Anomalous transport: from the quark gluon plasma to Weyl semi-metals on a superstring

Karl Landsteiner

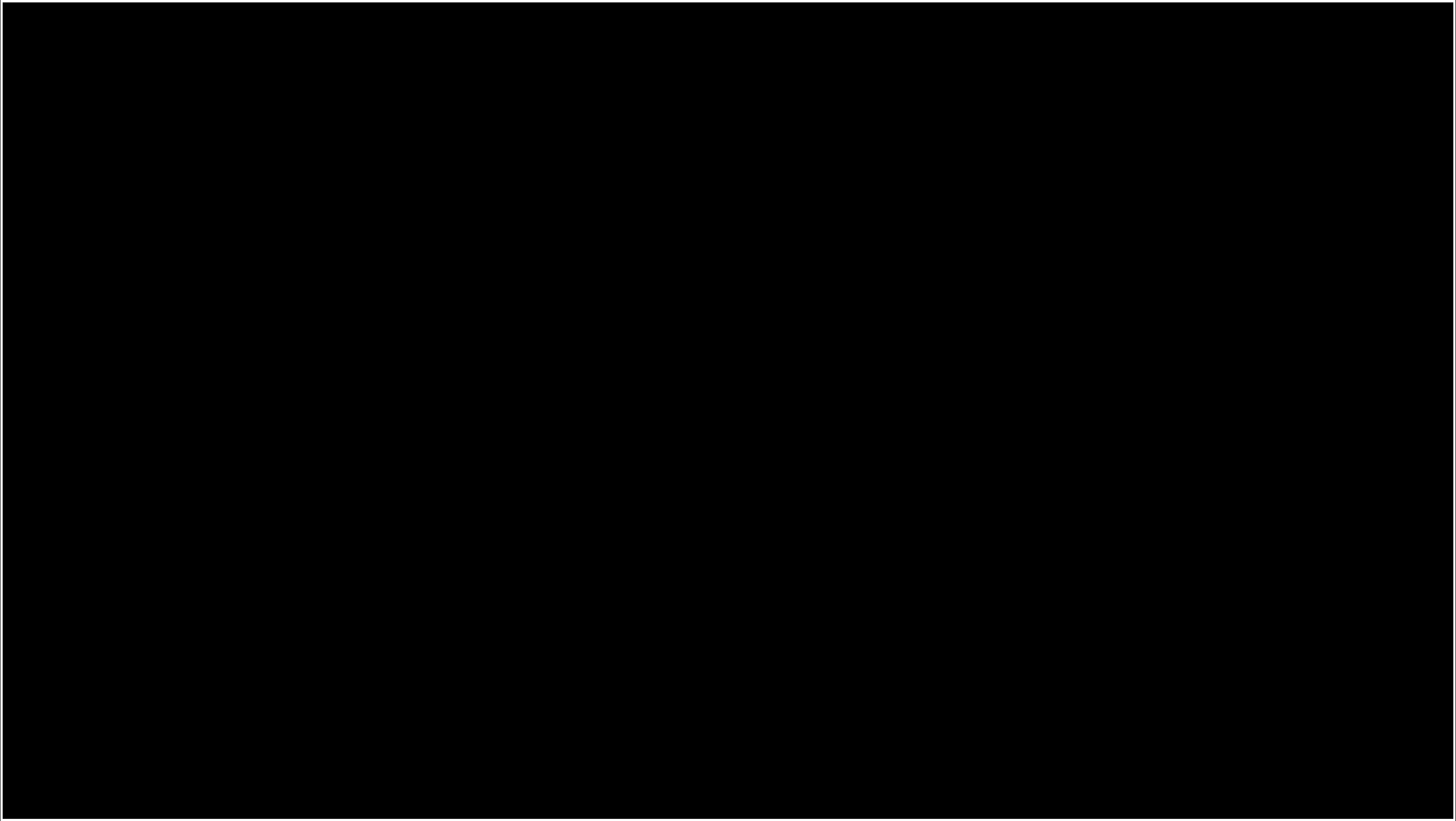
Instituto de Física Teórica UAM-CSIC

Postgraduate Meeting, IFT-Madrid, 18-11-2015

Outline

- Quark Gluon Plasma
- Flash review: AdS/CFT
- Anomalous transport model in AdS/CFT
- Map to Weyl semi-metals
- Conclusions

Quark gluon plasma



Quark gluon plasma

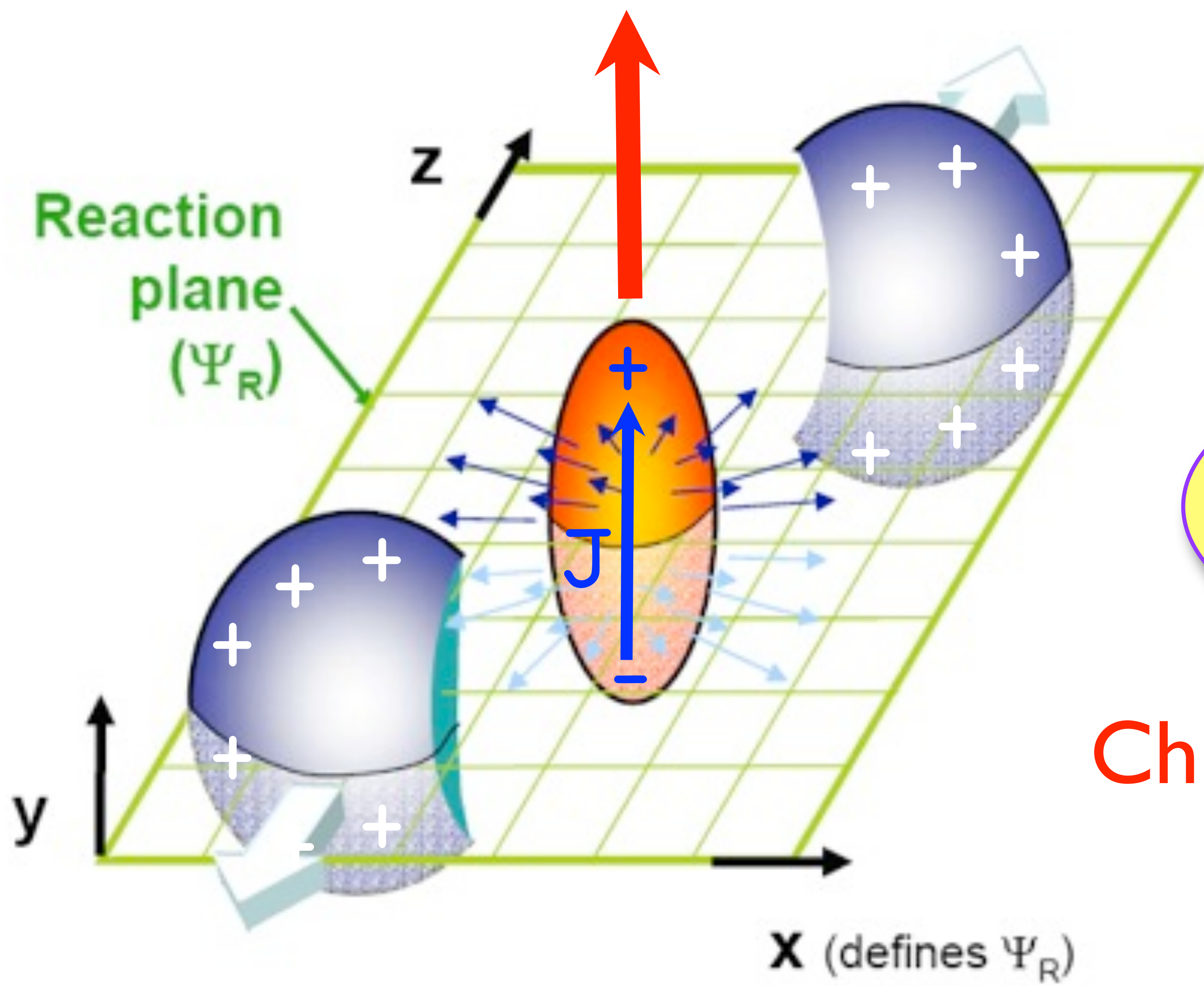
- QCD - confined quarks and gluons
- High T: de-confinement and plasma phase
- Smash nucleons agains each other (RHIC, LHC)
- Lowest specific viscosity known!
- Charge separation effect observed
- Possible explanation: Chiral magnetic effect
- $T \sim 3T_c$: deconfined but strongly coupled

Quark gluon plasma

strongest Magnetic field in the Universe 10^{15}T!!!

(QHE: 10 T)

(T ~ 10^{12} K)



$$\vec{J} = \frac{\mu_5}{2\pi^2}\vec{B}$$

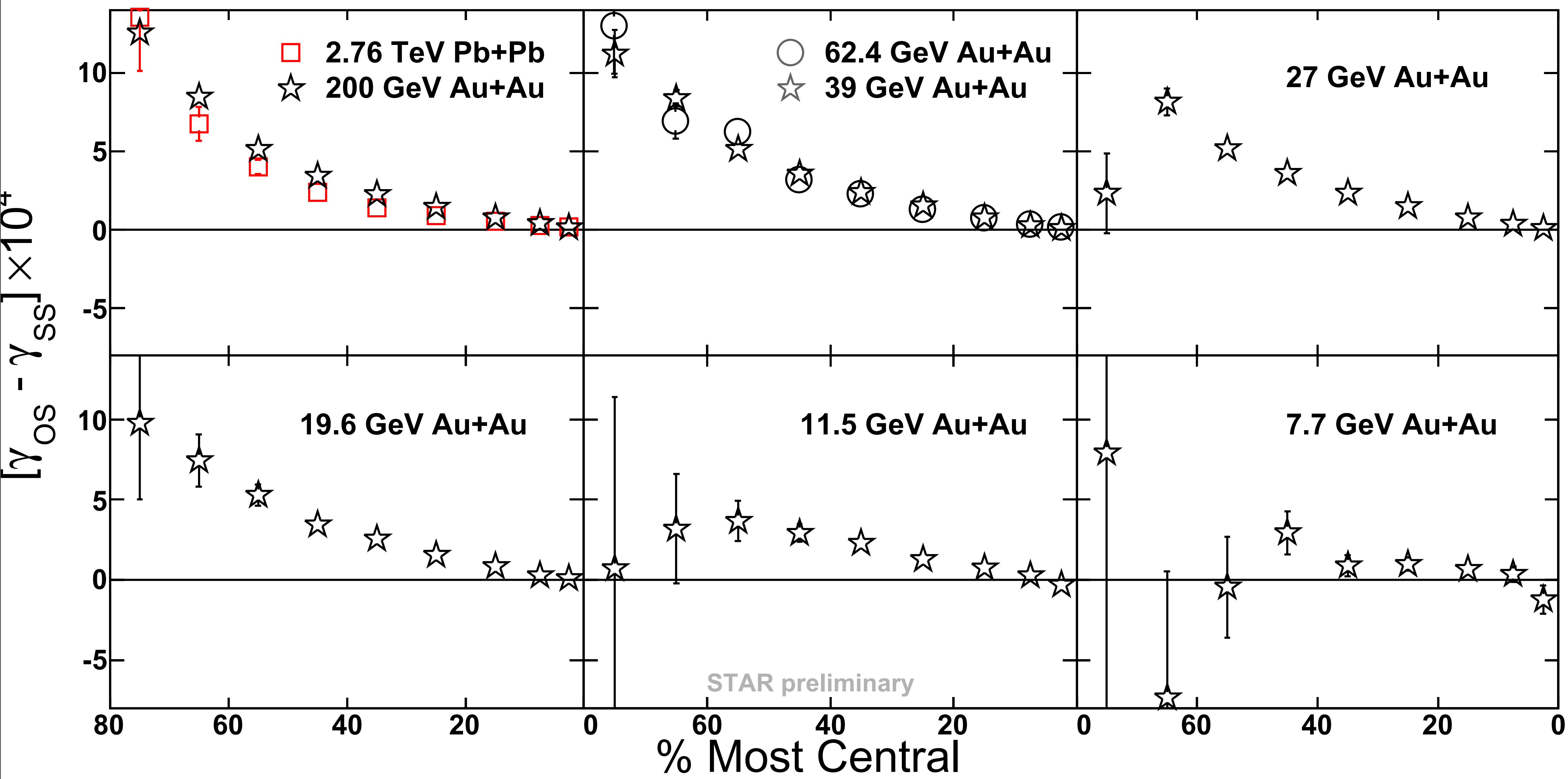
Chiral Magnetic Effect

[Fukushima, Kharzeev, McLarren]
[Fukushima, Kharzeev, Warringa]

Net chirality

- topological charge $Q_w = \frac{g^2}{32\pi^2} \int d^4x \, F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$
- axial anomaly (QCD) $\partial_\mu j^\mu_5 = 2m_f \langle \bar{\psi}_f i\gamma_5 \psi_f \rangle - \frac{N_f g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$
- topologically non trivial gauge field
- effective: axial chemical potential $\mu_5 \longleftrightarrow \Delta Q_5 = 2N_f Q_w$

Quark gluon plasma



Quark gluon plasma

- Strongly coupled QCD is difficult
- In need of strongly coupled toy model
- Comes in: AdS/CFT correspondence
- Famous result: $$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$ [Policastro, Son, Starinets] [Kovtun, Son, Starinets]
- Understand CME + relatives in AdS/CFT model

AdS/CFT

Motto:

"… if the gravitational field didn't exist, one could invent it for the purposes of this paper…"

"Theory of Thermal Transport Coefficients"
Luttinger Phys. Rev. 135, A1505, (1964)

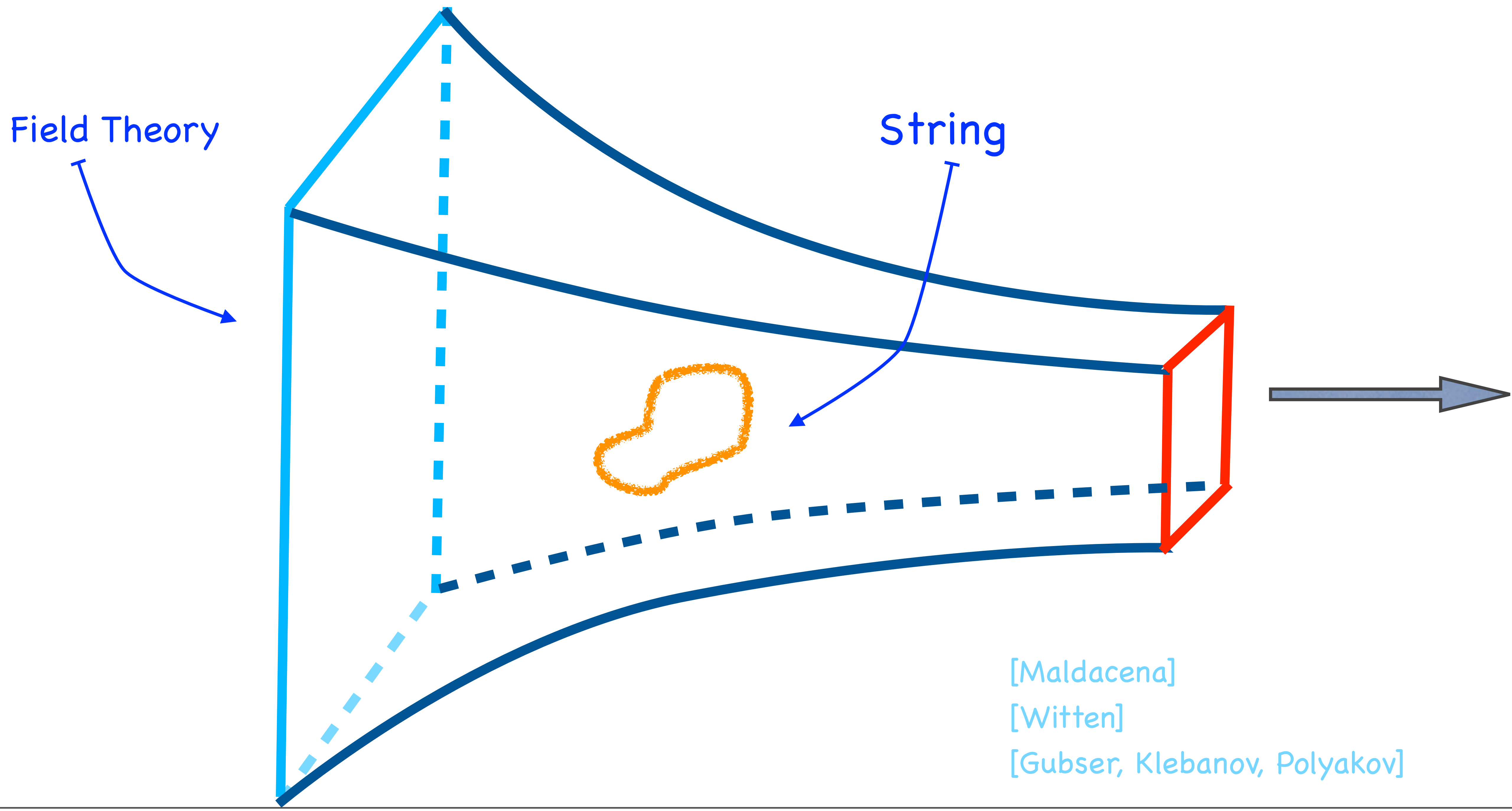
AdS/CFT

"... if the string theory didn't exist, one could invent it for the purposes of computing transport coefficients in strongly coupled theories..."

- Shear viscosity in QGP
- Relativistic 2nd oder hydrodynamics
- Relativistic superfluids
- Parity odd
- ....

AdS/CFT

$$ds^2 = \frac{r^2}{L^2}(dt^2 + d\vec{x}^2) + \frac{L^2 dr^2}{r^2}$$



[Maldacena]
[Witten]
[Gubser, Klebanov, Polyakov]

AdS/CFT

$$\int_{\Phi|_\partial=\Phi_0} D\Phi e^{iS[\Phi]} = e^{iZ[\Phi_0]}$$

$$\frac{\delta^n Z[\Phi_0]}{\delta\Phi_1(x_1)\cdots\delta\Phi_n(x_n)} = \langle O_1(x_1)\ldots O_n(x_n)\rangle$$

Path integral (string theory) on AdS is hard. In practice resort to semi classical limit:

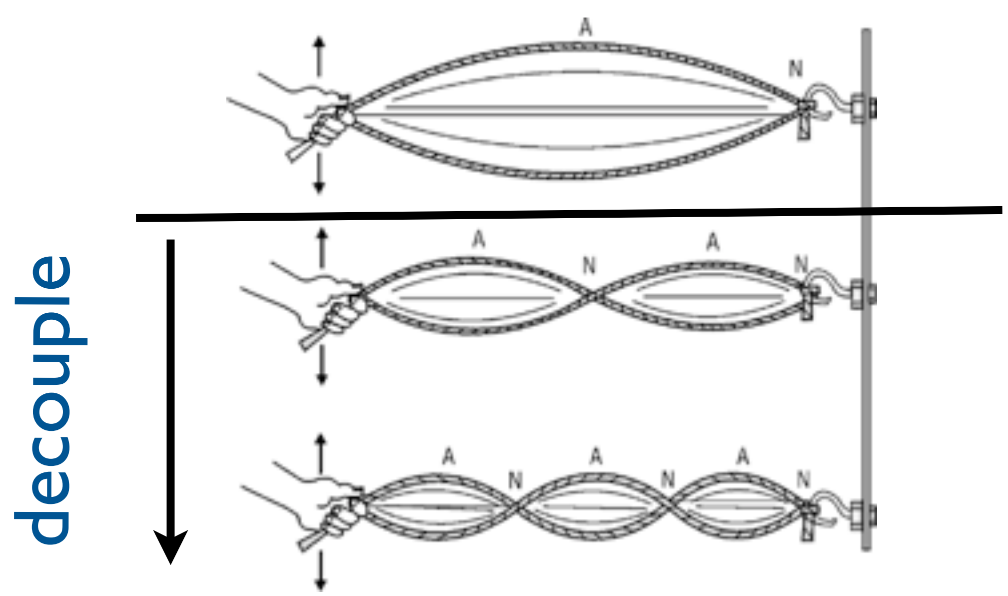
$$S_{grav}[\Phi_0] = Z[\Phi_0]$$

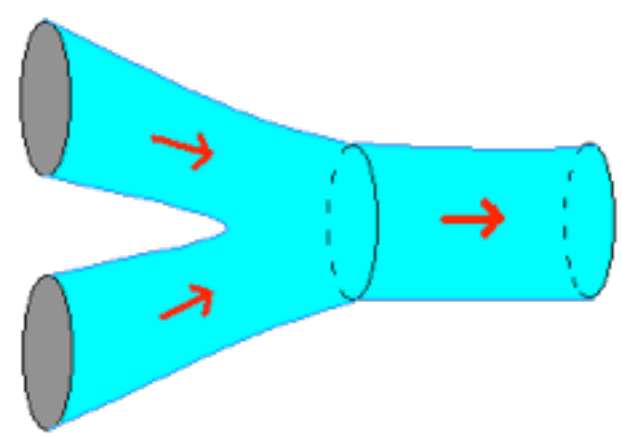
AdS/CFT

- N=4 SYM best understood example: $\{\mathcal{A}_\mu, \Psi^a_\alpha, \phi^I\}$
- All (4-d) fields are NxN matrices (adjoint rep)
- *N*=4 SYM is equivalent to IIB string theory on $AdS_5 \times S^5$

$$g^2_{YM} N = \frac{R^4}{\alpha'^2} \qquad \frac{1}{N} \propto g_s$$

- *semiclassical gravity limit = large N, large coupling*





AdS/CFT

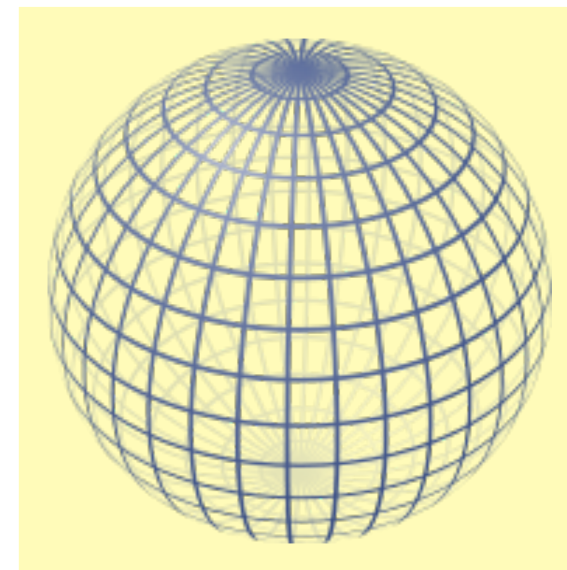
Dictionary

AdS	Field Theory
five dimensional	four dimensional
strongly coupled	weakly coupled
gravity	no gravity
metric	energy momentum tensor
gauge field	current
...	...

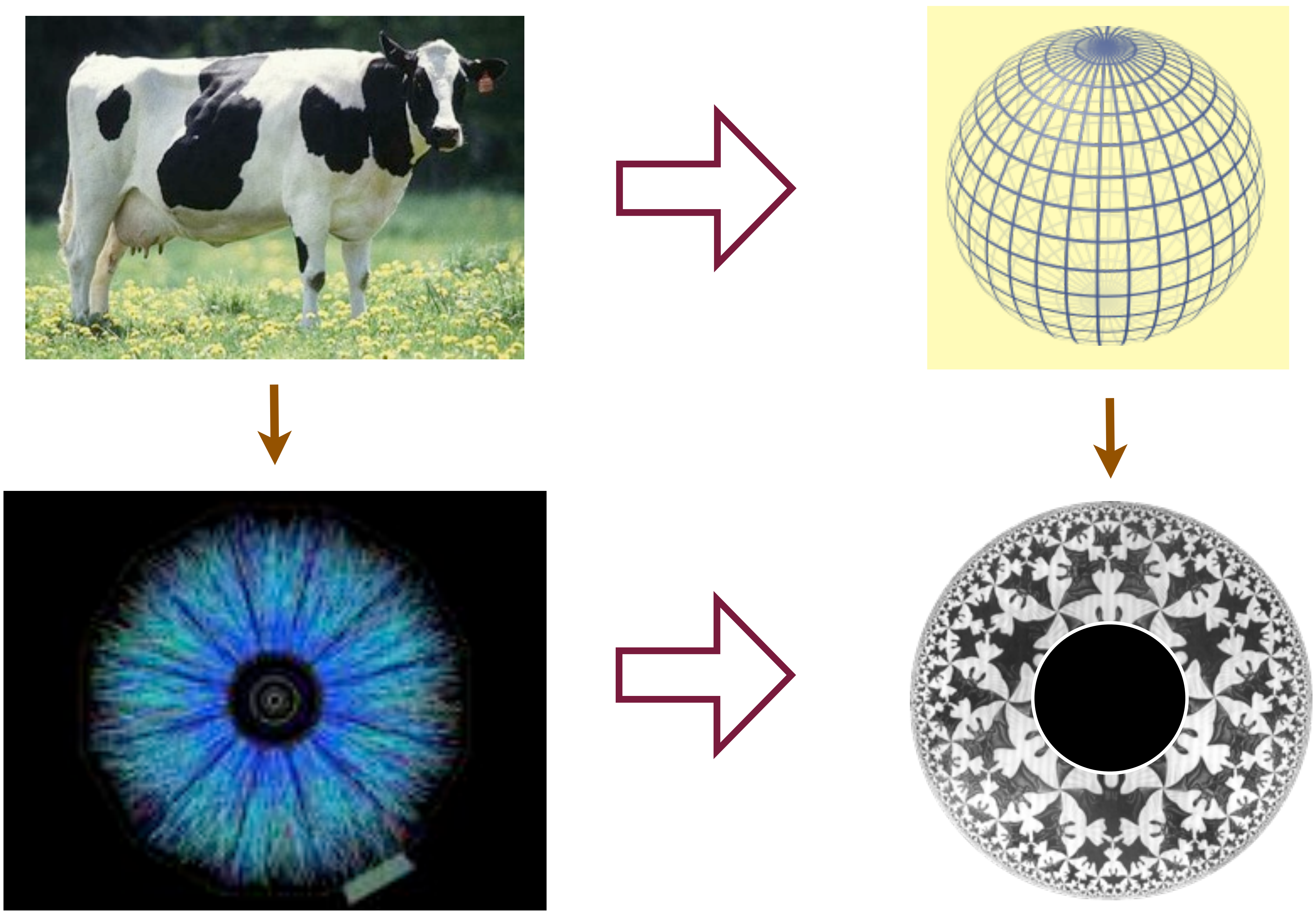
The Holographic QGP

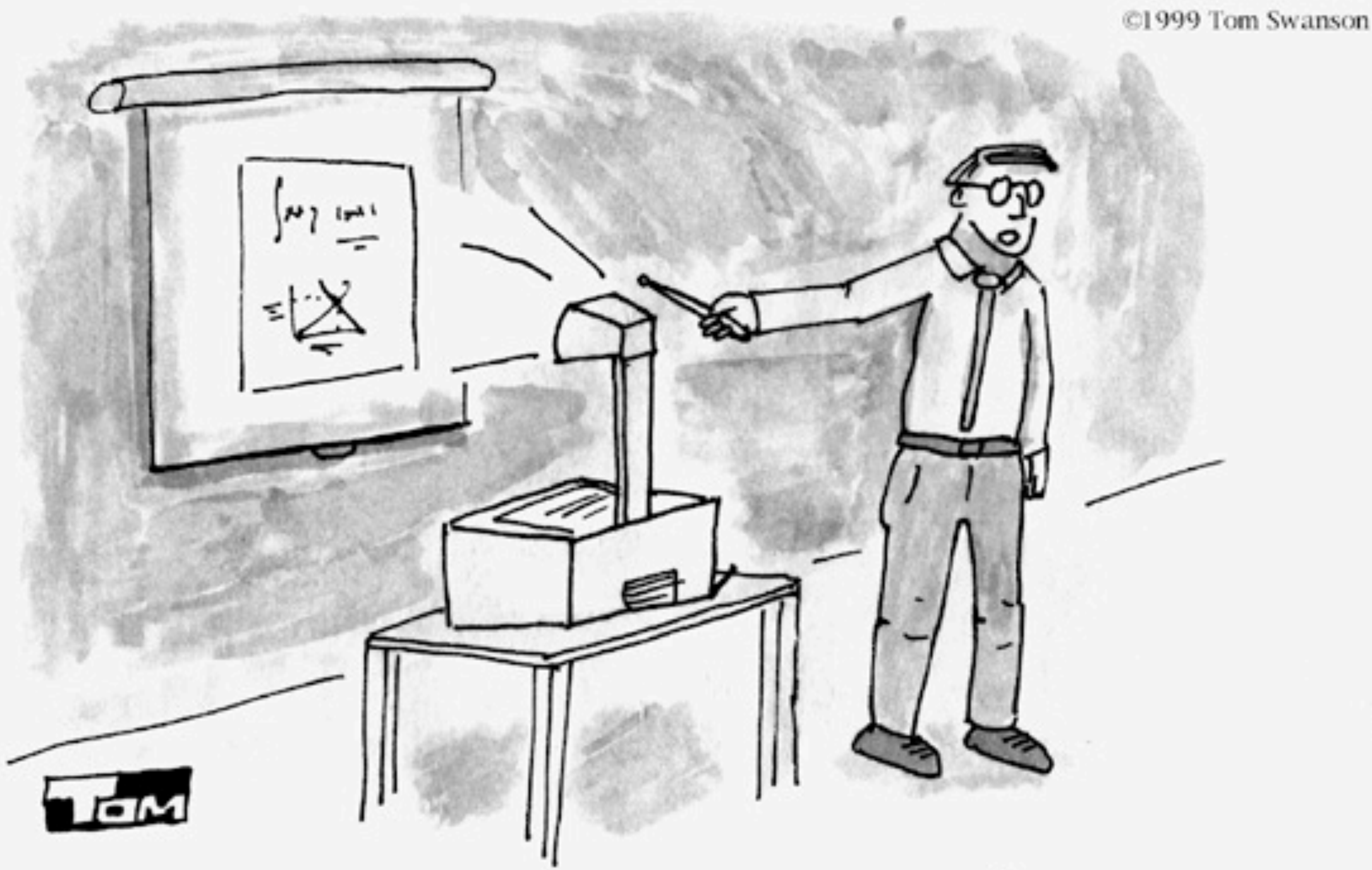
The Holographic QGP

The Holographic QGP



The Holographic QGP





ACTUALLY, THAT ASSUMPTION ISN'T REALLY NECESSARY. WE CAN SEE HERE THAT THE POINT-COW APPROXIMATION WORKS EQUALLY WELL.

Anomalous transport

- Chiral fermion: $\mathcal{H} = \pm\vec{\sigma}.\vec{p}$
- Classical U(1) symmetry broken by quantum effects

$$\partial_\mu J^\mu = c\,\epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda}$$

- Model anomaly in 4D via Chern-Simons term in 5D

$$S_{CS} = \int d^5x\,\epsilon^{MNPQR} A_M F_{NP} F_{QR}$$

- Gauge invariant up to boundary term = Anomaly (cfg. QHE)

$$\delta S_{CS} = \int_\partial d^4x\,\epsilon^{\mu\nu\rho\lambda}\,\lambda\, F_{\mu\nu} F_{\rho\lambda}$$

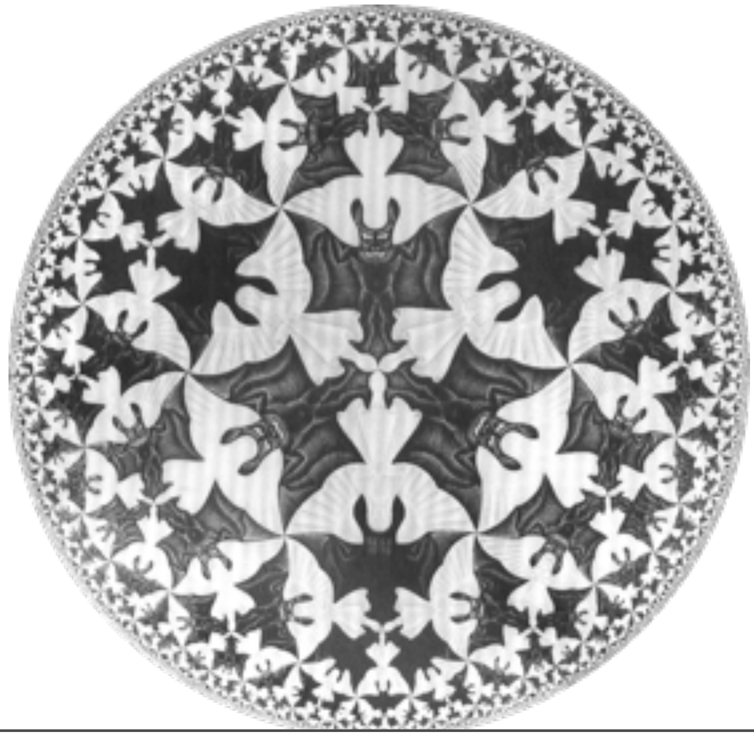
Anomalous transport

Our Model:

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

In finite T, μ state:
charged black hole in AdS



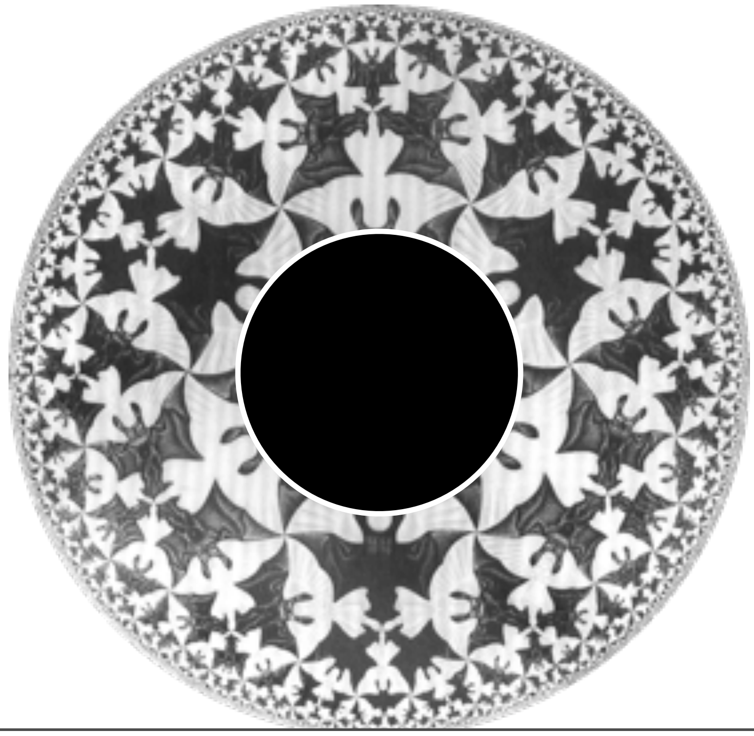
Anomalous transport

Our Model:

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

In finite T, μ state:
charged black hole in AdS



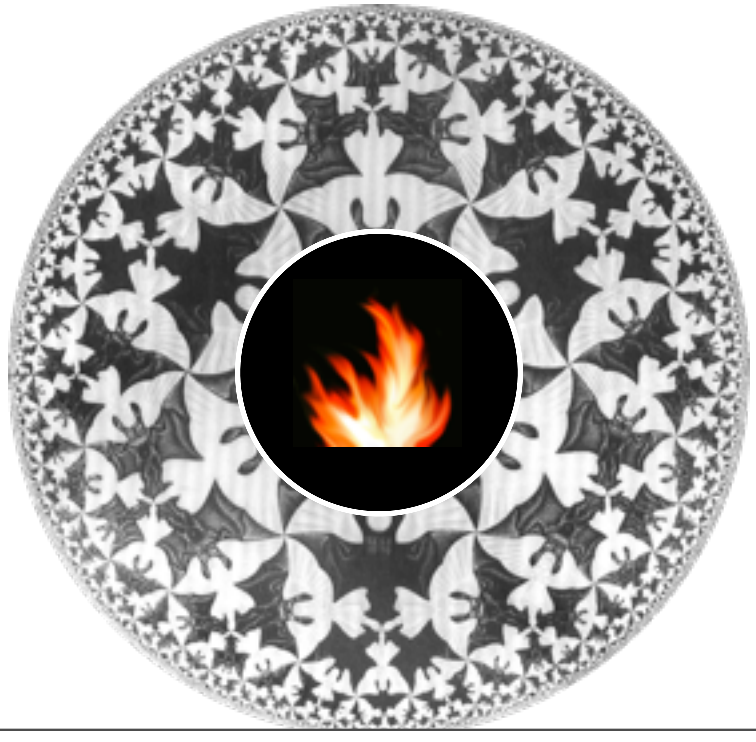
Anomalous transport

Our Model:

$$S_{EM} = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\left[R + 2\Lambda - \frac{1}{4}F_{MN}F^{MN}\right]$$

$$S_{CS} = \frac{1}{16\pi G}\int d^5x\,\epsilon^{MNPQR}A_M\left(\frac{\kappa}{3}F_{NP}F_{QR} + \lambda R^A{}_{BNP}R^B{}_{AQR}\right)$$

In finite T, μ state:
charged black hole in AdS



Anomalous transport

- Compute response to magnetic field and rotation

electric current:

$$\vec{J} = \left(\frac{\mu_5}{2\pi^2} - \frac{A_0^5}{2\pi^2}\right)\vec{B} + \frac{\mu\mu_5}{4\pi^2}\vec{\omega}$$

axial current:

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \frac{\mu^2 + \mu_5^2}{4\pi^2} + \frac{T^2}{12}\vec{\omega}$$

energy current:

$$\vec{J}_\epsilon = \frac{3\mu_5\mu^2 + \mu_5^3}{6\pi^2}\vec{B} + \frac{T^2\mu_5}{6}\vec{\omega}$$

μ = chemical potential μ_5 = axial chemical potential T = temperature A_0^5 = axial gauge field

[Erdmenger, Haack, Kaminski, Yarom], [Banerjee, Bhattacharya, Bahattacharya, Dutta Loganayagam, Surowka], [K.L., Megias, Melgar, Pena-Benitez]

Anomalous transport

- Compute response to magnetic field and rotation

electric current:

$$\vec{J} = \left(\frac{\mu_5}{2\pi^2} - \frac{A_0^5}{2\pi^2}\right)\vec{B} + \frac{\mu\mu_5}{4\pi^2}\vec{\omega}$$

axial current:

$$\vec{J}_5 = \frac{\mu}{2\pi^2}\vec{B} + \frac{\mu^2+\mu_5^2}{4\pi^2} + \frac{T^2}{12}\vec{\omega}$$

energy current:

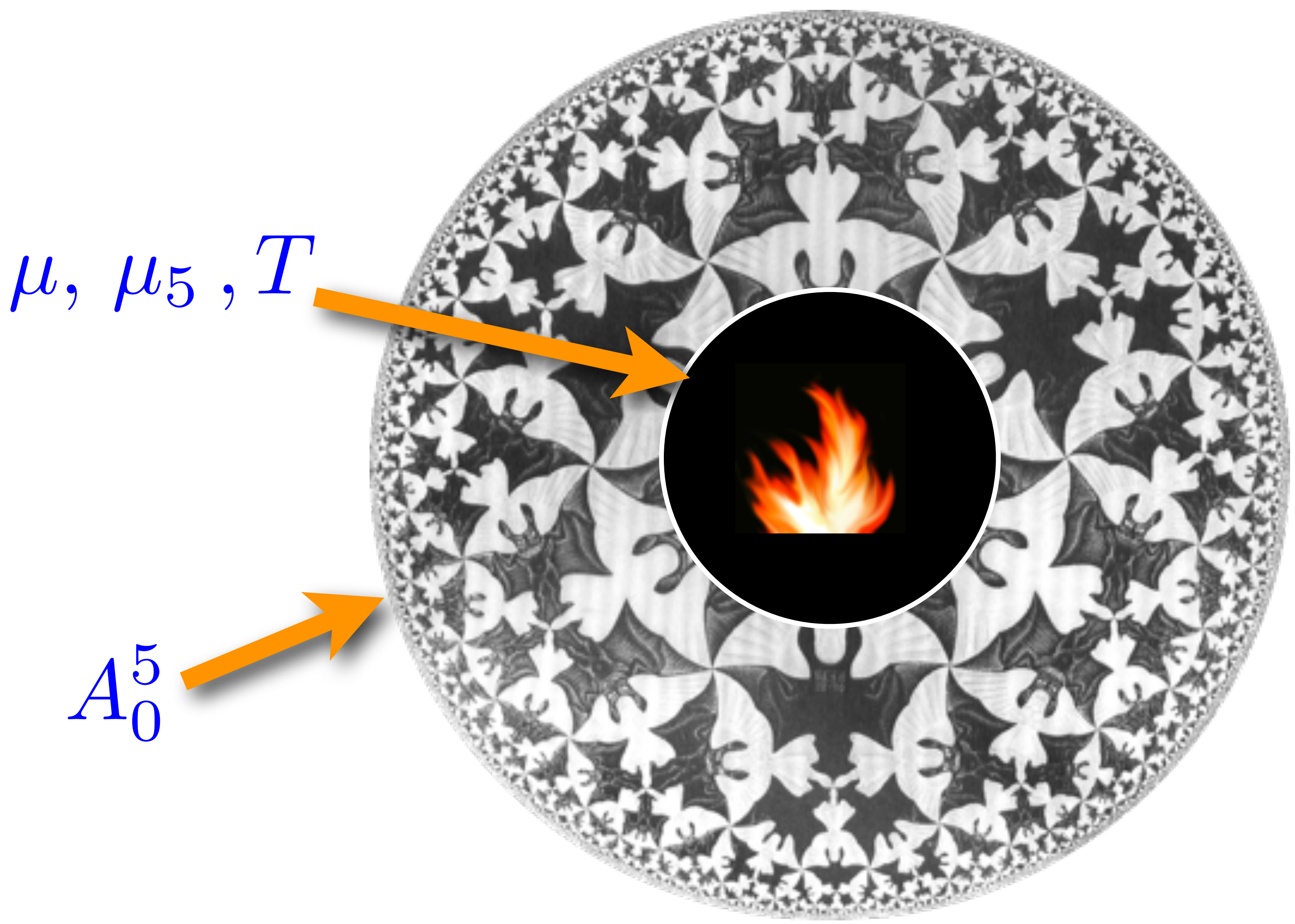
$$\vec{J}_\epsilon = \frac{3\mu_5\mu^2+\mu_5^3}{6\pi^2}\vec{B} + \frac{T^2\mu_5}{6}\vec{\omega}$$

$$(D_\mu J^\mu)^a = \epsilon^{\mu\nu\rho\lambda}\left(\frac{d^{abc}}{32\pi^2}F^b_{\mu\nu}F^c_{\rho\lambda} + \frac{b_a}{768\pi^2}R^\alpha{}_{\beta\mu\nu}R^\beta{}_{\alpha\rho\lambda}\right)$$

Anomalous transport

Gauge fields vs state variables

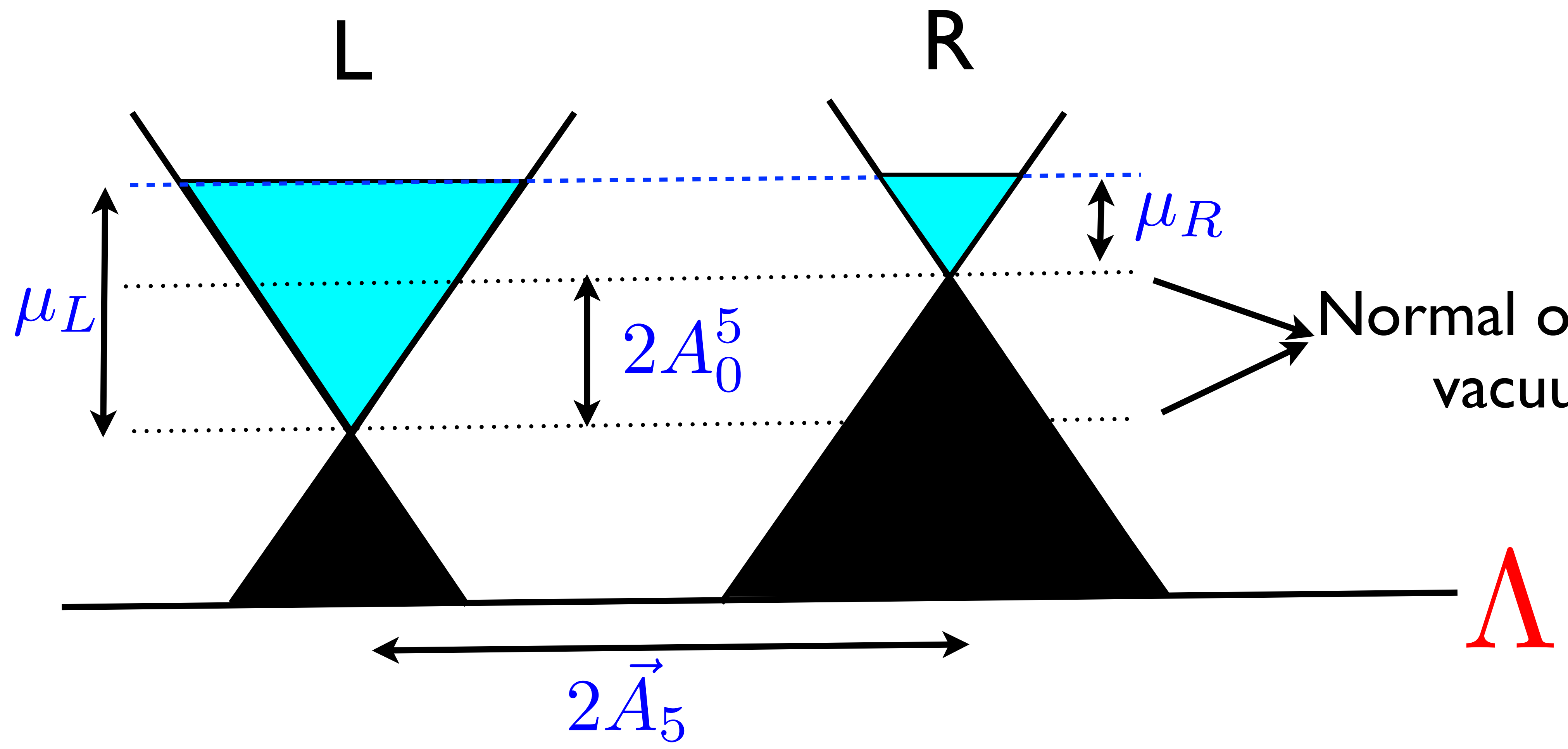
- μ, μ_5, T are state variables, determined by interior of AdS
- A_0^5 is a boundary condition for AdS, a coupling in field theory



[Rebhan, Schmitt, Stricker],
[Gynther, K.L.,Pena-Benitez,Rebhan]

Map to WSMs

Map to WSMs



$$\mu_5 = \frac{1}{2}(\mu_L - \mu_R) \qquad \mu = \frac{1}{2}(\mu_R + \mu_L)$$

CME: $$\vec{J} = \frac{1}{2\pi^2}\left(\mu_5 - A_0^5\right)\vec{B} = 0$$

No CME in equilibrium!

Map to WSMs

$$\mathcal{L}_{\text{eff}} = \bar{\psi}\gamma^{\mu}\left(i\partial_{\mu} - \gamma_5 b_{\mu}\right)\psi$$

spatial variation = axial magnetic field

- Edge state (Fermi arcs) = LLL of axial magnetic field
- Exotic response patterns (?)

$$\vec{J} = \frac{\mu}{2\pi^2}\vec{B}_5 \qquad \vec{J}_5 = \left(\frac{\mu_5}{2\pi^2} - \frac{A_0^5}{6\pi^2}\right)\vec{B}_5$$

$$\vec{J}_{\epsilon} = \cdots + \frac{T^2}{12}\vec{B}_5 \qquad \vec{J}_5 = \frac{1}{6\pi^2}\vec{A}_5 \times \vec{E}_5$$

[M. Chernodub, A. Cortijo, A. Grushin, K.L., M.A.H. Vozmediono]

Negative Magnetoresistivity

CME + Ohms law: $$\vec{J} = \sigma \vec{E} + \frac{\mu_5}{2\pi^2}\vec{B}$$

Axial anomaly: $$\partial_t \rho_5 = \frac{1}{2\pi^2}\vec{E}.\vec{B}$$

Susceptibility: $$\rho_5 = \chi_5 \mu_5$$

$$\vec{J} = \left(\frac{i}{\omega}\frac{B^2}{4\pi^4\chi_5} + \sigma\right)\vec{E}$$

$$\frac{1}{x+i\epsilon} = \mathcal{P}\frac{1}{x} - i\delta(x)$$

[Nielssen, Ninoyima], [Son, Spivak]

Negative Magnetoresistivity

CME + Ohms law: $$\vec{J} = \sigma \vec{E} + \frac{\mu_5}{2\pi^2}\vec{B}$$

Axial anomaly: $$\partial_t \rho_5 = \frac{1}{2\pi^2}\vec{E}.\vec{B} \; - \frac{1}{\tau}\rho_5$$

Susceptibility: $$\rho_5 = \chi_5 \mu_5$$

$$\vec{J} = \left(\tau \frac{B^2}{4\pi^4 \chi_5} + \sigma \right) \vec{E}$$

In real life axial charge is not conserved even for vanishing electric or magnetic fields. Decay time τ

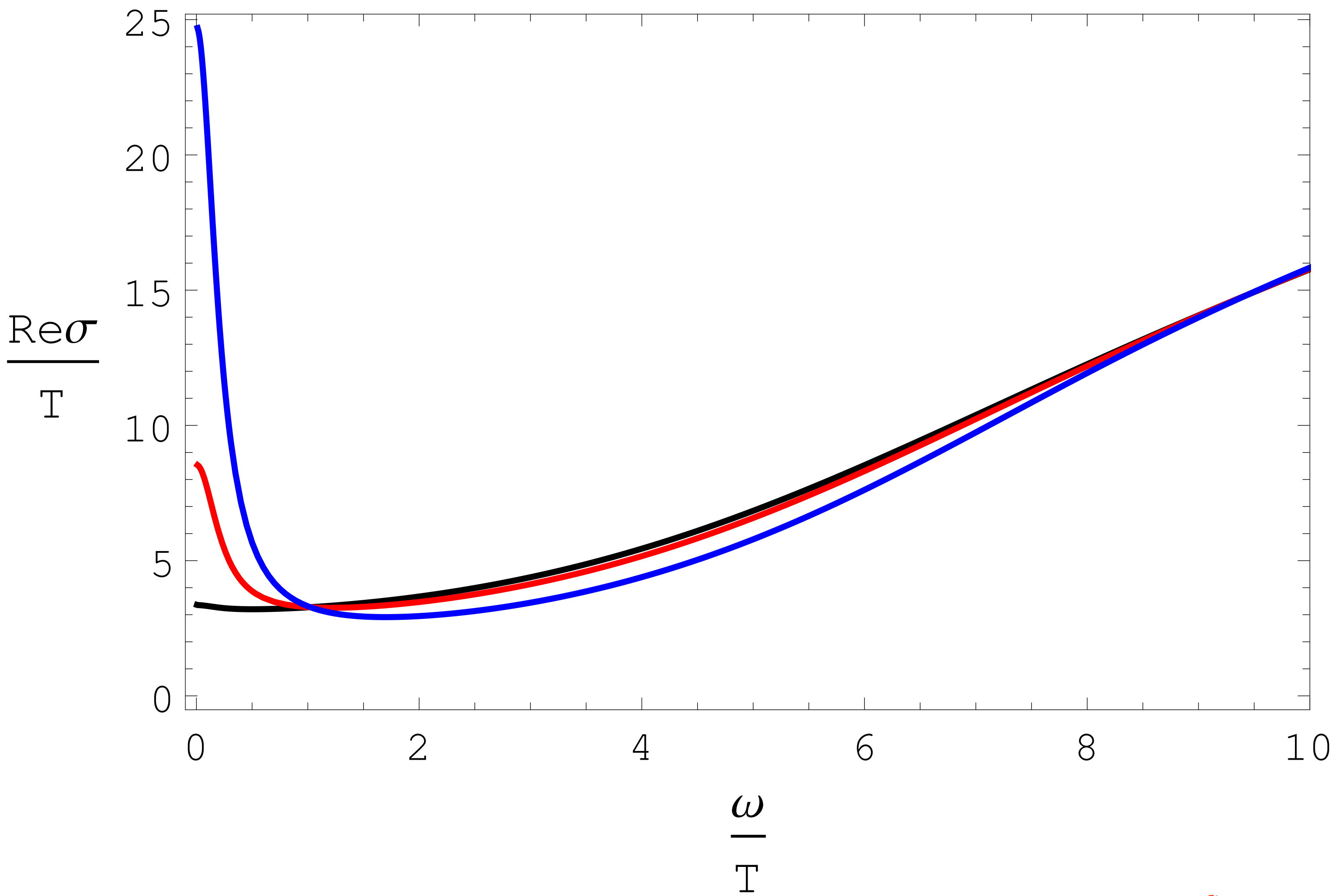
[Nielssen, Ninoyima], [Son, Spivak]

Negative Magnetoresistivity

$$\vec{J} = \left(\tau \frac{B^2}{4\pi^4 \chi_5} + \sigma \right) \vec{E}$$

- quadratic for small B-field
- old argument: linear at large B-field
  susceptibility dominated by LLL
- what does holography say?

NMR in Holography:



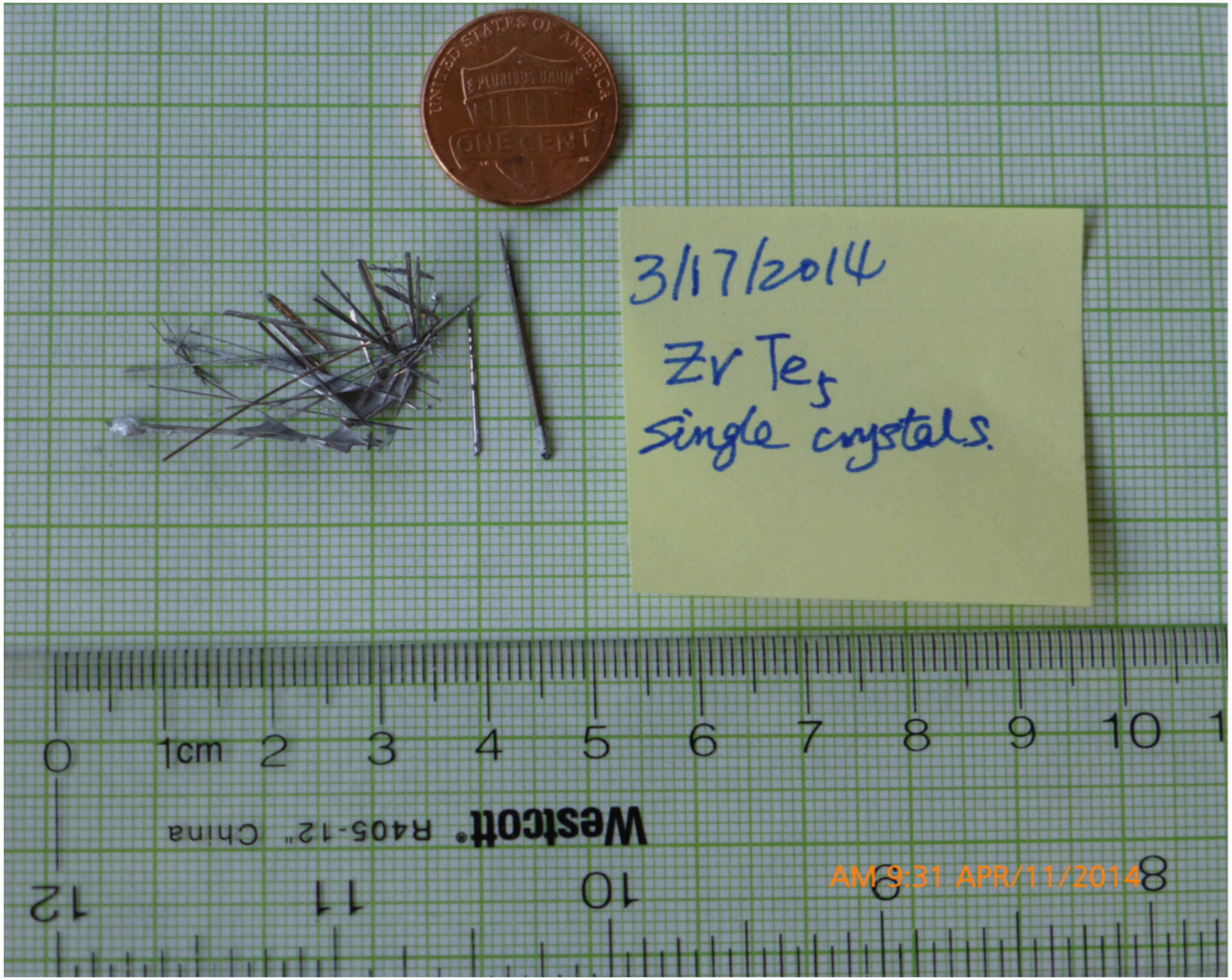
- optical conductivity at finite magnetic field
- very strong but finite peak
- sum rule

$$\frac{d}{dB}\int d\omega \Re(\sigma(\omega)) = 0$$

$$\sigma_{\rm DC} = \pi T + \frac{32 B^2 \alpha^2}{\pi^3 T^3 q^2 \phi_0^2}$$

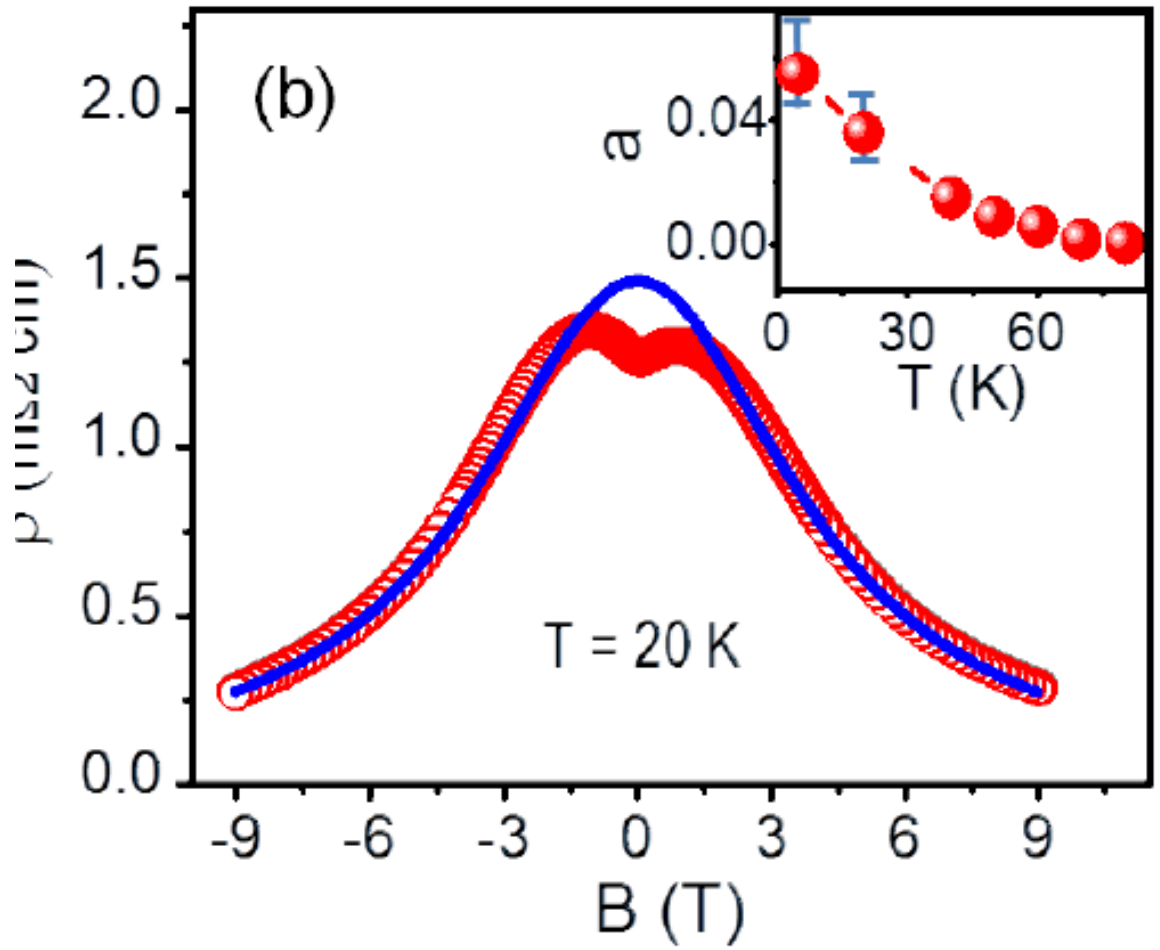
[K.L. Yan Liu, Ywaen Sun] [Jimenez-Alba, K.L., Liu, Sun]

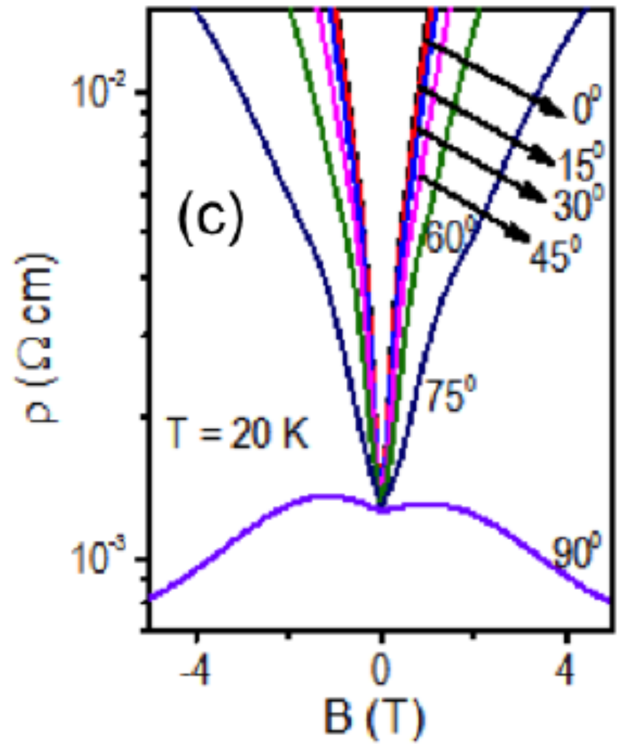
Recent Experiments



[D. Kharzeev: Lectures at Schladming Winterschool 2015]

Recent Experiments

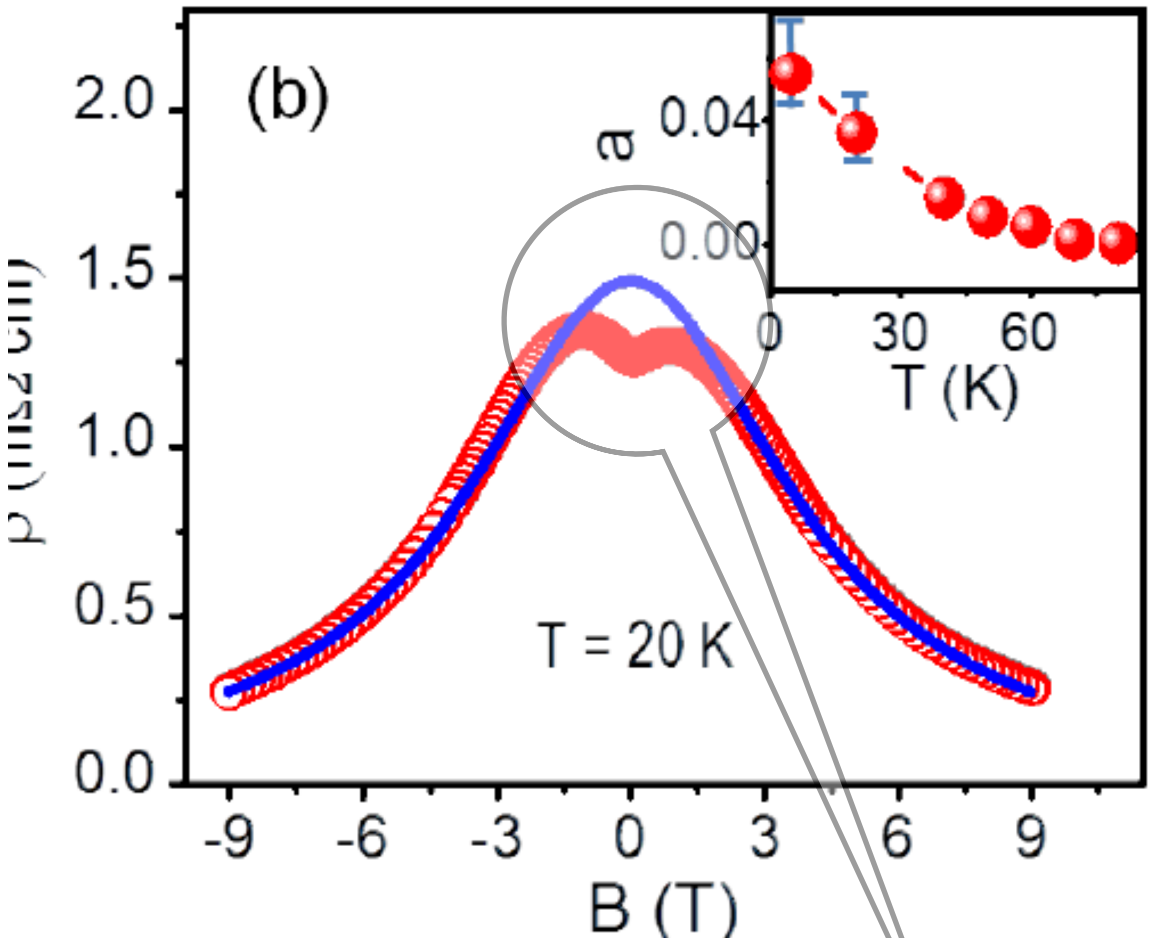


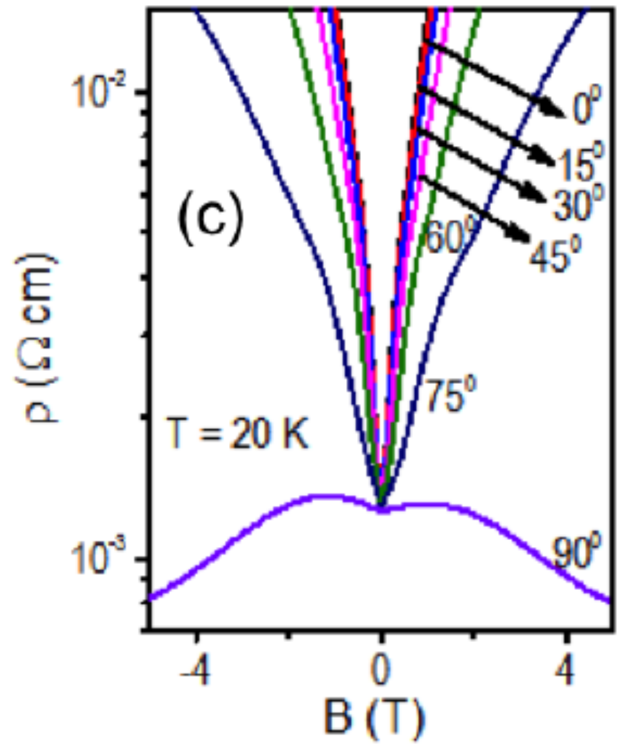


fits to 1/(a+b B^2)

B^2 behaviour!

Recent Experiments





fits to 1/(a+b B^2)

B^2 behaviour!

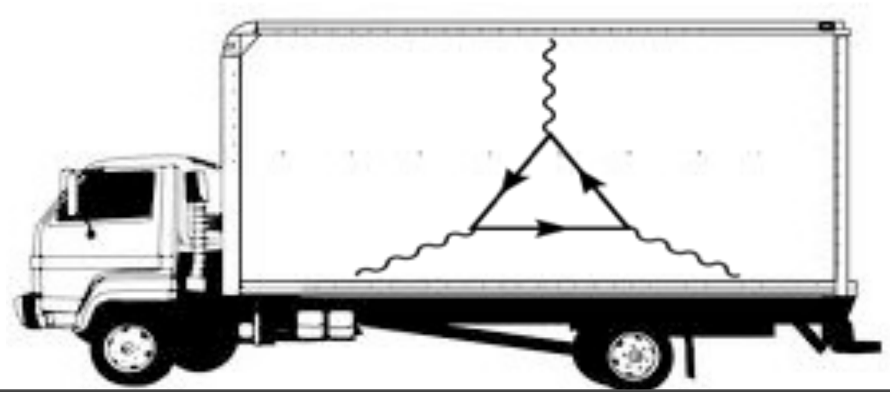
"Schmutzphysik"
(dirt physics)

Summary

- AdS is QFT
- Effective toy models for strongly coupled systems
- Particularly suited for transport in relativistic systems
- Anomalies
- (Non)-renormalization theorems
- Possible input for WSM physics?
- Gravitational anomaly measurable in a metal
- More exotic response patters observable in WSMs?
  (axial magnetic or electric fields)

Summary

- AdS is QFT
- Effective toy models for strongly coupled systems
- Particularly suited for transport in relativistic systems
- Anomalies
- (Non)-renormalization theorems
- Possible input for WSM physics?
- Gravitational anomaly measurable in a metal
- More exotic response patters observable in WSMs?
  (axial magnetic or electric fields)



Thank You!