Locality in AdS and for AdS black holes

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Mostly based on
PRD 86, 026004 (1204.0126) with Kabat, Lifschytz and Roy;
PRD 90 8, 086005 (1408.0415);
PRD 91, 086004 (1411.4657) with Xiao;
and 1505.03895 (to appear in PRD) with Roy

Motivation

- To understand how locality in a (particular) theory of Quantum Gravity breaks down. To what extent locality is satisfied.

  Holographic theory like AdS/CFT helps us to fulfill that purpose in an elegant way.

  The problem boils down to finding local bulk physics in terms of another local theory that we know and understand, namely boundary CFT.

- What does bulk locality have to say about black hole information problem.

Outline

- Review of the 'bulk' scalar field construction at leading order in large N expansion.

  Use this framework to extend the prescription to various directions:

- Generelization to spin- 1, 2 and arbitrary integer spin s.
- Local operators in terms of fields on a cut-off surface instead of at the conformal boundary. Connection with similar construction in dS (also for higher integer spins) and to holographic RG framework.
- Order by order at $1/N$ expansions and finite N effects. Operators in the black hole background; especially inside the horizon.
- Black hole information problem and discussions.

Review

Identification of normalizable and non-normalizable modes:

- GKPW prescription for Euclidean signature.

  Gubser, Klebanov, Polyakov; Witten 1998

- Normalizeable modes in Lorentzian spacetime. A 'Transfer function' was constructed which had support over the entire boundary. We will be working in this framework.

  Balasubramanian et al.; Banks et al. 1998.; Bena 1999

Basic idea is to write local bulk fields (commuting at the spacelike separation) in terms of integrating boundary operators over some boundary region. So, may be we can write something like

$$\phi(x,z) = \int_{boundary} dx' \, K(x,z|x')\mathcal{O}(x')$$

with boundary integral over a tractable range?

Review (Cont'd)

Turns out that it is possible. For example, for a free massive scalar

$$\phi(t,x,z) \sim \int_{t'^2+y'^2<z^2} dt' d^{d-1}y' (\sigma z')^{\Delta-d} \mathcal{O}(t+t', x+iy')$$

For bulk-boundary AdS covariant distance

$$\sigma(z,x|z',x') = \frac{z^2+z'^2+(x-x')^2}{2zz'}$$

and boundary operator with dimension Δ. Here we will always work in Poincaré coordinates for AdS_{d+1} with metric

$$ds^2 = G_{MN}dX^M dX^N = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

Hamilton, Kabat, Lifschytz, Lowe 2005.

Some comments

- For higher dimensional AdS spaces and for free fields studied so far, the integration is over finite boundary region (spacelike separated from the bulk field) provided we complexify boundary spacetime.

  Hamilton et al. 2006, 2007.

- For timelike boundaries of AdS, solving bulk fields in terms of boundary is not a standard Cauchy problem and one can think of this construction as arising from starting with a retarded Green's function in dS.
- Other equivalent compromising alternatives are certainly possible. For example, for scalar fields, momentum space representations.

  Papadodimas and Raju (PR) 2013-2015

Comments (Cont'd)

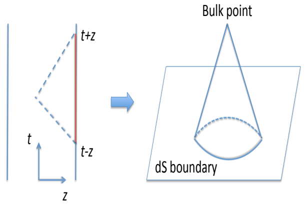


Figure: The smearing function support after translating to complexified coordinates (although here shown for 2-dimensional bulk). In terms of dS, it could be thought of as the retarded Green's function.

Spins 1 and 2 and higher

Next step would naturally be generalization for free and interacting gauge fields.

Kabat, Lifschytz, Roy, DS 2012

Some Issues:

- Gauss constraint must somehow disturb the construction to some extent.

  Heemskerk et al. 2012
  Kabat, Lifschytz: 2012.

- Problem with lower dimension: $\mathrm{AdS}_{d+1\leq 3}$ as in lower dimensions, the normalizable modes themselves are fluctuating at the boundary.

Spins 1 and 2 and higher (Cont'd)

- For scalars or any integer spins (HS) (upon a suitable "holographic gauge" choice for $s \geq 1$), one can denote the bulk fields in Poincaré patch in terms of smeared boundary operators in the following way (for free fields)

$$\Phi_{\mu_1\ldots\mu_s} = \frac{A(s,\Delta,d)}{z^s} \int_{\mathcal{B}^d \text{ with radius } z} dt' d^{d-1}\mathbf{y}' \left(\frac{z^2 - t'^2 - |\mathbf{y}'|^2}{z}\right)^{s-2} \mathcal{O}_{\mu_1\ldots\mu_s}(t+t', \mathbf{x}+i\mathbf{y}')$$

Field behaves like $z^{\Delta - s} = z^{d-2}$ near the boundary.

Kabat, Lifschytz, Roy and DS 2012; DS and Xiao 2014

- The prescription is slightly different for spin- 1. For spins $s \geq 1$ the locality conditions are satisfied as per Gauss law.

Spins 1 and 2

In general dimensions, the bulk-boundary correlator is

$$\langle z A_\mu(t,\mathbf{x},z) j_\nu(0)\rangle = \frac{\Gamma(d/2)}{2\pi^{d/2}}\left(\frac{d-2}{d-1}\,\eta_{\mu\nu} I_1 - \frac{1}{2(d-1)(d-2)}\,\partial_\mu\partial_\nu I_2\right)$$

where

$$I_n = \int_{t'^2+|\mathbf{y}'|^2=z^2} dt' d^{d-1}y' \,\frac{1}{\left(-(t+t')^2+|\mathbf{x}+i\mathbf{y}'|^2\right)^{d-n}}$$

- Even though I_1 is only singular on the bulk lightcone, I_2 turns out to be singular in both bulk and boundary light cone.
- On the other hand, $\langle Fj\rangle$ correlators are only singular in bulk light cone.

Spins 1 and 2 (Cont'd)

Similar to the Maxwell case, using

$$\langle T_{\mu\nu}(x) T_{\alpha\beta}(0)\rangle = X_{\mu\nu\alpha\beta}\frac{1}{(x^2)^d} + Y_{\mu\nu\alpha\beta}\frac{1}{(x^2)^{d-1}} + Z_{\mu\nu\alpha\beta}\frac{1}{(x^2)^{d-2}}$$

we find

$$z^2\langle h_{\mu\nu}(t,\mathbf{x},z) T_{\alpha\beta}(0)\rangle = X_{\mu\nu\alpha\beta}J_0 + Y_{\mu\nu\alpha\beta}J_1 + Z_{\mu\nu\alpha\beta}J_2$$

where

$$J_n = \frac{1}{\mathrm{vol}(B^d)}\int_{t'^2+|\mathbf{y}'|^2<z^2} dt' d^{d-1}y' \frac{1}{\left(-(t+t')^2+|\mathbf{x}+i\mathbf{y}'|^2\right)^{d-n}}$$

- It turns out J_1 and J_2 is again singular on both bulk and boundary light cone.
- Upon calculating correlator with Weyl tensor we find, they only depend on I_0 and I_1 found previously and hence they lead to causal commutator.

Some comments

- For all cases discussed so far, from the form of smearing function it's clear that in the boundary limit we go to a localized operator at the boundary.
- For AdS_3 Maxwell, the bulk gauge field is proportional to a fluctuating gauge field at the boundary and the dictionary here is not well understood.
- The smearing distribution is pretty clever. For example, for AdS_3, it already knows that the boundary conserved current is dual to the Chern-Simons gauge fields.

  Jensen 2010
- In all cases, the smearing distributions is fixed and have the correct AdS isometric properties.

Cut-off surface, dS etc.

- One can also construct bulk operators as smeared boundary operators residing on a cut-off surface $z = z_0$.

DS 2014

$$\Phi(z,x) = \int d^d x' K_1(x'|x,z,z_0)\phi_c(x',z_0) + \int d^d x' K_2(x'|x,z,z_0)\, j_c$$

with some complicated K_1 and K_2. Recover AdS result as $z_0 \to 0$ upon using normalizeable condition.

Two uses:

- Derive the local bulk field at any stage of holographic RG flow in the bulk. Heemskerk et al.; Faulkner et al. 2010
- Connection to a similar construction for de Sitter/CFT for scalars and HS. A bulk operator there is obtained by smearing two sets of operators appropriately.

DS and Xiao 2014

$1/N$ Smearing Function

- Add higher dimension operators: Bilocal operators or equivalently tower of higher dimensional operators to restore locality for scalar fields at the level of 3 point function.

$$\phi(z,x) = \int dx' K_\Delta(z,x|x')\mathcal{O}_\Delta(x') + \sum_l a_l \int dx' K_{\Delta_l}(z,x|x')\mathcal{O}_{\Delta_l}(x')$$

Kabat, Lifschytz, Lowe, 2011; Kabat, Lifschytz, 2012

- Clearly indicates how locality can break down as we go to finite N. Not enough higher dimensional operators to recover locality.
- Similar story for gauge fields. Locality is satisfied to the extent of non-locality introduced by Gauss constraints.

Kabat, Lifschytz 2012, 2013, 2015

Operators behind black hole horizon

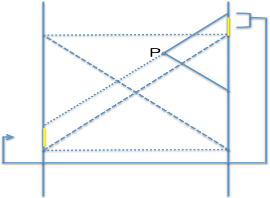


Figure: Subhorizon operator construction as thermofield double. The construction is similar in Rindler and BTZ and can be extended to eternal AdS-Schwarzschild.

This construction works for infinite N as the CFT correlators decay at large times, even though the smearing blows up.

$$\Phi(R) = \int_{s.l} dy\; K(R|y)\, \mathcal{O}^R_\Delta(y) + \int_{t.l} dy'\; K(R|y') f(\Delta, R, y')\, \mathcal{O}^L_\Delta(y')$$

$$= \int_{s.l} dy\; K(R|y)\, \mathcal{O}^R_\Delta(y) + \int_{-\infty}^{-y'_{min}} dy'\; K(R|y') f(\Delta, R, -y')\, \mathcal{O}^R_\Delta(y'^0 - i\beta/2)$$

Finite N Smearing Function

- The finite N effects can be incorporated in the smearing function only if they are cutoff at a timescale $t_{max} \sim \frac{\beta S}{\Delta}$. This is because, at finite N the CFT correlation function starts to show small fluctuations of at least of size $e^{-S/2}$ at the time scale t_{max}, which when convolved with smearing function, blows up to give meaningless expressions.
  Barbon-Rabinovici '03, '14; Hamilton et al. '07; Kabat-Lifschytz '13

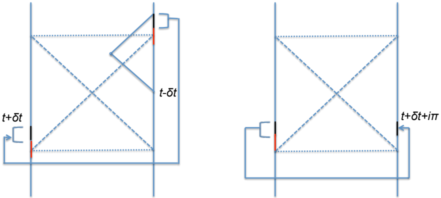


Figure: The second figure shows an alternate formulation using the mapping of left side smearing function to the right boundary. Only true for thermofield states inside a correlator.

Finite N Smearing Function (Cont'd)

- This cutoff prescription induces exponentially small non-locality which might indicate a firewall for large BHs in AdS, after the Page time.

$$\phi_{modified} = \phi_{semiclassical} - e^{-dS/2\Delta}\mathcal{O}$$

- This is one way of showing that these large time scales can play an important role in the bulk non-locality, which is otherwise invisible in general. But this can be interpreted as a 'coarse graining' to obtain an approximate interior semiclassical description even at finite N.
- However, this firewall could still be an artifact of working with the semiclassical saddle points. It could be that in a full quantum gravity path integrals, late time integrals are dominated by geometries without a horizon and hence circumventing the necessity of firewalls.

Soloduhkin 2005, Germani and DS 2015

One sided BHs at infinite and finite N

This construction can also be extended to BHs formed by collapse processes using certain combinations of retarded Green's propagator. First done in this context by Lowe and Roy in 2008 for shell collapse in AdS_2.

Roy and DS 2015

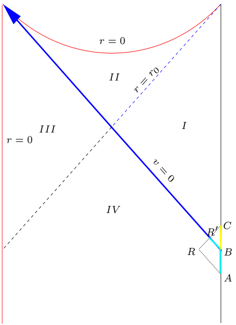


Figure: AdS_{d+1}- Vaidya geometry for a null-shell collapse.

At finite N, one then just need to implement the finite time cut-offs. But note that one should also impose an initial time cut-off until eigenstate thermalization time to implement a proper coarse graining.

Roy and DS 2015

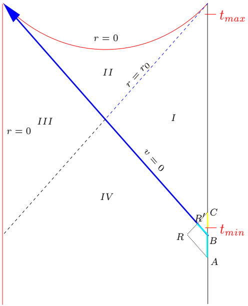


Figure: AdS$_{d+1}$- Vaidya construction with cut-offs.

One sided BHs at infinite and finite N (Cont'd)

The construction for region II is bit more involved. At infinite N, the convolutions are the following:

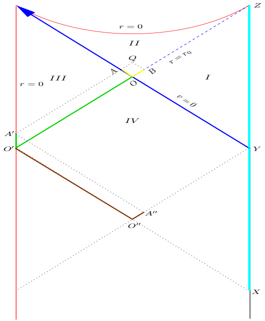


Figure: Smearing of points inside horizon for AdS_{d+1}- Vaidya geometry.

One sided BHs at infinite and finite N (Cont'd)

One then might again naively try to implement the time-cutoffs for finite N like below:

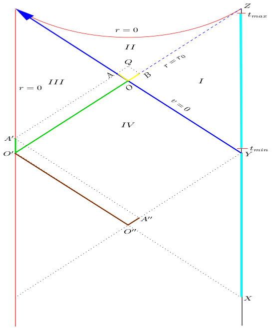


Figure: Smearing of points inside horizon for AdS_{d+1}- Vaidya geometry with cut-offs.

Problems and Solutions

- However, by doing so, we are directly encountering the trans-Planckian problem. Some solutions have been proposed before, but one needs to be careful.

Jacobson 1991, 1996

- The trans-Planckian collision sets in after time of order $\log S$. Before that the Green's function method should work. But after this time (also known as scrambling time), the interior is well approximated as eternal geometry. So, it seems, one can implement the finite N construction for eternal geometries then.
- The construction is easiest for region I (just empty AdS) and a bit more involved in region III.

Some comments

- Computing the convolution of the Green's functions are not possible in dimensions higher than AdS_3 as only numerical solutions are known. However, the construction can be done for BTZ explicitly.

  Banerjee, Papadodimas (?), DS; in progress

- The construction discussed so far doesn't require any state dependence as in PR and is only background dependent. However, as shown before, after Page time, when an observer can in principle measure a commutator inside a large point correlator, the non-locality can be order 1. This is not possible by a low energy observer. This is like ignoring the edge effects in PR construction.
- PR requires state dependence as they avoid the large time or small ω divergences of infinite N construction. Whether it's good or bad is still an open issue.

  Marolf, Polchinski 2015

Well-defined projects

- Extension of HS construction to three-point level? Any possible Weyl like "gauge invariant" HS fields?
- Interactions and finite N effects for dS construction. Implications for dS/CFT?
- Exploring in further details the relation between cut-off smearing construction and holographic RG?
- $\mathcal{O}\left(1/N^2\right)$?

Heemskerk et al. 2009, Penedones 2010, Bekaert et al. 2014

THANK YOU

AdS_3 Rindler

$$ds^2 = -(r^2-1)dt^2 + (r^2-1)^{-1}dr^2 + r^2dx^2$$

$$\phi(t,x,r) = \int d\omega dk e^{-(i\omega t - ikx)} a_{\omega k} f_{\omega k}(r)$$

$$\phi \sim r^{-\Delta}\mathcal{O}, \quad f_{\omega k}(r) \sim r^{-\Delta}$$

So,

$$a_{\omega k} = \int e^{(i\omega t - kx)}\mathcal{O}$$

Then $K = F.T.\{f_{\omega k}\}$ as $\phi = \int K\mathcal{O}$. But turns out $f_{\omega k}$ blows up exponentially as $k \to \pm\infty$.

Spins 1 and 2- Cont'd

We set the *holographic gauge* $A_z = 0$ which seems to be the easiest choice.

- For $\text{AdS}_{d+1>3}$, the field $\phi_\mu = zA_\mu$ satisfy the usual scalar wave equation with mass $m^2R^2 = 1 - d$ which leads to its conformal dimension $\Delta = d - 1$ which still satisfies BF bound.

Breitenlohner, Freedman 1982

The smearing integral is given by

$$
\begin{aligned}
zA_\mu &\sim \int_{t'^2+|y|'^2=z^2} dt' d^{d-1}y' j_\mu(t+t', x+iy') \\
&= \frac{1}{\text{vol}(S^{d-1})} \int dt' d^{d-1}y' \delta(\sigma z') j_\mu(t+t', x+iy')
\end{aligned}
$$

Spins 1 and 2- Cont'd

Similar story for metric perturbation. Considering linearized perturbation of AdS metric

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + g_{\mu\nu}dx^\mu dx^\nu\right)$$

$$\text{with} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \frac{z^2}{R^2}h_{\mu\nu}; \quad g_{zz} = g_{z\mu} = 0$$

In general $z^2 h_{\mu\nu}$ satisfy massless scalar wave equation. For $\text{AdS}_{d+1>3}$,

$$z^2 h_{\mu\nu} = \frac{1}{\text{vol}(B^d)}\int_{t'^2+|y|'^2<z^2} dt'dy'\,T_{\mu\nu}(t+t', x+iy')$$

Hologram on a Cutoff Surface

- Construct massive bulk scalar field Φ in terms of operators on a cutoff surface at the large N limit. We again use Poincaré coordinates.
- Remember that near the AdS boundary ($z \to 0$) the field has two distinct behaviors ($\nu = \Delta - \frac{d}{2}$)

$$\Phi(z, x) = \frac{\phi_b(x)}{2\nu} z^{\Delta} + z^{d-\Delta} j(x)$$

where j and ϕ_b are defined via

$$j(x) = z^{-d+\Delta}\Phi(z, x)|_{z\to 0} \qquad \text{and}$$
$$\phi_b(x) = z^{-2\nu} z\partial_z (z^{-d+\Delta}\Phi)|_{z\to 0} \leftrightarrow \mathcal{O}(x)$$

- We then similarly define

$$j_{cut}(x, z_0) = z^{-d+\Delta}\Phi(z, x)|_{z\to z_0} \qquad \text{and}$$
$$\phi_{b,cut}(x, z_0) = z^{-2\nu} z\partial_z (z^{-d+\Delta}\Phi)|_{z\to z_0}$$

Hologram on a Cutoff Surface- Cont'd

- The correct prescription turns out to be

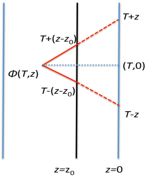
$$\Phi(z,x) = \int d^d x' K_1(x'|x,z,z_0)\phi_{b,cut}(x',z_0) + \int d^d x' K_2(x'|x,z,z_0) j_{cut}(x',z_0)$$

  with some complicated K_1 and K_2, which are of course also functions of Δ, d and the cutoff surface location z_0.

- The expressions are simplest for massless scalars in AdS_2 where the final result for the bulk operator becomes ($z_0 = z/m$)

$$\phi(T,z) = \frac{1}{2}\left[j_{cut}(T+(m-1)z_0, z_0) + j_{cut}(T-(m-1)z_0, z_0)\right] + \frac{1}{2}\int_{T-(z-z_0)}^{T+(z-z_0)} dT' \phi_{b,cut}(T', z_0)$$

Hologram on a Cutoff Surface- Cont'd



This gives the correct $\mathrm{AdS}_2/\mathrm{CFT}_1$ prescription, as we take z_0 to zero. As we take $z_0 \to 0$ limit, the current $j_{cut} \to 0$.

Hologram on a Cutoff Surface- Cont'd

For integer m, it gives us the usual expected value of the bulk field in terms of cutoff surface boundary values once we impose normalizeability.

$$\Phi(T, z) = \sum_{i=1}^{m} \Phi(T_i, z_0) = \sum_{i=1}^{m} j_{cut}(T_i, z_0), \quad \text{with}$$

$$T_1 = T + (z - z_0), \quad T_{i+1} = T_i - 2z_0, \quad \ldots \quad T_m = T - (z - z_0)$$

which is a special case for massless scalars in AdS_2 with $z_0 = z/m$.

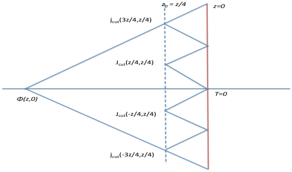


Figure: Smearing function for AdS_2/ CFT_1 ($m = 4$) for cut-off surface $z_0 = z/4$ contains four delta functions as shown in the figure.

Use1: Smearing in dS

- Analytic continuation $z \to \eta, \quad x^i \to ix^i$ and $R_{AdS} \to iR_{dS}$ takes AdS to flat patch dS with time η. Considering normalizable modes in AdS, leads to taking only +ve/-ve frequency modes in dS.
- So if normalizeability condition is not imposed, then analytic continuation between AdS and dS is visible. Taking $z_0 \to 0$ limit gives a bulk dS to boundary prescription in terms of smearing two sets of operators.
- Similar construction can be carried out in de Sitter (dS) spacetime. In particular one way to construct local bulk operators in dS is to smear two sets of CFT operators.
- In particular, for dS_{d+1}, one obtains (generalizeable to higher spins)

$$\Phi(\eta, \mathbf{x}) = A(\Delta, d) \int_{|x'|<\eta} d^d\mathbf{x}' \tilde{K}_1(\eta, \mathbf{x}') \mathcal{O}_\Delta(\mathbf{x} + \mathbf{x}')$$
$$+ B(\Delta, d) \int_{|x'|<\eta} d^d\mathbf{x}' \tilde{K}_2(\eta, \mathbf{x}') \mathcal{O}_{d-\Delta}(\mathbf{x} + \mathbf{x}')$$

Use2: Connection to Holographic RG Framework

- In the standard holographic RG prescription,

$$Z = \int \mathcal{D}\tilde{\phi}(z_0, x)\Psi_{IR}(z_0, \tilde{\phi})\Psi_{UV}(z_0, \tilde{\phi})$$

where Ψ_{IR} and Ψ_{UV} arises from integrating out relevant bulk fields against the exponential of the relevant part of the bulk action:

$$\Psi_{\substack{IR \\ UV}} = \int \mathcal{D}\phi|_{\substack{z>z_0 \\ z<z_0}} e^{-\kappa^{-2}\mathcal{S}|_{\substack{z>z_0 \\ z<z_0}}}$$

Faulkner et al. 2010 and Heemskerk et al. 2010

- If one considers the UV factor is a local Gaussian for a single bulk scalar, i.e.

$$\Psi_{UV}(z_0, j_{cut}) = \exp\left\{-\frac{1}{2h\kappa^2}\int d^d x \left(j_{cut}(x, z_0) + g(x)\right)^2\right\}$$

Then after the UV part is integrated out, one effectively induces double trace operators.

Connection to Holographic RG Framework- Cont'd

It was postulated that

$$\Psi_{IR}(z_0, j_{cut}) = \int \mathcal{D}M|_{kz_0<1} \exp\left\{-S_0 + \frac{1}{\kappa^2}\int d^d x\, j_{cut}\mathcal{O}_i\right\}$$

which is usually interpreted as the wavefunction for the QFT with a cut-off. From the bulk point of view, one can simply treat the j_{cut}'s appearing on the exponential of Ψ_{IR} to be the on-shell operators and the $\mathcal{O}_i$'s as their sources. Then after integrating the UV part,

$$\frac{\delta}{\delta\mathcal{O}}\frac{\delta}{\delta\mathcal{O}}Z|_{\mathcal{O}\to 0} = \int \mathcal{D}\tilde{\phi}\langle j_{cut}j_{cut}\rangle \Psi_{UV}[\tilde{\phi}] \propto \langle j_{cut}j_{cut}\rangle$$

(modulo some factors of h and κ). Thus one can simply recover the cut-off bulk locality or relate the cut-off surface field theory correlators with the bulk partition function while flowing in holographic RG. UV-cutoff CFT will not give us local bulk fields.