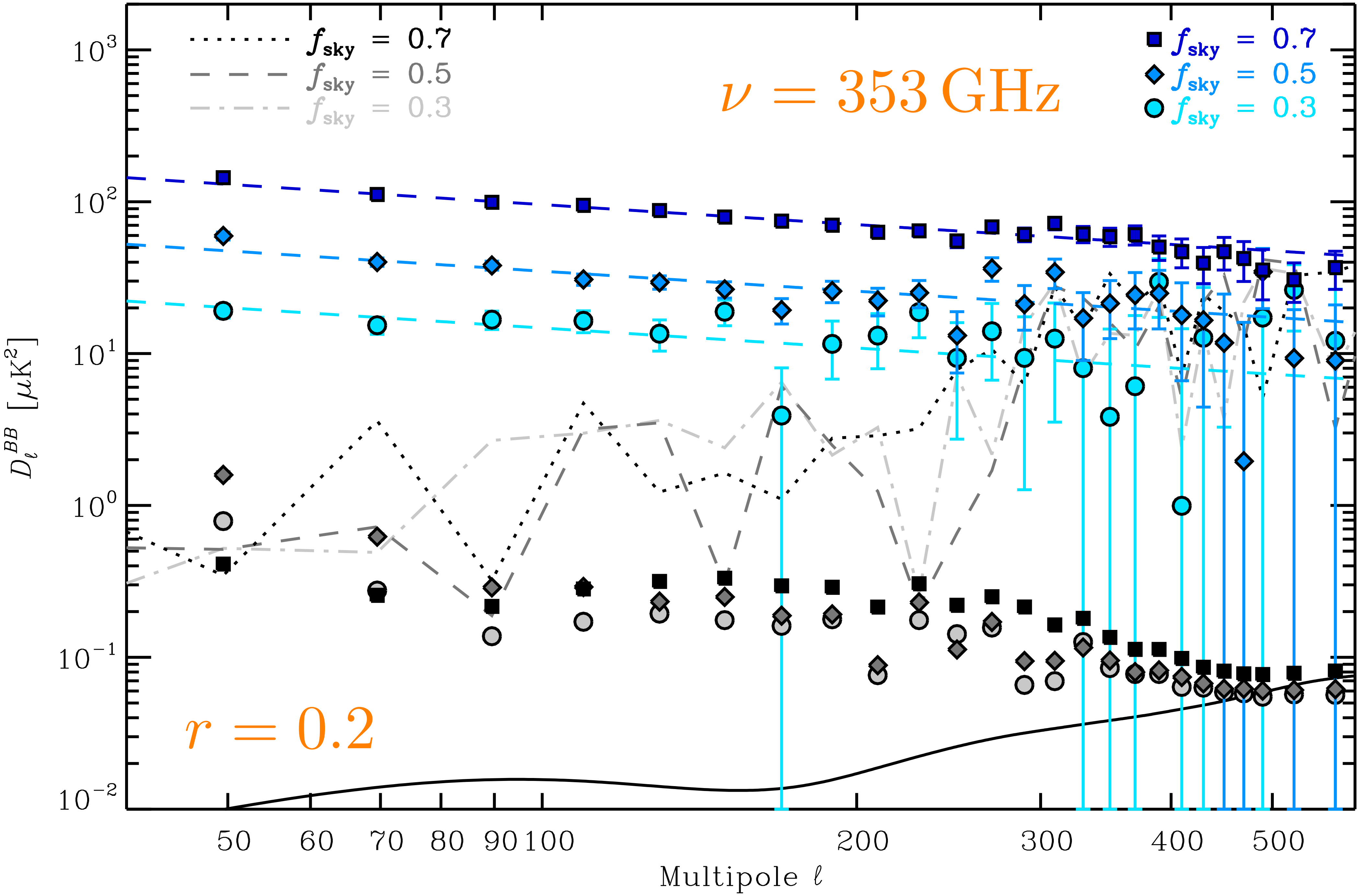
Planck 2013 I

Foreground Contamination



Planck 2013 I

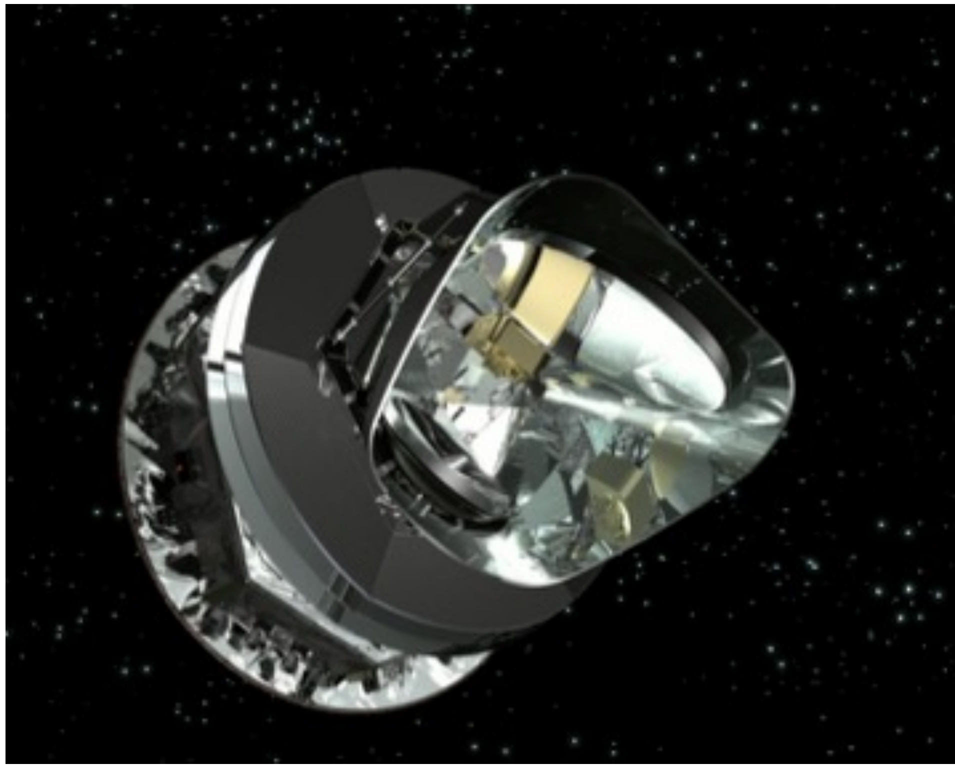
Foreground Contamination



Planck 2014 XXX

Experiments Specifications

Mission	Channel (GHz)	FWHM (arcmin)	ΔT ($\mu K_{\mathrm{CMB}}\cdot$arcmin)	ΔP ($\mu K_{\mathrm{CMB}}\cdot$arcmin)
	30	32.7	203.2	287.4
	44	27.9	239.6	338.9
	70	13.0	221.2	298.7
	100	9.9	31.3	44.2
Planck	143	7.2	20.1	33.3
	217	4.9	28.5	49.4
	353	4.7	107.0	185.3
	535	4.7	1100	-
	857	4.4	8300	-
	45	23.3	5.25	9.07
	75	14.0	2.73	4.72
	105	10.0	2.68	4.63
	135	7.8	2.63	4.55
	165	6.4	2.67	4.61
	195	5.4	2.63	4.54
	225	4.7	2.64	4.57
CORE	255	4.1	6.08	10.5
	285	3.7	10.1	17.4
	315	3.3	26.9	46.6
	375	2.8	68.6	119
	435	2.4	149	258
	555	1.9	227	626
	675	1.6	1320	3640
	795	1.3	8070	22200



ESA 2025?

CMB likelihood

$$-2\ln\mathcal{L}^{\mathrm{CMB}} = \sum_\ell (2\ell+1) f_{sky}\left[\ln\left(\frac{C_\ell^{BB}}{\hat{C}_\ell^{BB}}\right) - \frac{\hat{C}_\ell^{BB}}{C_\ell^{BB}} - 3\right.$$

Jaffe et al. ‘99

Verde et al. ‘06

$$\left. + \ln\left(\frac{C_\ell^{TT}C_\ell^{EE} - (C_\ell^{TE})^2}{\hat{C}_\ell^{TT}\hat{C}_\ell^{EE} - (\hat{C}_\ell^{TE})^2}\right) - \frac{\hat{C}_\ell^{TT}C_\ell^{EE} + C_\ell^{TT}\hat{C}_\ell^{EE} - 2C_\ell^{TE}\hat{C}_\ell^{TE}}{C_\ell^{TT}C_\ell^{EE} - (C_\ell^{TE})^2}\right]$$

$$C_\ell = C_\ell^{th} + N_\ell + R_\ell^F$$

$C_\ell^{\mathcal{AB}} = \mathrm{Measured}$

$\hat{C}_\ell^{\mathcal{AB}} = \mathrm{Theoretical}$

$$R_\ell^F = \sigma_F C_\ell^F$$

$\sigma_F \leq 0.1\, Planck$

$\sigma_F \simeq 0.01\, COrE$

Foreground Spectra

Synchrotron:

$$C_\ell^S = A_S \left(\frac{\ell}{\ell_S}\right)^{\alpha_S} \left(\frac{\nu}{\nu_S}\right)^{2\beta_S}$$

$\nu_S = 0.408\,\mathrm{GHz}$ $\quad \ell_S = 100$ $\quad \alpha_S = -2.5 \pm 0.02$ $\quad \beta_S = -3.00 \pm 0.05$

Foreground Spectra

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$\nu_S = 0.408\,\mathrm{GHz}$ $\ell_S = 100$ $\alpha_S = -2.5 \pm 0.02$ $\beta_S = -3.00 \pm 0.05$

Dust:

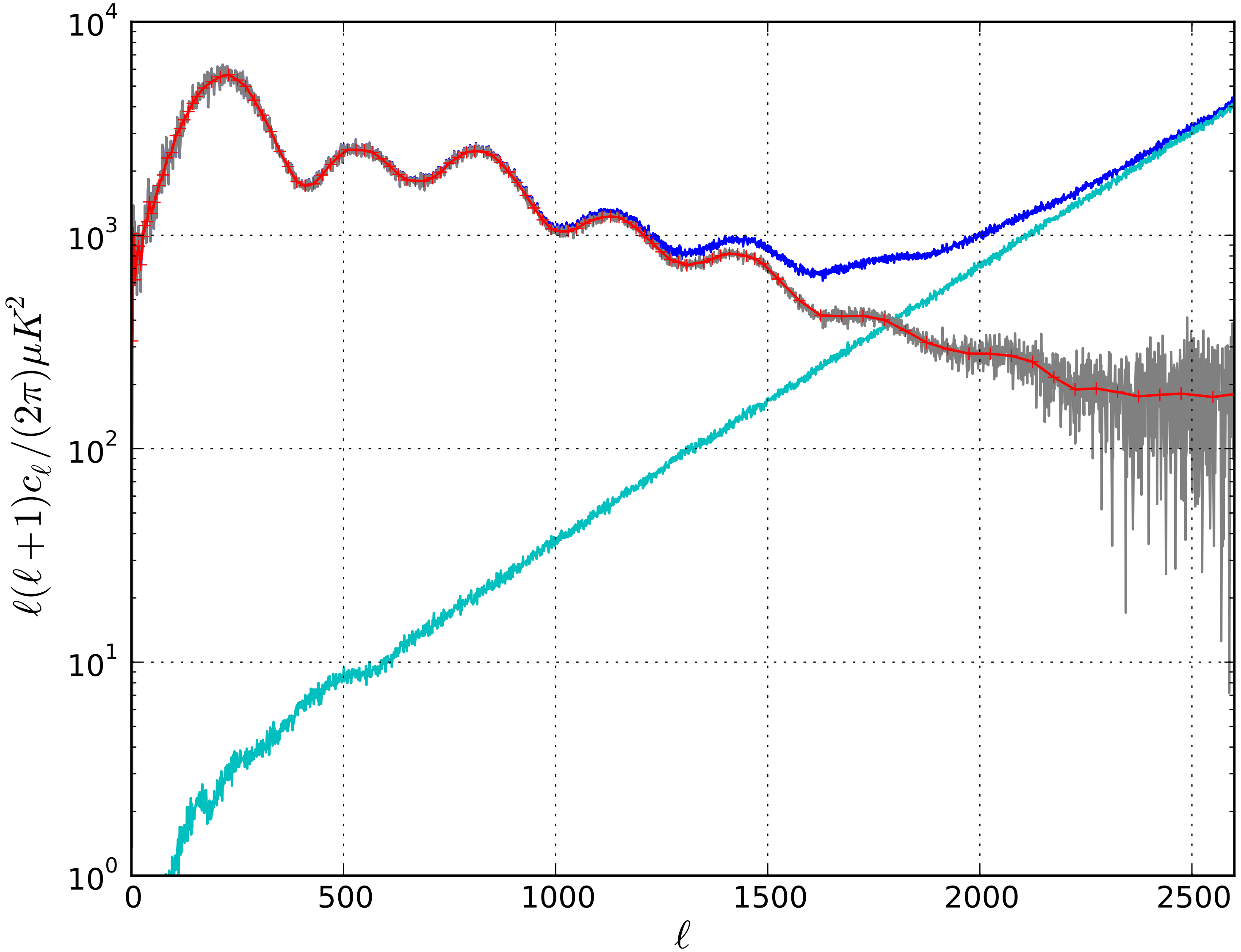
$$C_\ell^D = A_D \left(\frac{\ell}{\ell_D}\right)^{\alpha_D} \left(\frac{\nu}{\nu_D}\right)^{2\beta_D - 4} \left(\frac{B_\nu(T_D)}{B_{\nu_D}(T_D)}\right)^2$$

$\nu_D = 353\,\mathrm{GHz}$ $\ell_D = 100$ $\alpha_D = -2.4 \pm 0.02$ $\beta_D = 1.51 \pm 0.01$

Noise Spectrum

$$N_\ell^{\mathcal{AB}} = \sigma^{\mathcal{A}} \sigma^{\mathcal{B}} \delta_{\mathcal{AB}} \exp\left(\ell\left(\ell+1\right)\frac{\theta^2_{\mathrm{FWHM}}}{8\ln 2}\right)$$

Recall, $\theta = 180^\circ/\ell$



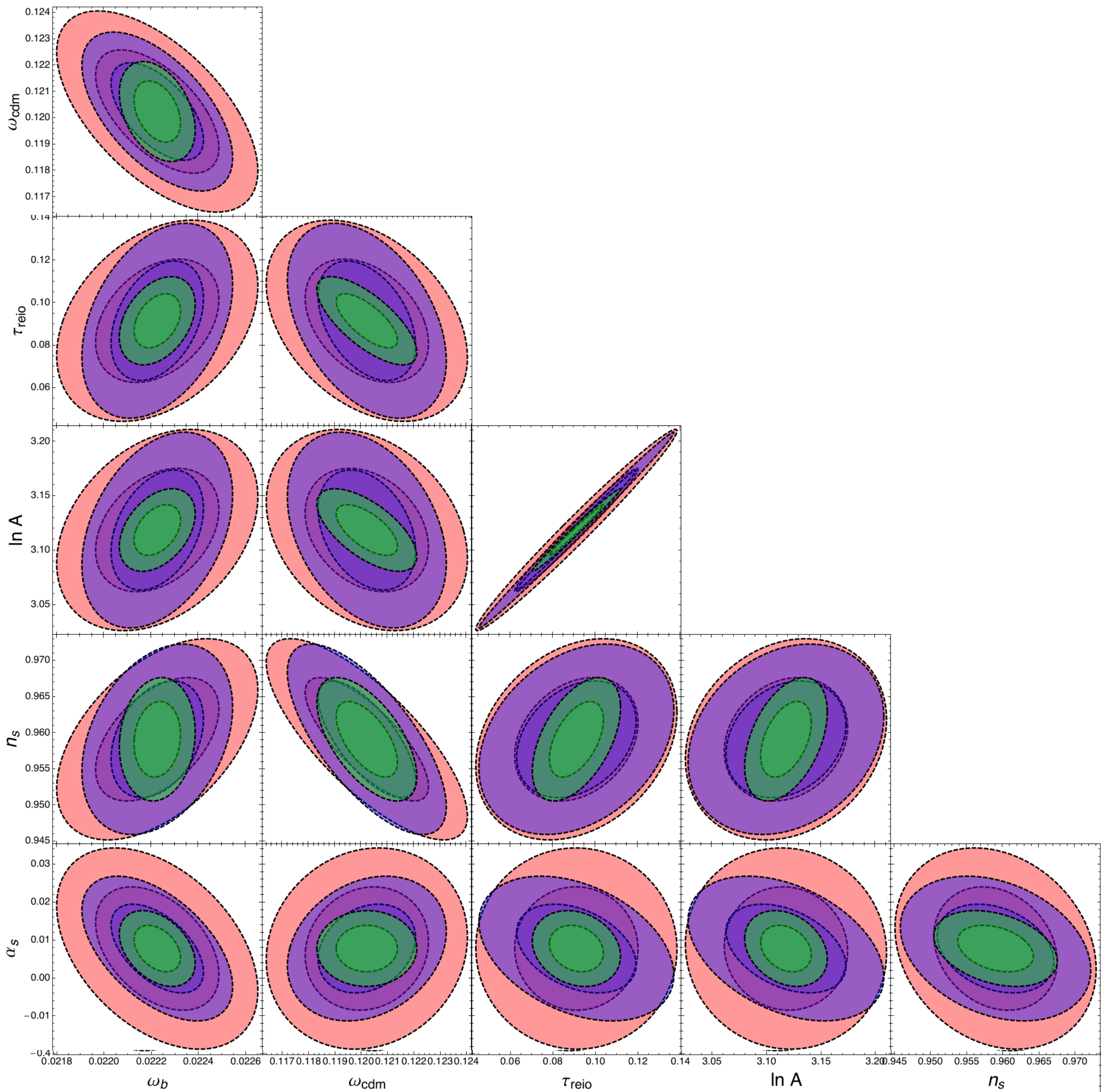
Forecast: Fisher matrix formalism

$$F_{ij} \equiv -\frac{\partial^2 \mathcal{L}}{\partial\theta_i \partial\theta_j}\Big|_{\theta=\theta_{fid}}$$

Cramér-Rao $\sigma_{\theta_i} \geq F_{ii}^{-1}$

$$C_{ij} = F_{ij}^{-1}$$

Consistency check



Implications of Inflation

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$$k_* = 0.05\,\mathrm{Mpc}^{-1}$$

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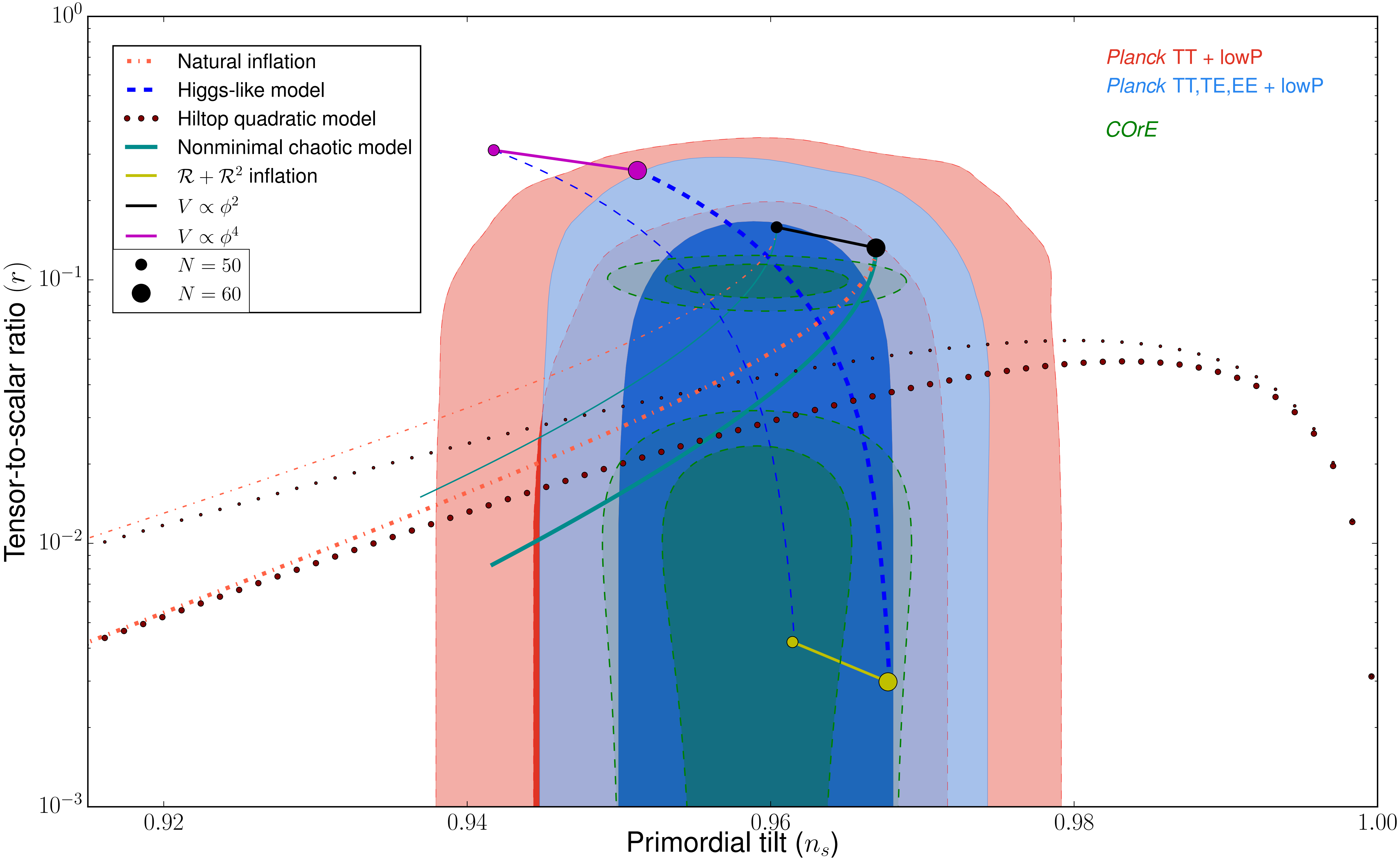
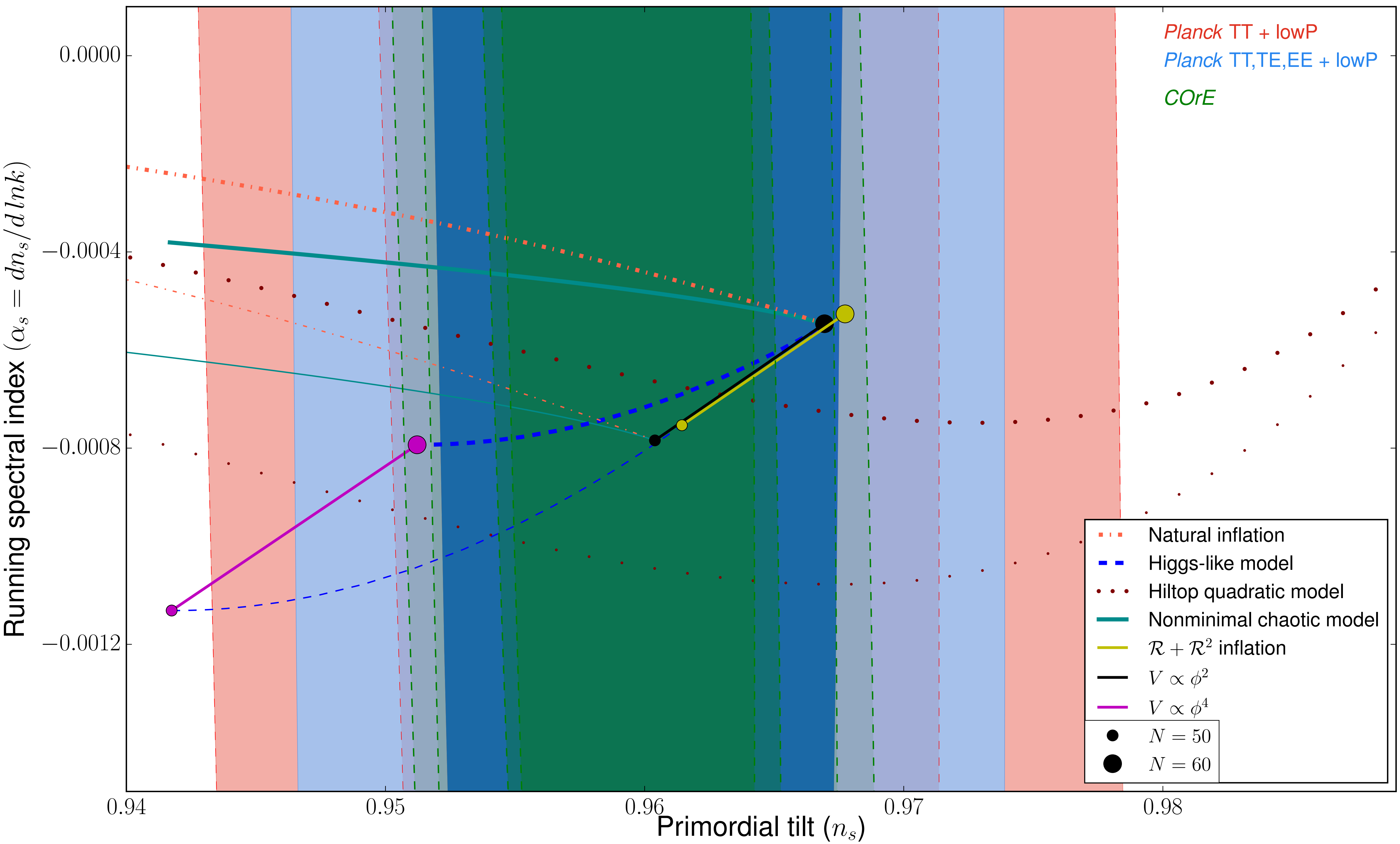
Results (r, n_s)



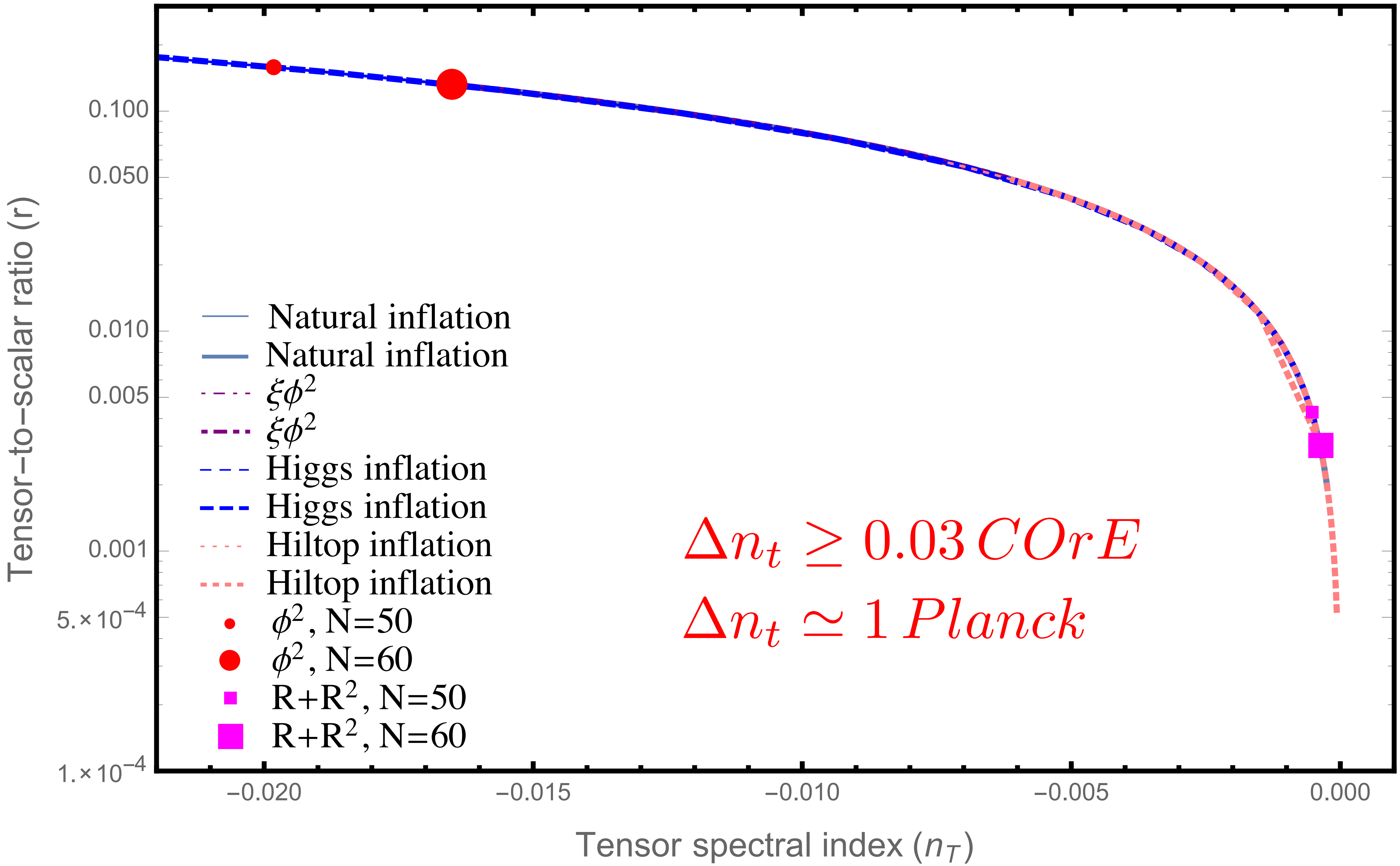
Escudero, Ramirez, Boubekeur, Guisarma, Mena '15

Results (α_s, n_s)



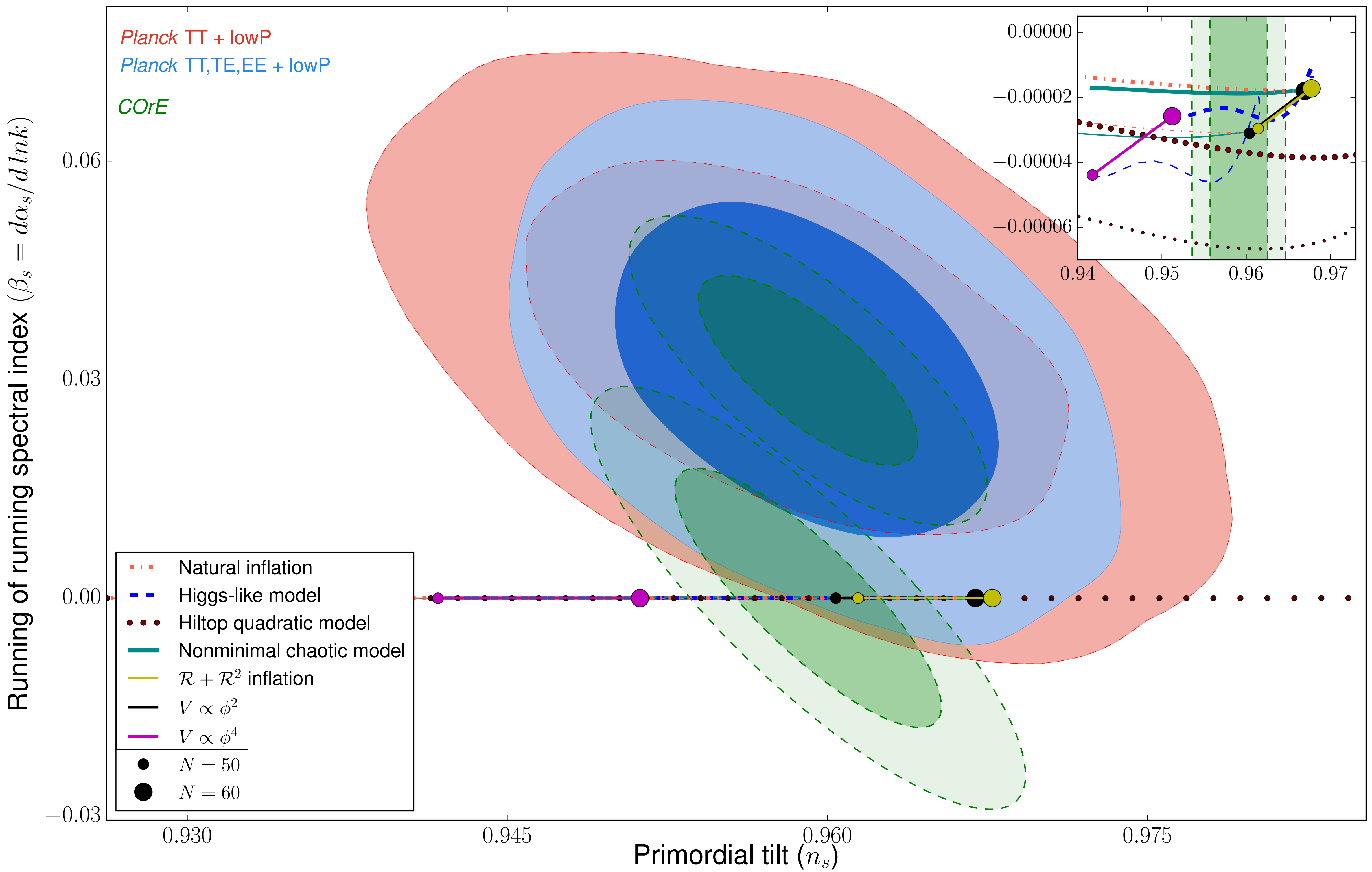
Escudero, Ramirez, Boubekeur, Guisarma, Mena ‘15

Results (r, n_t)



Escudero, Ramirez, Boubekeur, Guisarma, Mena ‘15

Results (β_s, n_s)



Escudero, Ramirez, Boubekeur, Guisarma, Mena '15

Conclusions

1) CMB anisotropies:

Powerful tool

Observational limitations

2) Results:

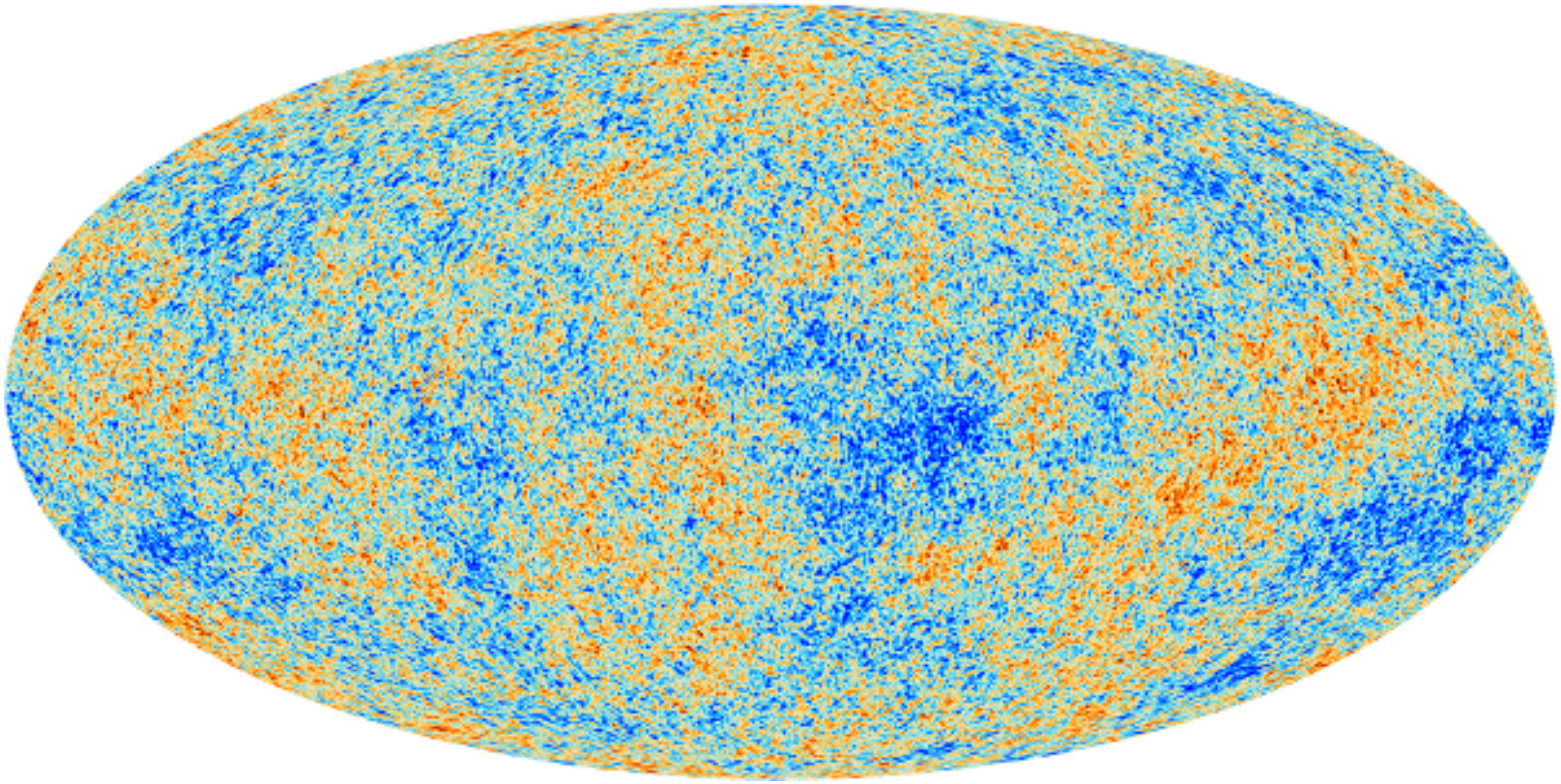
n_s r β_s n_t α_s

3) Prospects:

a) Study the anomaly: Dark Energy, non-standard scenarios?

b) Large Photometric Survey? Basse et al. ‘15

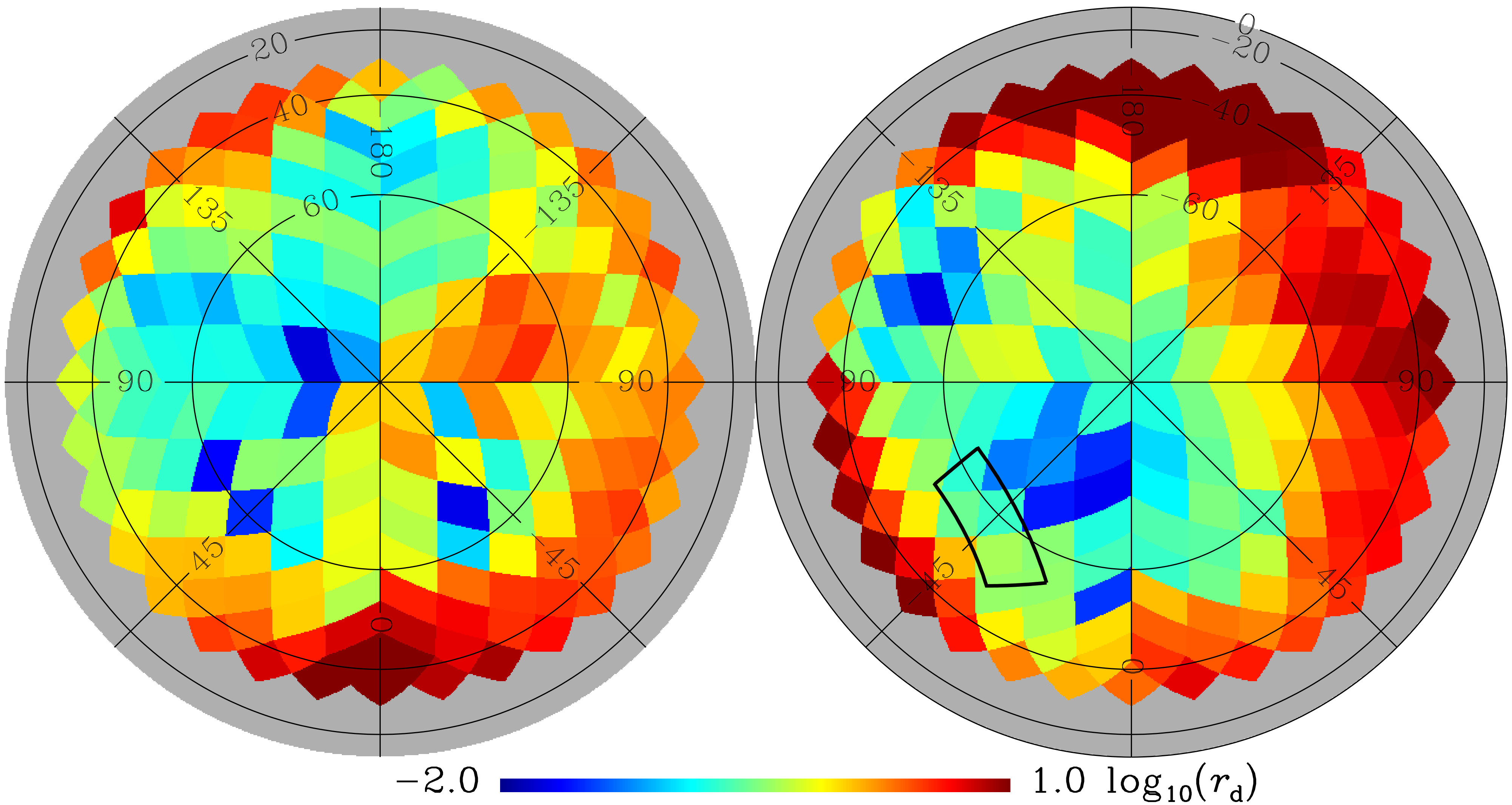
Thanks for your attention!



CMB Polarization: Kosowski; Annals Phys. 246 (1996) 49-85; arXiv:9501045
CMB Likelihood: Jaffe et al. AIP Conf. Proc. 476, 249 (1999); arXiv:0306506
CMB Forecast: Verde et al. JCAP 0601, 019 (2006); arXiv:0506036
Galaxy Survey: Basse et al. JCAP 1506 06, 042 (2015); arXiv:0506036
Future Mission: COrE Satellite; arXiv:1102.2181
Foregrounds: Planck intermediate results. XXX; arXiv:1409.5738
Inflation: Planck 2015 XX "Constraints on Inflation"; arXiv:1502.02114

BICEP2: Dust Confusion

$r = 1$



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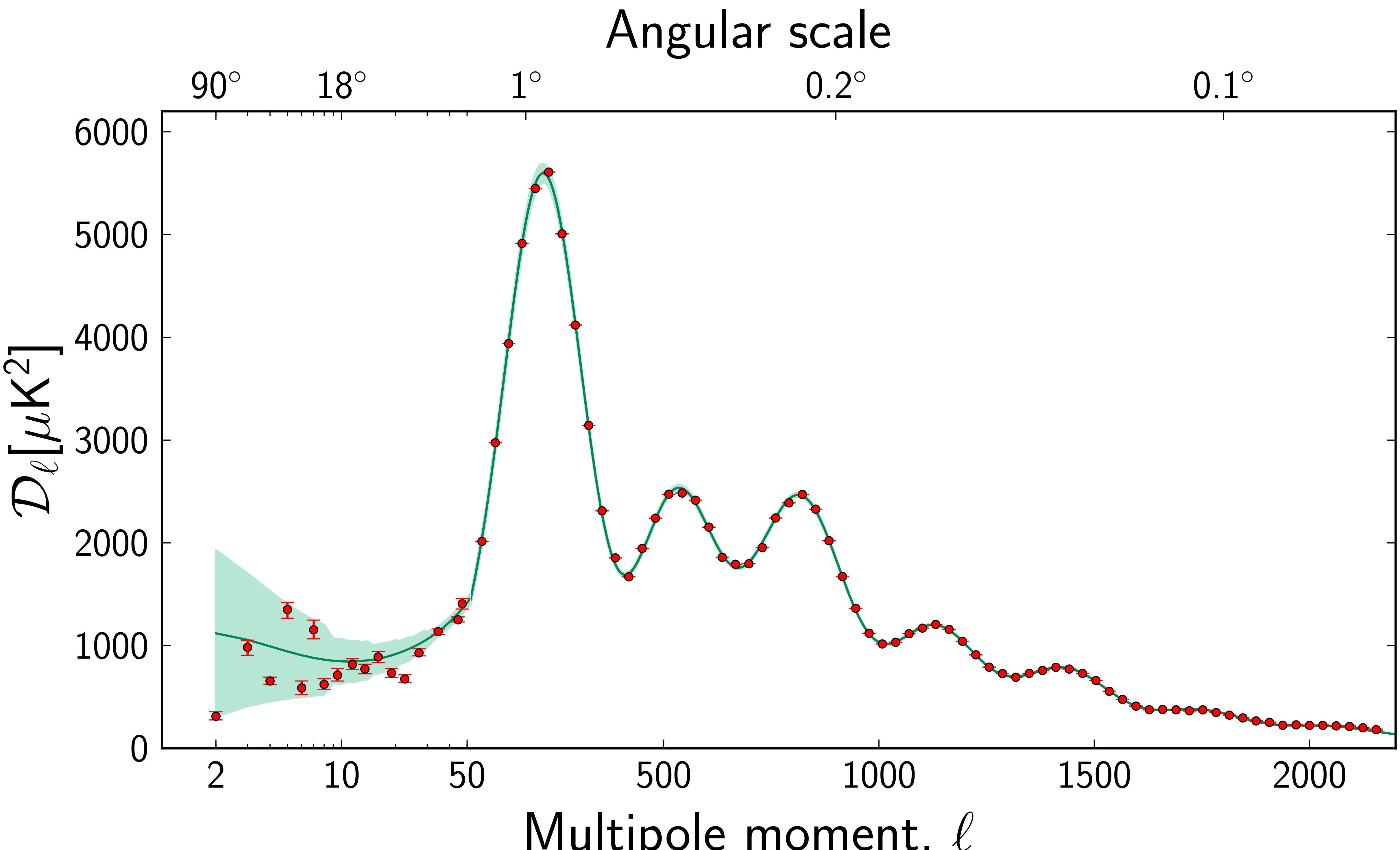
Planck Data



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