Lifshitz quasinormal modes
and
relaxation from holography

Based on arXiv 1503.07457

by Stefan Vandoren and Watse Sybesma

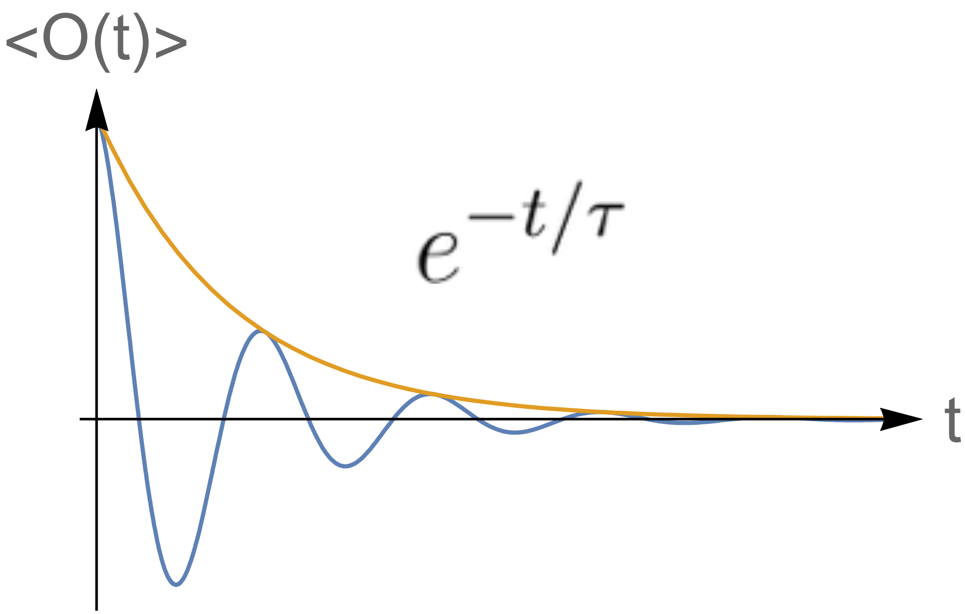
Utrecht University

18-11-2015 in Madrid

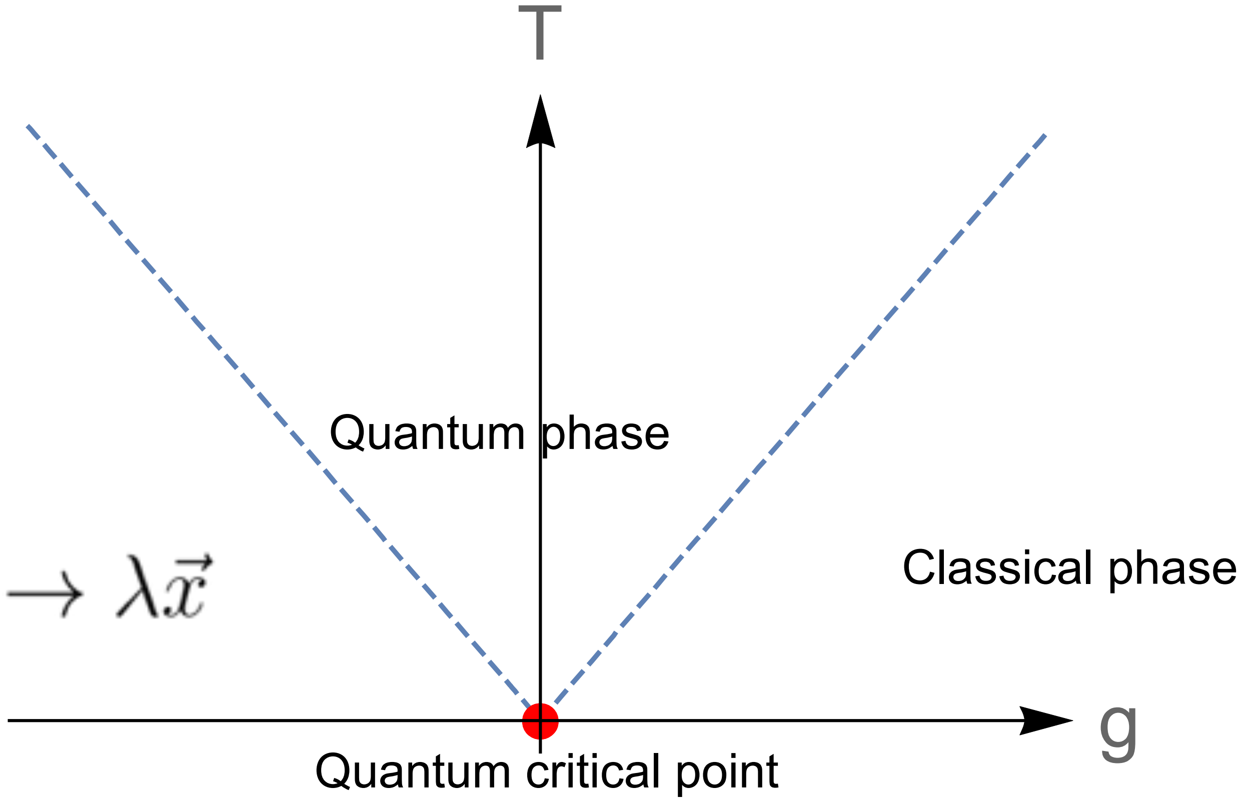
Motivation

- How do interacting systems equilibrate or relax after a perturbation?

$$< \mathcal{O} > \sim \mathrm{Re}[e^{-i\omega t}] = e^{-|\omega_{\mathrm{im}}|t} \cos(\omega_{\mathrm{re}} t)$$



Motivation



Lifshitz invariance:

$$t \to \lambda^z t \,, \qquad \vec{x} \to \lambda \vec{x}$$

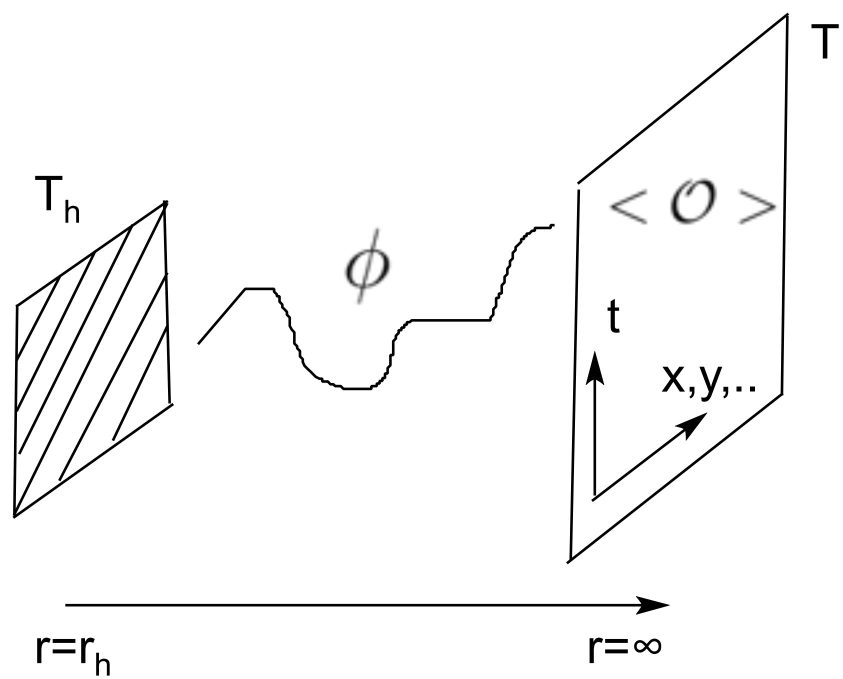
Possible approach:

$$\mathcal{L} = (\partial_0 \phi)^2 - \kappa \left(\nabla^2 \phi\right)^2 - c\phi^{2\frac{d+1}{d-3}}$$

$$< \mathcal{O} > = < \phi\phi >$$

Goal

Obtaining τ of $< \mathcal{O} >$ with scaling dimension Δ
by computing the quasinormal modes of ϕ in the gravitational bulk



Lifshitz gravitational bulk

- Solve this wave equation in Lifshitz background

$$\Box\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi\right) = m^2\phi$$

- Lifshitz black brane line element

$$ds^2 = \frac{1}{r^2 f(r)}dr^2 - f(r)r^{2z}dt^2 + r^2 d\vec{x}^2_{d-1}$$

$$t \to \lambda^z t\,, \quad \vec{x} \to \lambda\vec{x}$$

$$f(r) = 1 - \left(\frac{r_h}{r}\right)^{d+z-1} \qquad 4\pi T = (d+z-1)r_h^z$$

Not unique

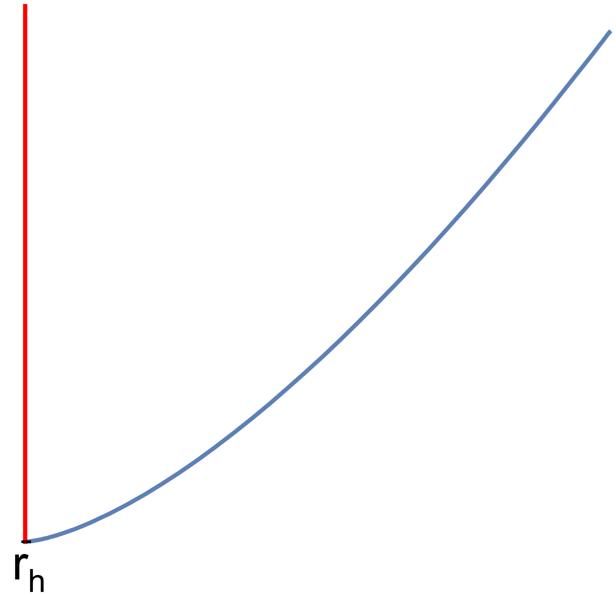
K. Balasubramanian et al, P.A. Gonzalez et al, A. Giacomini et al, E. Abdalla et al, ..

Solving the wave equation for qnms

- Radial Ansatz

$$\phi = r^{\frac{d-1}{2}}\phi(r)e^{-i\omega t+i\vec{k}\cdot\vec{x}} \qquad \partial_* = r^{z+1}f(r)\partial_r$$

$$\left[\partial_*^2 + \omega^2 - V(r,\vec{k},d,z,m,T)\right]\phi = 0$$



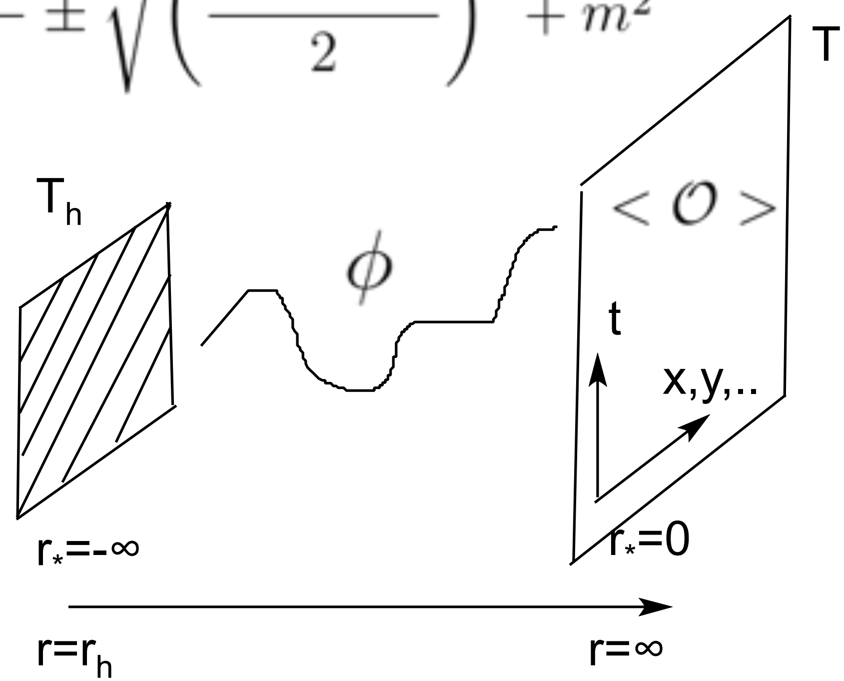
Solving the wave equation for qnms

$$\left[\partial^2_{r_*} + \omega^2 - V(r, \vec{k}, d, z, m, T)\right] \phi = 0$$

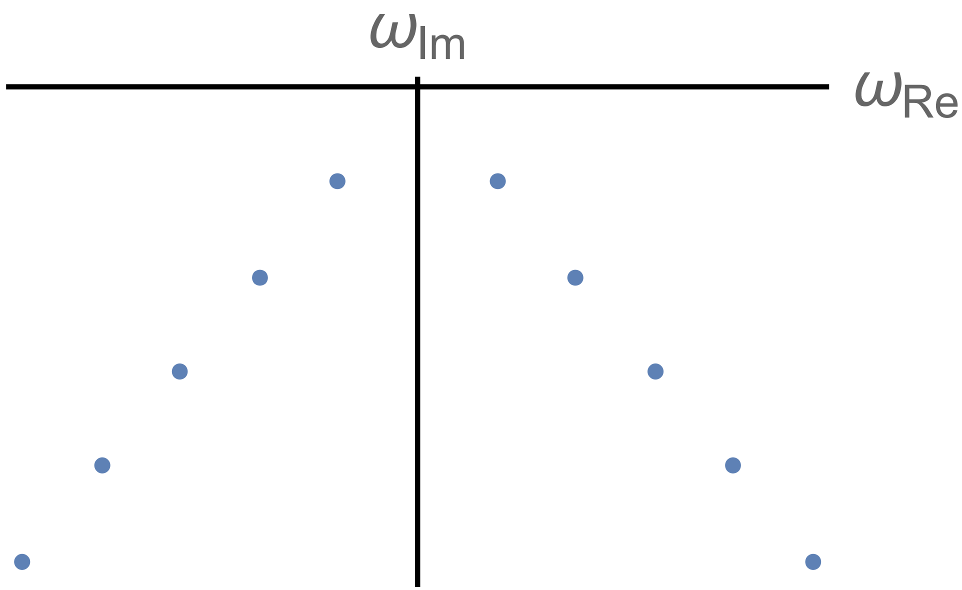
Asymptotically for $r \to r_h$ we require $\phi \sim e^{\pm i\omega r_* - i\omega t}$

Asymptotically for $r \to \infty$ we require $\phi \sim Ar^{-\Delta_-} + Br^{-\Delta_+}$

$$\Delta_\pm = \frac{d+z-1}{2} \pm \sqrt{\left(\frac{d+z-1}{2}\right)^2 + m^2}$$



Results and Outlook

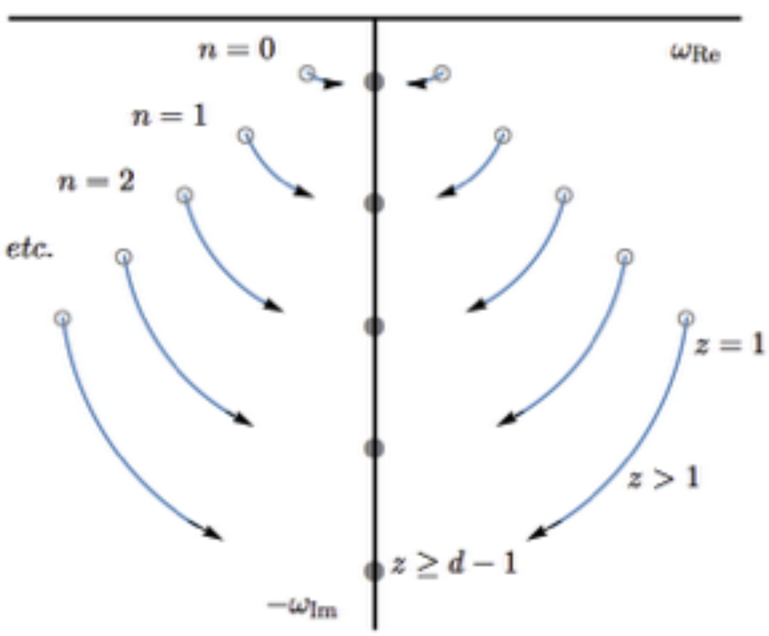


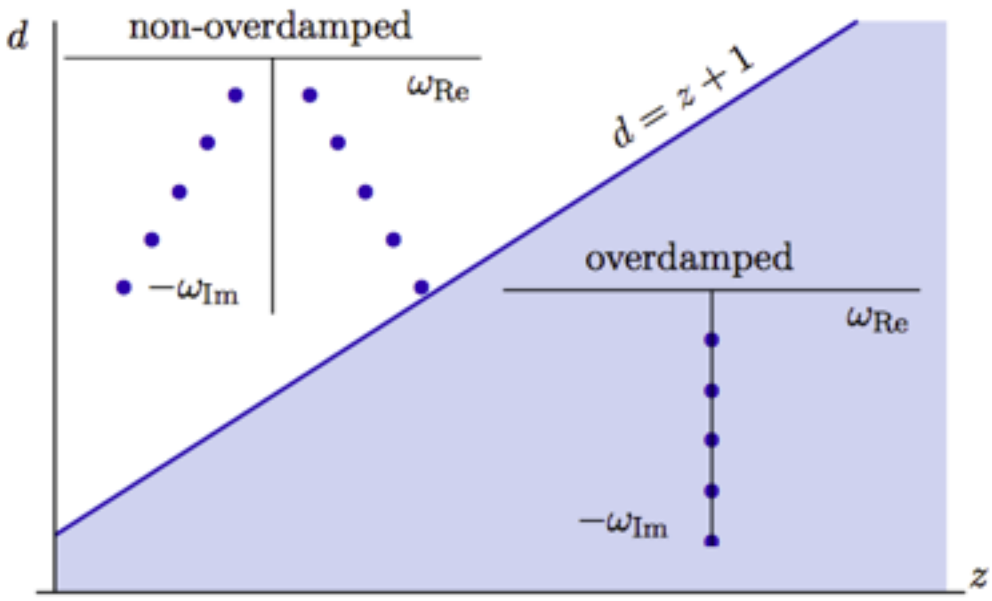
$$\left|e^{-i\omega_n t}\right| = e^{(\omega_n)_{\text{Im}} t} \equiv e^{-t/\tau}$$

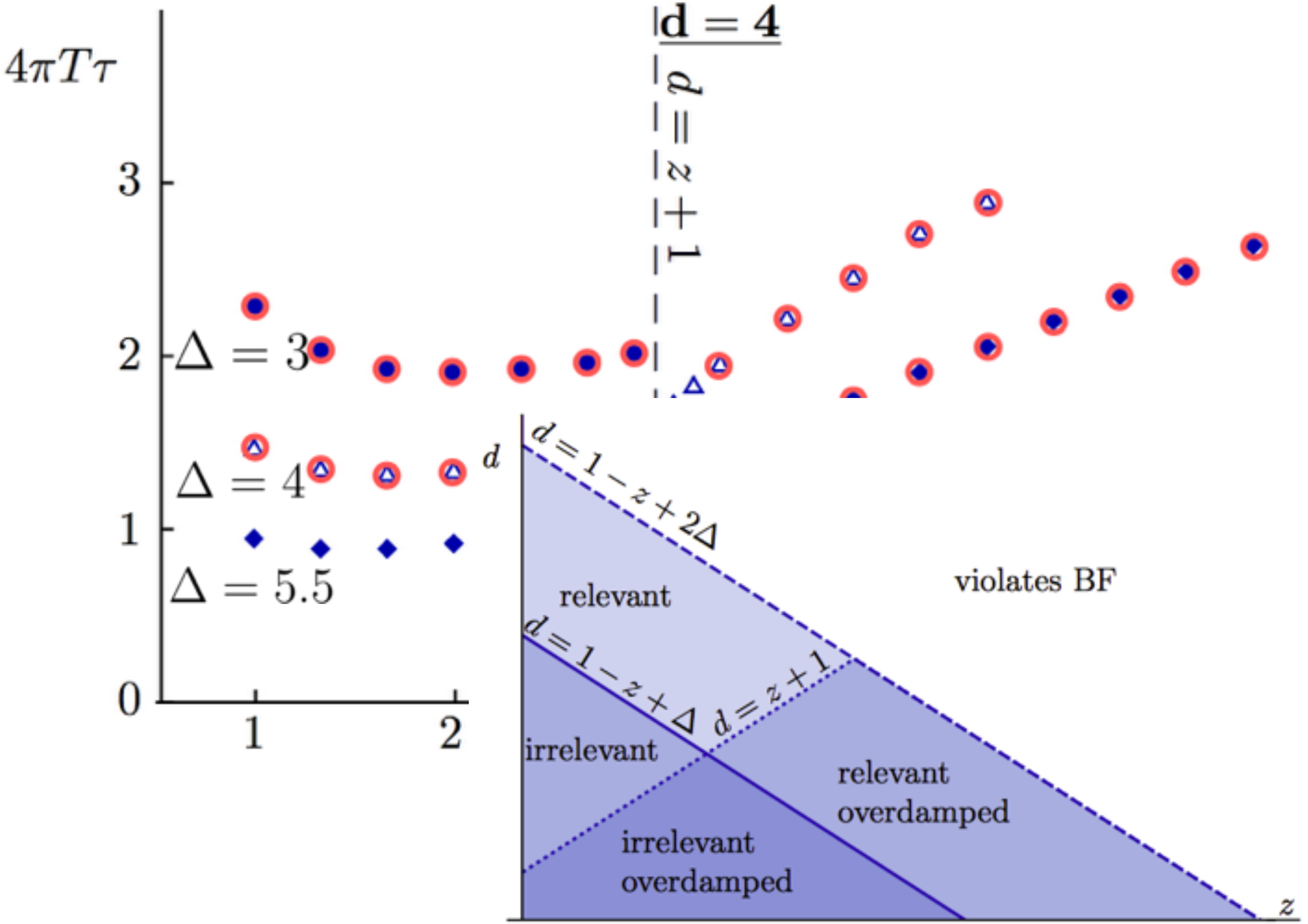
For $d = z + 1$ and $k = 0$ one is able to solve analytically and find $\frac{\omega_n}{2\pi T} = -i\left(2n + \frac{\Delta}{z}\right)$ or $\tau = \frac{z}{2\pi T \Delta}$

Results and Outlook

- In other cases we solve numerically







$4\pi T\tau$
3
2
1
0
1
2
$\Delta = 3$
$\Delta = 4$
$\Delta = 5.5$
d = 4
$d = z + 1$
d
z
$d = 1 - z + 2\Delta$
$d = 1 - z + \Delta$
$d = z + 1$
violates BF
relevant
irrelevant
relevant overdamped
irrelevant overdamped

Results and Outlook

- We obtained relaxation time τ using holography
- $d = z + 1$ is special in Lifshitz theories
- use other black brane
- including back reaction to check the validity of the linear approach
- investigate hydrodynamic quantities for spin 1 and spin 2

Muchas gracias for your attention!

Results and Outlook

- For a d-dimensional theory with Lifshitz scaling

$$\omega \sim |k|^z$$

- Volume occupied per allowed state

$$\Omega(k) \sim k^{d-1}$$

- Density of states is found to be

$$D(\omega) = \frac{d\Omega(k(\omega))}{d\omega} = \frac{d\Omega(k)}{dk}\frac{dk(\omega)}{d\omega} \sim \omega^{\frac{d-2}{z}}\omega^{\frac{1-z}{z}} = \omega^{\frac{d-(z+1)}{z}}$$