New AdS/CFT duals through non-Abelian T-duality

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Outline

Motivation

- AdS/CFT duality: AdS_d Super Gravity solutions are dual to strongly coupled Conformal Field Theories in $d-1$ dimensions.
- Examples of CFTs include $\mathcal{N}=4$ SYM and other interesting supersymmetric models.
- Finding new AdS solutions can help understand the properties of the dual CFTs.
- However, new solutions are not always at hand.
- Non-Abelian T-duality has proven to be a powerful solution-generating technique in the context of SuGra.

Non-Linear Sigma Model formalism of T-duality

- 2-dim field theory (NLSM) associated to a string propagating in a $D = 10$ target space:

$$S = \int d^2\sigma \, (G_{mn} + B_{mn}) \, \partial_+ X^m \partial_- X^n \,, \qquad m, n = 0, 1, \ldots, D-1$$

- Assume $U(1)$ isometry, rewrite in adapted coordinates $X^m = (\theta, X^\mu)$ where $\mu = 1, \ldots, D-1$ such that

$$S = \int d^2\sigma \, \{ G_{\theta\theta} \partial_+ \theta \partial_- \theta + (G_{\mu\theta} + B_{\mu\theta}) \partial_+ X^\mu \partial_- \theta$$
$$+ (G_{\mu\theta} - B_{\mu\theta}) \partial_+ \theta \partial_- X^\mu + (G_{\mu\nu} + B_{\mu\nu}) \partial_+ X^\mu \partial_- X^\nu \}$$

is invariant under $\theta \to \theta + \varepsilon$.

Finding the T-dual theory

- (Rocec & Verlinde, 91) Gauge isometry $\partial_\pm \to \partial_\pm + A_\pm$ and add Lagrange multiplier $\tilde{\theta}$ for flat connection: $+\int \tilde{\theta}(\partial_+ A_- - \partial_- A_+)$:

$$S = \int d^2\sigma \, \{ G_{\theta\theta} A_+ A_- + (G_{\mu\theta} + B_{\mu\theta}) \partial_+ X^\mu A_- \\ + (G_{\mu\theta} - B_{\mu\theta}) A_+ \partial_- X^\mu + (G_{\mu\nu} + B_{\mu\nu}) \partial_+ X^\mu \partial_- X^\nu \\ + \tilde{\theta}(\partial_+ A_- - \partial_- A_+) \Big\} \qquad (\theta = 0 \text{ gauge})$$

- Solving for $\tilde{\theta} \implies A = \partial\theta$ (if no holes), recover original theory.
- Solving for $A = A(\partial\tilde{\theta}, \partial X^\mu) \implies$ dual action ($\neq$ original action):

$$\hat{S} = \int d^2\sigma \, (\hat{G}_{mn} + \hat{B}_{mn}) \, \partial_+ \hat{X}^m \partial_- \hat{X}^n \,, \qquad \hat{X}^m = (\tilde{\theta}, X^\mu)$$

T-duality NS-NS sector tranformation rules

- New theory with different coefficients:

$$\hat{S} = \int d^2\sigma \, (\hat{G}_{mn} + \hat{B}_{mn}) \, \partial_+ \hat{X}^m \partial_- \hat{X}^n \, , \qquad \hat{X}^m = (\tilde{\theta}, X^\mu)$$

- Buscher rules: algebraical relations

$$\hat{G}_{mn}(G_{mn}, B_{mn}) \, , \qquad \hat{B}_{mn}(G_{mn}, B_{mn})$$

- Scale inversion for the dualised direction: $\hat{G}_{\theta\theta} = 1/G_{\theta\theta}$.
- Dilaton gets shifted from path integral considerations:

$$\Phi \longrightarrow \hat{\Phi} = \Phi - \frac{1}{2} \ln G_{\theta\theta}$$

Remark

This technique is not restricted to String Theory!

Non-trivial world-sheets in T-duality

- Upon solving for A, partial integration is performed on the Lagrange multiplier term
$$+\int \tilde{\theta}(\partial_+ A_- - \partial_- A_+)$$
- Integration over non-trivial world-sheet topologies yield holonomies
$$\oint_\gamma A \neq 0$$
- We can cancel out A holonomies choosing adequate periodicity for $\tilde{\theta}$ (multivalued!):
$$\oint_\gamma d\tilde{\theta} = 2\pi n\,, \qquad n \in \mathbb{Z}$$
- Works for any genus (any power of g_s in perturbation theory) $\Longrightarrow$

T-duality can be a symmetry of full (perturbative) string theory!

Results and properties of T-duality

- Also transformation rules for R-R fields.
- (Bergshoeff, Hull, Ortín'95) Reduce to a unique $\mathcal{N} = 2$, $d = 9$ SUGRA.
- Toroidal compactification of T-duals on circles of radii R and R':

string on circle of radius R, momentum n	$\longleftrightarrow$	string on circle of radius $R' = 1/R$, winding mode n

- Chirality is switched for the $U(1)$ isometry group: IIA $\longleftrightarrow$ IIB
- SUSY preserved if $\partial_\theta \epsilon = 0$.
- Invertible transformation: two T-dualities provide original background.

Non-Abelian T-duaility (NATD)

- Same idea as before, but take non-Abelian isometry group G for dualization.
- No general formulation: NS-NS and R-R sector transformation rules known on a case-by-case basis.
- May also change chirality of the theory:

$$\dim G = 2n+1 \quad \Longleftrightarrow \quad IIB \longleftrightarrow IIA$$

- We will focus on $G = SU(2)$ ($\dim G = 3$) $\quad \Longrightarrow \quad IIB \longleftrightarrow IIA$

Differences w.r.t. the Abelian case

- Non-invertible transformation: two NATDs don't yield the original background.
- Non-commutativity ruins the argument for compact dual backgrounds:

$$\int \tilde{\theta} \left(\partial_+ A_- - \partial_- A_+ - [A_+, A_-] \right)$$

- Consequence: not shown to be full symmetry of string (genus) perturbation theory, just tree-level symmetry.
- No reason for a compact manifold after NATD (problem for dual CFT interpretation).

NATD as a solution-generating symmetry

- D-brane origin for NATD backgrounds only speculative!
- NATD-generated CFTs might not survive α' or $1/N$ corrections $\implies$ only SuGra or $N \to \infty$ solutions.
- But they can be different to the CFTs dual to the original backgrounds!

$\mathcal{N} = 2$ IIA NATD-T dual of ABJM

- ABJM: $\mathcal{N} = 6$ IIA $AdS_4 \times \mathbb{CP}^3$ with magnetic F_2 (flavour $D6$-branes) and electric F_4 (colour $D2$-branes).
- Take $G = SU(2)$ NATD $\rightarrow$ IIB.
- We want to uplift to M-theory: additional T-duality $\rightarrow$ IIA AdS_4 with both F_2 and F_4 with electric and magnetic components.
- Finding D-brane configurations possibly originating this background require some considerations: determine which branes are BPS, compute their number,. . .

Dealing with the non-compact direction

- NATD Lagrange multipliers living on $SU(2)$ Lie algebra:

$$\text{compact } S^3 \longrightarrow \text{non-compact } \mathbb{R}^3 \approx \mathbb{R}^+ \times S^2 \equiv \{\rho\} \times S^2$$

- Holonomies around non-contractible loops must satisfy:

$$\frac{1}{4\pi^2}\left|\int_{S^2} B_2\right| \in [0,1] \qquad \Longrightarrow \begin{cases} B_2 & \to B_2 + n\, Vol(S^2) \\ \rho & \in [n\pi, (n+1)\pi[\end{cases}$$

  where we consider $B_2 \sim -\rho\, Vol(S^2)$ in a near-singularity approx.
- We need to define large gauge transformations (LGT)!

Possible brane configurations

- How does LGT affect dual CFT interpretation?
- Rank of CFT gauge groups $\longleftrightarrow$ number N_p of colour Dp-branes.
- We find BPS $N_6 \neq 0$, $N_4 = nN_6$ and also $N_{NS5} \sim \int H_3 = 1$ for each $\rho \in [n\pi, (n+1)\pi)$ interval.
- Colour D6 for $\rho \in [0, \pi)$ but (D4,D6) bound state - NS5 intersection for $n \neq 0$, interpreted as D4-branes being created inside the D6 each time a NS5 is crossed.

M-theory solution

- After uplift to M-theory, G_4 is purely magnetic (no M2 branes possible).
- It's the 2nd explicit solution found for the general classification of (Gauntlett et al.'06).
- $\mathcal{F} \sim N^{3/2}$ similar to ABJM, as expected from NATD-T.
- However, M2's always give rise to $N^{3/2}$, with or without M5's.
- Wrapped M5's yield $\mathcal{F} \sim N^3$ in known examples.
- Two possible ways out of this apparent contradiction:
  1. Still no proof that wrapped M5's necessarily yield $\mathcal{F} \sim N^3$.
  2. (Gauntlett et al.'06) classification only requires solutions to satisfy necessary conditions to be arising from M5 wrapped on 3-cycles.

1. Motivation
2. Abelian T-duality
3. NATD
4. AdS/CFT examples
   - $\mathcal{N}=2$ AdS_4
   - $\mathcal{N}=(0,4)$ $AdS_3\times S^2$
5. Conclusions

Classifying $\mathcal{N}=(0,4)$ $AdS_3 \times S^2$ solutions in M-theory

- (Kim, Kim, Kim '07) gave an AdS_3 analogue of the Toda eq. for AdS_5 (Lin, Lunin, Maldacena '04).
- Classification for 1/4-BPS ($\mathcal{N}=(0,4)$) 2D SCFTs with $SU(2)$-structure for the internal manifold.
- Simplifying Ansatz for M-theory flux:

$$G_4 = \mathcal{H} \wedge \mathrm{vol}(S^2)$$

- We intend to find example of the classification via NATD + uplift.
- Start from half-BPS IIB solution $AdS_3\times S^3\times S^3\times S^1$.
- NATD IIA solution $AdS_3\times S^3\times S^2\times\mathbb{R}\times S^1$ is 1/4-BPS and has $F_0=m\neq 0$.
- Lucky trick: we can apply T-duality in two different $U(1)$ isometry directions without breaking SUSY and find a $m=0$ IIA solution.
- Uplift to M-theory:

$$1/4\text{-BPS } AdS_3\times S^2 \longleftrightarrow \mathcal{N}=(0,4)\ \text{2D SCFT}$$

$$G_4=\mathcal{A}_1\wedge\text{vol}(AdS_4)+\mathcal{H}\wedge\text{vol}(S^2)+\mathcal{G}$$

Summary and open problems

- NATD is a powerful tool for generating new AdS/CFT solutions which are hardly reachable by other means.
- As a con, CFTs found by this method are not guaranteed to exist beyond the large N limit.
- General issue: CFT interpretation of the non-compact NATD direction.
- Specific results and questions for the M-theory solutions:
  1. $\mathcal{N}=2$ AdS_4: understand its origin, possibly doesn't stem from wrapped M5-branes.
  2. $\mathcal{N}=(0,4)$ $AdS_3 \times S^2$: wider classification supporting a general G_4 flux (work in progress).

Thanks!