

# BEYOND THE COSMOLOGICAL PRINCIPLE

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# BEYOND THE COSMOLOGICAL PRINCIPLE

- Subject of the thesis
  - To study cosmological phenomena assuming inhomogeneity and/or anisotropy of the underlying geometry.
- Papers:
  - P. Sundell and I. Vilja, *Mod. Phys. Lett. A* 29, 1450053 (2014).
  - P. Sundell, E. Mörtzell and I. Vilja, *JCAP*08(2015)037.
  - P. Sundell and T. Koivisto, [arXiv:1506.04715](https://arxiv.org/abs/1506.04715).
  - P. Sundell and I. Vilja, work in progress.

# BEYOND THE COSMOLOGICAL PRINCIPLE

- Motivation
  - The standard cosmological model is  $\Lambda$ -Cold Dark Matter ( $\Lambda$ CDM) model. What is  $\Lambda$  or dark energy? Do we really need either of those or can we explain things otherwise?
  - The observed asymmetries in the Cosmic Microwave Background (CMB) anisotropy field are not predicted by the cosmological standard model.
- I studied these issues separately

# CAN A VOID MIMIC THE $\Lambda$ IN $\Lambda$ CDM?

- $\Lambda$  was introduced to explain the supernova observations. However, the origin of  $\Lambda$  is unknown and is more generally known as the dark energy problem
- In void models,  $\Lambda$  or dark energy is not needed and the supernovae are explained by inhomogeneous matter distribution.

# THE LEMAÎTRE-TOLMAN-BONDI MODEL

- We assumed the Lemaître-Tolman-Bondi model

- Only radial inhomogeneity

- Only dust

- The Einstein equations yield

$$ds^2 = -dt^2 + \frac{R_{,r}^2(r, t)}{1 + 2e(r)r^2} dr^2 + R^2(r, t)(d\theta^2 + \sin^2 \theta d\phi^2)$$

$$H_T^2(r, t) = H_0^2(r) \left[ \Omega_M(r) \left( \frac{R_0(r)}{R(r, t)} \right)^3 + \Omega_K(r) \left( \frac{R_0(r)}{R(r, t)} \right)^2 \right]$$

- We assumed the model has the  $\Lambda$ CDM luminosity distance

$$D_A = \frac{1}{H_0^F (1+z) \sqrt{\Omega_k^F}} \sinh \left\{ \sqrt{\Omega_k^F} \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_m^F (1+\tilde{z})^3 + \Omega_k^F (1+\tilde{z})^2 + \Omega_w^F (1+\tilde{z})^w + \Omega_\Lambda^F}} \right\}$$

- And homogeneous bang time  $t_b(r) = 0$  and early universe

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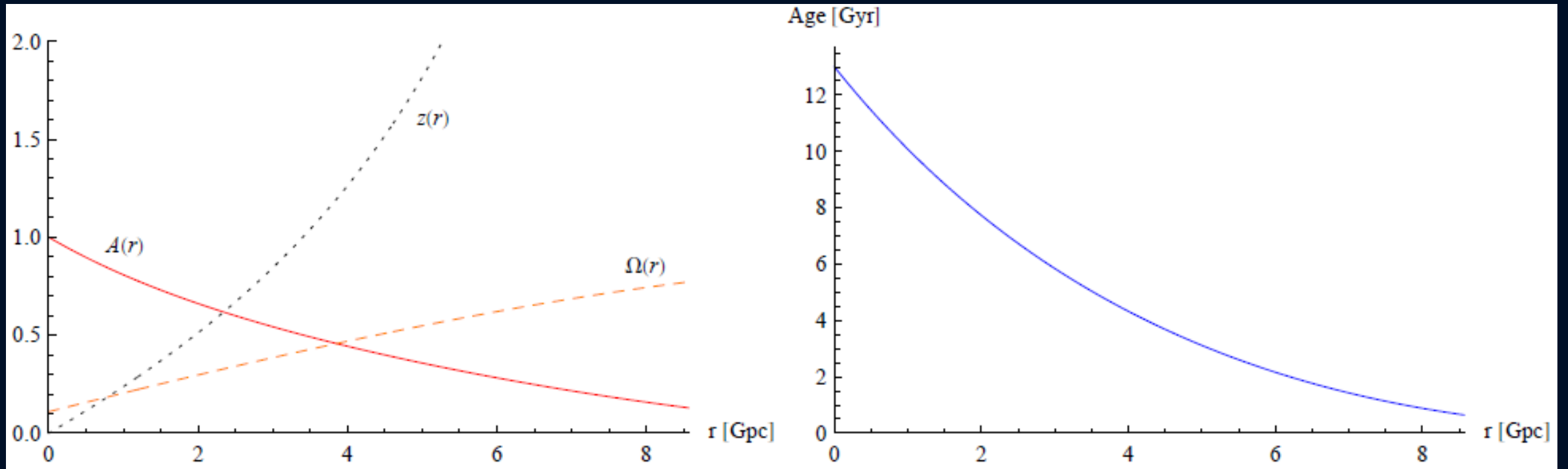
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# THE LEMAÎTRE-TOLMAN-BONDI MODEL

The solution up to redshift around 7 is

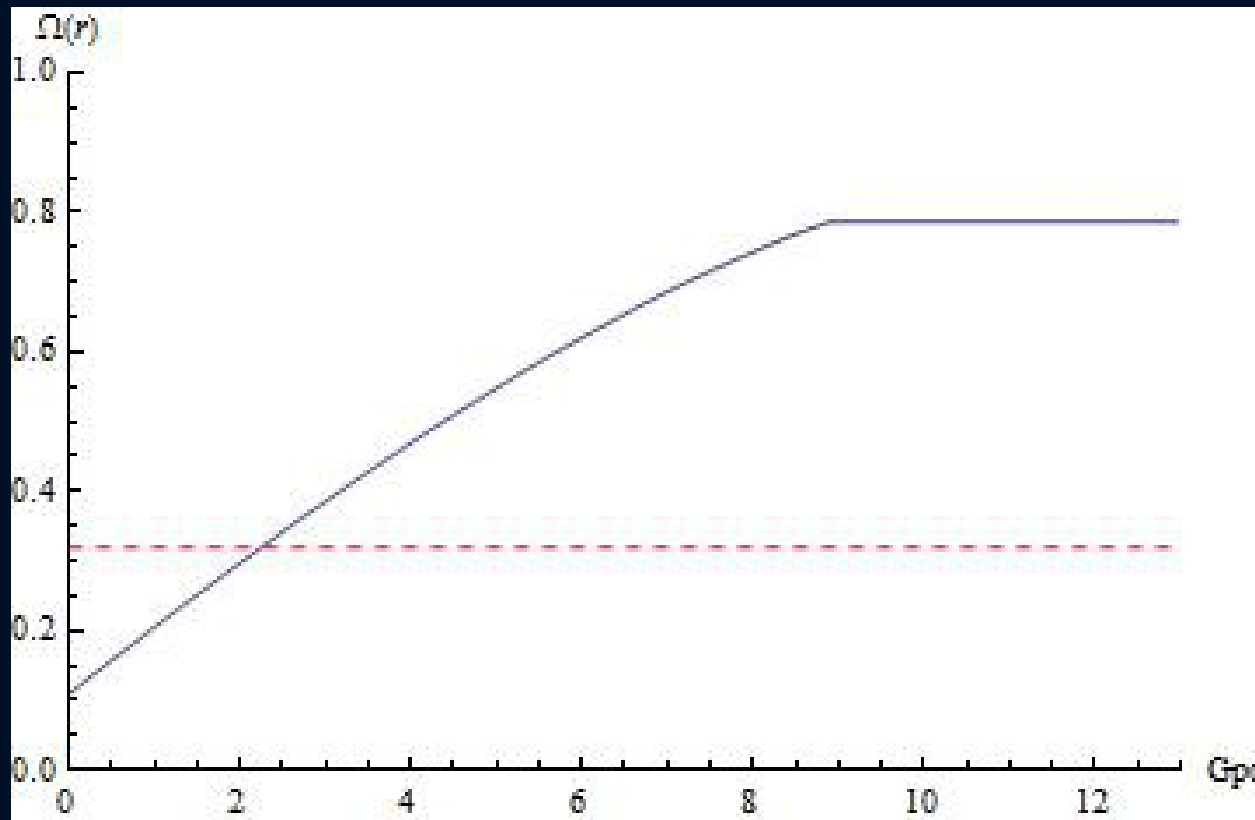


For larger redshifts the model cannot describe the  $\Lambda$ CDM luminosity distance without hitting to a shell crossing singularity. No matter, we actually do not need to copy that luminosity distance anymore.



# THE LEMAÎTRE-TOLMAN-BONDI MODEL

An example of our void



a void

$\Lambda$ CDM matter distribution

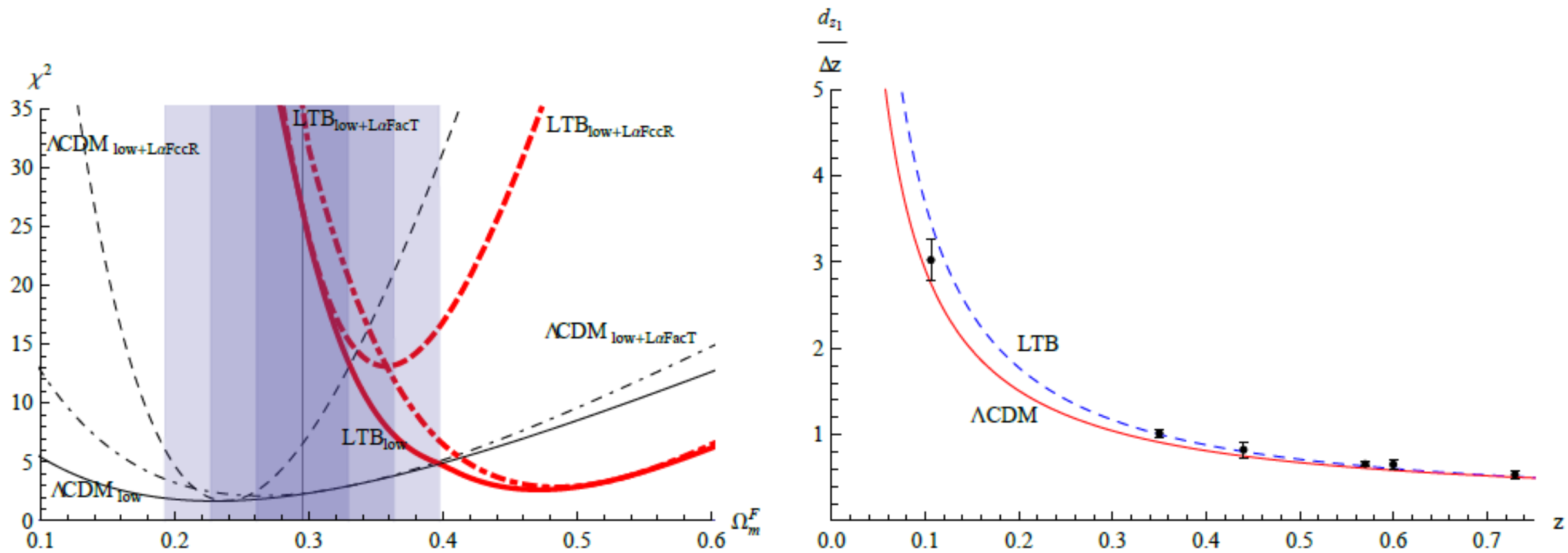
Milky Way's disk is approximately 30 kpc in diameter

# COMPARISON BETWEEN LTB AND $\Lambda$ CDM MODELS

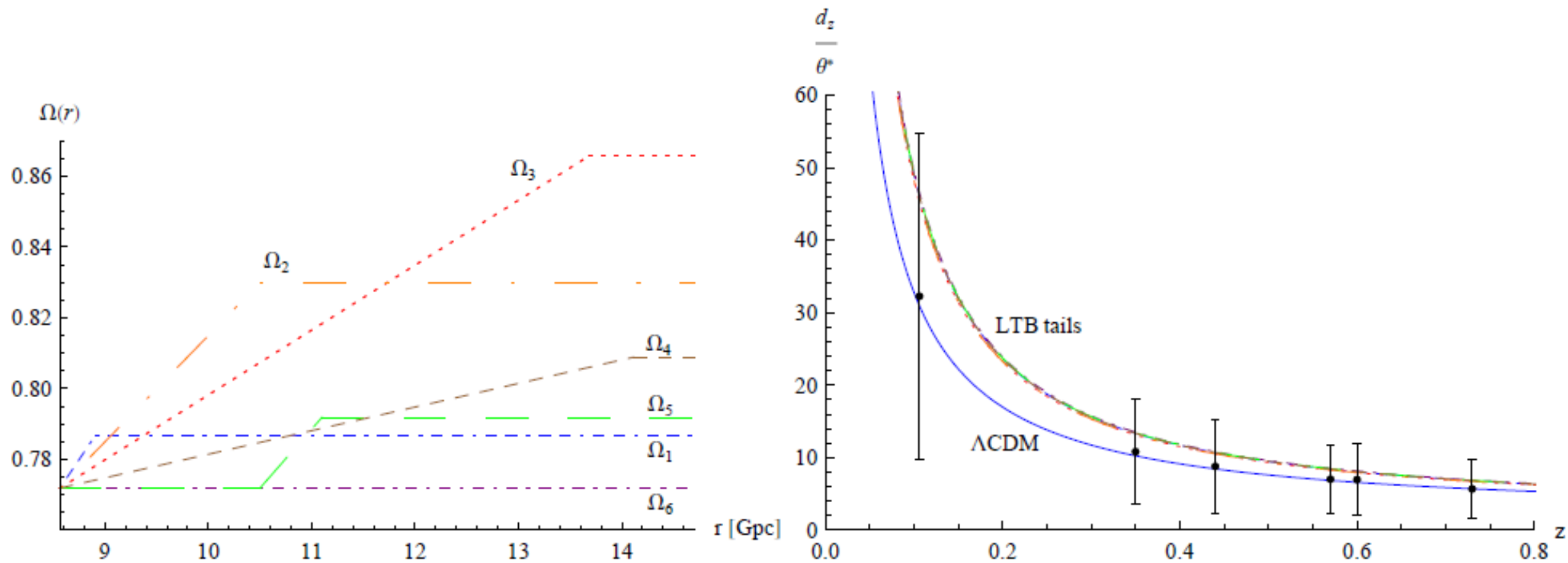
- We compared the properties of the void and the  $\Lambda$ CDM models
  - Local growth in time differ considerably, the further the two locations are from each other the greater the difference
- Inspired by these results, we investigated local structure growth
  - Baryon acoustic oscillations (BAO)
  - First CMB peak

# CAN A VOID MIMIC THE $\Lambda$ IN $\Lambda$ CDM?

- We used the low redshift BAO data and BAO data from the Lyman  $\alpha$  forest.
  - We were the first to fit the BAO data from the Lyman  $\alpha$  forest in the context of void models
- We used Planck data
- We discussed about the local Hubble data
- Remember that the model fits equally well to the supernova data as the  $\Lambda$ CDM model



**Figure 3.** The left panel: Different  $\chi^2(\Omega_m^F)$  curves for different models and data sets are presented. The black vertical line in the middle of the vertical contours indicates the preferred  $\Omega_m^F$  according to the SN data [34], whereas the vertical contours indicates 1-3  $\sigma$  deviations from the best fit value. See Table 2 for numerical values. The right panel: The (black) points and bars represent observations and their  $1\sigma$  errors, the red solid curve represents the  $\Lambda\text{CDM}$  model's prediction and the dashed blue curve represent the LTB model's prediction. For both models,  $\Omega_m^F = 0.32$  and  $\Delta z$  is chosen to correspond  $\text{LaFccR}$ . Colors available online.



**Figure 4.** The LTB and the  $\Lambda$ CDM models with  $H_0^F = 67.1$  and  $\Omega_m^F = 0.32$ . In the left panel several tails are plotted. In the right panel, (black) points and bars are observed  $d_z/\theta^*$  values and their  $1\sigma$  error bars, the solid blue curve represents the  $\Lambda$ CDM prediction and non-solid colored curves represent the LTB models with tails, the curve color and style indicates the corresponding left panel tails. The LTB curves can be arranged from the worst to the best fit as follows:  $\Omega_6$ ,  $\Omega_5$ ,  $\Omega_1$ ,  $\Omega_4$ ,  $\Omega_2$ , and  $\Omega_3$ . Colors available online.

# CAN A VOID MIMIC THE $\Lambda$ IN $\Lambda$ CDM?

- The results:
  - The low redshift BAO + SN strongly favours the  $\Lambda$ CDM model
  - The combined BAO + SN data favours the  $\Lambda$ CDM model even more
    - The discrepancy that appears between the  $\Lambda$ CDM model and the BAO features of the Lyman  $\alpha$  forest exists between the void model and the data, too
  - Our parametrisation allows us to satisfy the local Hubble observations independently on other observations.
  - CMB is more compatible with the  $\Lambda$ CDM model

# CAN A VOID MIMIC THE $\Lambda$ IN $\Lambda$ CDM?

## What did we learn?

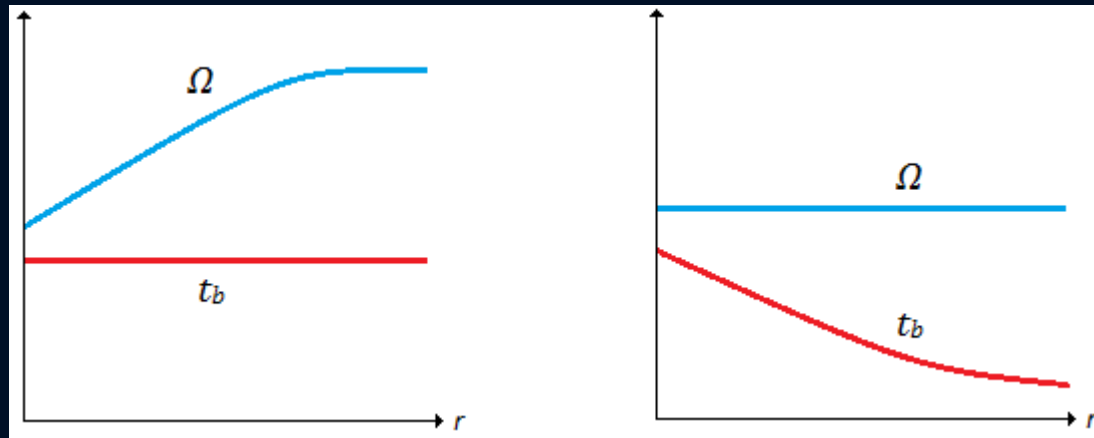
- To obtain observationally more viable models, one needs to include more degrees of freedom by renouncing
  - The homogeneous early universe assumption
  - The homogeneous bang time assumption
  - The negligible effects of pressure assumption
  - The spherical symmetry assumption

# INHOMOGENEOUS COSMOLOGICAL MODELS AND FINE-TUNING OF THE INITIAL STATE

- What if the redshift observed from one object would not change in time. Then its luminosity distance would stay constant too, because

$$D_A = \frac{1}{H_0^F (1+z) \sqrt{\Omega_k^F}} \sinh \left\{ \sqrt{\Omega_k^F} \int_0^z \frac{d\tilde{z}}{\sqrt{\Omega_m^F (1+\tilde{z})^3 + \Omega_k^F (1+\tilde{z})^2 + \Omega_w^F (1+\tilde{z})^w + \Omega_\Lambda^F}} \right\}$$

- It is shown that the above luminosity distance can be produced by either varying the matter density or the bang time.



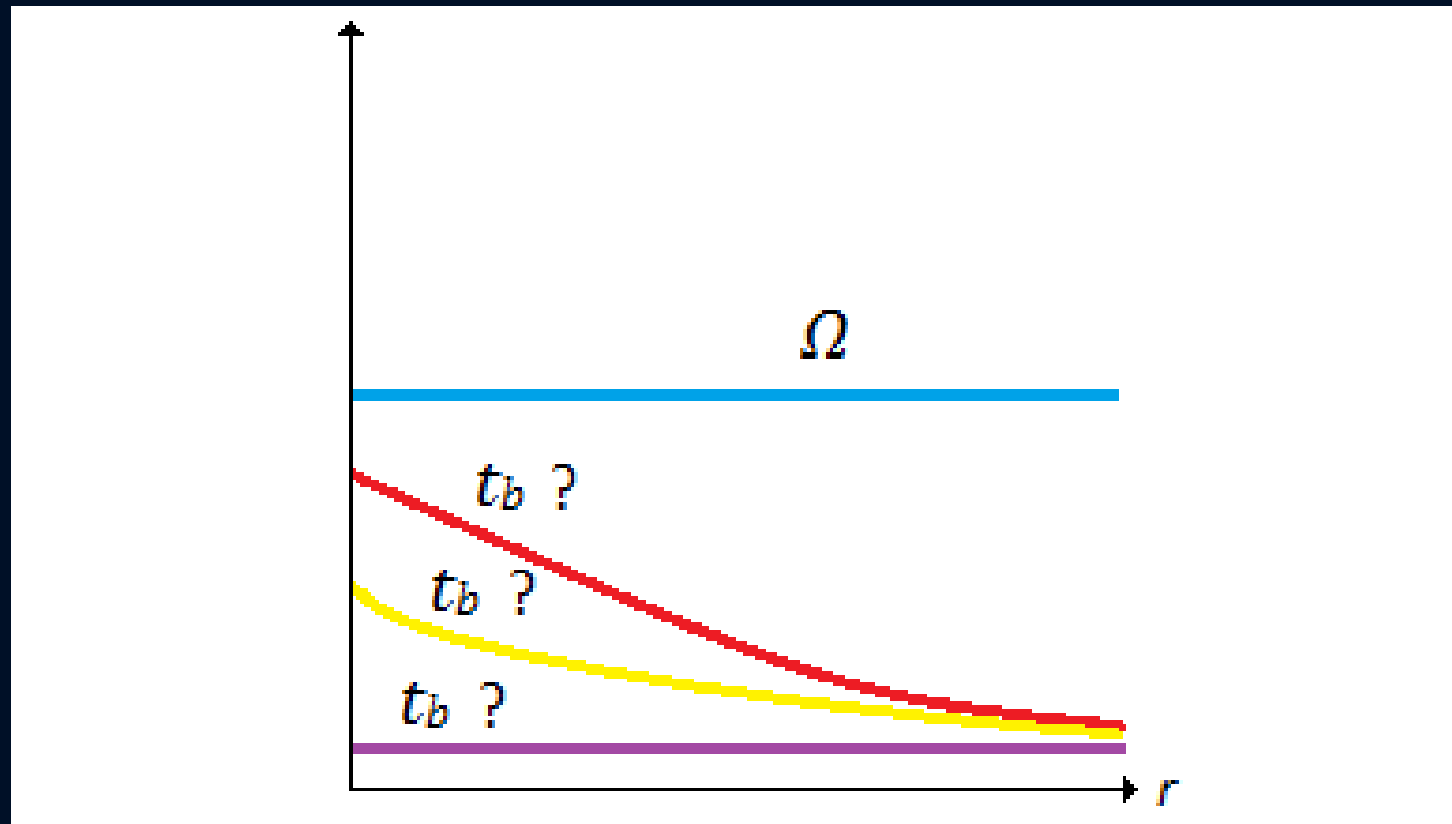


# THE UNDERLYING IDEA

- Consider a stable fixed point in the dynamical systems theory. Albert starts an experiment. He follows an object and when it is close enough to the fixed point he can no longer see it moving. Erwin walks into the room and sees the object still. Erwin does not know how long the object has stayed still, i.e. he does not know the time of the beginning of the experiment.
- Here, we do not see movement but redshift, so constant redshift corresponds to "stabilized state". If the redshift indeed does not change in time, we with our SN observations are the Erwin of the reality.

# THE UNDERLYING IDEA

- If this holds at every  $r$ , then we do not know where the bang time curve actually is...

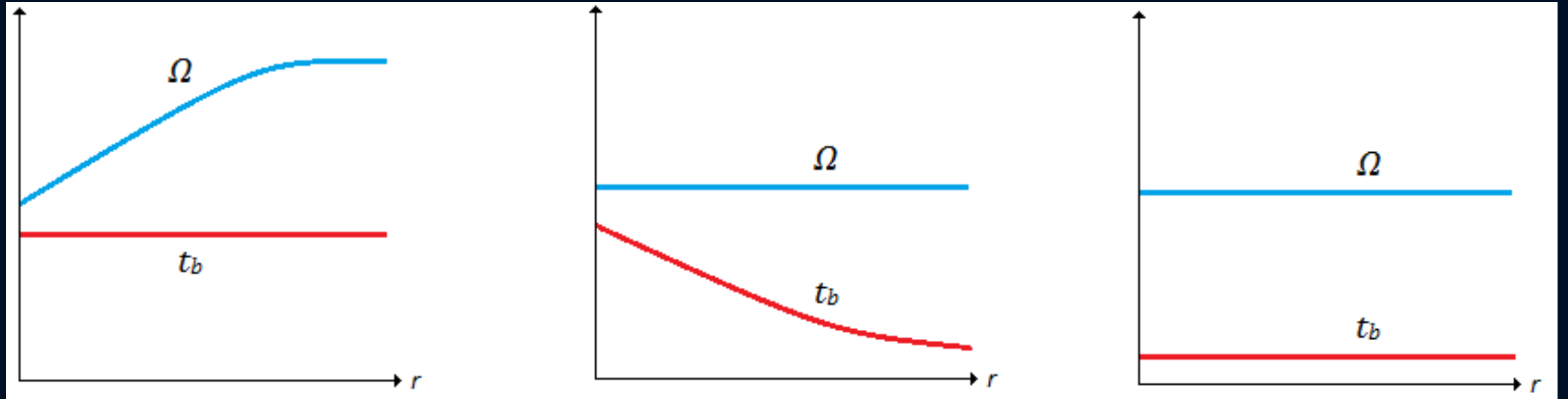


# WHAT WE DID

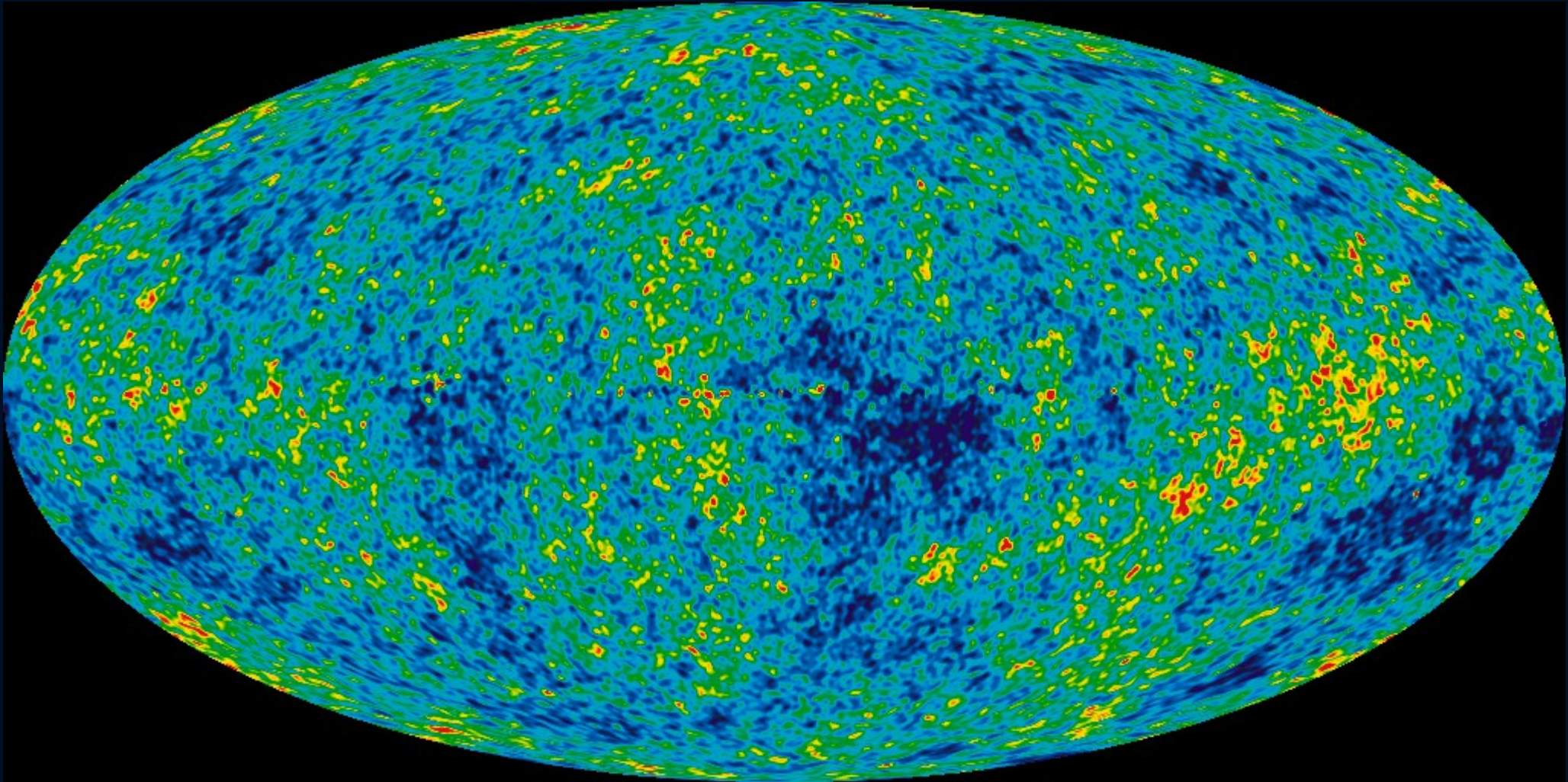
- We considered this possibility in the LTB models.
  - We investigated the general conditions to be satisfied in order to this scenario to take place
  - We gave the conditions also in the homogeneous limit
  - We showed that this scenario can not take place in realistic homogeneous situations

# THE PROMISE...

- Structure formation is similar everywhere
- Fits well to supernovae
- Observers location no longer fine-tuned



# Cosmic microwave background radiation



NASA / WMAP Science Team

# THE ANOMALIES OF THE CMB

- Several statically anisotropic features have been found in the data analysis of the CMB sky
- 1996 COBE-DMR found lack of angular two point correlation in CMB
- WMAP confirms
- WMAP found power spectrum asymmetry between opposite hemispheres
- Planck confirms

# THE ANOMALIES OF THE CMB

- It is debated if these asymmetries are statistically significant  
=> Do not rule  $\Lambda$ CDM out
- But anomalies are found by many different groups
- Suggests that there is a better fitting model

# AXISYMMETRIC BIANCHI IX MODEL ALLOWING ENERGY FLUX

Line element

$$ds^2 = -dt^2 + e^{2\alpha(t)+2\beta(t)} [(\omega^1)^2 + (\omega^2)^2] + e^{2\alpha(t)-4\beta(t)} (\omega^3)^2,$$

Where the one forms are

$$\omega^1 = \sin \psi d\theta - \sin \theta \cos \psi d\phi,$$

$$\omega^2 = \cos \psi d\theta + \sin \theta \sin \psi d\phi,$$

$$\omega^3 = \cos \theta d\phi + d\psi.$$

Energy-momentum tensor

$$T_b^a = [\rho(t) + p(t)]u^a u_b + p(t)g_b^a + q^a u_b + u^a q_b$$

Four velocity

$$u^a = (\cosh[\lambda(t)], 0, 0, \sinh[\lambda(t)]e^{2\beta(t)-\alpha(t)})$$

Equation of state

$$p(t) = w(t)\rho(t)$$

Einstein equations

$$E_b^a = T_b^a$$



# THE ANOMALIES OF THE CMB

Then, we rewrote the equations using physically more appealing quantities (than  $\alpha$  or  $\beta$  and their time derivatives). That is

- The Ricci scalar of the 3-dimensional constant time hypersurfaces
- The covariant definitions of the kinematical quantities in the 1+3 formalism
  - The shear scalar
  - The vorticity scalar
  - Expansion scalar

Finally we defined the dimensionless quantities:

$$\Sigma(t) \equiv \frac{\sigma(t)}{\sqrt{3} \cosh[\lambda(t)] H(t)}, \quad K(t) = -\frac{{}^3R(t)}{6H(t)^2}, \quad \mathcal{V}(t) = \frac{\omega(t)}{\sinh[\lambda(t)] H(t)}, \quad \Omega(t) = \frac{\rho(t)}{3H(t)^2}$$

and assumed the average expansion  $\alpha$  to count time, i.e.  $\frac{f'(t)}{H(t)} = \frac{f'(t)}{\alpha'(t)} = f'(\alpha)$

# WE FOUND THE EINSTEIN EQUATIONS TO BE

$$W \equiv \left(-w + 1 + \frac{w+1}{\cosh(2\lambda)}\right)/2 \quad \text{“tilt”}$$

$$\Sigma'(\alpha) = -1 + 2K(\alpha) + \Sigma(\alpha)^2 + (\epsilon - 3)\Sigma(\alpha) + \mathcal{V}(\alpha)^2 + \Omega(\alpha),$$

Shear

$$\Omega'(\alpha) = \Omega(\alpha) \left( 2\epsilon + 2\Sigma(\alpha) - 4 + \frac{1 - 3w - 2\Sigma(\alpha)}{W(\alpha)} \right) - \frac{\Omega(\alpha)}{W(\alpha)} W'(\alpha),$$

Fluid

$$\mathcal{V}'(\alpha) = \mathcal{V}(\alpha) (\epsilon - 4\Sigma(\alpha) - 1),$$

Vorticity

$$K'(\alpha) = -2 \{ K(\alpha) [-\epsilon + 1 + \Sigma(\alpha)] + \Sigma(\alpha) \mathcal{V}(\alpha)^2 \},$$

Curvature

$$\epsilon = \frac{1}{2} [3w - 1] \Omega(\alpha) - K(\alpha) + 2 + \Sigma^2(\alpha),$$

Acceleration

$$1 = W\Omega(\alpha) + K(\alpha) + \Sigma^2(\alpha).$$

Generalized Friedmannian

# THE ACCELERATION PARAMETER

$$\epsilon \equiv -H'(t)/H(t)^2$$

$\epsilon = 1$	in the curvature filled universe,
$\epsilon = 3$	in the shear filled universe,
$\epsilon = 2 - \frac{1}{1 + \operatorname{sech}(2\lambda)}$	in the dust filled universe,
$\epsilon = 2$	in the radiation filled universe,
$\epsilon = 0$	in the dark energy filled universe.

# DYNAMICAL SYSTEMS ANALYSIS - THE MOST INTERESTING FIXED POINTS

- THE ONE(S) WITH VORTICITY

$$\begin{aligned} \mathcal{V}^* &= \pm \frac{3}{2} \sqrt{\frac{(2-3w)w + 8(W-1)W + 1}{(1-5W)^2}} \\ K^* &= \frac{3((2-3w)w + 8(W-1)W + 1)}{4(1-5W)^2} \\ \Omega^* &= -\frac{3(w-6W+1)}{(1-5W)^2} \\ \Sigma^* &= \frac{3w+2W-1}{10W-2} \end{aligned}$$

- THE ONE THAT ACCELERATES ANISOTROPICALLY

$$\begin{aligned} V^* &= 0 \\ K^* &= 0 \\ \Omega^* &= \frac{3(w-1)(3w-4W+1)}{W(-3w+2W+1)^2} \\ \Sigma^* &= \frac{2-2W}{-3w+2W+1} \end{aligned}$$

# THE MOST INTERESTING FIXED POINT

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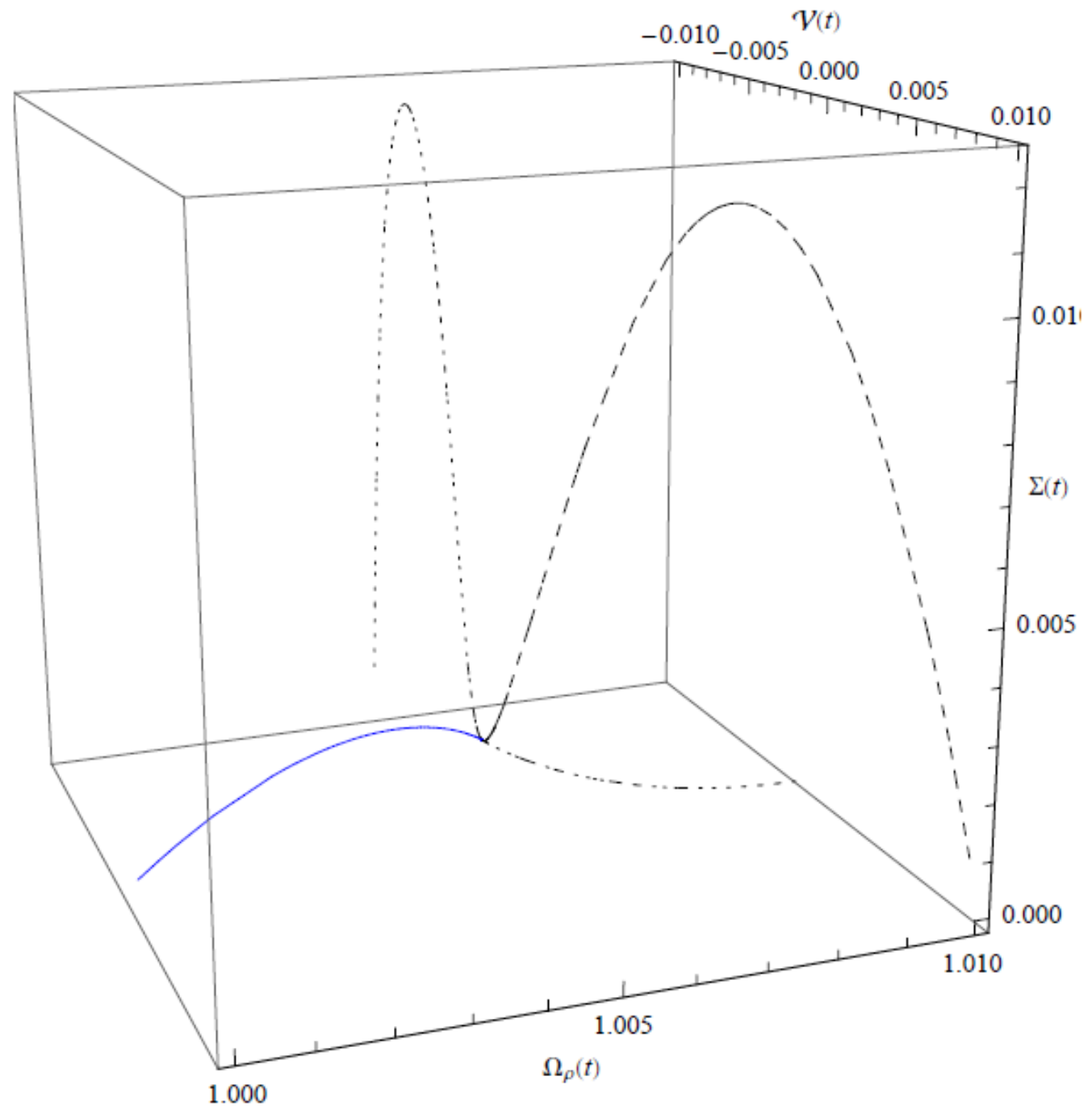
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# ANISOTROPIC INFLATION

$$w = -0.99 \text{ and } \lambda = 4,$$

$$\epsilon \approx 0.005$$

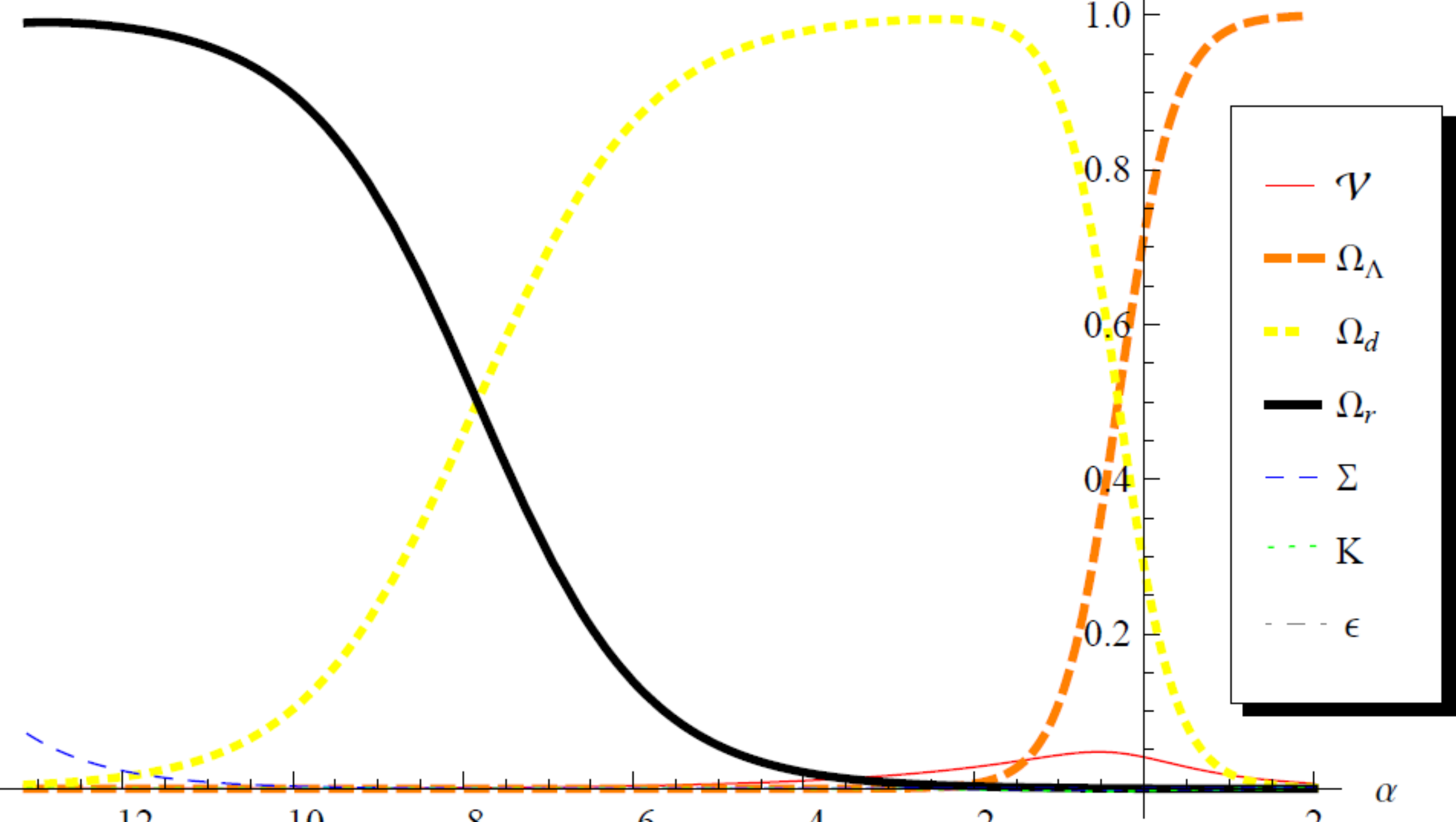
$$\mathcal{V}^* = 0, \Sigma^* \approx 1.7 \times 10^{-3} \text{ and } \Omega^* \approx 1.005$$



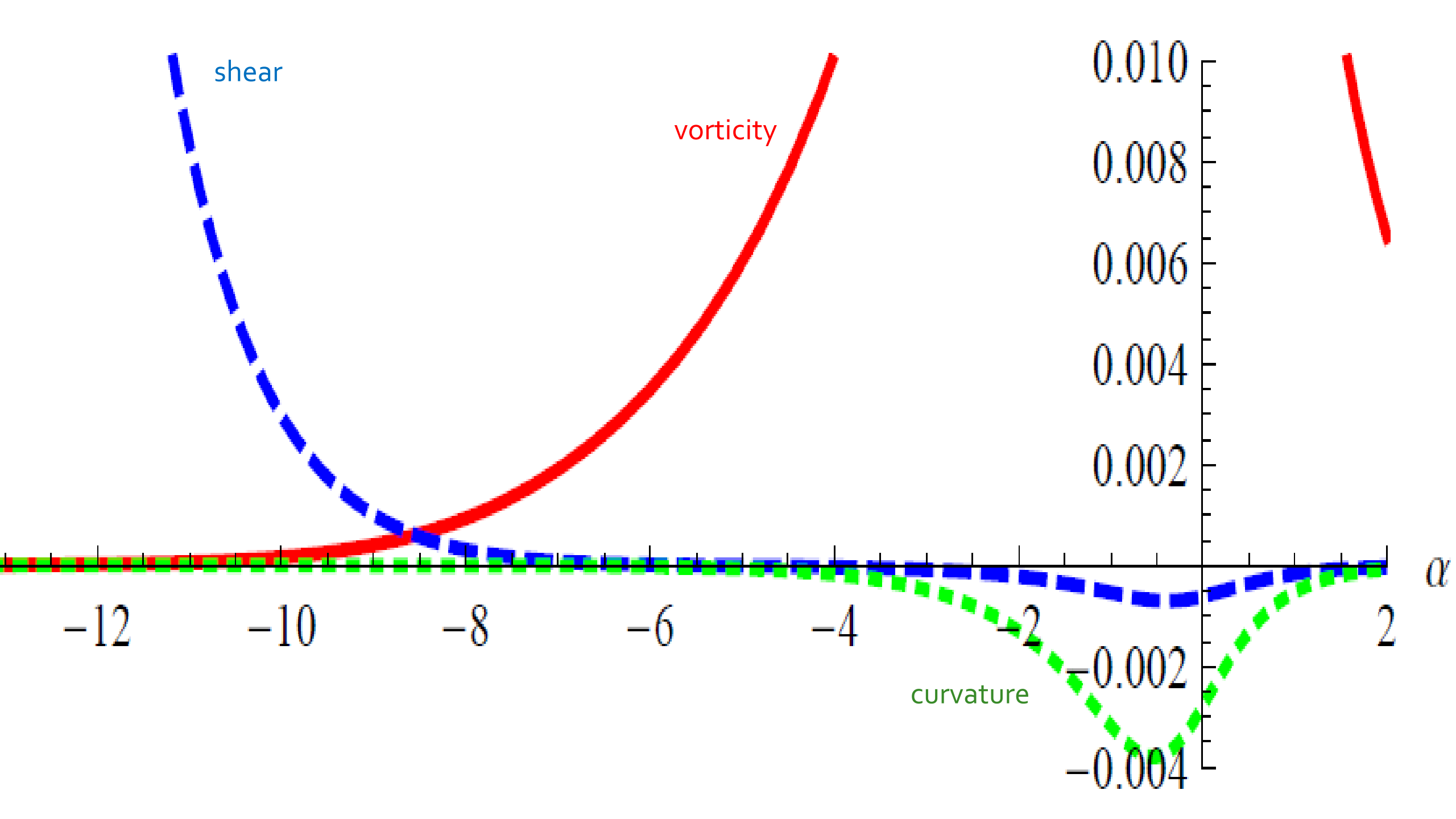
# ANISOTROPIC COSMOLOGY

- The fluid is composed of dust, radiation and dark energy
- Only dark energy is tilted and  $w = -1$
- The tilt is very small  $\lambda = 1/100$ ,
- Only dark energy experiences energy conduction

	$\Omega_\Lambda$	$\Omega_d$	$\Omega_r$	$\Sigma$	$\mathcal{V}$	$K$
Initially	$1 \times 10^{-19}$	$5 \times 10^{-3}$	0.99	$\approx 7 \times 10^{-2}$	$1 \times 10^{-5}$	$-1 \times 10^{-10}$
At the Decoupling $\sim$	$1 \times 10^{-9}$	0.7	0.3	$9 \times 10^{-5}$	$2 \times 10^{-3}$	$-6 \times 10^{-6}$
Present Time $\sim$	0.7	0.3	$1 \times 10^{-4}$	$-6 \times 10^{-4}$	$4 \times 10^{-2}$	$-3 \times 10^{-3}$





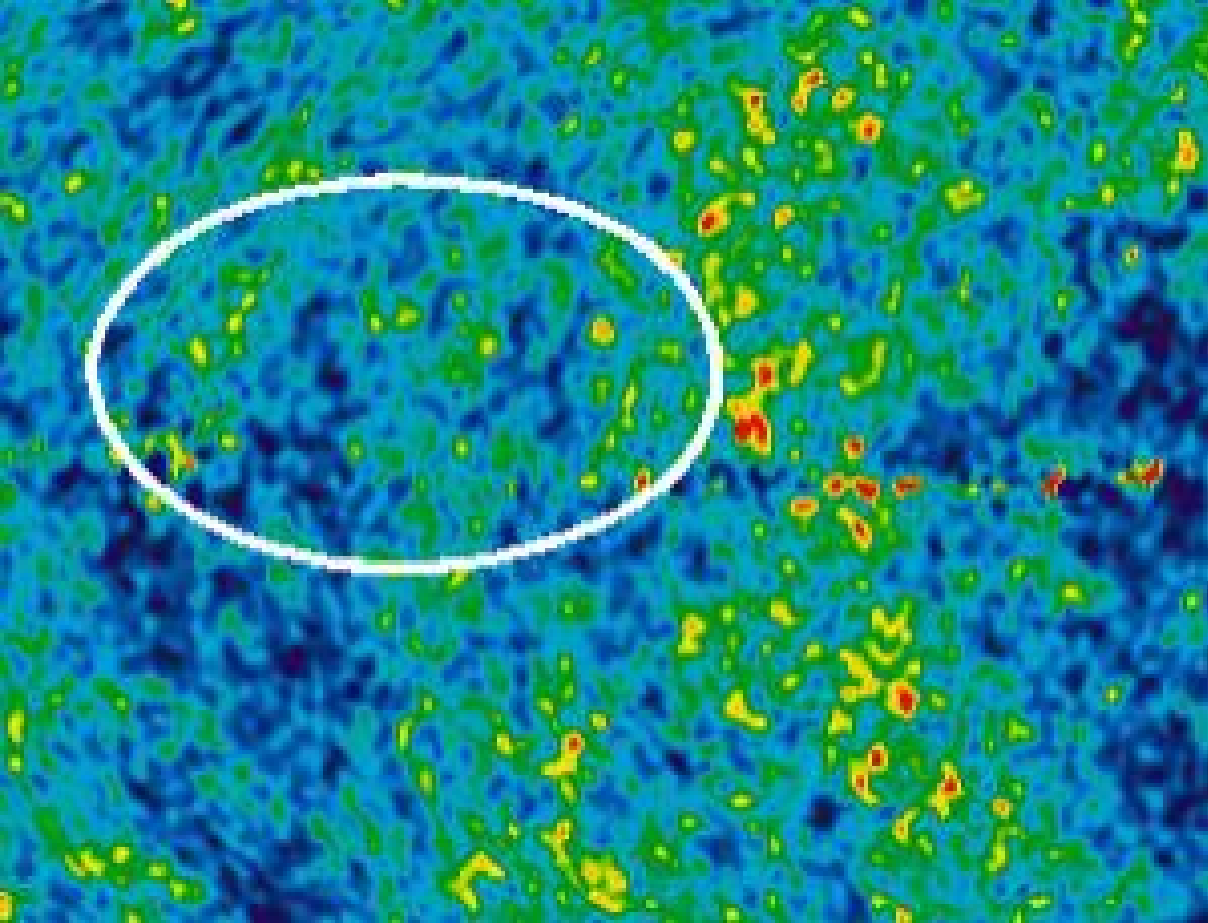


# CONCLUSIONS

- The model can explain the anomalies of CMB
- The model differs from the  $\Lambda$ CDM model only what we assume from of dark energy
  - It is slightly tilted
  - dark energy experiences energy conduction
- The model does not require any further fine-tuning than the  $\Lambda$ CDM model!
- We showed tilted fluid allowing energy flux gives a counter-example to cosmic no-hair conjecture

# FINAL REMARKS

- The  $\Lambda$ CDM model is not perfectly compatible with several observations
- Inhomogeneity and anisotropy offer a natural generalization
- Inhomogeneities can explain some observed features (at least partly)
- Anisotropies appear viable explanations for the CMB anomalies (at least partly)



THANK YOU!

Peter Sundell  
University of Turku

Image: NASA/WMAP  
Science Team