Generalized geometric vacua with eight supercharges

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Outline

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Motivation

- **Einstein gravity:**

  $\longrightarrow$ Gravity

  $\longrightarrow$ Symmetries: Diffeomorphisms $GL(4, \mathbb{R})$

  $\longrightarrow$ suitable formalism: (Riemannian) Geometry

- **Supergravity:**

  $\longrightarrow$ Gravity + p-forms + Fermions

  $\longrightarrow$ Symmetries: T-duality, U-duality

  $\longrightarrow$ suitable formalism: Generalized Geometry

$\Longrightarrow$ Describe supergravity in a purely geometric way!

GR review

Pure Einstein gravity:

- Objects: particles
- Charges: momenta described by vectors ξ

  $\Longrightarrow$ Space of charges = Tangent bundle $T \ni \xi$
- Symmetries: diffeomorphisms $\delta_\xi g = \mathcal{L}_\xi g$
  - Natural group action: $GL(d)$
  - Lie derivative $(\mathcal{L}_u w)^m = u^n \partial_n w^m - (\partial_n u^m) w^n$

The "arena" of Generalized Geometry

Supergravity $\longrightarrow$ gravity (g_{mn}) + p-forms ($A_{m_1 \dots m_p}$)

- Objects: Extended (Strings + D_{p-1}-branes)

- Charges: momenta described by vectors ξ
  + electric charges described by (p-1)-forms λ
  + magnetic charges described by the dual forms $\tilde{\lambda}$

$$\Longrightarrow \text{Space of charges :} \quad E = \underbrace{T}_{\ni \xi} \oplus \underbrace{G}_{\ni \lambda\,,\,\tilde{\lambda}}, \quad V = \begin{pmatrix} \xi \\ \lambda \\ \tilde{\lambda} \end{pmatrix}$$

$\longrightarrow$ Enlargement of the tangent structure!

$\longrightarrow$ How to do geometry on E?

Generalized Geometry

- Symmetries: diffeomorphisms $\delta_\xi g = \mathcal{L}_\xi g,\ \delta_\xi A = \mathcal{L}_\xi A$
  + gauge tran/tions $\delta_\lambda g = 0, \quad \delta_\lambda A = d\lambda$
- $F = dA$ globally defined but A might change (recall Dirac string)

$$A' - A = d\lambda \Longrightarrow \quad e^A \cdot V \quad \text{covariant gen. vectors}$$

- Generalization of Lie derivative: Dorfman derivative

$$L_V \tilde{V} = (V \cdot D)\tilde{V} - (D \times V)|_{ad}\tilde{V}$$

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The setup

Supergravity compactified on a d-dimensional manifold:

$$\mathcal{M}_{10} = \mathcal{M}_{ext} \times \mathcal{M}_d$$

metric: $$ds^2_{10} = e^{2A(x)} ds^2_{ext}(y) + ds^2_d(x)$$

spinors: $$\epsilon_{10} = \zeta_{ext} \otimes \chi_d + c.c.$$

fluxes: $$F_{10} = F_d + \text{vol}_{ext} \wedge \tilde{F}_d$$

- $\mathcal{M}_{ext}$ maximally symmetric
- ζ_{ext} parametrizes preserved susy (focus on 8 supercharges)

Supersymmetry conditions

A supersymmetric background satisfies

$$\text{Gravitino:} \qquad \delta\Psi_M^{(10)} = \begin{pmatrix} \delta\Psi_\mu^{(10)} \\ \delta\Psi_m^{(10)} \end{pmatrix} = 0$$

$$\text{Dilatino:} \qquad \delta\lambda = 0$$

$\Longrightarrow$

$$[\nabla + (\text{Fluxes})_I]\,\chi = 0 \quad \text{Internal gravitino}$$

$$[m + (\text{Fluxes})_E]\,\chi = 0 \quad \text{External gravitino}$$

$$[(\text{Fluxes})_D]\,\chi = 0 \quad \text{Dilatino}$$

What do we generalize?

The Calabi-Yau story (d=6):

$$\text{Fluxes} = 0 \quad \Longrightarrow \quad \nabla_m \chi = 0$$

Build bilinears:

$$\text{Kähler structure:} \quad \omega_{(2)} = \chi^\dagger \gamma_{(2)} \chi$$

$$\text{Complex structure:} \quad \Omega_{(3)} = \chi^{c\dagger} \gamma_{(3)} \chi$$

susy background $\Longrightarrow$ $d\omega = d\Omega = 0$ (closure conditions)

$\Longrightarrow$ Calabi-Yau manifold $\longrightarrow$ well studied case

$\Longrightarrow$ Well-known effective theory $\longrightarrow D = 4, \mathcal{N} = 2$ (ungauged) sugra

Seek for generalization of the closure conditions when fluxes $\neq 0$

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Generalized Complex Geometry

Supergravity NSNS sector $\longrightarrow$ (g_{mn}, B_{mn}, ϕ)

- Generalization of tangent space* $T \to \underbrace{T}_{\text{momenta}} \oplus \underbrace{T^*}_{\text{windings}}$
- Natural group action: $O(d,d)$ (T-duality)
- Construct bispinors $\longrightarrow$ $\Phi_1 = \chi_1 \chi_2^\dagger, \quad \Phi_2 = \chi_1 \chi_2^{c\dagger}$

  and identify with forms: $\Phi_{(n)} = \sum_n \chi^\dagger \gamma_{(n)} \chi$
- Natural $O(d,d)$ spinor bilinear Mukai pairing:

$$\langle \Phi_1, \Phi_2 \rangle = \Phi_1 \wedge s(\Phi_2)|_{\text{top}}$$

- For even d, they define complex structures on $T \oplus T^*$.

*Hitchin 02, Gualtieri 04

CGC closure conditions

susy equations for χ $\Longrightarrow$

$$d_H \Phi = \text{obstructions}$$

$$d_H = d + H \wedge = e^{-B} d e^{B}$$

where

$$\text{obstructions} = \begin{cases} F_{RR} & \text{RR fluxes} \\ dA & \text{warping} \\ m & \text{AdS curvature} \end{cases}$$

$\Longrightarrow$ Really (to be)-closed object = "Dressed" spinor: $\Phi^D = e^B \Phi$

$\Longrightarrow$ Pure spinors inherit the patching on $T \oplus T^*$.

Link to effective sugra

Effective theory data written in terms of T-duality invariants (as in CY)

$\longrightarrow$ For purely NSNS backgrounds:

- Moduli kinetic terms given by Kähler potential

$$e^K = i\langle \Phi^D, \bar{\Phi}^D \rangle = i\langle \Phi, \bar{\Phi} \rangle$$

- Moduli potential given by Killing prepotentials

$$\mathcal{P} \sim \langle \Phi^D, d\Phi^D \rangle = \langle \Phi, d_H \Phi \rangle$$

$\longrightarrow$ Seek for U-duality invariants when RR fluxes are included

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Exceptional Generalized Geometry

Full Bosonic Sector (NSNS +RR) $\longrightarrow$ $(g_{mn}, B_{mn}, \phi, C_{m_1 \dots m_p})$

- Generalization of tangent space*:

$$T \to \underbrace{T}_{\text{momenta}} \oplus \underbrace{T^*}_{\text{windings}} \oplus \underbrace{\wedge^{\pm} T^*}_{\text{D-charges}} \oplus \underbrace{\wedge^5 T^*}_{\text{NS5-charge}} \oplus \underbrace{\wedge^5 T}_{\text{KKmon-charge}}$$

- Natural group action: E_{d+1} (U-duality)

- Differential structure

$\longrightarrow$Identifying $GL(d) \subset E_{d+1}$, we embed $D_M = \begin{cases} \partial_m & , M = m \\ 0 & , \text{otherwise} \end{cases}$

$\longrightarrow$Then construct Dorfman

$$L_V \tilde{V} = (V \cdot D)\tilde{V} - (D \times V)|_{ad} \tilde{V}$$

*Hull 07, Pacheco Waldram 08

Construction of the structures

$$(\chi_1, \chi_2) \quad \longrightarrow \quad (\theta_1, \theta_2) \quad \longrightarrow \quad (J_{a0}, K_0) \quad \longrightarrow \quad (J_a, K)$$

H-structure:

- $J_{a0} = (\sigma_a)^{ij} \theta_i \otimes \theta_j^*, \quad SU(2)_R$ triplet
- adjoint of group $E_{d+1} \quad \longrightarrow$ generalized tensor
- associated to hypermultiplet moduli

V-structure:

- $K_0 = \epsilon^{ij} \theta_i \otimes \theta_j, \quad SU(2)_R$ singlet
- fundamental of $E_{d+1} \quad \longrightarrow$ generalized vector
- associated to vector-multiplet moduli

$\longrightarrow J_a = e^A J_{a0},\ K = e^A K_0$ are the "dressed" structures (recall $\Phi^D = e^B \Phi$)

The (conjectured) closure equations

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Condition	Generalised Closure	=	AdS deformation
H-	$DJ_a + \epsilon_{abc}\mathrm{tr}(J_b DJ_c)$	=	$\lambda_a s(K)$
V-	$L_K K$	=	0
$(\mathrm{H} \cap \mathrm{V})_{\text{comp.}}$	$L_K J_a$	=	$\epsilon_{abc}\lambda_b J_c$

where:

- $s(K)$ is related to the group invariant
- $SU(2)$ vector λ_a : parametrizes cosmological constant

*Graña Louis Sim Waldram 09, Ashmore Waldram 15

Concrete application for d=5

d=5 $\longrightarrow$ U-duality group = $E_{6(6)} \supset \begin{cases} USp(8) \\ SL(6) \times SL(2) \end{cases}$

Spinor embedding:

$$(\chi_1, \chi_2) \quad \longrightarrow \quad \theta_1^\alpha = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}, \theta_2^\alpha = \begin{pmatrix} \chi_1^c \\ \chi_2^c \end{pmatrix}, \quad \text{fund. of } USp(8)$$

$$K_0^{\alpha\beta} = [\epsilon^{ij}\theta_i^\alpha\theta_j^\beta]_{\text{tr}=0}, \quad (J_{a0})^\alpha{}_\beta = (\sigma_a)^{ij}\theta_i^\alpha\theta_j^{*\beta}$$

Derivative and gauge fields embedding:

By identifying $GL(5) \subset SL(6) \times SL(2)$, e.g. $\partial_m = D_{m6}$, in the $(\bar{\mathbf{15}}, \mathbf{1})$

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Applications to AdS/CFT

General goal:
Describe marginal deformations of SCFT's using EGG

$\longrightarrow$ Success of GCG in this direction (e.g. describing β-deformations) *

Marginal deformations in $\mathcal{N} = 1$ SCFTs

Two classes:

- Kähler deformations $\sim \int d^4\theta\, V$ correspond to δK
- Superpotential deformations $\sim \int d^2\theta\, \mathcal{O}$ correspond to δJ_a

$\longrightarrow$ Field theory analysis by †
$\longrightarrow$ Recover field theory results by "solving" the exceptional integrability conditions (future work)

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*Minasian, Petrini, Zaffaroni 2006
†Green Komargodski Seiberg Tachikawa Wecht 10

Summary:

- Generalized Geometry is the geometric language to describe the supergravity degrees of freedom.
- Supersymmetric vacua can be described by the generalized "closure" of appropriate spinor bilinears.

Future directions:

- Study of moduli spaces for the effective theory.
- Applications to AdS/CFT (marginal couplings)

Thank you for your attention!