Gaussian interferometric power as a measure of continuous variable Non-Markovianity

Leonardo A.M. Souza[1,2], Himadri Shekhar Dhar[3], Manabendra Nath Bera[4,3], Pietro Liuzzo-Scorpo[2], Gerardo Adesso[2]

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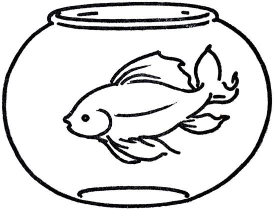

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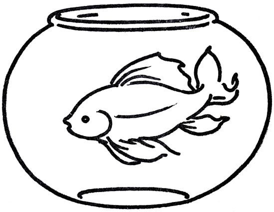

Memoryless dynamics!

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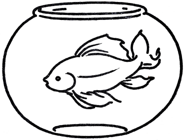
$$p(x_n, t_n \mid x_{n-1}, t_{n-1}; \ldots ; x_0, t_0) = p(x_n, t_n \mid x_{n-1}, t_{n-1}) \quad \forall\ t_n \in I\ .$$

No memory of the past values of X (x_n does not have a history).



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Integrating over x_2 on both sides and dividing by $p(x_1,t_1)$ we obtain **Chapman-Kolmogorov equation**

$$\boxed{K(x_3,t_3;x_1,t_1) = \int dx_2 K(x_3,t_3|x_2,t_2)K(x_2,t_2|x_1,t_1)}$$

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Φ_{t,t_0} describes a **Markovian dynamics** if

$$\boxed{\Phi_{t_2,t_0} = \tilde{\Phi}_{t_2,t_1}\Phi_{t_1,t_0}} \quad \forall t_2 > t_1 > t_0$$

where $\tilde{\Phi}_{t_2,t_1}$ is CPTP, i.e. the intermediate map is a legit quantum evolution.

Φ_t describes a Markovian dynamics if it is **CP-divisible**.

So, to summarize:

Whenever a dynamical quantum map Φ_t cannot be written as a composition of legit (CPTP) quantum maps we have a **Non-Markovian** dynamics!

Essential references:

[1] Angel Rivas Vargas, *Open Quantum Systems and Quantum Information Dynamics*, *PhD thesis* (2011)
[2] A. Rivas, S. F. Huelga, M. B. Plenio, *PRL* 105, 050403 (2010)
[3] A. Rivas, S. F. Huelga, M. B. Plenio, *Rep. Prog. Phys.* 77, 094001 (2014)
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What are gaussian states?

A gaussian state ρ describing n modes (with annihilation operators $\{\hat{a}_k\}_{k=1\dots n}$) can be defined by the quadrature vector $\hat{\mathbf{O}} = \{\hat{q}_1, \hat{p}_1, \dots, \hat{q}_n, \hat{p}_n\}$, where

$$\hat{q}_k = \frac{\hat{a}_k + \hat{a}_k^\dagger}{\sqrt{2}} \quad \text{and} \quad \hat{p}_k = \frac{\hat{a}_k - \hat{a}_k^\dagger}{\sqrt{2}i}$$

The quadratures obey the canonical commutation relations $[\hat{O}_i, \hat{O}_j] = i\Omega_{ij}$, where Ω_{ij} is the element of the symplectic form

$$\mathbf{\Omega} = \bigoplus_1^n \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} .$$

It is completely characterized by its first and second statistical moments of the quadrature vector:

$$D_j = \mathrm{Tr}(\rho \hat{O}_j)$$
$$\sigma_{ij} = \mathrm{Tr}\left(\rho\{(\hat{O}_i - D_i), (\hat{O}_j - D_j)\}_+\right)$$

The positivity condition, $\rho \geq 0$ is translated by the *bona fide* condition

$$\sigma + i\Omega \geq 0 .$$

Evolution

Unitary operation on the state ρ corresponds to real symplectic $\mathbf{S}$ transformation on the first and second moments:

$$\rho' = \hat{U}\rho\hat{U}^{\dagger} \quad \rightarrow \begin{cases} \mathbf{D} = \mathbf{S}\mathbf{D} \\ \boldsymbol{\sigma}' = \mathbf{S}\boldsymbol{\sigma}\mathbf{S}^{T} \end{cases} \quad .$$

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In general, a dynamical map Φ_t acting on ρ corresponds to two $2n \times 2n$ matrices $(X(t), Y(t))$ acting on $\boldsymbol{\sigma}$ as

$$\boldsymbol{\sigma}(0) \ \rightarrow \boldsymbol{\sigma}(t) = X(t)\boldsymbol{\sigma}X(t)^T + Y(t)\ .$$

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$(X(t), Y(t))$ describe a legit quantum (gaussian) evolution if and only if

$$Y(t) + i\Omega - iX(t)\Omega X(t)^T \geq 0 \quad \forall \ t$$

Gaussian Interferometric Power

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- The information on the black-box generator is provided to the two parties only after the transformation allowing for optimal measurement to be performed on the output.
- The objective of the interferometric setup is to deduce the unknown phase ϕ with the maximum possible precision.

Assuming a large number κ of copies of the probing state ρ_{AB} are initially prepared we have a bound on the precision with which we can estimate the parameter ϕ given by the Cramer-Rao bound

$$\kappa \Delta\phi^2 \geq \frac{1}{\mathcal{F}(\phi_{AB}^{\phi})}$$

where $\mathcal{F}(\phi_{AB}^{\phi})$ is the Quantum Fisher Information defined as

$$\mathcal{F}(\phi_{AB}^{\phi}) = -2 \lim_{d\phi \to 0} \frac{\partial^2 F(\rho_{AB}^{\phi}, \rho_{AB}^{\phi+d\phi})}{\partial (d\phi)^2}$$

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The GIP of a two-mode Gaussian state is defined as

$$\mathcal{Q}_B^G(\rho_{AB}) = \frac{1}{4} \inf_{\hat{H}_B} \mathcal{F}(\rho_{AB}^{\phi})$$

quantifies the guaranteed precision allowed by a given probing state ρ_{AB} in the estimation of the parameter ϕ with incomplete prior knowledge of the local generator $\hat{H}_B$.

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About GIP:

[6] D. Girolami *et al.*, *PRL* 112, 210401 (2014)
[7] G. Adesso, *PRA* 90, 022321 (2014)
[8] M. N. Bera, *arXiv*:1406.5144

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- We know that an evolution Φ_t is Markovian if $\Phi_{t_2,t_0} = \tilde{\Phi}_{t_2,t_1}\Phi_{t_1,t_0}$ with the intermediate map $\tilde{\Phi}_{t_2,t_1}$ being CPTP.

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- We have a discord-type correlation quantity $\mathcal{Q}_B^G(\rho_{AB})$ monotonically non-increasing under local CPTP operations.

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Well, but then...

$$\mathcal{D}(t) = \frac{d}{dt}\mathcal{Q}_B^G(\rho_{AB}(t)) > 0 \quad \Rightarrow \text{Non-Markovian evolution!}$$

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We define a Non-Markovianity **witness**

$$\boxed{\mathcal{N}_Q^{\boldsymbol{\sigma}}(\Phi) = \int_{\mathcal{D}(t)>0} \mathcal{D}(t)dt}$$

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However, it is worth notice that the most remarkable aspect of characterizing Non-Markovianity through gaussian GIP is its ability to witness Non-Markovian dynamics of a local Gaussian channel by using two-mode probes which exhibit quantum correlations beyond entanglement.

Example: Damping channel
Let's consider the Gaussian channel characterized by the following equation:

$$\frac{d\rho}{dt} = \alpha \frac{\gamma(t)}{2} (2\hat{a}\rho\hat{a}^\dagger - \{\hat{a}^\dagger \hat{a}, \rho\}_+)$$

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The covariance matrix is mapped into

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where $x(t) = \alpha\int_0^t 2\gamma(s)ds$.

It can be easily shown that if $\gamma(t) \geq 0\ \forall t$, then the intermediate map $\tilde{\Phi}_{t_2,t_1}$ is completely positive and hence the dynamics is **markovian**.

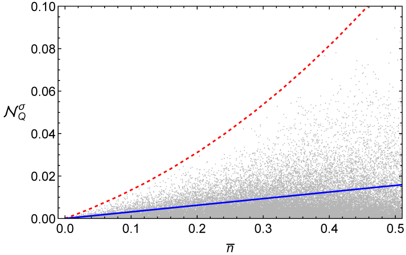
We choose for illustrative purpouses the following damping coefficient

$$\gamma(t) = \begin{cases} \frac{1}{2} e^{-t/10} \sin t & \text{if } t < 5\pi/2 \\ \frac{1}{2} e^{-\pi/4} & \text{if } t \geq 5\pi/2 \end{cases} .$$

so that $\gamma(t) < 0$ for $\pi < t < 2\pi$.

Hence

$$\mathcal{N}_Q^{\sigma}(\Phi) = \mathcal{Q}_B^G(\rho_{AB}(t = 2\pi)) - \mathcal{Q}_B^G(\rho_{AB}(t = \pi))$$



- We have defined the Non-Markovianity of a map Φ_t as the violation of the complete positivity of the intermediate map $\tilde{\Phi}_{t_2,t_1}$.
- We introduced a witness of gaussian Non-Markovianity based on revivals of a discord-type correlation quantity, of metrological relevance, namely Gaussian Interferometric Power.
- Remarkably, this witness allows to witness Non-Markovianity using non-entangled probes.

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