The LHC, Cosmology and the Origin of Scales

Alberto Salvio

Universidad Autónoma de Madrid (UAM)

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Based on

- Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia (JHEP) arXiv:1307.3536
- Salvio, Strumia (JHEP) arXiv:1403.4226
- Kannike, Hütsi, Pizza, Racioppi, Raidal, Salvio, Strumia (JHEP) arXiv:1502.01334
- Salvio, Mazumdar (Phys. Lett. B) arXiv:1506.07520

Introduction

The stability bound

Inflation and the Standard Model

Models without fundamental masses and inflation

Conclusions

Experimental situation

- *Discovery of a Higgs boson in 2012* at the Large Hadron Collider (LHC). It weights $M_h \approx 125$ GeV

- So far no significant deviation from the Standard Model (SM) at the electroweak (EW) scale. Although beyond the SM (BSM) physics is not yet excluded!

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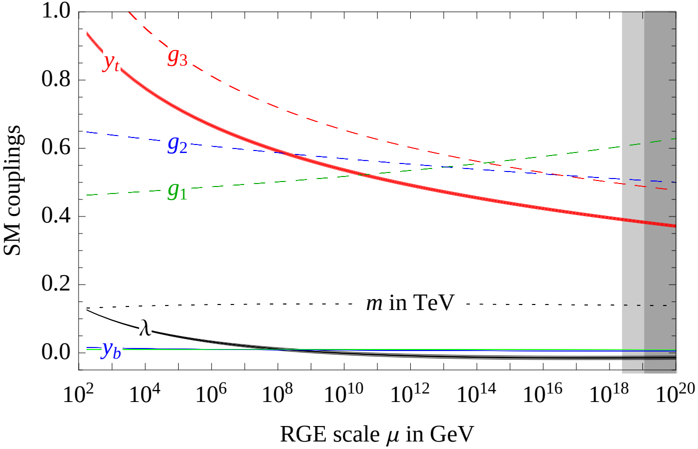
Indeed, we are in the era of precision cosmology!
(e.g. Planck, Bicep2/Keck, Bicep3/Keck, SKA, ...)

Can we really extrapolate the SM?

The consistency seems ok: some couplings seem to diverge as a function of the energy μ (Landau poles), but above $M_{\rm Pl}$



Solutions of the renormalization group equations (RGEs) of the most relevant SM parameters (defined in the $\overline{MS}$ scheme ...)

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Still there are unsolved problems:

- **Dark matter**
- **(small) neutrino masses**
- **Baryon asymmetry**
- **Understanding of EW symmetry breaking, why $m \ll M_{\rm Pl}$?, ...**

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Nevertheless, there are extensions that solve these phenomenological problems and do not invalidate all the SM predictions at very high energies …

Origin of the stability bound

- In simple terms

$$\lambda h^4$$

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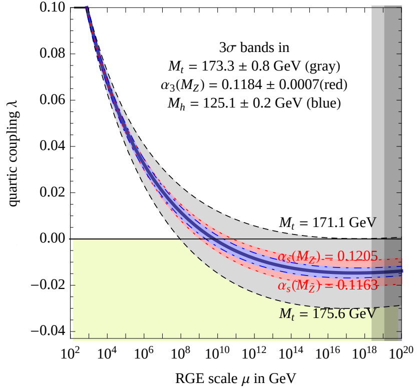
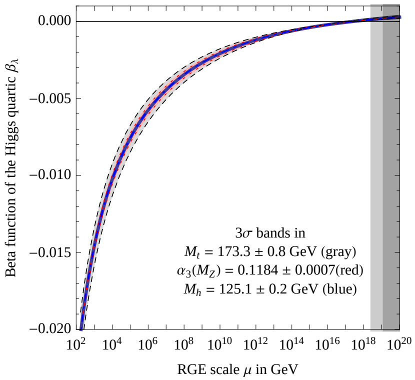
Precise "running" of λ and its β-function $\left(\beta_\lambda \equiv \frac{d\lambda}{d\ln\mu}\right)$





Result for the stability bound

$$M_h > 129.6\,\mathrm{GeV} + 2.0(M_t - 173.34\,\mathrm{GeV}) - 0.5\,\mathrm{GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\mathrm{th}}\,\mathrm{GeV}$$

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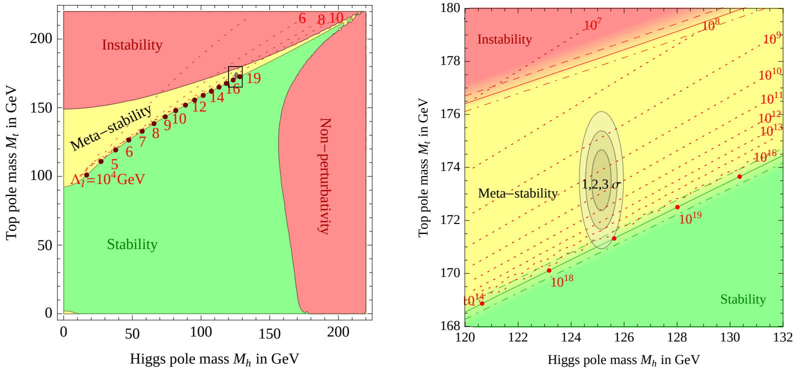
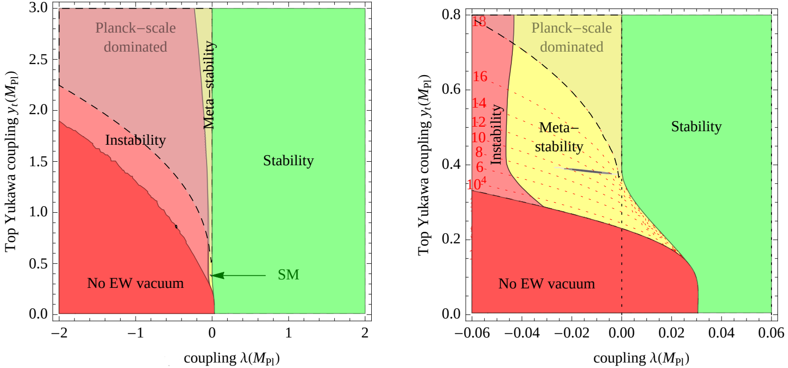
- Phase diagram of the SM:



The SM phase diagram in terms of Planck scale couplings

$y_t(M_{\mathrm{Pl}})$ versus $\lambda(M_{\mathrm{Pl}})$

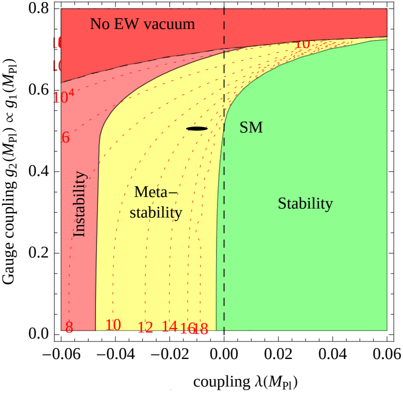


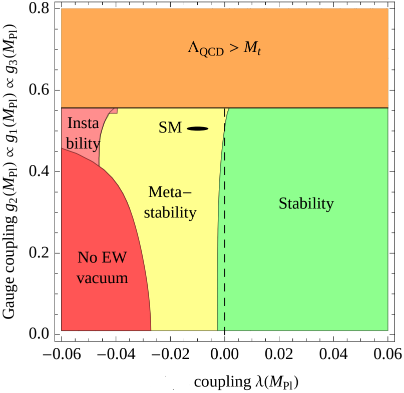
"Planck-scale dominated" corresponds to $\Lambda_I > 10^{18}$ GeV

"No EW vacuum" corresponds to a situation in which λ is negative at the EW scale

The SM phase diagram in terms of Planck scale couplings

Gauge coupling g_2 at $M_{\rm Pl}$ versus $\lambda(M_{\rm Pl})$

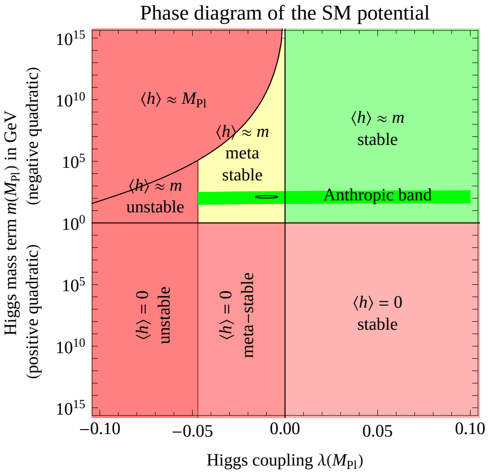




Left: $g_1(M_{\rm Pl})/g_2(M_{\rm Pl}) = 1.22$, $y_t(M_{\rm Pl})$ *and* $g_3(M_{\rm Pl})$ *are kept to the SM value*

Right: *a common rescaling factor is applied to* g_1, g_2, g_3. $y_t(M_{\rm Pl})$ *is kept to the SM value*

The SM phase diagram in terms of potential parameters



If $\lambda(M_{\rm Pl}) < 0$ there is an upper bound on m requiring $\langle h \rangle \neq 0$ at the EW scale.

This bound is, however, much weaker than the anthropic bound of
[Agrawal, Barr, Donoghue, Seckel (1997); Schellekens (2014)]

Meta-stability

- Is it worrisome?

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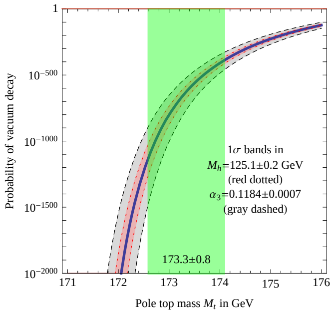

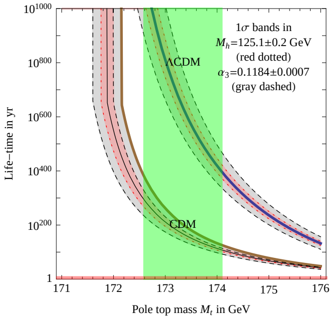

Left: *The probability that EW vacuum decay happened in our past light-cone.*

Right: The life-time of the EW vacuum, with 2 different assumptions for future cosmology: universes dominated by the cosmological constant (ΛCDM) or by the dark matter (CDM).

Inflation

[Brout, Englert, Gunzig (1978); Guth (1981); Linde (1982); Albrecht, Steinhardt (1982)]

What it can solve: horizon, flatness, monopole problems

To solve these problems inflation should last enough $\rightarrow$ lower bounds on

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_{\rm in})}\right) \equiv \text{number of } e\text{-foldings}$$

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How it is implemented (slow-roll inflation):

- we assume some scalar field φ (the inflaton)
- at some early time the potential $U(\varphi)$ is large, but flat enough
- $\to$ the Hubble $H \equiv a^{-1}da/dt$ changes slowly $\to a(t) \approx a(t_{\rm in})e^{Ht}$

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For each model we can compute parameters that are observable (e.g. by PLANCK):

- the scalar amplitude A_s,
- its spectral index n_s
- and the tensor-to-scalar ratio $r = A_t/A_s$

Inflation in the SM (the inflaton is h)

[Barvinsky, Kamenshchik, Mishakov (1996); Bezrukov, Shaposhnikov (2008)]

- The model:

$$\mathscr{L} = \mathscr{L}_{\rm EH} + \mathscr{L}_{\rm SM} - \xi |H|^2 R$$

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SM potential

The potential is $$U \equiv \frac{V}{\Omega^4} = \frac{\bar{M}_{\rm Pl}^4 \lambda (h^2 - v^2)^2}{4(\bar{M}_{\rm Pl}^2 + \xi h^2)^2}$$

Ω^2 corresponds, by definition, to what multiplies R in $\mathscr{L}$
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$$U \ll \bar{M}_{\rm Pl}^4 \quad \text{(such that quantum Einstein gravity effects are small)}$$

ξ generically is very large

- Perturbative unitarity is violated at an energy close to the inflationary one:
[Burgess, Lee, Trott (2009); Barbon, Espinosa (2009); Hertzberg (2010); Burgess, Lee, Trott (2010); Burgess, Patil, Trott (2014)]

Initial conditions for the Higgs field

Anothe condition to have inflation is that $\overline{\Pi} = \frac{d\varphi}{dt}(t_{\rm in})$ is small enough .
However, it is *not* natural: any $|\overline{\Pi}| \ll \bar{M}_{\rm Pl}^2$ is allowed

In Higgs inflation one usually requires $\overline{\Pi} \ll \sqrt{U} \sim 10^{-5}\bar{M}_{\rm Pl}^2$ and $\overline{\chi} \sim \bar{M}_{\rm Pl}$

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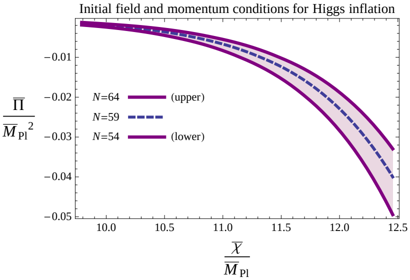
Classical level

Even if we start from a $\overline{\Pi}^2 \gg U$ we can compensate with a slightly larger initial value of the Higgs field, $\overline{\chi}$.



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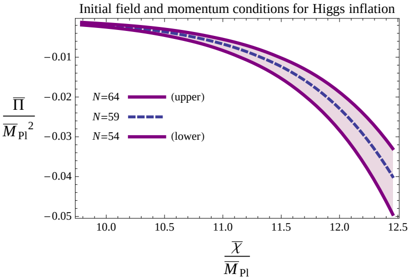
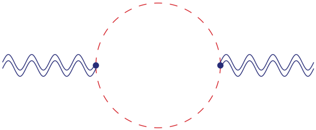


Quantum level

A large value of ξ can generate higher order terms in the Lagrangian: R^2, R^3, ...

For example this diagram generates R^2 $\longrightarrow$



The absence of fundamental masses is a key to inflation

For any scalar s

$$U(s) = \frac{\lambda_S s^4}{(2\xi_S s^2)^2} \bar{M}_{\rm Pl}^4 = \frac{\lambda_S}{4\xi_S^2} \bar{M}_{\rm Pl}^4$$

Some motivations for models without fundamental masses (agravity)

Motivation 1: inflation

As we saw, the potential of a scalar s in agravity is

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$\rightarrow$ **inflation!**

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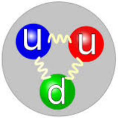
Motivation 2: origin of mass and EW symmetry breaking

Most of the mass of the matter we see has a dynamical origin

Example: the proton mass $\longrightarrow$



Is it possible to generate all the mass dynamically? If yes, with $m \ll M_{\rm Pl}$?

Agravity scenario

The general agravity Lagrangian:

$$\mathscr{L} = \frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \mathscr{L}_{\rm SM}^{\rm adim} + \mathscr{L}_{\rm BSM}^{\rm adim}$$

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Non-gravitational sector

- $\mathscr{L}_{\rm SM}^{\rm adim}$ is the SM $\mathscr{L}$ (without $m^2|H|^2/2$ plus $-\xi_H|H|^2R$):
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adding a scalar s $\rightarrow \mathscr{L}_{\rm BSM}^{\rm adim} = ... + \lambda_{HS}s^2|H|^2/2 - \xi_S s^2 R/2$

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One can generate the EW and Planck scales such that their hierarchy is stable under quantum corrections (including gravity effects)! ... This requires $\lambda_{HS} \ll 1$ ...

Quantum Agravity is renormalizable

(clear from absence of fundamental scales)

However, looking at the spectrum:

- **(i)** massless graviton
- **(ii)** scalar z with mass $M_0^2 \sim \frac{1}{2} f_0^2 \bar{M}_{\rm Pl}^2$
- **(iii)** massive graviton with mass $M_2^2 = \frac{1}{2} f_2^2 \bar{M}_{\rm Pl}^2$ and negative norm, but with energy bounded from below

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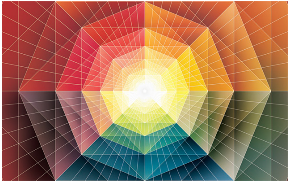
Is there a dark side of quantum mechanics?

negative norms

positive norm

Absence of fundamental scales = “Classical Scale Invariance”



For Quanta Magazine (Simons Foundation)

Scale invariance is broken by quantum corrections

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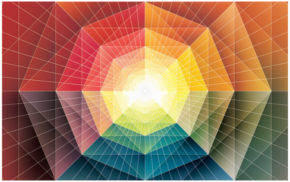

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Previous literature on classical scale invariance ... *standing on the shoulder of giants!*

[Alexander-Nunneley, Carone, Chang, Chun, Englert, Fatelo, Foot, Gastmans, Gerard, Hambye, Heikinheimo, Hempfling, Henz, Hill, Hur, Iso, Jaeckel, Jung, Karananas, Khoze, Ko, Kobakhidze, Lee, Meissner, McDonald, Nicolai, Ng, Okada, Orikasa, Pawlowski, Pilaftsis, Quiros, Raidal, Racioppi, Ramos, Rodigast, Spannowsky, Spethmann, Strumia, Tkachov, Truffin, Tuominen, Volkas, Wetterich, Weyers, Wu]

Dynamical generation of $\langle s \rangle$

Two basic conditions:

$$\begin{cases} \lambda_S(\langle s \rangle) & \simeq & 0 & \leftrightarrow & \text{nearly vanishing cosmological constant (dark energy)} \\ \frac{d\lambda_S}{ds}(\langle s \rangle) & \simeq & 0 & \leftrightarrow & \text{minimum condition (it fixes } \langle s \rangle\text{)} \end{cases}$$

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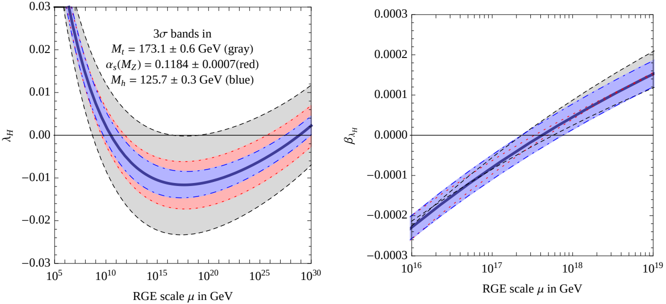
It is possible to satisfy these conditions as they are realized in the physics we know (the SM)!



Predictions for inflation

- The minimal realistic model has at least 3 scalars:

h, s and "a graviscalar" z
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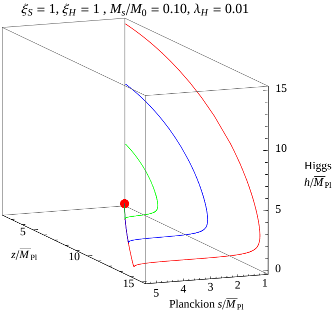
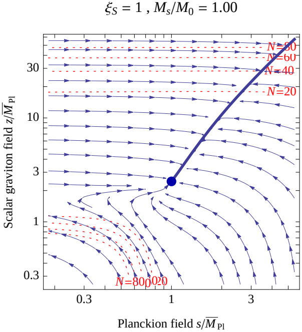
M_s = mass of *s*
M_0 = mass of *z*





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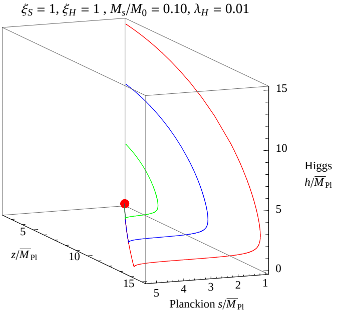
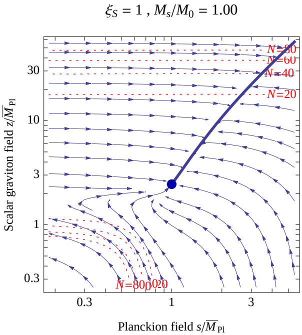
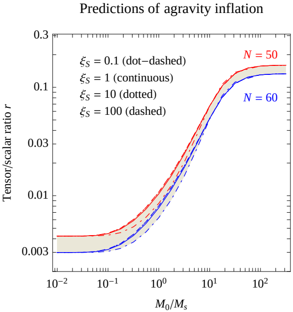
h, *s* and "a graviscalar" *z* (from $f(R) = R^2$)

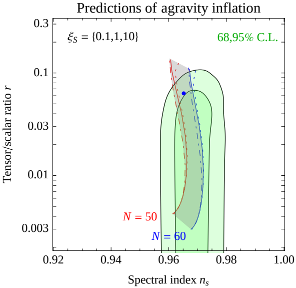
M_s = mass of s
M_0 = mass of z





- **left:** when $M_s \ll (\gg) M_0$, the inflaton is s (z)
- **right:** comparison with a global fit of Planck and BICEP2/Keck





Other virtues

This scenario also

- leads to successful reaheating
- has natural dark matter candidates and can account for neutrino masses

Conclusions

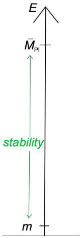
1. *The SM is compatible with data (so far), but for sure it has to be extended. There are nevertheless extensions that solve its phenomenological problems and do not invalidate all its predictions at very high energies (See Point* **3.***)*

2. *We presented the currently most precise SM stability bound. Data indicate some tension: the EW vacuum is metastable (life-time $>$ than the age of the universe) although absolute stability is still possible. We also discussed inflation in the SM.*

3. *A rationale for inflation and a dynamical origin of mass can be obtained in models of all interactions (including gravity) where fundamental scales are absent: agravity.*

   We did not show it here, but in agravity one also has

   - *dark matter and neutrino masses,*
   - *a hierarchy between the EW and Planck scales that is stable under quantum corrections.* $\longrightarrow$



Thank you very much for your attention!

Extra slides

Qualitative origin of the stability bound

$$V_{\text{eff}} = V + V_1 + V_2 + \dots$$

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2, \quad V_1(h) = \frac{1}{(4\pi)^2}\sum_i c_i m_i(h)^4 \left(\ln \frac{m_i(h)^2}{\mu^2} + d_i\right), \quad \dots$$

where $h^2 \equiv 2|H|^2$ and c_i and d_i are ~ 1 constants

By substituting bare parameters $\rightarrow$ renormalized ones

$\Longrightarrow \dfrac{\partial V_{\text{eff}}}{\partial \mu} = 0$ and one is free to choose μ to improve perturbation theory

- Since at large fields, $h \gg v$, we have $m_i(h)^2 \propto h^2$, we choose $\mu^2 = h^2$, then

$$V_{\text{eff}}(h) = \frac{\lambda(h)}{4}\left(h^2 - v(h)^2\right)^2 + \dots = -\frac{m(h)^2}{2}h^2 + \frac{\lambda(h)}{4}h^4 + \dots$$

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So for $h \gg v$

$$V_{\text{eff}}(h) \approx \frac{\lambda(h)}{4} h^4$$

- M_h contributes positively to $\lambda \to$ lower bound on M_h
- y_t contributes negatively to the running of $\lambda \to$ upper bound on M_t

Procedure to extract the stability bound

Steps of the procedure

- $V_{\rm eff}$, including relevant parameters
- RGEs of the relevant couplings
- Values of the relevant parameters (also called *threshold corrections* or *matching conditions*) at the EW scale (e.g. at M_t) ...

Finally impose that $V_{\rm eff}$ at the EW vacuum is the absolute minimum!

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State-of-the-art loop calculation

- Two loop $V_{\rm eff}$ including the leading couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop RGEs for $\{\lambda, y_t, g_3, g_2, g_1\}$ and one loop RGE for $\{y_b, y_\tau\}$ ...
- Two loop values of $\{\lambda, y_t, g_3, g_2, g_1\}$ at M_t ...

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Previous calculations

[Cabibbo, Maiani, Parisi, Petronzio (1979); Casas, Espinosa, Quiros (1994, 1996); Bezrukov, Kalmykov, Kniehl, Shaposhnikov (2012); Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia (2012)]

Input values of the SM observables

(used to fix relevant parameters: λ, y_t, g_1, g_2)

$$M_h = 125.15 \pm 0.24\ \mathrm{GeV}$$

[CMS Collaboration (2013, 2014); ATLAS Collaboration (2013, 2014); average from Giardino, Kannike, Masina, Raidal and Strumia (2014)]

M_W	$=$	$80.384 \pm 0.014\ \mathrm{GeV}$	Mass of the W boson [1]
M_Z	$=$	$91.1876 \pm 0.0021\ \mathrm{GeV}$	Mass of the Z boson [2]
M_h	$=$	$125.15 \pm 0.24\ \mathrm{GeV}$	(source already quoted)
M_t	$=$	$173.34 \pm 0.76 \pm 0.3\ \mathrm{GeV}$	Mass of the top quark [3]
$V \equiv (\sqrt{2}G_\mu)^{-1/2}$	$=$	$246.21971 \pm 0.00006\ \mathrm{GeV}$	Fermi constant [4]
$\alpha_3(M_Z)$	$=$	0.1184 ± 0.0007	$SU(3)_c$ coupling (5 flavors) [5]

[1] TeVatron average: FERMILAB-TM-2532-E. LEP average: CERN-PH-EP/2006-042

[2] 2012 Particle Data Group average, pdg.lbl.gov

[3] ATLAS, CDF, CMS, D0 Collaborations, arXiv:1403.4427. Plus an uncertainty $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ because of non-perturbative effects [Alekhin, Djouadi, Moch (2013)]

[4] MuLan Collaboration, arXiv:1211.0960

[5] S. Bethke, arXiv:1210.0325

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Step 1: effective potential

RG-improved tree level potential (V)

Classical potential with couplings replaced by the running ones

One loop (V_1)

$V_{\rm eff}$ depends mainly on the top, W, Z, h and Goldstone squared masses in the classical background h: in the Landau gauge ... they are

$$t \equiv \frac{y_t^2 h^2}{2}, \quad w \equiv \frac{g_2^2 h^2}{4}, \quad z \equiv \frac{(g_2^2 + 3g_1^2/5)h^2}{4}, \quad m_h^2 \equiv 3\lambda h^2 - m^2, \quad g \equiv \lambda h^2 - m^2$$

$\to (4\pi)^2 V_1$ is (in a suitable renormalization scheme, called $\overline{\rm MS}$)

$$\frac{3w^2}{2}\left(\ln\frac{w}{\mu^2} - \frac{5}{6}\right) + \frac{3z^2}{4}\left(\ln\frac{z}{\mu^2} - \frac{5}{6}\right) - 3t^2\left(\ln\frac{t}{\mu^2} - \frac{3}{2}\right) + \frac{m_h^4}{4}\left(\ln\frac{m_h^2}{\mu^2} - \frac{3}{2}\right) + \frac{3g^2}{4}\left(\ln\frac{g}{\mu^2} - \frac{3}{2}\right)$$

In order to keep the logarithms in the effective potential small we choose

$$\mu = h$$

Indeed, t, w, z, m_h^2 and g are $\propto h^2$ for $h \gg m$

Two loop (V_2)

It is very complicated, but always depend on t, w, z, m_h^2, g plus g_i

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Step 2: running couplings

For a generic parameter p we write the RGE as

$$\frac{dp}{d\ln\mu^2} = \frac{\beta_p^{(1)}}{(4\pi)^2} + \frac{\beta_p^{(2)}}{(4\pi)^4} + \ldots$$

They were computed before in the literature up to three loops

(very long and not very illuminating expressions at three loops)

One loop RGEs for λ, y_t^2, g_i^2 and m^2

$$\begin{aligned}
\beta_\lambda^{(1)} &= \lambda\left(12\lambda + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}\right) - 3y_t^4 + \frac{9g_2^4}{16} + \frac{27g_1^4}{400} + \frac{9g_2^2g_1^2}{40}, \\
\beta_{y_t^2}^{(1)} &= y_t^2\left(\frac{9y_t^2}{2} - 8g_3^2 - \frac{9g_2^2}{4} - \frac{17g_1^2}{20}\right), \\
\beta_{g_1^2}^{(1)} &= \frac{41}{10}g_1^4, \quad \beta_{g_2^2}^{(1)} = -\frac{19}{6}g_2^4, \quad \beta_{g_3^2}^{(1)} = -7g_3^4, \\
\beta_{m^2}^{(1)} &= m^2\left(6\lambda + 3y_t^2 - \frac{9g_2^2}{4} - \frac{9g_1^2}{20}\right)
\end{aligned}$$

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Step 3: threshold corrections

$$
\begin{aligned}
\lambda(M_t) &= 0.12604 + 0.00206\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) - 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.00030_{\mathrm{th}}\\
\frac{m(M_t)}{\mathrm{GeV}} &= 131.55 + 0.94\left(\frac{M_h}{\mathrm{GeV}} - 125.15\right) + 0.17\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) \pm 0.15_{\mathrm{th}}\\
y_t(M_t) &= 0.93690 + 0.00556\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00042\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.00050_{\mathrm{th}}\\
g_2(M_t) &= 0.64779 + 0.00004\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) + 0.00011\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}}\\
g_Y(M_t) &= 0.35830 + 0.00011\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right) - 0.00020\,\frac{M_W - 80.384\,\mathrm{GeV}}{0.014\,\mathrm{GeV}}\\
g_3(M_t) &= 1.1666 + 0.00314\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} - 0.00046\left(\frac{M_t}{\mathrm{GeV}} - 173.34\right)
\end{aligned}
$$

The theoretical uncertainties on these quantities are much lower than those used in previous determinations of the stability bound

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Tunneling probability

The probability of creating a bubble of the absolute minimum in $dV\,dt$ **was found by** *[Kobzarev, Okun, Voloshin (1975); Coleman (1977); Callan, Coleman (1977)]*

$$d\wp = dt\,dV\;\Lambda_B^4\,e^{-S(\Lambda_B)}$$

$S(\Lambda_B) \equiv$ the action of the bounce of size $R = \Lambda_B^{-1}$, given by $S(\Lambda_B) = \dfrac{8\pi^2}{3|\lambda(\Lambda_B)|}$

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h inflation: classical analysis

The part of S that depends on $g_{\mu\nu}$ and H *only* $\longrightarrow$

$$S_{gH} = \int d^4x\sqrt{-g}\left[-\left(\frac{\bar{M}_{\rm Pl}^2}{2} + \xi|H|^2\right)R + |D_\mu H|^2 - V(H)\right]$$

The non-minimal coupling can be eliminated through a *conformal* transformation ...

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{2\xi|H|^2}{\bar{M}_{\rm Pl}^2}$$

In the unitary gauge, where the only scalar field is the radial mode $h \equiv \sqrt{2|H|^2}$

$$S_{gH} = \int d^4x\sqrt{-\hat{g}}\left[-\frac{\bar{M}_{\rm Pl}^2}{2}\hat{R} + K\frac{(\partial h)^2}{2} - \frac{V}{\Omega^4}\right]$$

where $K \equiv (\Omega^2 + 6\xi^2 h^2/\bar{M}_{\rm Pl}^2)/\Omega^4$ and we set the gauge fields to zero.

The h kinetic term can be made canonical through $h = h(\chi)$ defined by

$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/\bar{M}_{\rm Pl}^2}{\Omega^4}}$$

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$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/\bar{M}_{\rm Pl}^2}{\Omega^4}}$$

This is what we want in order to have slow-roll ...

Thus, χ feels a potential $\quad U \equiv \dfrac{V}{\Omega^4} = \dfrac{\lambda(h(\chi)^2 - v^2)^2}{4(1+\xi h(\chi)^2/\bar{M}_{\rm Pl}^2)^2} \overset{h > \bar{M}_{\rm Pl}/\sqrt{\xi}}{\approx} \dfrac{\lambda}{4\xi^2}\bar{M}_{\rm Pl}^4$

h inflation: classical analysis

All parameters can be fixed through experiments and observations ...

ξ can be fixed requiring the normalization of *[Planck Collaboration (2015)]*

$$A_s(h_{\rm in}) \approx (2.14 \pm 0.05) \times 10^{-9}$$

$h_{\rm in}$ is fixed by requiring $$N = \int_{h_{\rm end}}^{h_{\rm in}} \frac{U}{\bar{M}_{\rm Pl}^2} \left(\frac{dU}{dh}\right)^{-1} \left(\frac{d\chi}{dh}\right)^2 dh \approx 59$$

[Garcia-Bellido, Figueroa, Rubio (2009); Bezrukov, Gorbunov, Shaposhnikov (2009)]

and $h_{\rm end}$ is the field value at the end of inflation: $\epsilon(h_{\rm end}) \approx 1$

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and $h_{\rm end}$ is the field value at the end of inflation: $\quad \epsilon(h_{\rm end}) \approx 1$

This leads to $\xi \approx 4.7 \times 10^4 \sqrt{\lambda}$ and indicates that xi has to be large ...

h inflation: quantum analysis

Two regimes

- small fields: $h \ll \bar{M}_{\rm Pl}/\xi$ (the SM is recovered)
- large fields: $h \gg \bar{M}_{\rm Pl}/\xi$ (chiral EW action with VEV set to $h/\Omega \approx \bar{M}_{\rm Pl}/\sqrt{\xi}$) $\rightarrow$ decoupling of h in the inflationary regime

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State-of-the-art calculation of the bound on M_h to have inflation

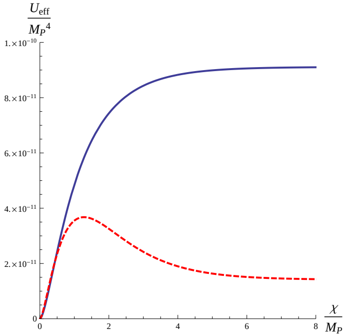
- Two loop effective potential $U_{\rm eff}$ in the inflationary regime including the effect of ξ and the leading SM couplings $= \{\lambda, y_t, g_3, g_2, g_1\}$
- Three loop SM RGE from the EW scale up to $\bar{M}_{\rm Pl}/\xi$ for $\{\lambda, y_t, g_3, g_2, g_1\}$ ...
- Two loop RGE for the same SM couplings and one loop RGE for ξ in the chiral EW theory
- Two loop threshold corrections at the top mass, for these SM couplings

Bound on M_h to have h inflation

Derivation

1. We fix ξ as in the classical case, but with U replaced by $U_{\rm eff}$.
   ... this already gives $\xi_{\rm inf} \equiv \xi(\bar{M}_{\rm Pl}/\sqrt{\xi_t})$, where conventionally $\xi_t = \xi(M_t)$

2. If M_h is too small (or M_t is too large) we go from the blue behavior to the red one! When the slope is negative the Higgs cannot roll towards the EW vacuum



We set the th. errors to zero and the input parameters to the central values, except M_t:

- *Solid line:* $M_t = 171.43\,GeV$ *(ξ fixed as described above)*
- *Dashed line:* $M_t = 171.437\,GeV$ *($\xi_t = 300$)*

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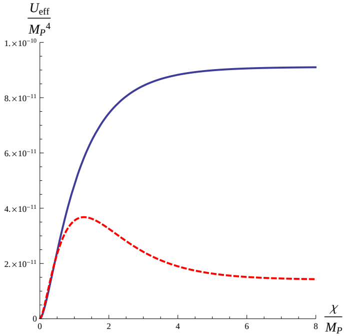

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Result (bound to have h inflation)

$$M_h > 129.4\,\mathrm{GeV} + 2.0(M_t - 173.34\,\mathrm{GeV}) - 0.5\,\mathrm{GeV}\,\frac{\alpha_3(M_Z) - 0.1184}{0.0007} \pm 0.3_{\rm th}\,\mathrm{GeV}$$

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Violation of perturbative unitarity

Consider the scattering "Higgs Higgs $\to$ Higgs Higgs" mediated by a graviton

At tree-level and for large ξ the leading contribution comes from the $-\xi|H|^2 R$ term:

$$-\frac{\xi}{\bar{M}_{\mathrm{Pl}}}|H|^2(\partial_\mu\partial_\nu h_{\mu\nu} - \eta_{\mu\nu}\partial^2 h_{\mu\nu})$$

The amplitude in the rest frame is then

$$\mathcal{A}(E) = \frac{6\xi^2 E^2}{\bar{M}_{\mathrm{Pl}}^2}$$

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$\to$ unitarity is violated at

$$E \sim \frac{\bar{M}_{\rm Pl}}{\xi}$$

This is typically smaller than the energy during inflation

$$E_{\rm inf} \sim \left(\frac{\lambda}{4\xi^2}\bar{M}_{\rm Pl}^4\right)^{1/4}$$

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Quantum agravity

Quantum effects are mostly encoded in the RGEs ...

They are important to obtain n_s and r and to dynamically generate $\bar{M}_{\rm Pl}$ and m

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They are important to obtain n_s and r and to dynamically generate $\bar{M}_{\rm Pl}$ and m

The most general agravity can be parameterized by the following $\mathcal{L}$

$$\frac{R^2}{6f_0^2}+\frac{\frac{1}{3}R^2-R_{\mu\nu}^2}{f_2^2}-\frac{\left(F_{\mu\nu}^A\right)^2}{4}+\frac{(D_\mu\phi_a)^2}{2}-\frac{\xi_{ab}}{2}\phi_a\phi_b R-\frac{\lambda_{abcd}}{4!}\phi_a\phi_b\phi_c\phi_d+\bar{\psi}_j i\not{D}\psi_j-Y_{ij}^a\psi_i\psi_j\phi_a+\text{h.c.}$$

We obtain the RGEs of this renormalizable quantum field theory:

$$\beta_p \equiv \frac{dp}{d\ln\mu} \qquad \text{(of all parameters } p\text{)}$$

Without gravity this was done before *[Machacek and Vaughn (1983,1984,1985)]*

We include gravity

Results for RGEs

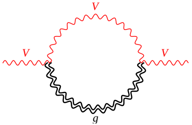
Gauge couplings

Their contributions to the RGEs cancel!

This was previously noticed in *[Narain, Anishetty (2013)]*

Possible explanation:
the graviton is not charged

Possible new gravity contributions



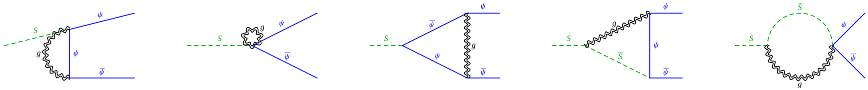
(Rainbow)

(Seagull)

Yukawa couplings

We find the one-loop RGE (where $C_{2F} \equiv t^A t^A$ and $t^A \equiv$ "fermion gauge generators"):

$$(4\pi)^2 \frac{dY^a}{d\ln\mu} = \frac{1}{2}(Y^{\dagger b} Y^b Y^a + Y^a Y^{\dagger b} Y^b) + 2 Y^b Y^{\dagger a} Y^b + Y^b \mathrm{Tr}(Y^{\dagger b} Y^a) - 3\{C_{2F}, Y^a\} + \frac{15}{8} f_2^2 Y^a$$



All remaining RGEs

We also computed the RGEs for λ_{abcd} ξ_{ab} f_0 and f_2

RGEs for the quartic couplings

Tens of Feynman diagrams contribute to these RGEs ... we obtain

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_{abcd}}{d\ln\mu} \quad = \quad & \sum_{\text{perms}} \Big[\frac{1}{8}\lambda_{abef}\lambda_{efcd} + \frac{3}{8}\{\theta^A,\theta^B\}_{ab}\{\theta^A,\theta^B\}_{cd} - \mathrm{Tr}\, Y^a Y^{\dagger b} Y^c Y^{\dagger d} + \\
& + \frac{5}{8} f_2^4 \xi_{ab}\xi_{cd} + \frac{f_0^4}{8}\xi_{ae}\xi_{cf}(\delta_{eb} + 6\xi_{eb})(\delta_{fd} + 6\xi_{fd}) \\
& + \frac{f_0^2}{4!}(\delta_{ae} + 6\xi_{ae})(\delta_{bf} + 6\xi_{bf})\lambda_{efcd} \Big] + \lambda_{abcd}\Big[\sum_k (Y_2^k - 3C_{2S}^k) + 5f_2^2\Big],
\end{aligned}
$$

where the first sum runs over the 4! permutations of *abcd* and the second sum over $k = \{a, b, c, d\}$, with Y_2^k and C_2^k defined by

$$
\mathrm{Tr}(Y^{\dagger a} Y^b) = Y_2^a \delta^{ab}, \quad \theta^A_{ac}\theta^A_{cb} = C_{2S}^a \delta_{ab}
$$

(θ^A are the scalar gauge generators)

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RGEs for the quartic couplings: SM case

For the SM H plus a complex scalar singlet S the RGEs become:

$$
\begin{aligned}
(4\pi)^2 \frac{d\lambda_S}{d\ln\mu} &= 20\lambda_S^2 + 2\lambda_{HS}^2 + \frac{\xi_S^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_S)^2\right] + \lambda_S\left[5f_2^2 + f_0^2(1+6\xi_S)^2\right] \\
(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} &= -\xi_H\xi_S\left[5f_2^4 + f_0^4(6\xi_S+1)(6\xi_H+1)\right] - 4\lambda_{HS}^2 + \lambda_{HS}\left\{8\lambda_S + 12\lambda_H + 6y_t^2\right. \\
&\quad \left. +5f_2^2 + \frac{f_0^2}{6}\left[(6\xi_S+1)^2 + (6\xi_H+1)^2 + 4(6\xi_S+1)(6\xi_H+1)\right]\right\} \\
(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} &= \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2 g_2^2 + \frac{27}{200}g_1^4 - 6y_t^4 + 24\lambda_H^2 + \lambda_{HS}^2 + \frac{\xi_H^2}{2}\left[5f_2^4 + f_0^4(1+6\xi_H)^2\right] \\
&\quad +\lambda_H\left(5f_2^2 + f_0^2(1+6\xi_H)^2 + 12y_t^2 - 9g_2^2 - \frac{9}{5}g_1^2\right).
\end{aligned}
$$

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RGEs for the scalar/graviton couplings

Complicated calculation (but computer algebra helps!)

$$(4\pi)^2 \frac{d\xi_{ab}}{d\ln\mu} = \frac{1}{6}\lambda_{abcd}\,(6\xi_{cd}+\delta_{cd}) + (6\xi_{ab}+\delta_{ab})\sum_k\left[\frac{Y_2^k}{3}-\frac{C_{2S}^k}{2}\right] + -\frac{5f_2^4}{3f_0^2}\xi_{ab} + f_0^2\xi_{ac}\left(\xi_{cd}+\frac{2}{3}\delta_{cd}\right)(6\xi_{db}+\delta_{db})$$

For the SM H plus a complex scalar singlet S the RGEs become:

$$(4\pi)^2 \frac{d\xi_S}{d\ln\mu} = (1+6\xi_S)\frac{4}{3}\lambda_S - \frac{2\lambda_{HS}}{3}(1+6\xi_H) + \frac{f_0^2}{3}\xi_S(1+6\xi_S)(2+3\xi_S) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_S$$

$$(4\pi)^2 \frac{d\xi_H}{d\ln\mu} = (1+6\xi_H)(2y_t^2 - \frac{3}{4}g_2^2 - \frac{3}{20}g_1^2 + 2\lambda_H) - \frac{\lambda_{HS}}{3}(1+6\xi_S) + + \frac{f_0^2}{3}\xi_H(1+6\xi_H)(2+3\xi_H) - \frac{5}{3}\frac{f_2^4}{f_0^2}\xi_H$$

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RGE for the gravitational couplings

Huge calculation ... (computer algebra practically needed!!)

$$\begin{aligned}
(4\pi)^2 \frac{df_2^2}{d\ln\mu} &= -f_2^4\left(\frac{133}{10}+\frac{N_V}{5}+\frac{N_f}{20}+\frac{N_s}{60}\right)\\
(4\pi)^2 \frac{df_0^2}{d\ln\mu} &= \frac{5}{3}f_2^4+5f_2^2f_0^2+\frac{5}{6}f_0^4+\frac{f_0^4}{12}(\delta_{ab}+6\xi_{ab})(\delta_{ab}+6\xi_{ab})
\end{aligned}$$

Here N_V, N_f, N_s are the number of vectors, Weyl fermions and real scalars.

In the SM $N_V = 12$, $N_f = 45$, $N_s = 4$.

We confirmed the calculations of *[Avramidi (1995)]*
rather than those of *[Fradkin and Tseytlin (1981,1982)]*

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Natural dynamical generation of the electroweak scale

1) *Low energies* ($\mu < M_{0,2}$): agravity can be neglected and the SM RGE apply:

$$(4\pi)^2 \frac{dm^2}{d\ln\mu} = m^2 \beta_m^{\rm SM}, \qquad \beta_m^{\rm SM} = 12\lambda_H + 6y_t^2 - \frac{9g_2^2}{2} - \frac{9g_1^2}{10}$$

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2) *Intermediate energies* ($M_{0,2} < \mu < \bar{M}_{\rm Pl}$): Both m and $\bar{M}_{\rm Pl}$ appear and we find

$$(4\pi)^2 \frac{d}{d\ln\mu} \frac{m^2}{\bar{M}_{\rm Pl}^2} = -\xi_H[5f_2^4 + f_0^4(1+6\xi_H)] + \dots$$

The red term is a non-multiplicative potentially dangerous correction to m

$$m^2 \sim \bar{M}_{\rm Pl}^2 g^2, \quad \text{naturalness} \;\to\; f_2, f_0(1+6\xi_H)^{1/4} \sim \sqrt{\frac{4\pi m}{M_{\rm Pl}}} \sim 10^{-8}$$

These *ultraweak couplings* are preserved by the RGE even for $f_0 \gtrsim 10^{-5}$
if $\xi \approx -1/6$

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These *ultraweak couplings* are preserved by the RGE even for $f_0 \gtrsim 10^{-5}$ if $\xi \approx -1/6$

3) *Large energies* ($\mu > \bar{M}_{\rm Pl}$):

$$\lambda_{HS}|H|^2 s^2 \quad \rightarrow \quad m^2 = \lambda_{HS}\langle s\rangle^2$$

λ_{HS} can be naturally small (looking at the RGE of λ_{HS}):

$$\rightarrow \quad \lambda_{HS} \sim f_{0,2}^4$$

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Agravity inflation: inflaton identified with s

We identify inflaton = s by taking the other scalar fields heavy ...

Then we can easily convert s into a scalar s_E with canonical kinetic term and find

$$\epsilon \equiv \frac{\bar{M}_{\rm Pl}^2}{2}\left(\frac{1}{U}\frac{\partial U}{\partial s_E}\right)^2 = \frac{1}{2}\frac{\xi_S}{1+6\xi_S}\left(\frac{\beta_{\lambda_S}}{\lambda_S} - 2\frac{\beta_{\xi_S}}{\xi_S}\right)^2$$

$$\eta \equiv \bar{M}_{\rm Pl}^2 \frac{1}{U}\frac{\partial^2 U}{\partial s_E^2} = \frac{\xi_S}{1+6\xi_S}\left(\frac{\beta(\beta_{\lambda_S})}{\lambda_S} - 2\frac{\beta(\beta_{\xi_S})}{\xi_S} + \frac{5+36\xi_S}{1+6\xi_S}\frac{\beta_{\xi_S}^2}{\xi_S^2} - \frac{7+48\xi_S}{1+6\xi_S}\frac{\beta_{\lambda_S}\beta_{\xi_S}}{2\lambda_S\xi_S}\right)$$

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The slow-roll parameters are given by the β-functions ...

We can insert them in the formulae for the observable parameters A_s, n_s and r:

$$n_s = 1 - 6\epsilon + 2\eta, \qquad A_s = \frac{U/\epsilon}{24\pi^2\bar{M}_{\rm Pl}^4}, \qquad r = 16\epsilon$$

where everything is evaluated at about $N \approx 60$ e-foldings when the inflaton $s_E(N)$ was

$$N = \frac{1}{\bar{M}_{\rm Pl}^2}\int_0^{s_E(N)}\frac{U(s_E)}{U'(s_E)}ds_E$$

Agravity inflation: inflaton identified with s (analytic approximation)

$$\left\{\begin{array}{rcl} \lambda_S(s) & \approx & 0 \\ \beta_{\lambda_S}(s) & \approx & 0 \\ \xi_S(s)s^2 & = & \bar{M}_{\rm Pl}^2 \end{array}\right. \qquad \to \qquad \lambda_S(\mu\approx s)\approx\frac{b}{2}\ln^2\frac{s}{\langle s\rangle}\,, \qquad \underbrace{\xi_S(\mu)\approx\xi_S}_{\text{for simplicity}}$$

$b\equiv g^4/(4\pi)^4$ can be computed in any given model ...

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$$\to \quad \epsilon\approx\eta\approx\frac{2\xi_S}{1+6\xi_S}\frac{1}{\ln^2 s/\langle s\rangle}=\frac{2\bar{M}_{\rm Pl}^2}{s_E^2}$$

The Einstein-frame potential is nearly quadratic around its minimum:

$$U=\frac{\bar{M}_{\rm Pl}^4}{4}\frac{\lambda_S}{\xi_S^2}\approx\frac{M_s^2}{2}s_E^2 \qquad \text{with} \qquad M_s=\frac{g^2\bar{M}_{\rm Pl}}{2(4\pi)^2}\frac{1}{\sqrt{\xi_S(1+6\xi_S)}}$$

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Inserting s_E at $N\approx 60$ e-foldings, $s_E(N)\approx 2\sqrt{N}\bar{M}_{\rm Pl}$, ... **we obtain the predictions**

$$n_s\approx 1-\frac{2}{N}\approx 0.967,\qquad r\approx\frac{8}{N}\approx 0.13,\qquad A_s\approx\frac{g^4N^2}{24\pi^2\xi_S(1+6\xi_S)}$$

(remember inflaton $=s$). Such predictions are typical of quadratic potentials

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VEVs above $\bar{M}_{\rm Pl}$, $s_E\approx 2\sqrt{N}\bar{M}_{\rm Pl}$, are needed for a quadratic potential

Agravity predicts physics above $\bar{M}_{\rm Pl}$, and a quadratic potential is a good approximation, even at $s_E > \bar{M}_{\rm Pl}$, because coefficients of higher order terms are suppressed by extra powers of the loop expansion parameters, which are small at weak coupling

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Tricks to bring the theory in a more standard form

$$\frac{\mathscr{L}}{\sqrt{g}} = \frac{R^2}{6f_0^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \xi\frac{\varphi^2}{2}R + \mathscr{L}_{\rm matter}$$

Tricks to bring the theory in a more standard form

First, we can trade the R^2 term with a scalar field: consider an auxiliary field χ

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$$= \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} - \frac{f}{2}R - \frac{3f_0^2}{8}\chi^2 + \mathscr{L}_{\rm matter}$$

where $f = \chi + \xi\varphi^2$ and $\mathscr{L}_{\rm matter} = \frac{(D_\mu\varphi)^2}{2} - \frac{1}{4}F_{\mu\nu}^2 + \bar{\psi}i\not{D}\psi + (y\,\varphi\psi\psi + \text{h.c.}) - V(\varphi)$

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Second, perform a conformal transformation of the metric and the other fields:

$g_{\mu\nu}^E = g_{\mu\nu} \times f/\bar{M}_{\rm Pl}^2 \qquad \varphi_E = \varphi\times(\bar{M}_{\rm Pl}^2/f)^{1/2}, \qquad \psi_E = \psi\times(\bar{M}_{\rm Pl}^2/f)^{3/4}, \qquad A_{\mu E} = A_\mu$

$$\frac{\mathscr{L}}{\sqrt{g_E}} = \frac{\frac{1}{3}R_E^2 - R_{E\mu\nu}^2}{f_2^2} - \frac{1}{4}F_{E\mu\nu}^2 + \bar{\psi}_E i\not{D}\psi_E + (y\varphi_E\psi_E\psi_E + \text{h.c.}) - \frac{\bar{M}_{\rm Pl}^2}{2}R_E + \mathscr{L}_\varphi - U$$

where $\mathscr{L}_\varphi = \bar{M}_{\rm Pl}^2\left[\frac{(D_\mu\varphi)^2}{2f} + \frac{3(\partial_\mu f)^2}{4f^2}\right]$ and $U = \frac{\bar{M}_{\rm Pl}^4}{f^2}\left[V(\varphi) + \frac{3f_0^2}{8}\chi^2\right]$

By redefining $z = \sqrt{6f}$,

$$\mathscr{L}_\varphi = \frac{6\bar{M}_{\rm Pl}^2}{z^2}\frac{(D_\mu\varphi)^2 + (\partial_\mu z)^2}{2}, \qquad U(z,\varphi) = \frac{36\bar{M}_{\rm Pl}^4}{z^4}\left[V(\varphi) + \frac{3f_0^2}{8}\left(\frac{z^2}{6} - \xi_\varphi\varphi^2\right)^2\right]$$

Matching the scalar amplitude

1. **Planckion inflation** $(M_0 \gg M_s)$

$$n_s \approx 1 - \frac{2}{N} \overset{N\approx 60}{\approx} 0.967, \qquad r \approx \frac{8}{N} \overset{N\approx 60}{\approx} 0.13$$

The scalar amplitude $A_s = M_s^2 N^2/6\pi^2 \bar{M}_{\mathrm{Pl}}^2$ is reproduced for $M_s \approx 1.4 \times 10^{13}\mathrm{GeV}$

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2. **Graviscalar inflation (Starobinsky inflation)** $(M_0 \ll M_s)$

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The scalar amplitude $A_s = f_0^2 N^2 / 48\pi^2$ is reproduced for $f_0 \approx 1.8 \times 10^{-5}$

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So generically we have

$$f_0 \gtrsim 10^{-5}$$

Reheating

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The decay of I with mass M_I and width Γ_I reheats the universe up to a temperature

$$T_{\rm RH} = \left[\frac{90}{\pi^2 g_*}\,\Gamma_I^2 \bar{M}_{\rm Pl}^2\right]^{1/4},$$

where $g_* \sim 100$ is the number of relativistic degrees of freedom at $T_{\rm RH}$.

The inflaton I is in the hidden sector. How can it reheat the universe?

Reheating

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($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $\mathscr{D}_\mu$ is the dilatation current)

The theory is classically scale-invariant $\to$ a non-zero $\partial_\mu \mathscr{D}_\mu$ arises only at loop level:

$$\partial_\mu \mathscr{D}_\mu = \frac{\beta_{g_1}}{2g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2g_3} G_{\mu\nu}^2 + \beta_{y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

where $\ldots$ are BSM terms (they are relevant for DM production as we will see)

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---

leading decay from $\partial_\mu \mathscr{D}_\mu$: $\Gamma(I \to gg) \approx \dfrac{|\xi_S| g_3^4 M_s^3}{(4\pi)^5 \bar{M}_{\rm Pl}^2} \quad \to \quad T_{\rm RH} \overset{\xi_S \sim 1}{\sim} 10^7\,{\rm GeV}\left(\dfrac{M_s}{10^{13}\,{\rm GeV}}\right)^{3/2}$

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The Planckion s and the graviscalar z respectively couple to

$$\frac{\partial_\mu \mathscr{D}_\mu}{\bar{M}_{\rm Pl}/\sqrt{\xi_S}} \qquad \text{and} \qquad \frac{T_{\mu\mu}}{\bar{M}_{\rm Pl}},$$

($T_{\mu\mu}$ is the trace of the energy-momentum tensor and $\mathscr{D}_\mu$ is the dilatation current)

The theory is classically scale-invariant $\to$ a non-zero $\partial_\mu \mathscr{D}_\mu$ arises only at loop level:

$$\partial_\mu \mathscr{D}_\mu = \frac{\beta_{g_1}}{2g_1} Y_{\mu\nu}^2 + \frac{\beta_{g_2}}{2g_2} W_{\mu\nu}^2 + \frac{\beta_{g_3}}{2g_3} G_{\mu\nu}^2 + \beta_{y_t} H Q_3 U_3 + \beta_{\lambda_H} |H|^4 + \ldots,$$

where $\ldots$ are BSM terms (they are relevant for DM production as we will see)

---

leading decay from $\partial_\mu \mathscr{D}_\mu$: $\Gamma(I \to gg) \approx \dfrac{|\xi_S| g_3^4 M_s^3}{(4\pi)^5 \bar{M}_{\rm Pl}^2} \quad \to \quad T_{\rm RH} \overset{\xi_S \sim 1}{\sim} 10^7\,\text{GeV}\left(\dfrac{M_s}{10^{13}\,\text{GeV}}\right)^{3/2}$

leading decay from $T_{\mu\mu}$: $\Gamma(I \to h_E h_E) \approx \dfrac{(1+6\xi_H)^2|\xi_S|}{|1+6\xi_S|}\,\dfrac{M_I^3}{64\pi \bar{M}_{\rm Pl}^2} \quad \to \quad T_{\rm RH} \overset{\xi_S \sim 1}{\sim} 10^9\,\text{GeV}$

Dark matter

There should be fermions in the s-sector. Two types of candidates come to mind

Dark matter, neutrino masses and leptogenesis

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Fermions in the s-sector with no gauge interactions

- They can couple to the SM behaving as right-handed neutrinos N and generate the observed neutrino masses via NLH couplings. The right-handed neutrino masses can be generated by sN^2 terms. *[Alexander-Nunneley, Pilaftsis (2010)]*
- They can provide baryogenesis via leptogenesis.
- *[Dodelson, Widrow (1993)]* claimed that N may also account for DM in special cases

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Fermions in the s-sector which cannot couple to the SM

The lightest fermion in the s-sector is a stable DM candidate if it cannot couple to the SM sector (for example because it has gauge interactions under the inflaton sector).

Dark Matter: a Concrete example

A predictive model (no extra parameters)

Take a 2^{nd} copy of the SM and impose a Z_2 symmetry, spontaneously broken by the fact that the mirror Higgs field (S) has

$$\langle S \rangle \sim \bar{M}_{\mathrm{Pl}} \quad \text{while} \quad \langle H \rangle \sim M_W$$

Mirror SM particles (e.g. a mirror neutrino or electron) may be DM ...

Interactions between these candidates and the SM are suppressed by λ_{HS} ...

Dark matter abundance

Terms in $\partial_\mu \mathscr{D}_\mu$ and $T_{\mu\mu}$ lead to decays of the inflaton I into DM
(along the lines of the reheating calculation)

More in general the DM fermions can also get a mass M from another source.
Then such fermion masses would contribute to $\partial_\mu \mathscr{D}_\mu$ and to $T_{\mu\mu}$ as $M\bar{\Psi}\Psi$
(we are considering, for example, a Dirac mass term)

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By identifying the fermion Ψ with DM, its abundance is

$$\Omega_{\rm DM} \equiv \frac{\rho_{\rm DM}}{\rho_{\rm cr}} \approx \frac{0.110}{h^2} \times \frac{M}{0.40\text{eV}} \frac{\Gamma(I \to \rm DM)}{\Gamma(I \to \rm SM)}$$

having inserted the present Hubble constant $H_0 = h \times 100\,\rm km/sec\ Mpc$

Dark matter abundance: result

The observed DM abundance is reproduced for

$$M \approx (10 - 200)\mathrm{TeV} \left(\frac{M_I}{10^{13}\mathrm{GeV}}\right)^{2/3}$$

where the lower (higher) estimate applies if $\Gamma(I \to gg)$ ($\Gamma(I \to h_E h_E)$) dominates

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