BLACK PARADOX RELOADED

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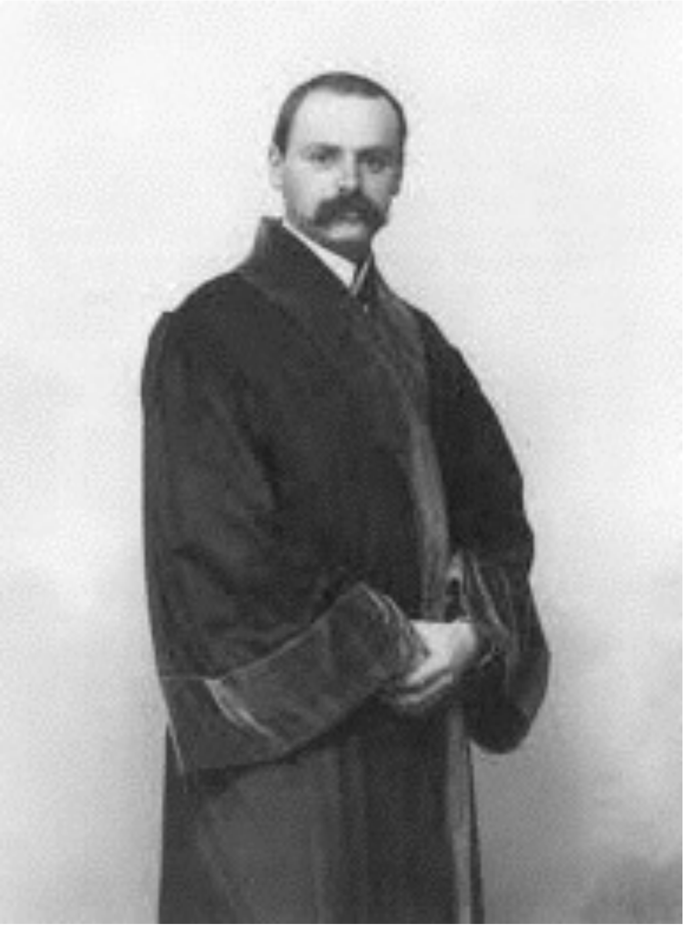
Black holes have contributed paradoxes to fundamental physics since the very beginning

$$ds^2 = -\left(1 - R/r\right)\, dt^2 + \frac{dr^2}{(1 - R/r)} + r^2\, d\Omega^2$$

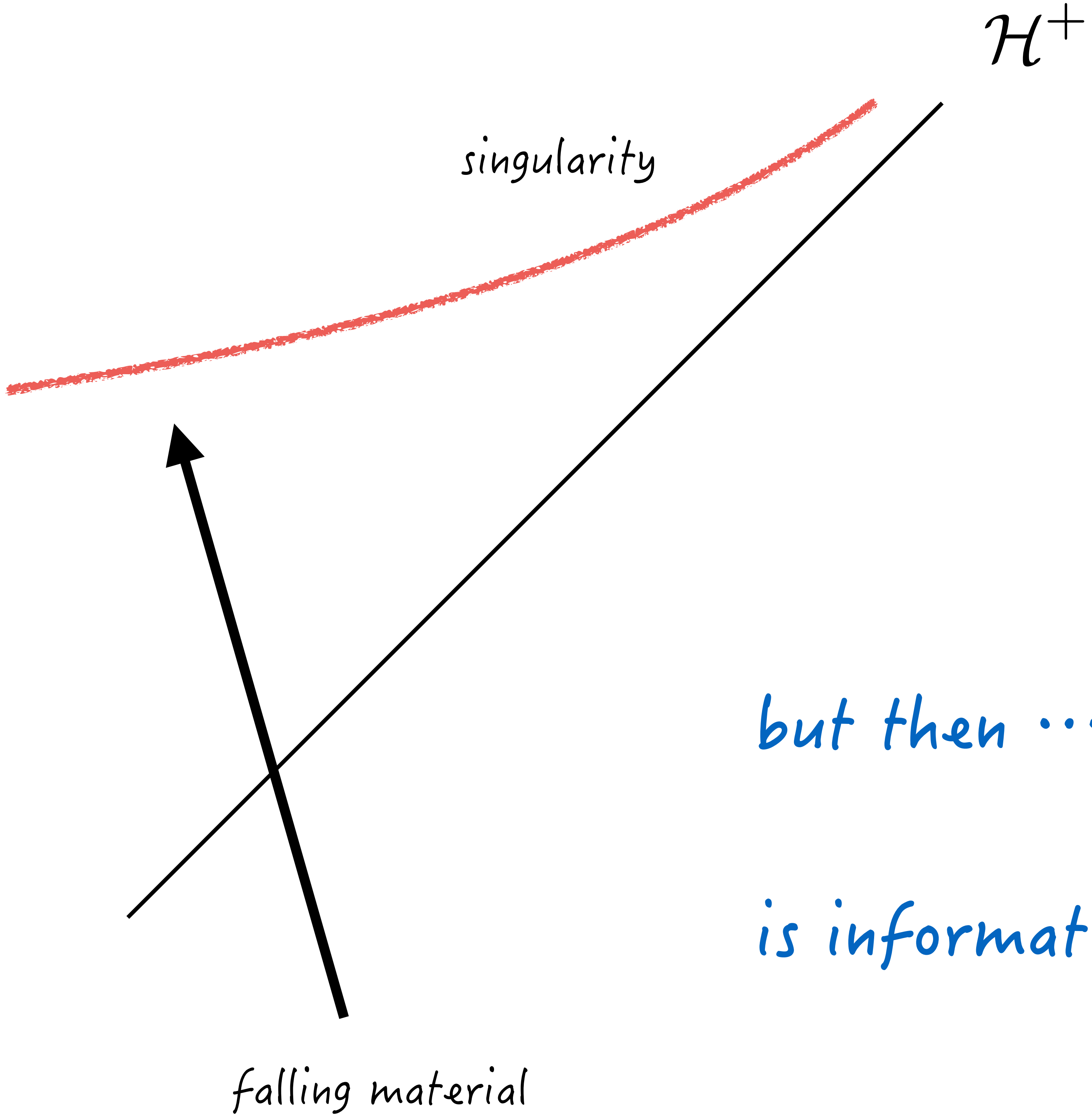
Schwarzschild



THE OLD PARADOX

Is the surface $r = R = 2GM$ singular?

NO! just an event horizon ···



Oppenheimer & Snyder

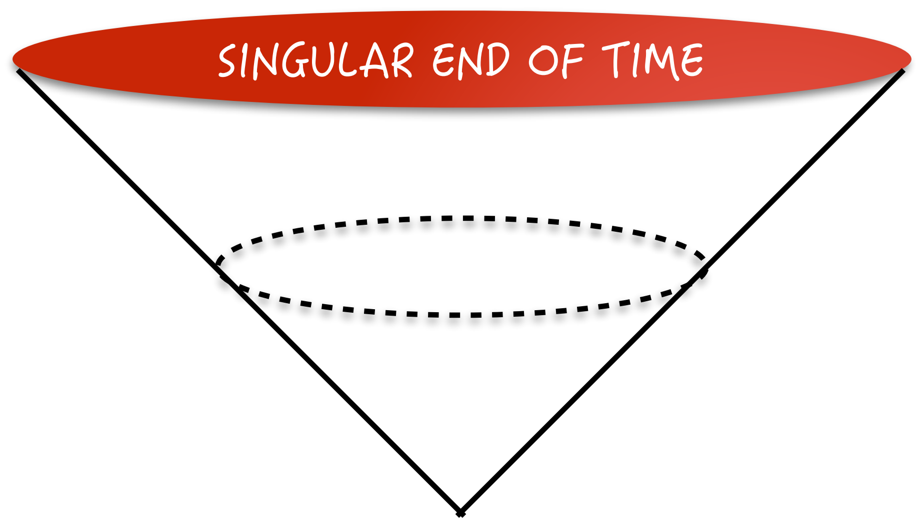
Kruskal

···

but then ···

is information lost forever?

The hole is the doomed region of space-time



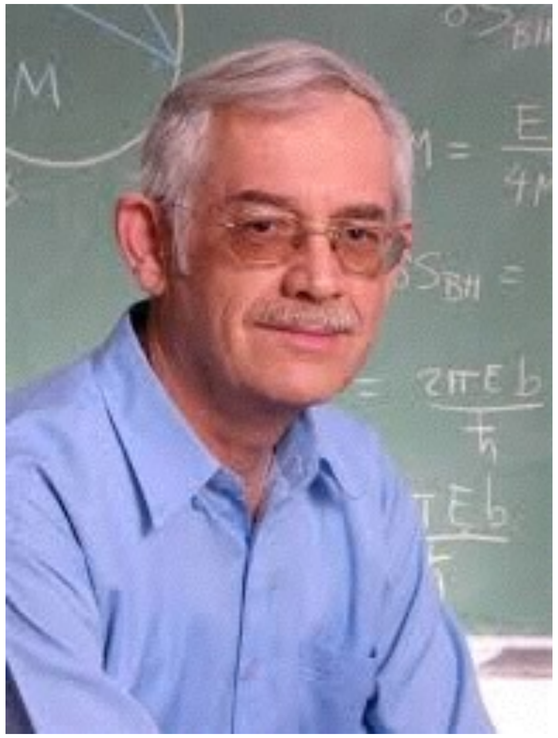
So, apparently the fate of the information depends on the fate of the singularity

Meanwhile …

Bekenstein found that the information CAPACITY should be FINITE

$$S \sim \frac{c^3 \mathrm{Area}_{\mathrm{horizon}}}{G\,\hbar}$$

Bekenstein



BEKENSTEIN'S ARGUMENT IN TWO LINES

Adding a two-level quantum system to a BH increases the dimensionality of its Hilbert space by a factor of 2

$$\Delta S_{\rm min} \sim \Delta \left(\log \dim \mathcal{H}_{\rm bh}\right)_{\rm min} \sim \log 2 \qquad\qquad \Delta M_{\rm min} \sim \omega_{\rm qbit} \sim \frac{1}{R}$$

$$\Rightarrow \quad \Delta A_{\rm min} \sim \Delta(R^2) \sim R\,\Delta R \sim R\,G\,\Delta M \sim G$$

$$\Rightarrow \quad \Delta S_{\rm min} \sim \frac{\Delta A_{\rm min}}{G}$$

$$S \sim \frac{A_H}{G}$$

So, the question was not why black holes should have an entropy. Rather, the question was why black holes would not have an INFINITE entropy

Bekenstein's argument gave an striking result: d.o.f. not extensive in 3-space!

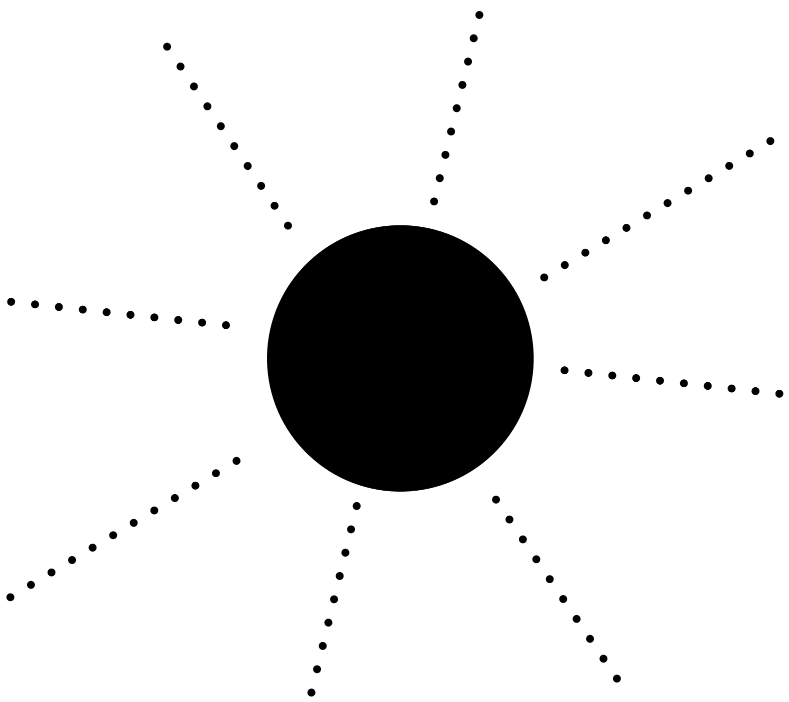
THE BEGINNING OF HOLOGRAPHY!

$$\log \dim \mathcal{H}_{\rm bh} \sim S \sim \frac{\rm Area}{\ell_P^2}$$

But $\cdots$ having an entropy, we must have a TEMPERATURE

$$T = \frac{\partial M}{\partial S} \sim \frac{1}{GM} \sim \frac{1}{R}$$

Famously found by Hawking

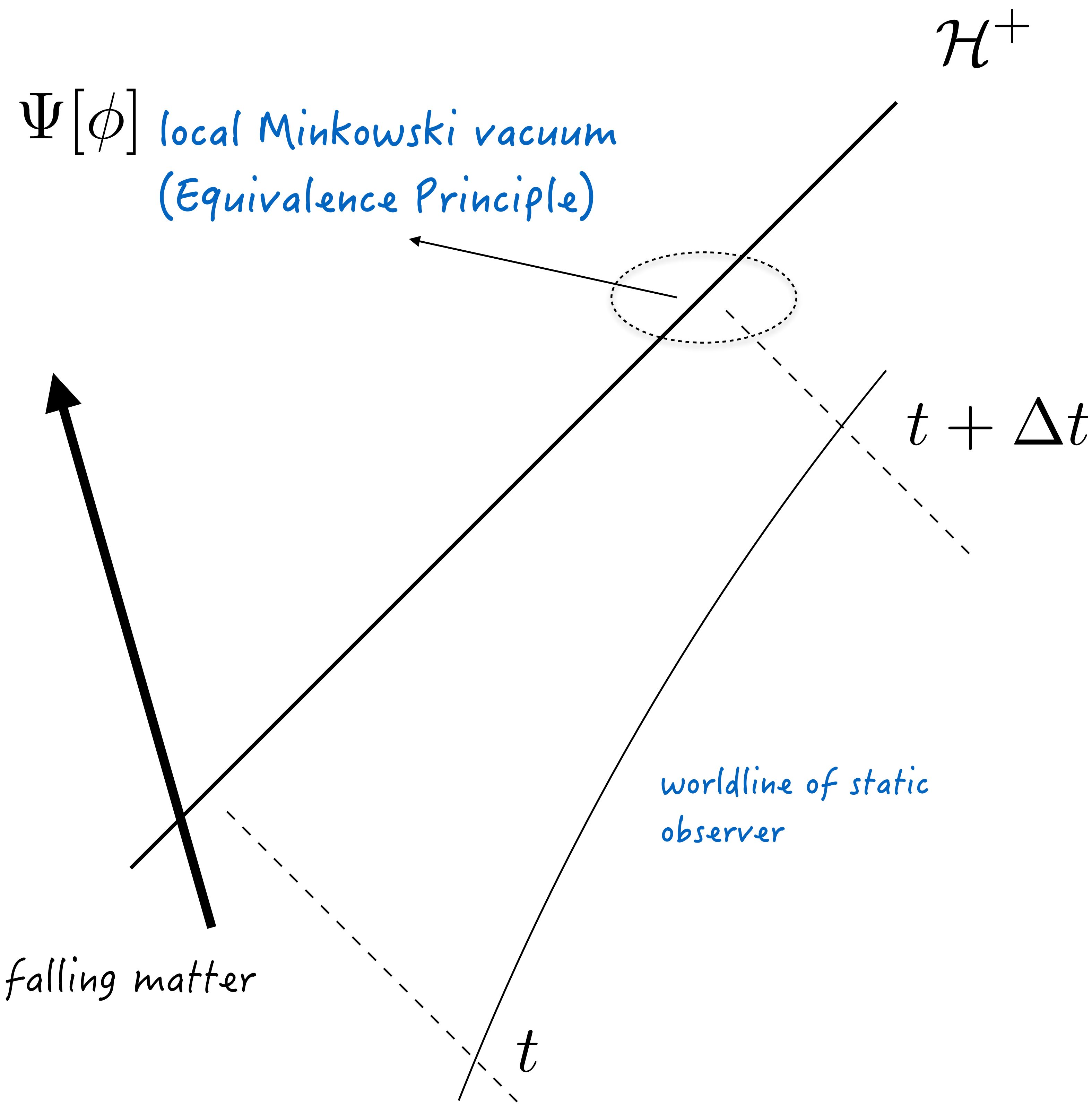


Hawking

$$T = \frac{1}{4\pi R} = \frac{1}{8\pi G M}$$

$$S = \frac{A_H}{4G}$$

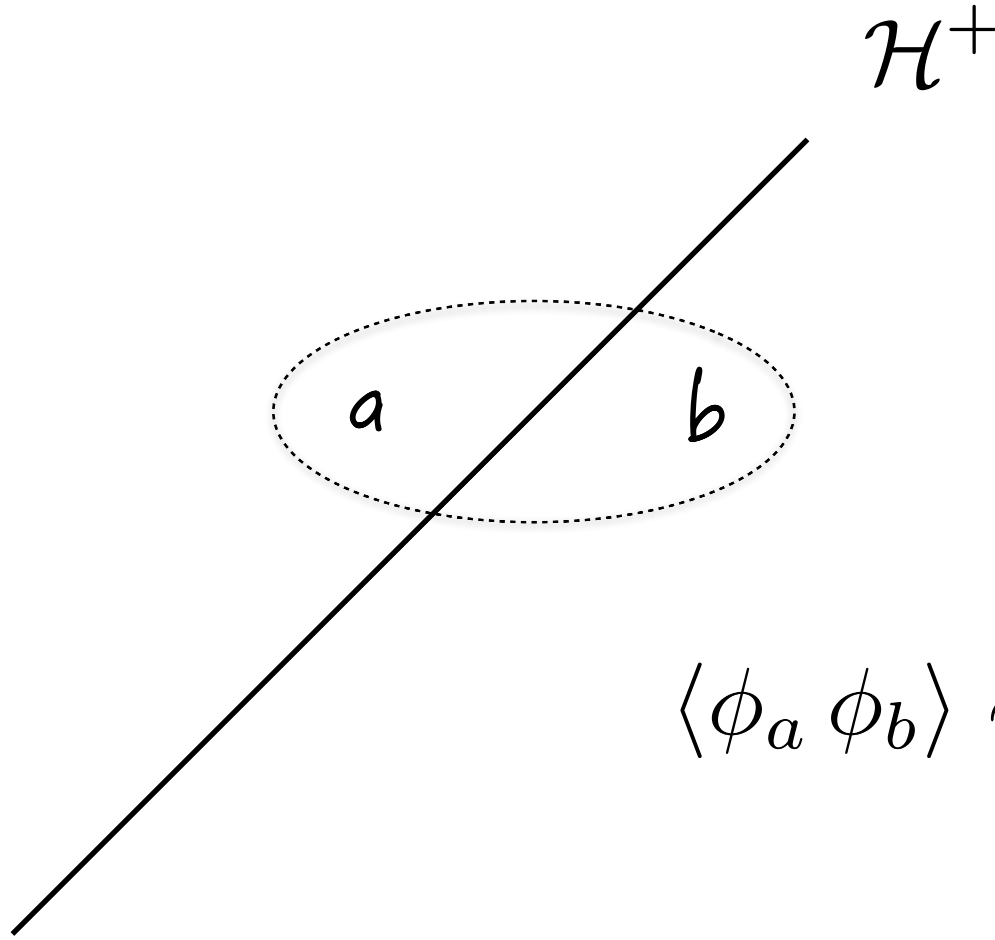
Hawking's calculation regards the radiation as a vacuum effect



effects of falling matter decay as

$$\exp(-\Delta t/R)$$

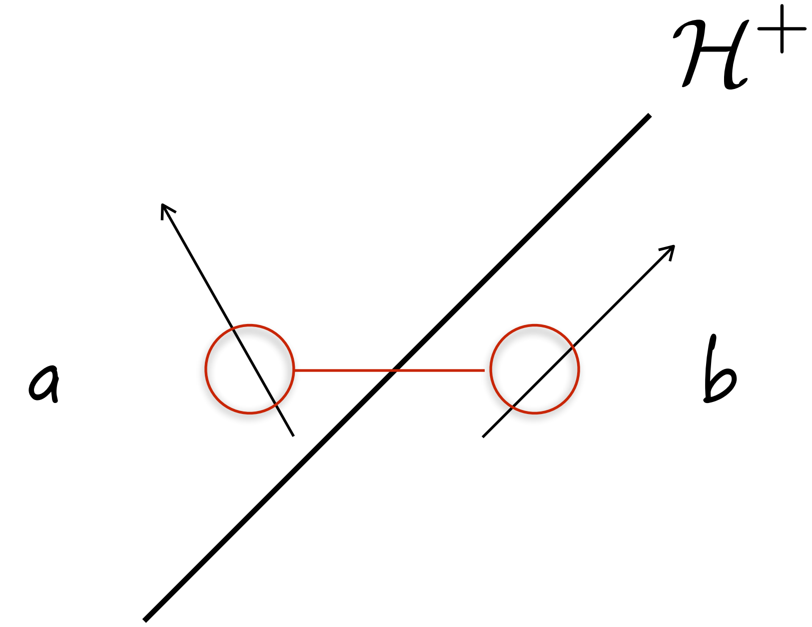
"no hair"



$$\langle \phi_a \, \phi_b \rangle \sim \frac{1}{|d_{ab}|^2}$$

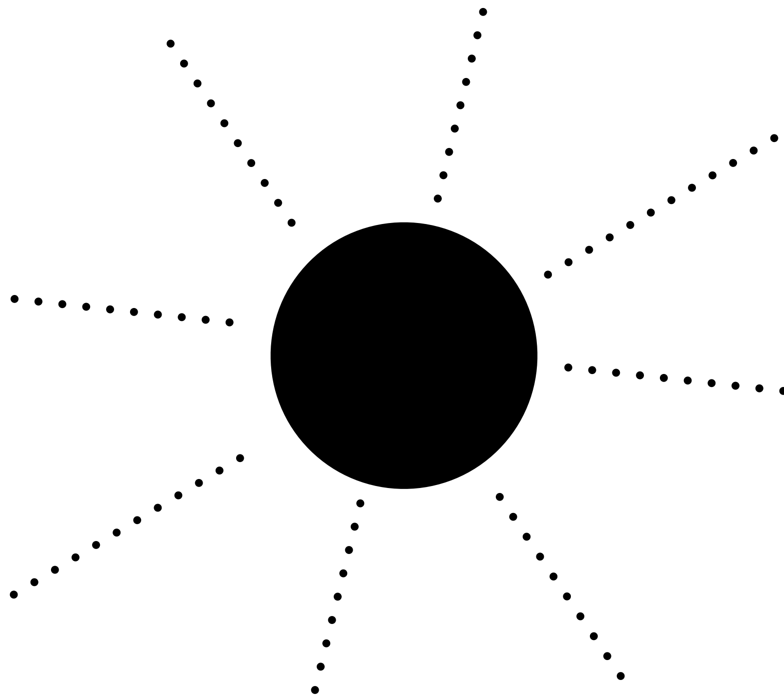
Equivalence Principle ⟺ smoothness of horizon

LARGE amount of ENTANGLEMENT between a and b



$$|a, b\rangle \propto \sum_{\omega} e^{-\omega/2T} \, |-\omega\rangle_a \otimes |\omega\rangle_b$$

$$\rho_b \propto \sum_{\omega} |\omega\rangle_b \, e^{-\omega/T} \, {}_b\langle\omega|$$



$$T \sim \frac{1}{R}$$

$$\omega_q \sim \frac{1}{R}$$

$$\lambda_q \sim R$$

$$\tau_q \sim R$$

$$\#_q \sim \frac{M}{\omega_q} \sim M\,R \sim S$$

$$\tau_{\mathrm{ev}} \sim \tau_q \cdot \#_q \sim R\,S$$

So, a black hole is a long-lived "resonance" with a very narrow width and a huge density of states

$$\Gamma \sim \frac{1}{\tau_{\rm ev}} \sim \frac{1}{RS} \ll M$$

$$\Omega(M) \propto e^S \sim \exp\left(4\pi^2 G M^2\right)$$

It is natural to treat the formation/evaporation as any other S-matrix process

Hawking's estimate is VERY accurate

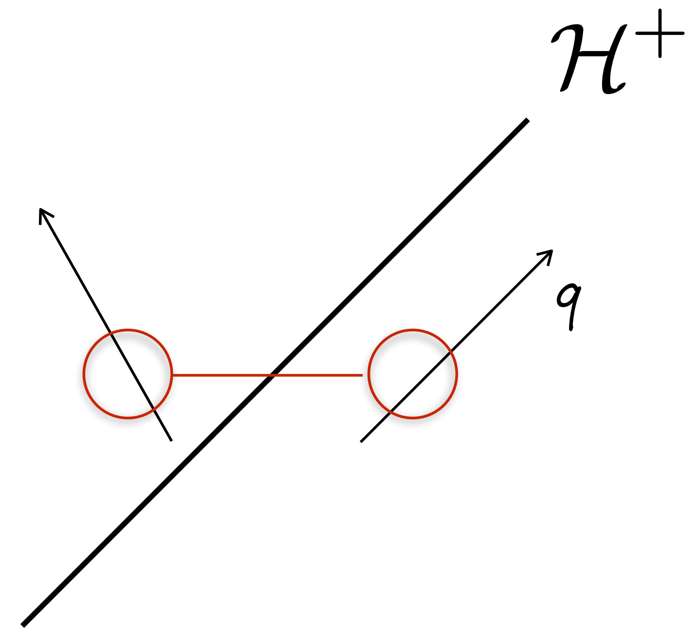
The effective expansion parameter for graviton loops is

$$\alpha_{\rm eff} = G\,\omega_q^2 \sim \frac{G}{R^2} \sim \frac{1}{S}$$

It is exact in the limit

$$G \to 0 \qquad M \to \infty \qquad R = 2GM = \text{fixed}$$

where $\alpha_{\rm eff} = \dfrac{1}{S} \to 0$



$$\rho_q \sim \sum_\omega |\omega\rangle\, e^{-\omega/T}\ \langle\omega| \quad +O(1/S) + O(e^{-Tt})$$

quantum corrections to the vacuum

"memory" from initial state

It is hard to keep track of information memory after the "scrambling time"

$$\tau_{\rm scram} \sim R \log S$$

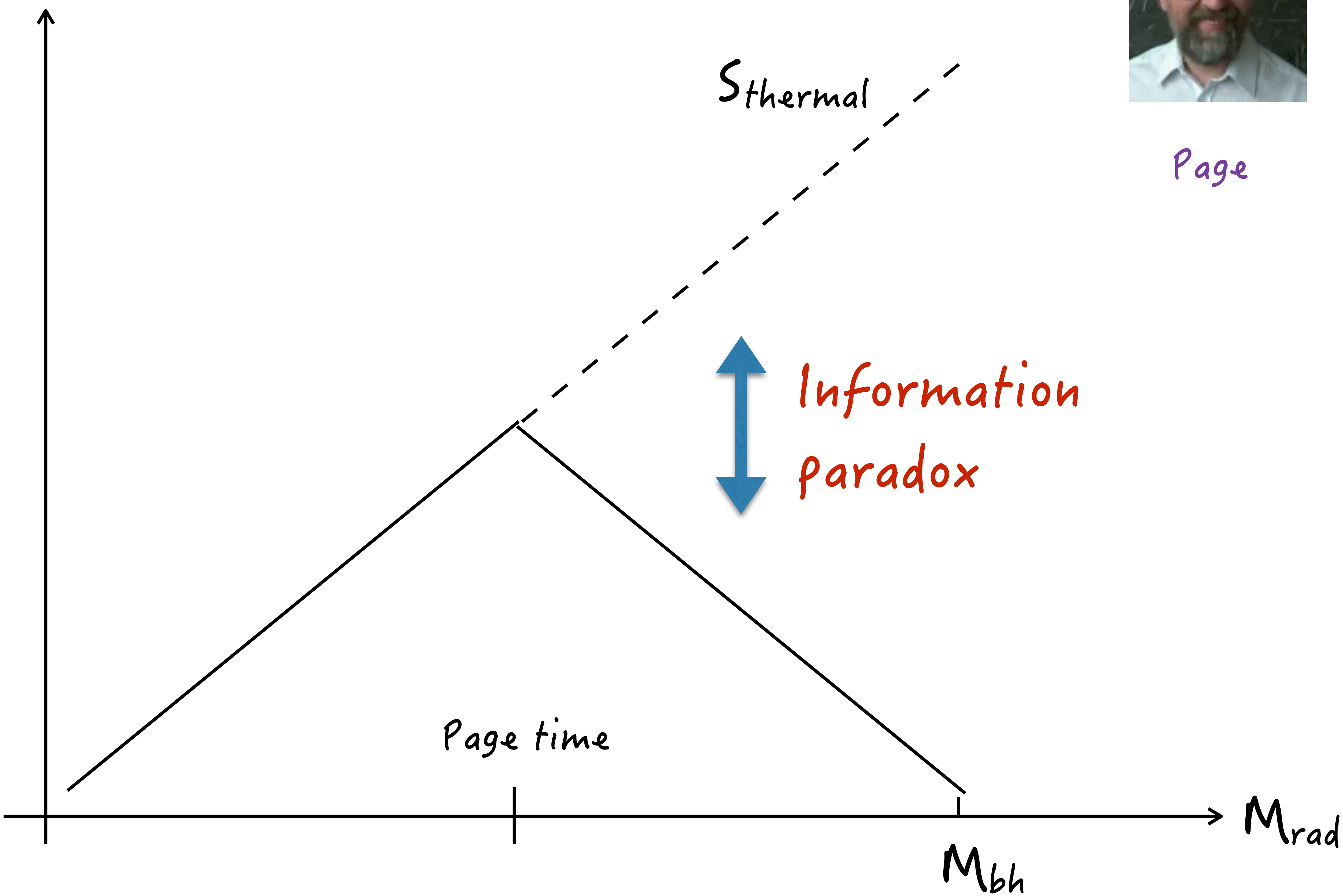
Each rad quantum is well approximated by a member of an EPR pair: radiation comes out in a highly mixed state

$$\Delta S\Big|_{\text{emission}} \sim \log 2 = O(1) \qquad +O(1/S) + O(e^{-Tt})$$

entropy production is roughly maximal

$$S_{\text{rad}} \sim \#_q \sim R\, M_{\text{rad}}$$

$$S_{\mathrm{rad}} = -\mathrm{Tr}\,\rho_{\mathrm{rad}}\,\log\,\rho_{\mathrm{rad}}$$

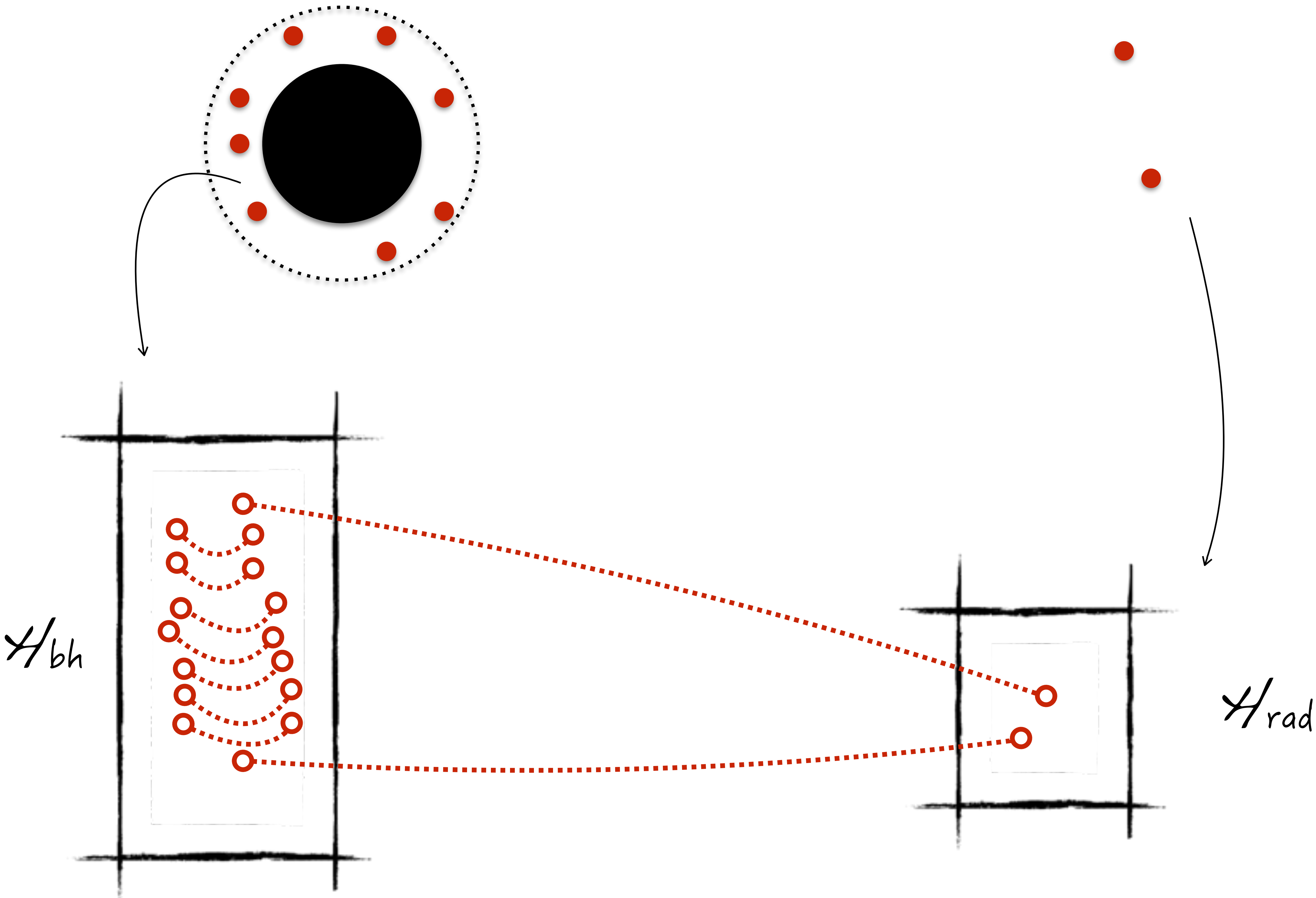


Page

This situation led to the

"BLACK HOLE WARS"

How can the entropy curve come down?



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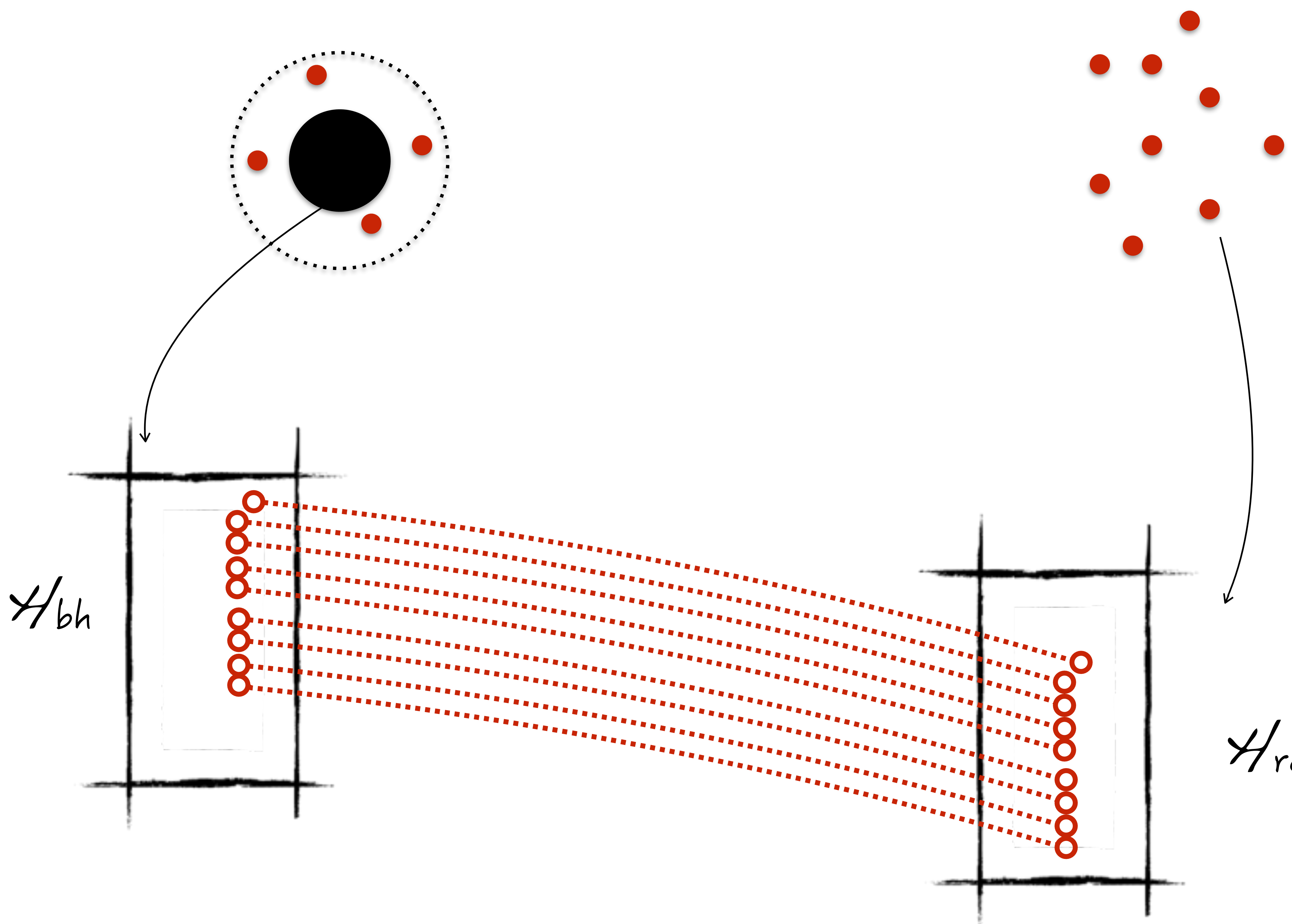
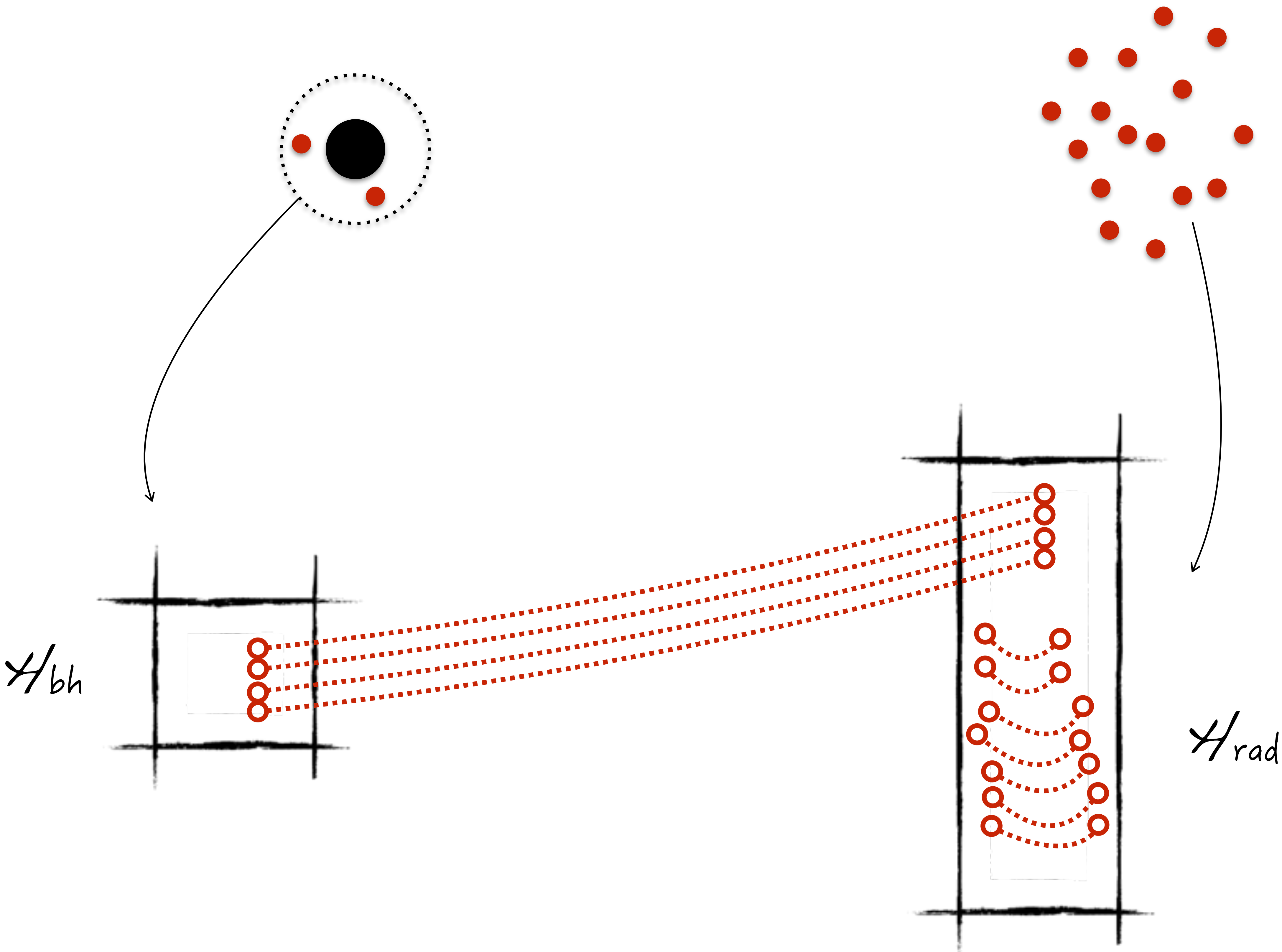



$\mathcal{H}_{bh}$
$\mathcal{H}_{rad}$

The "memory" term is only of size $\exp(-S)$ by the evaporation time, $\tau_{ev} \sim R\,S$, which is the time where the information MUST for sure begin to come out

But $\exp(-S) \sim \exp(-1/\alpha_{eff})$

which led to the suggestion that perhaps information restoration can only be understood in a fully non-perturbative treatment of gravity

This is actually "natural" ...

$$U^{+}\begin{pmatrix} 1 & & & 0 \\ & 0 & & \\ & & 0 & \\ & & & \ddots & \\ 0 & & & & 0 \end{pmatrix} U = e^{-S}\begin{pmatrix} \text{Random} \\ O(1)_{ij} \end{pmatrix}$$

$e^S \times e^S$ density matrix of pure state

random $e^S \times e^S$ unitary

$$\rho_{ij} = e^{-S}\delta_{ij} + O(e^{-S})_{ij}$$

Hawking estimate

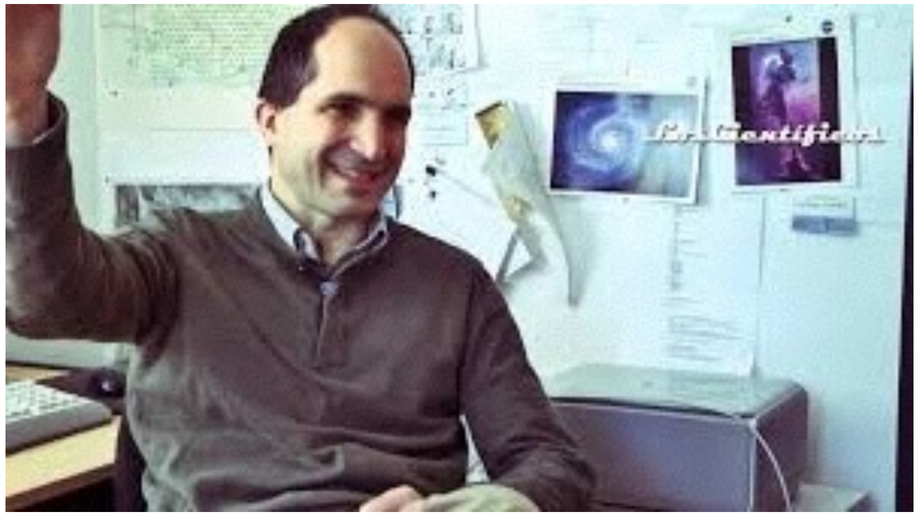
Nonperturbative correction of order $\exp(-1/\alpha_{eff})$

PURIFICATION AT PAGE TIME NOT EXPENSIVE!

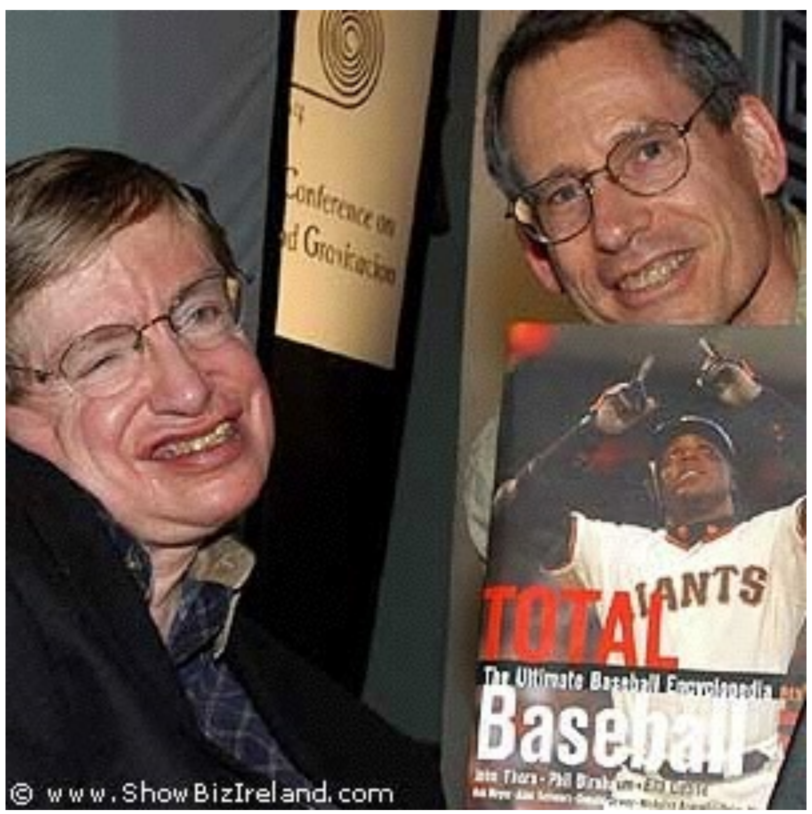
This scenario can be realised by enclosing black holes in a box, such as AdS space, and deriving general properties of correlation functions from AdS/CFT

Maldacena
Susskind
Barbon & Rabinovici
Hawking

Fine grained information after times of order S is encoded in exponential tails of size $\exp(-S)$ and long-time averages can be related to gravitational instanton transitions



South America wins the war?



This is all fine for "S-matrix questions" but ⋯

What about measuring the interior of the black hole?

Problem: if "self-entanglement" at horizon is lost after the Page time, what happens to Hawking's "detailed" argument?

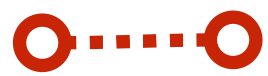
Is it possible that the horizon loses its smoothness?

Braunstein

Mathur

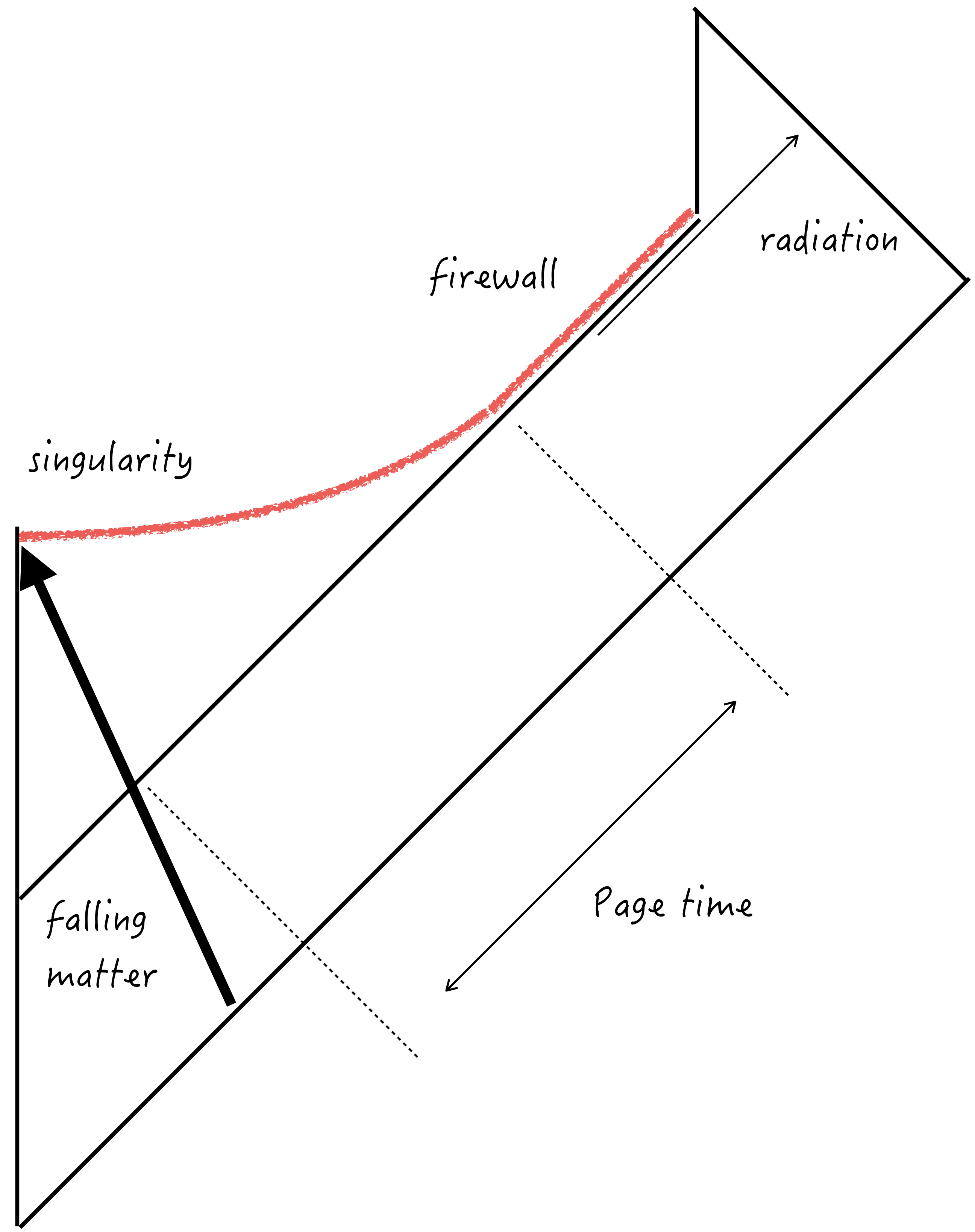
Amiheri, Marolf, Polchinski & Sully

"Disentangling" the ground state costs energy

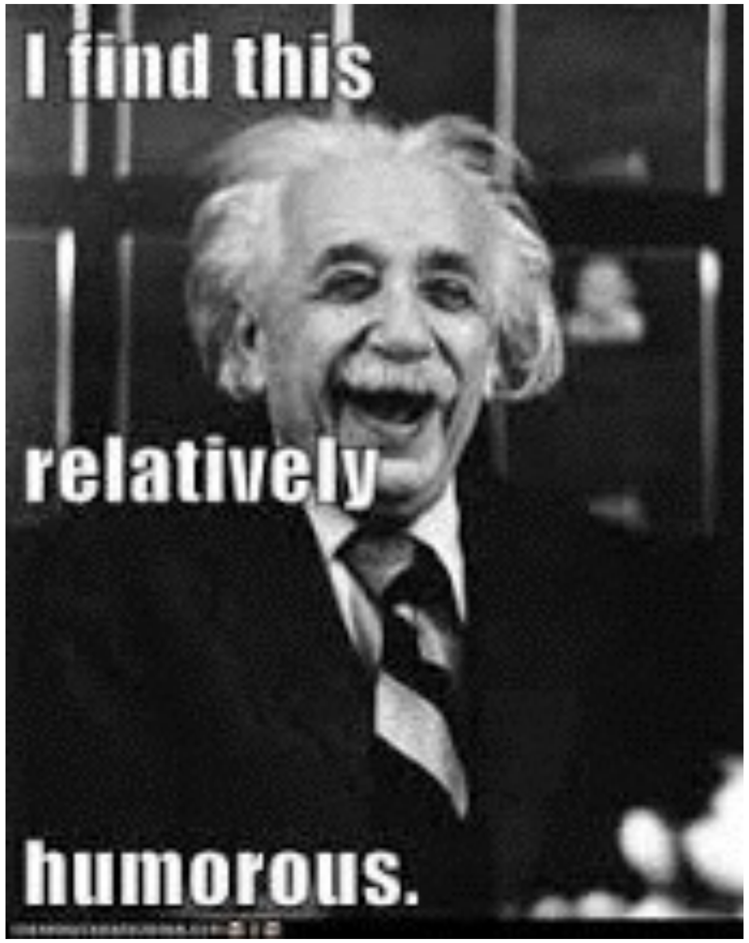
$E($  $) < E($  $)$

for each of the $A_H/4G$ qbits in the near-horizon

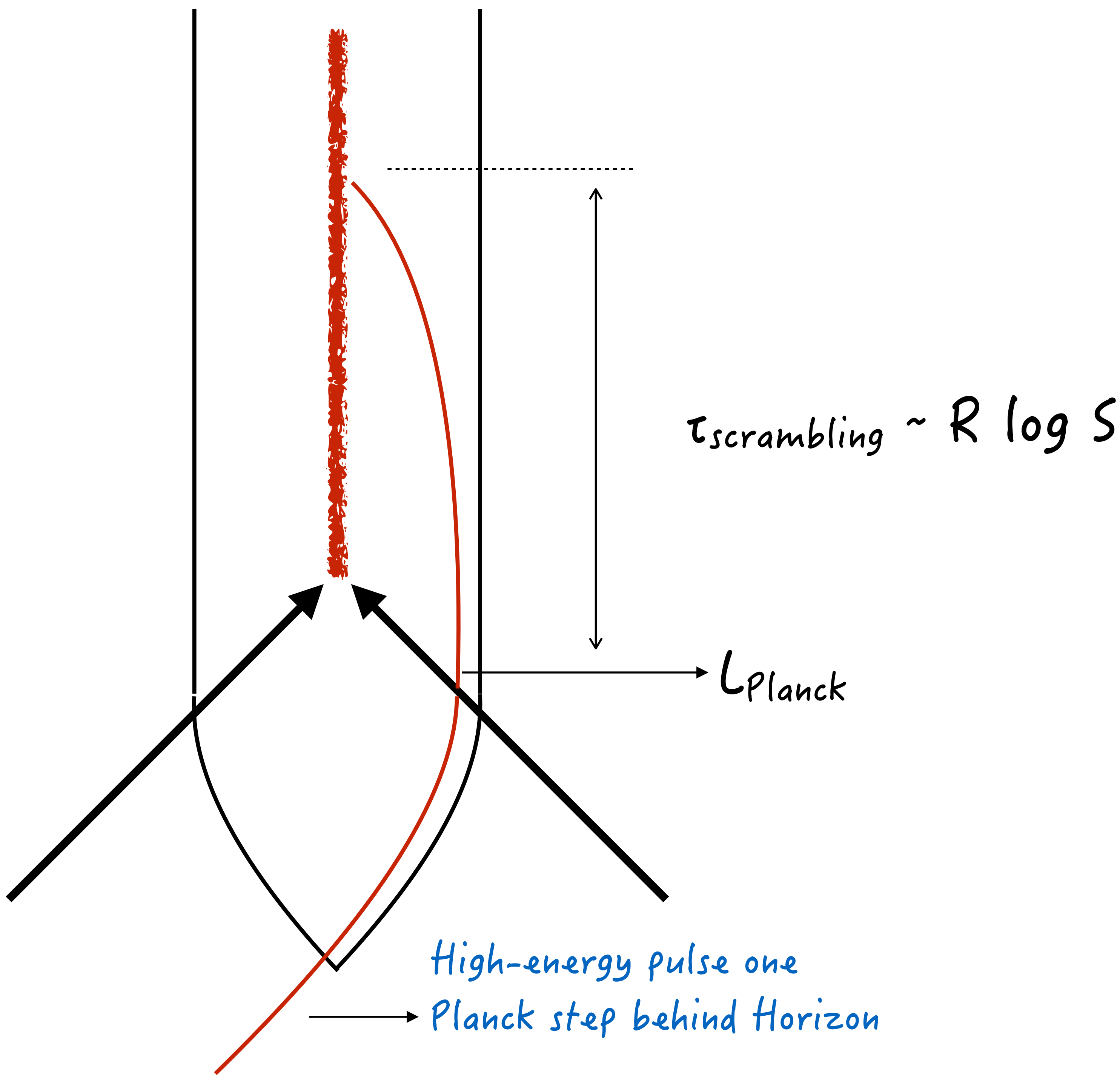
FIREWALLS?



IS THE $r=2GM$ SURFACE SINGULAR AFTER ALL?



Firewalls are difficult to make precise. Regularized versions do not "hold" onto the horizon ...



There would be much more to say ···

From black holes as postmodern neutron stars (FUZZBALLS)

Mathur et al

Bena, Warner et al

to a simple matter of ···

"Not only God plays dice. She also plays pin the tail of the quantum donkey"

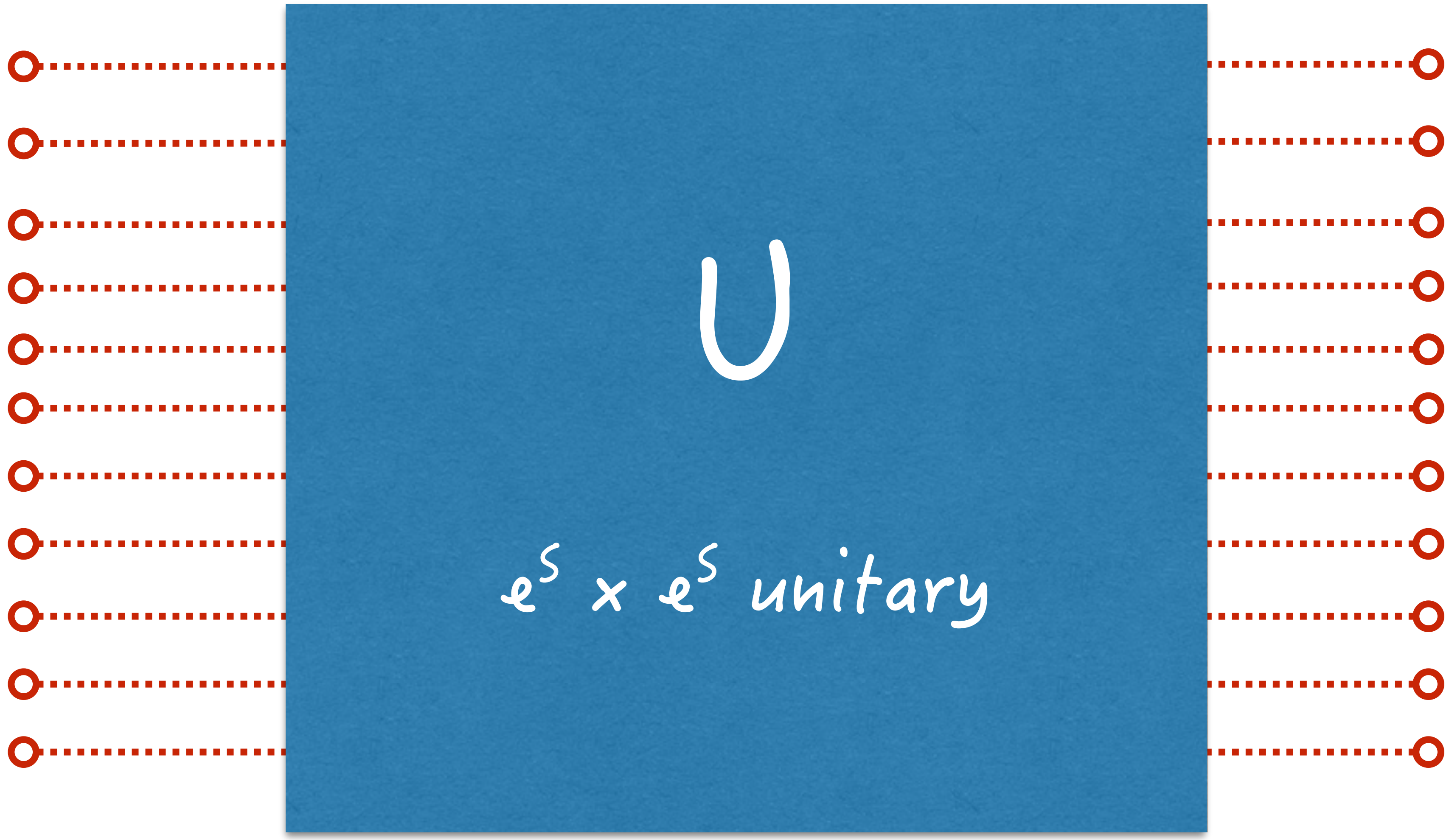
Papadodimas & Raju

We can try to distill the interior reconstruction problem by looking at the simplest possible situation

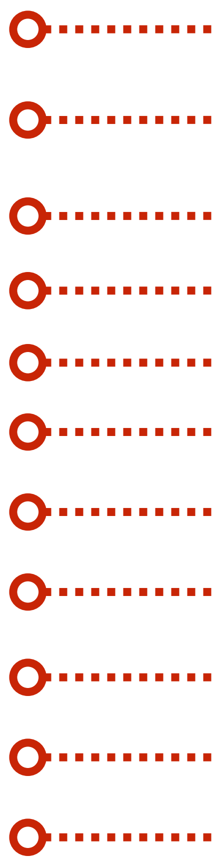
We just keep the maximal entanglement and remove the complications of the detailed evaporation dynamics

Simplest model : 25 qbits with maximal entanglement

Simplest model : 2S qbits with maximal entanglement

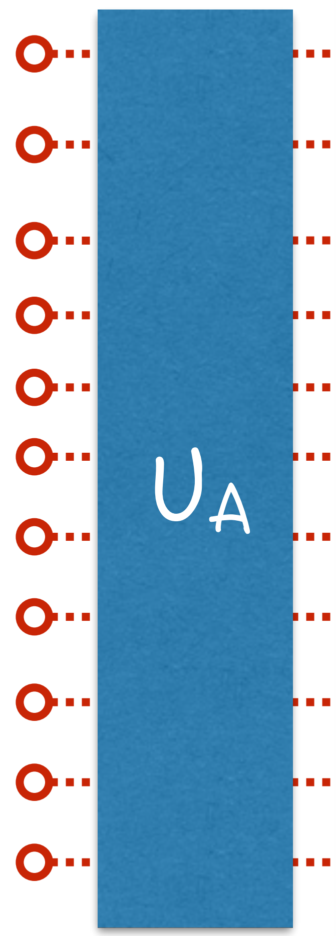


Simplest model : $2S$ qbits with maximal entanglement



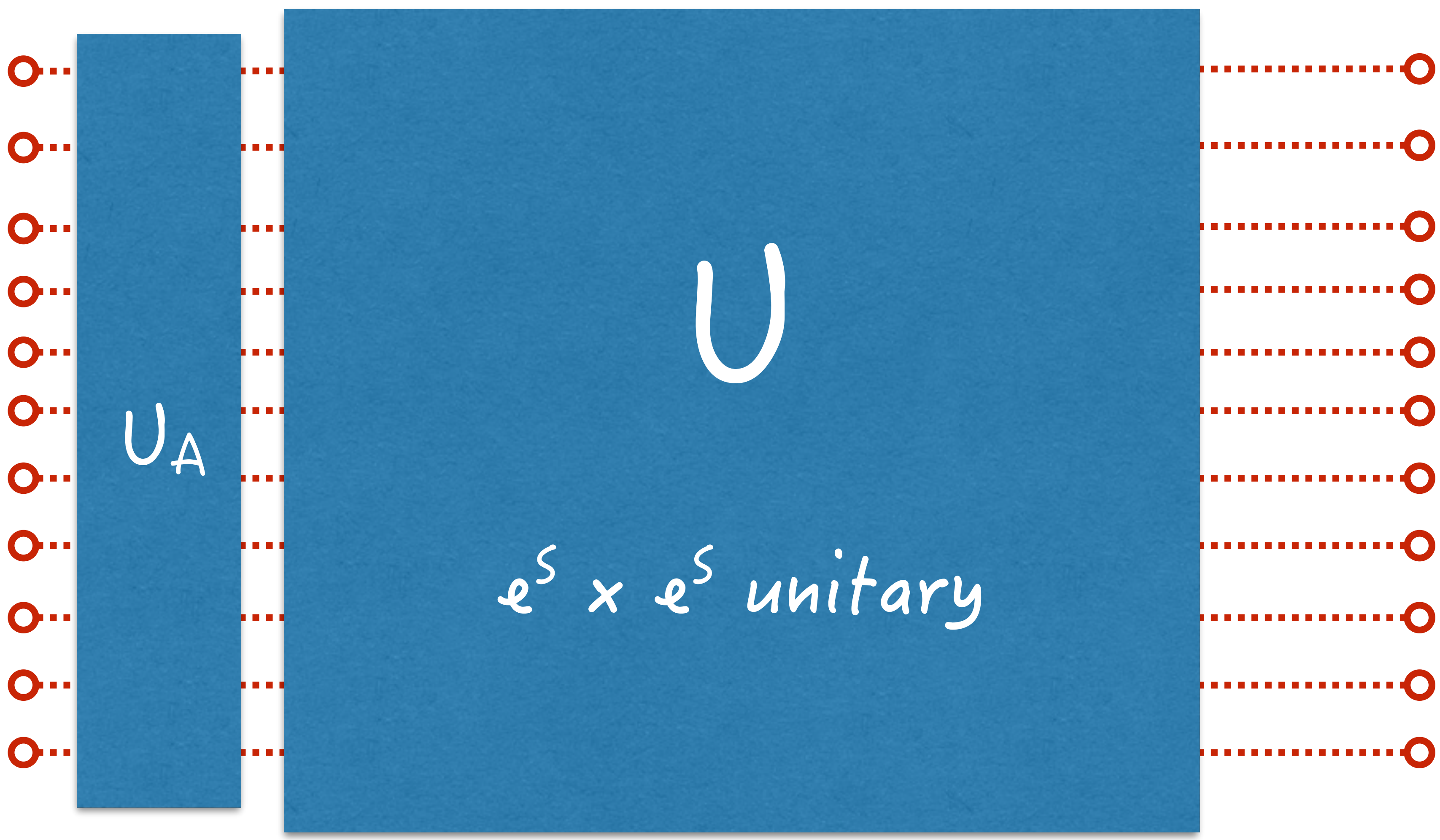
The half-state is totally mixed with entropy S

Simplest model : 2S qbits with maximal entanglement



Acting with unitaries on left or right does not change the total entanglement.

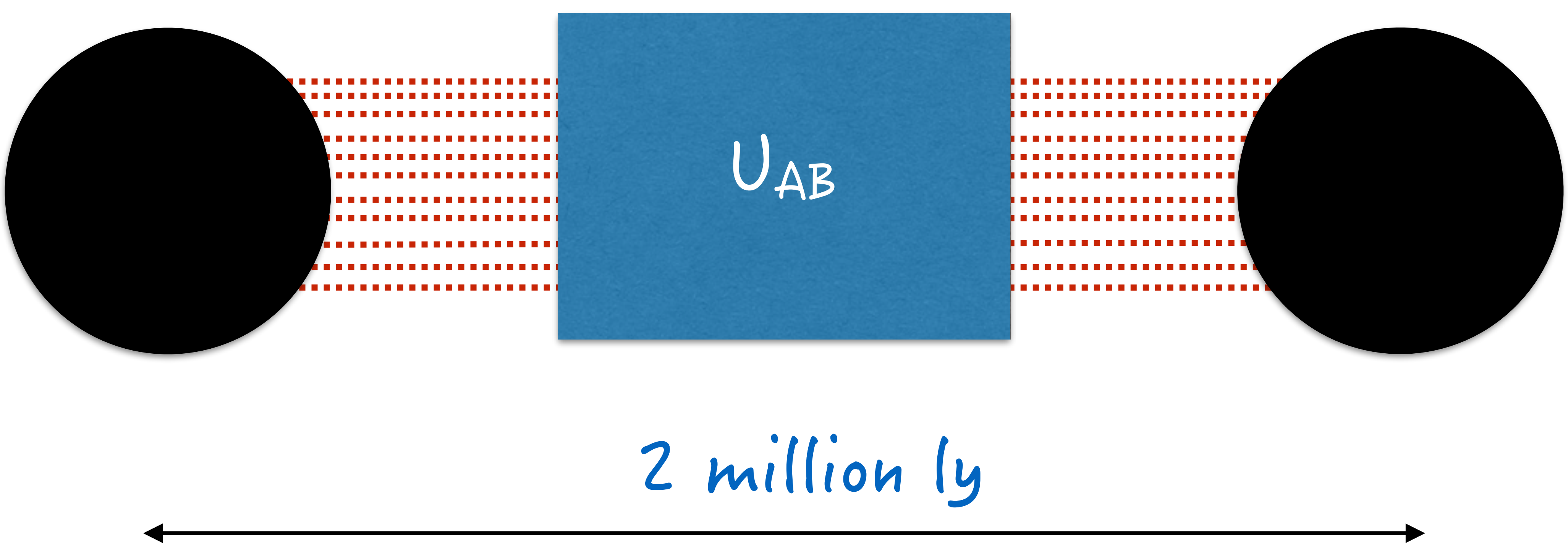
Simplest model : 2S qbits with maximal entanglement



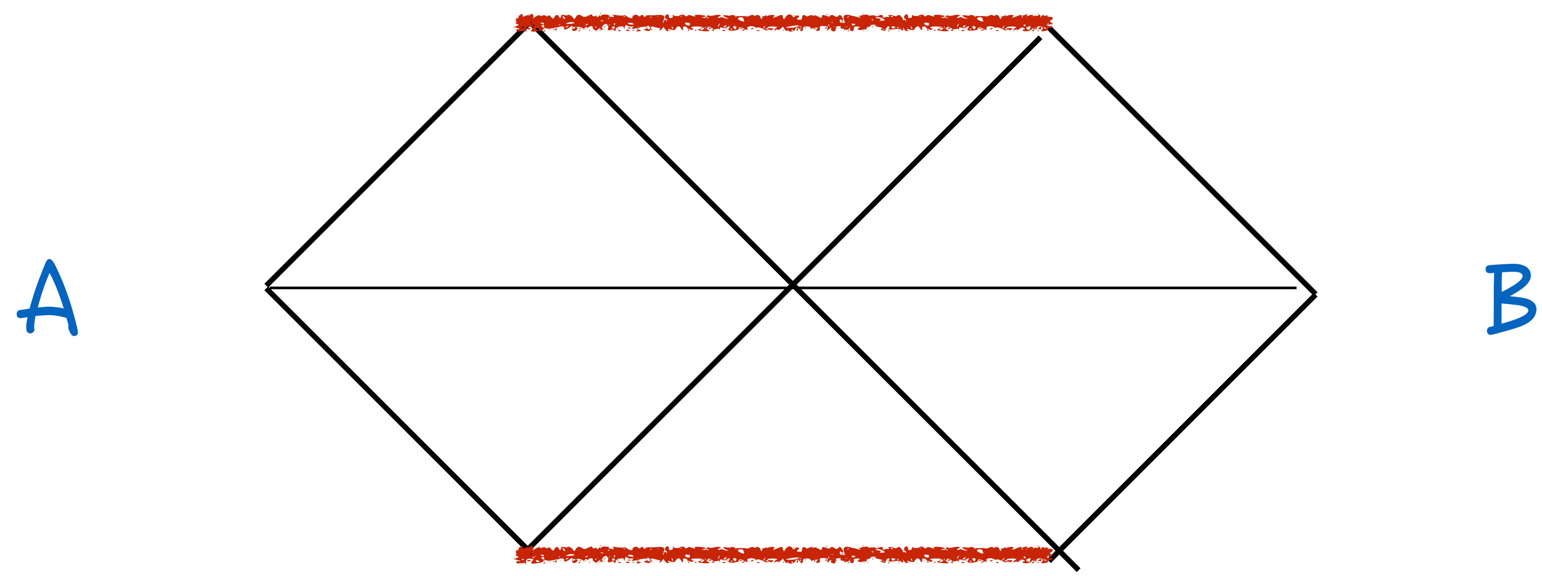
Rather, it redefines U, the entanglement structure

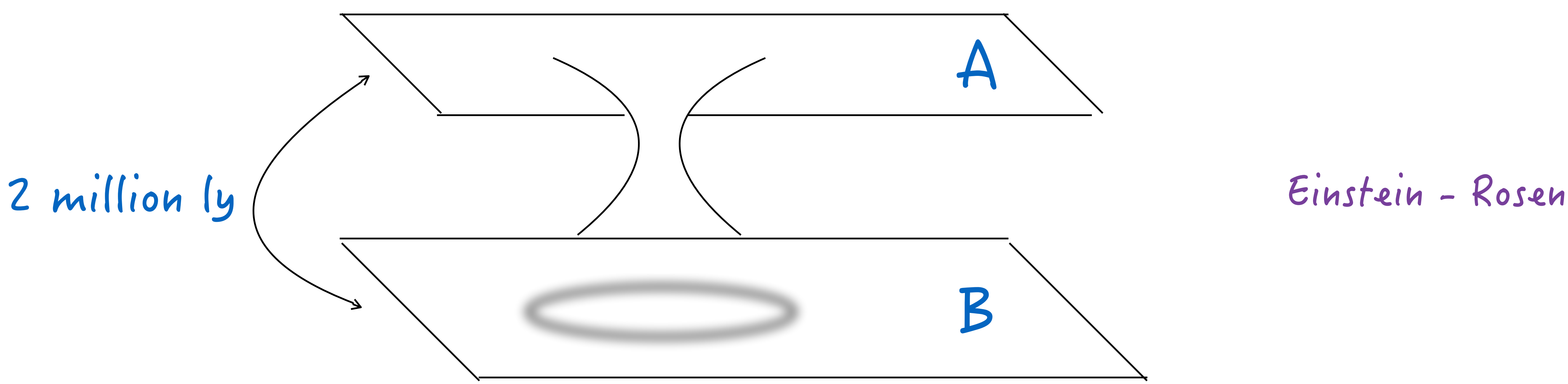
U_A and U_B may be defined so that we collapse the q-bits within a size of order $(G\,S)^{1/2}$

This would make two maximally entangled black holes, each with entropy S



There is a natural geometrical representation of such a situation





WHY?

State looks thermal on each side

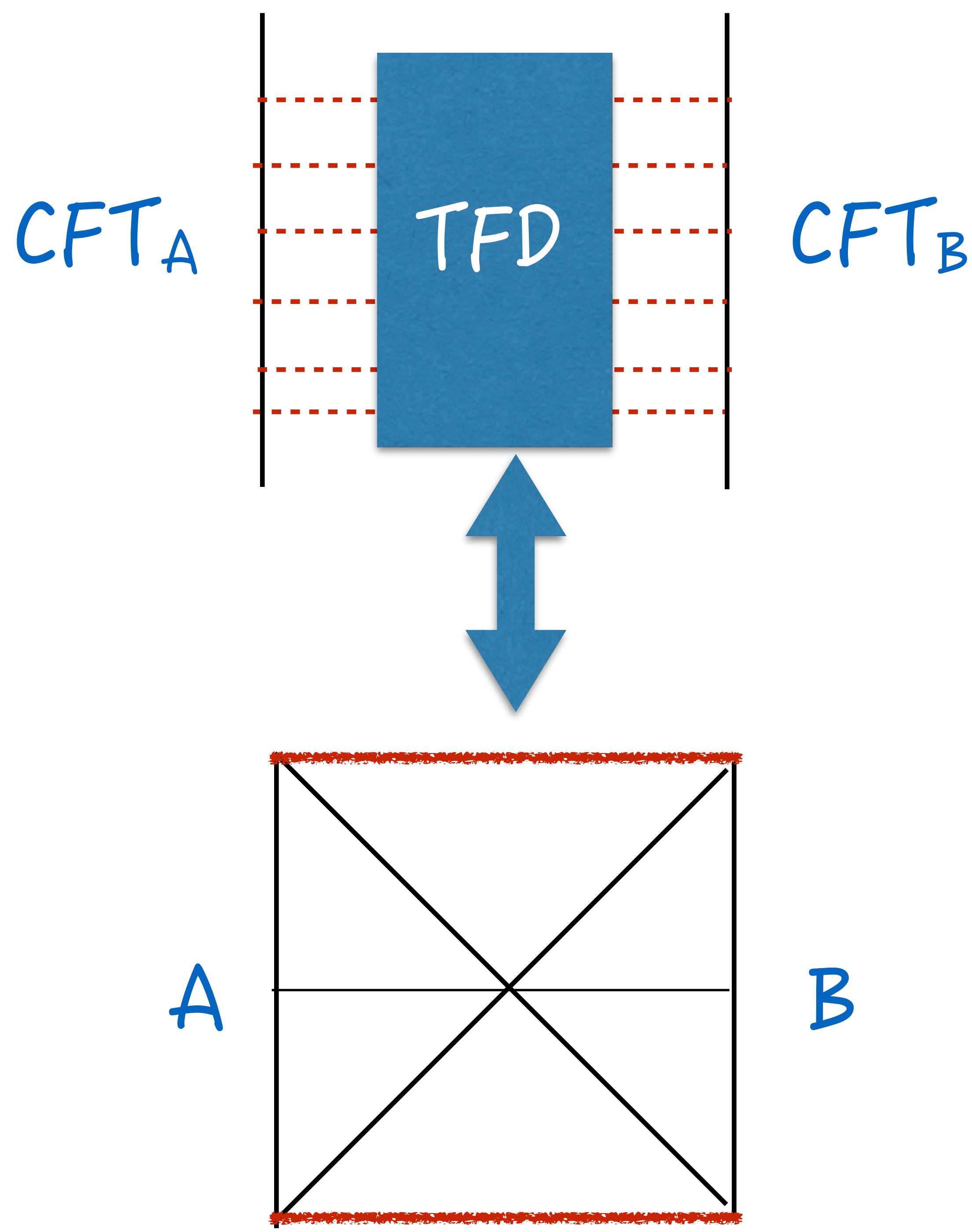
The bulk Hartle-Hawking state has explicitly the required entanglement structure (so called TFD)

$$|\mathrm{HH}\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_E e^{-E/2T} |E\rangle_A \otimes |E\rangle_B$$

Israel

$|E\rangle_{A,B}$ are energy eigenstates of the bulk Hamiltonians associated to the standard timelike Killing vectors

Maldacena elevated this to a non-perturbative statement in AdS/CFT, interpreting $|\varepsilon\rangle_{AB}$ as states in two decoupled CFTs



SLOGAN: **EPR = ER** Maldacena & Susskind

Notice that

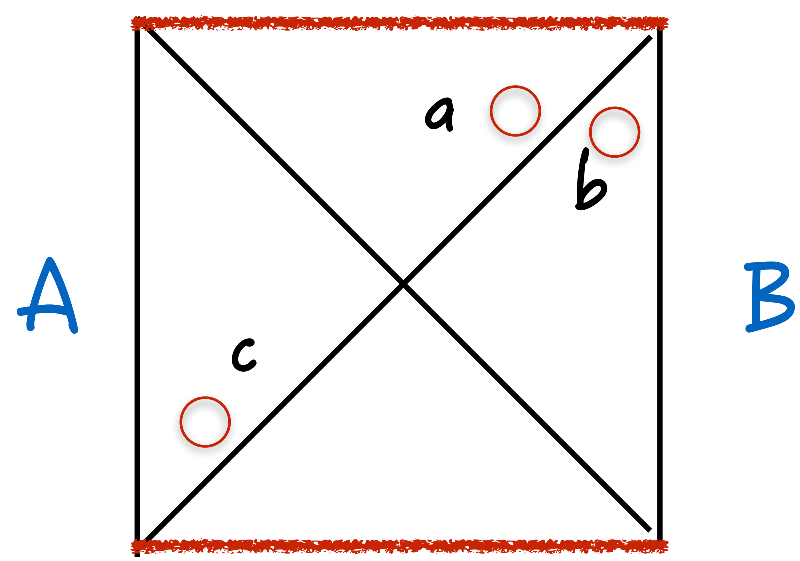
$$H = H_A + H_B$$

at the level of exact CFT Hamiltonians

This means that no signal can be exchanged between A and B in an exact sense

Equivalently, the wormhole is absolutely non-traversable

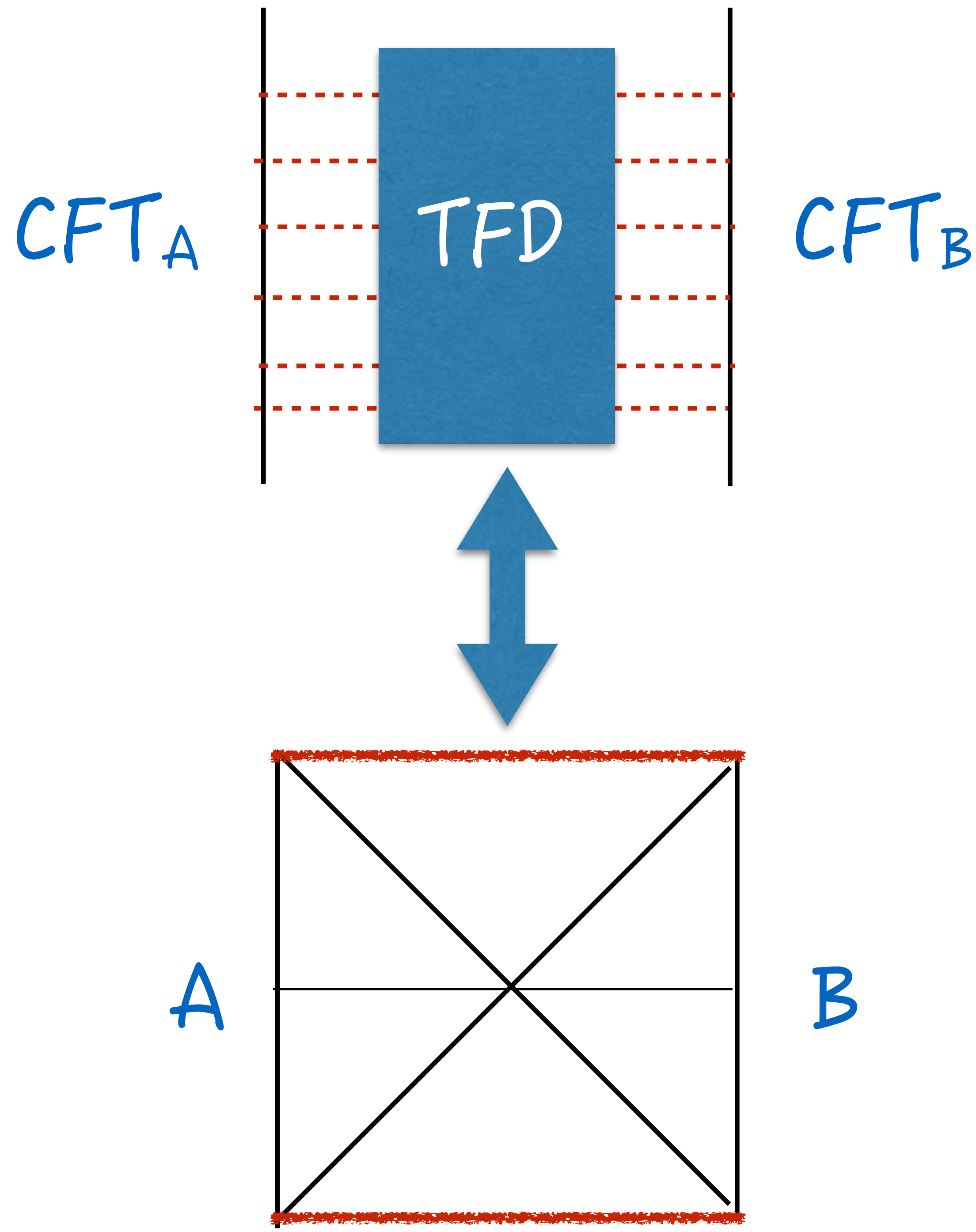
The remarkable fact about EPR=ER is that it provides a counterexample to the firewall solution!



CFT_A is the analog of the early radiation

A q-bit d.o.f. b can be maximally entangled with BOTH a and c, because a is in the future of c

The problem is, of course, how general is this "escape"



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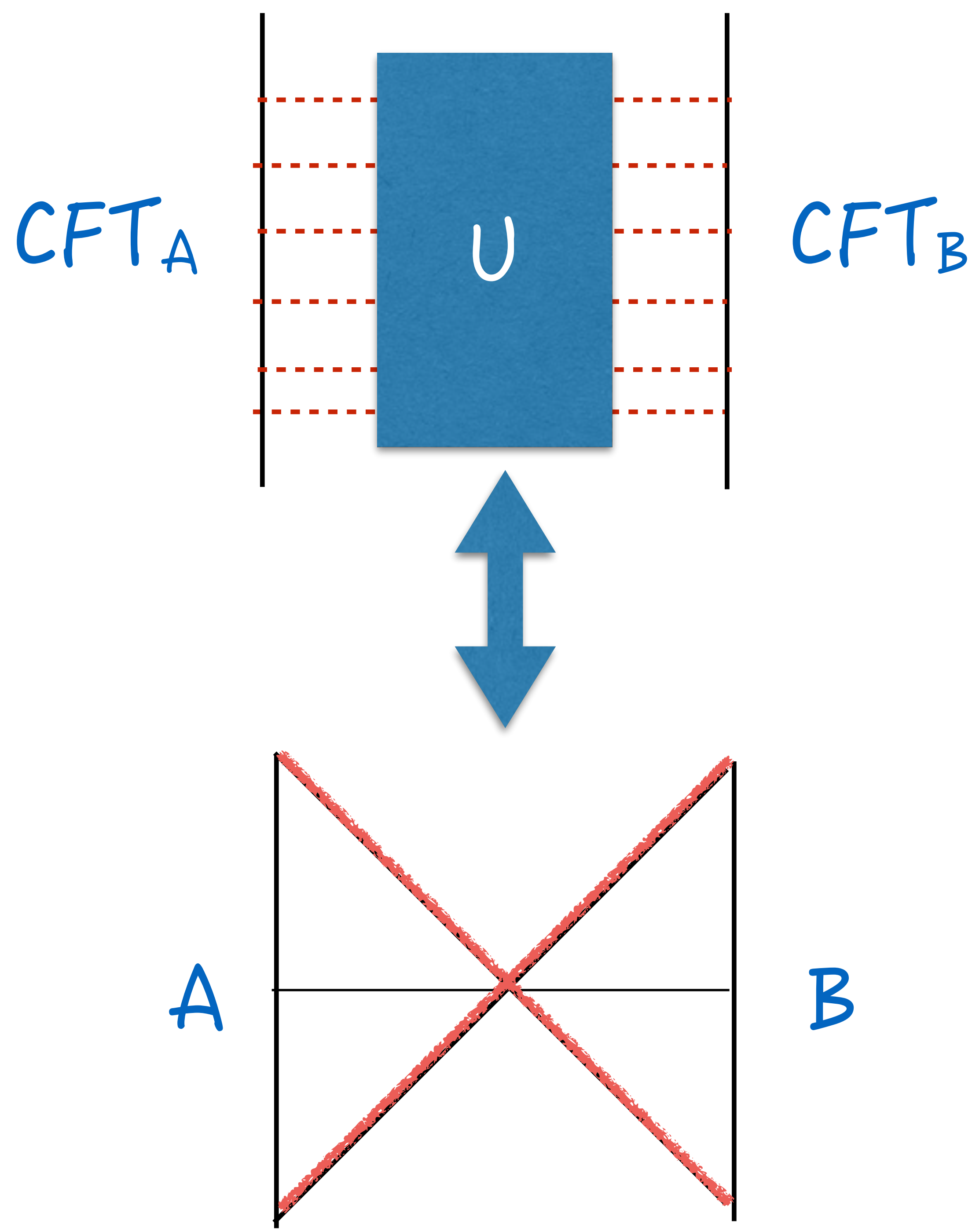

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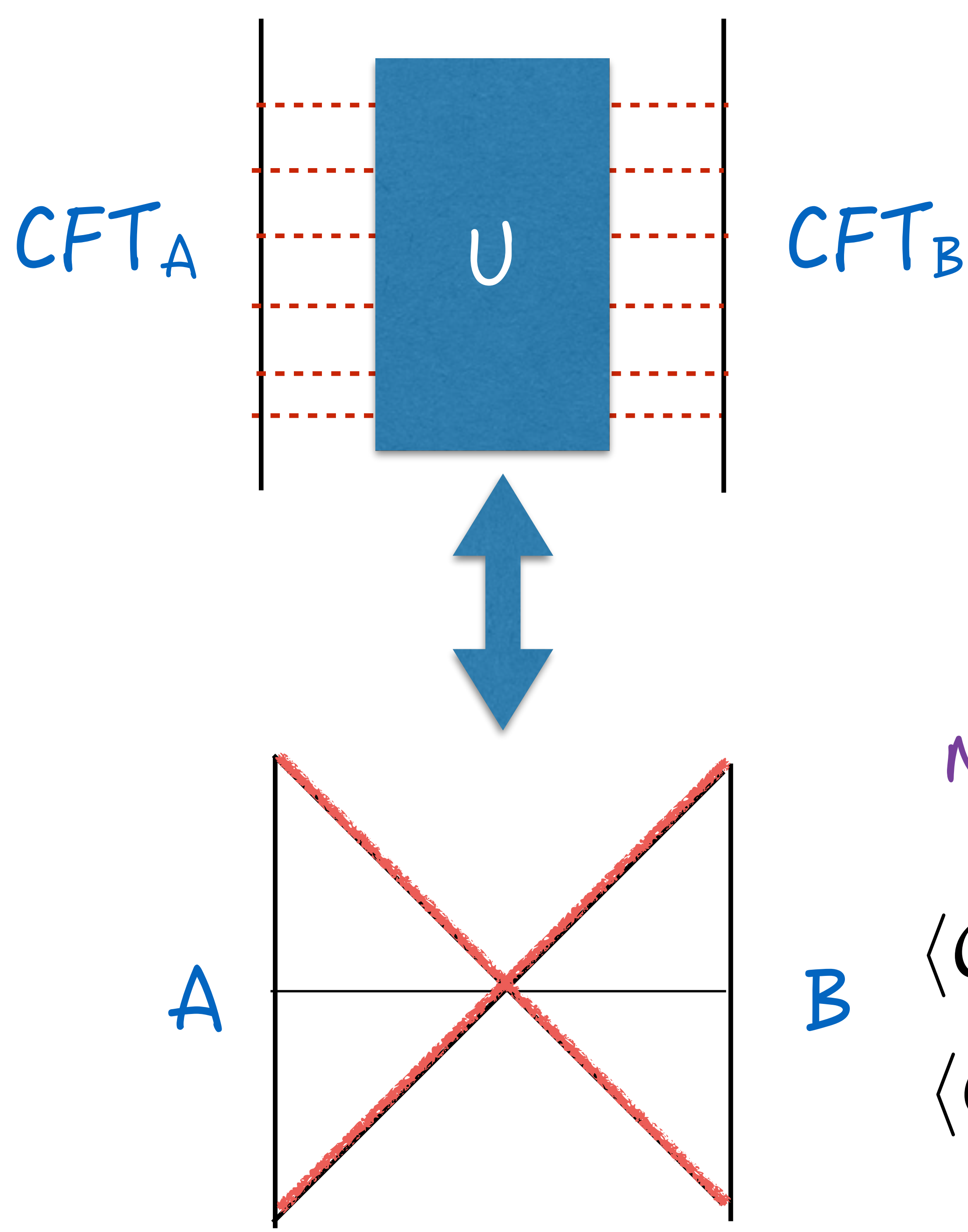
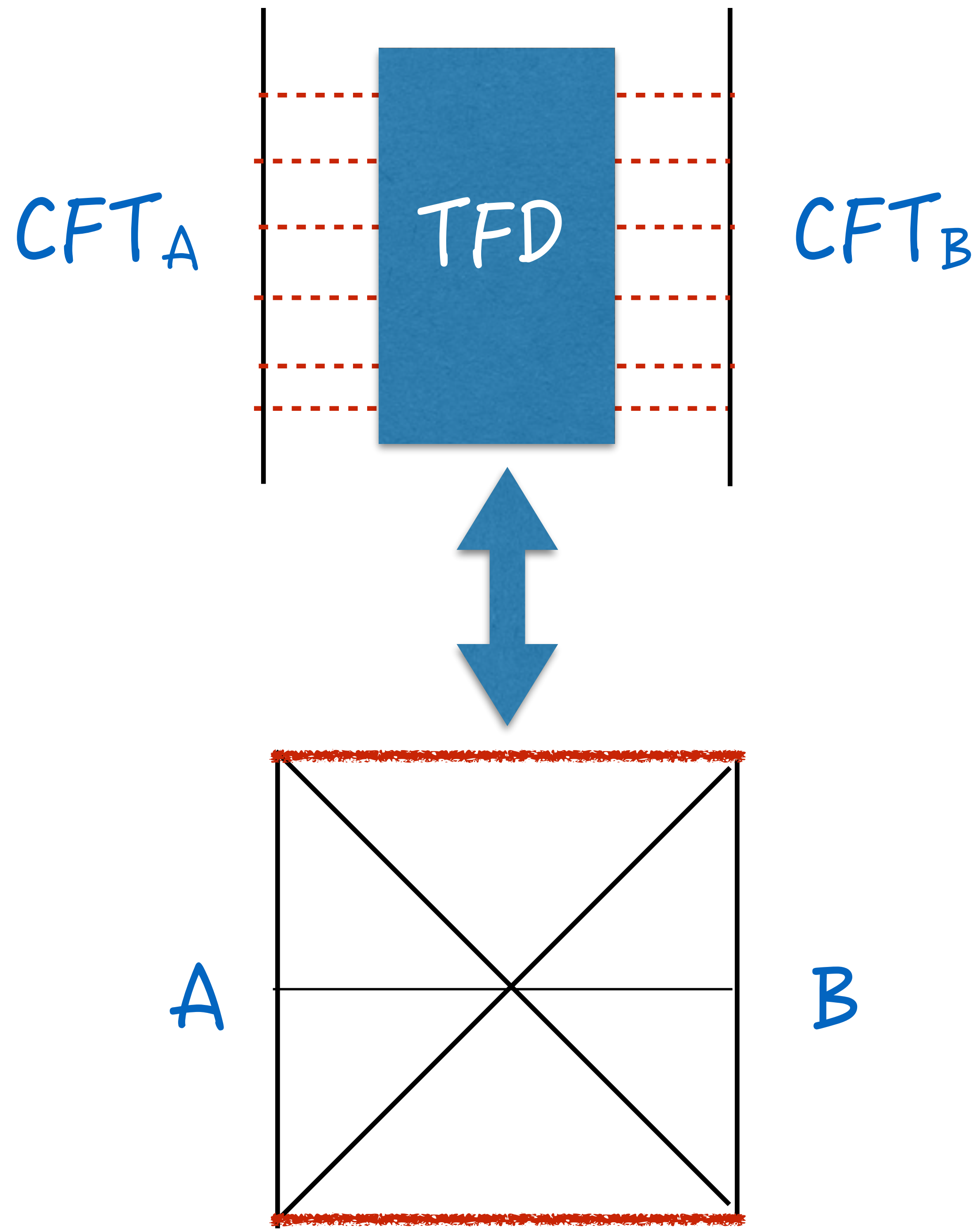


Marolf & Polchinski

$$\langle \mathcal{O}_A \, \mathcal{O}_B \rangle_{\rm TFD} \sim 1$$

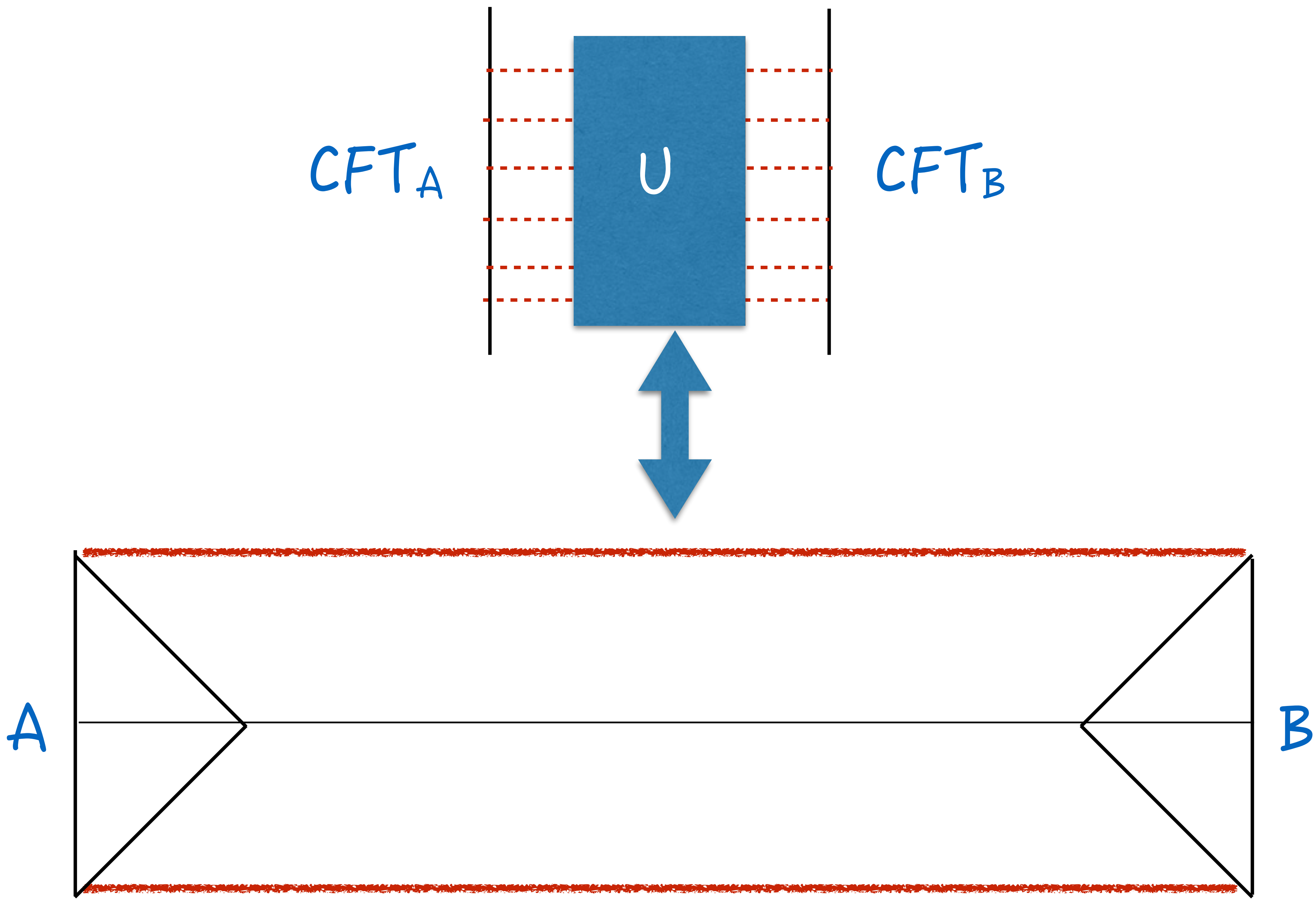
$$\langle \mathcal{O}_A \, \mathcal{O}_B \rangle_U \sim e^{-S}$$

A desperate out?



A desperate out?

Susskind et al

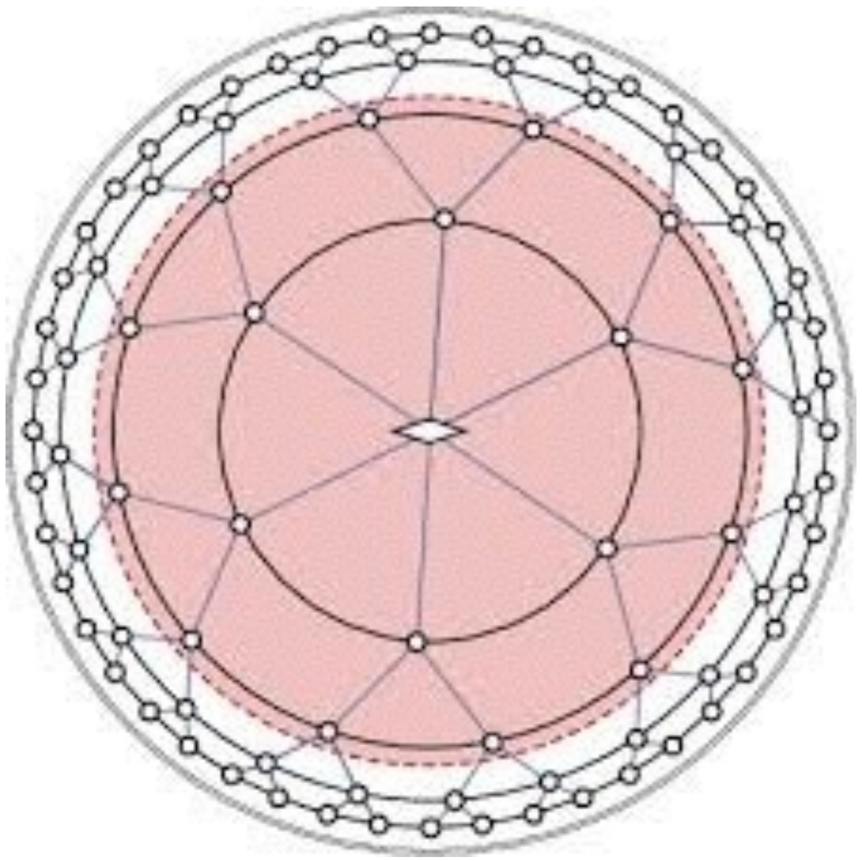


Wormhole volume = Computational complexity of U

The complexity of U can be parametrised by a tensor "entanglement network"

Complexity (U) = minimal size of quantum circuit

This connects with the AdS interpretation of tensor networks approximating CFT vacua



Vidal

Swingle

A new entry in the holographic dictionary?

ENTANGLEMENT ⟷ BULK CONNECTIVITY

COMPLEXITY ⟷ BULK VOLUME

PARTING SLOGANS

PARTING SLOGANS

Black hole paradoxes are not solved

PARTING SLOGANS

Black hole paradoxes are not solved

But they continue to bring new ideas on the table

PARTING SLOGANS

Black hole paradoxes are not solved

But they continue to bring new ideas on the table

Some of these ideas will "fly on their own"
and when they come back the old paradoxes will
look trivial to them

THANK YOU