Class. Quant. Grav. 32 (2015) 215017

Lancaster University

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

IV Postgraduate Meeting on Theoretical Physics
Madrid, Spain

Spyridon Talaganis

Supported by the Onassis Foundation

Onassis Foundation

Collaborators

- Tirthabir Biswas
- Anupam Mazumdar

Other people's work

- E.T. Tomboulis, arXiv: 9702146 [hep-th]
- L. Modesto, arXiv: 1107.2403 [hep-th]
- D .Anselmi, arXiv:1302.7100 [gr-qc]

Aim

- Our aim is to construct a UV-finite theory of quantum gravity that is not plagued by pathologies such as ghosts
- Towards that end, we consider a scalar field theory toy model
- Based on that, can we formulate a complete theory of quantum gravity?

Degree of Divergence in GR

- The superficial degree of divergence in d dimensions is $D = Ld + 2(V - I)$
- L is the number of loops, V is the number of vertices and I is the number of internal propagators
- Use the topological relation $L = 1 + I - V$
- In four dimensions, we get $D = 2 + 2L$
- The superficial degree of divergence keeps increasing as L increases

Renormalizability of GR

- Einstein-Hilbert action:

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}\, R$$

- Pure gravity is renormalizable at 1-loop order
- 1 new counterterm required at 2-loop order

Renormalizability of GR

- Stelle (1977) has shown that fourth-order pure gravity is renormalizable!

$$S = -\int \mathrm{d}^4x\,\sqrt{-g}\,\left(\alpha R_{\mu\nu}R^{\mu\nu} - \beta R^2 + \kappa^{-2}\gamma R\right)$$

$$\text{where} \quad \gamma = 2 \quad \& \quad \kappa^2 = 32\pi G$$

- We do not have to include $\int \mathrm{d}^4x\,\sqrt{-g}R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$

because of the Gauss-Bonnet topological invariance in four dimensions:

$$\int \mathrm{d}^4x\,\sqrt{-g}\,\left(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu nu}R^{\mu\nu} + R^2\right)$$

vanishes in Minkowski spacetime

Ghosts

- Unfortunately, Stelle's theory, as higher-derivative theories generically do, contains ghosts (poles in the propagator with negative residue); specifically, a massive spin-2 ghost

$$\Pi(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{p^2+m^2}$$

where $\quad m^2 > 0$

- Unitarity is violated
- We want to get rid of the ghost

Non-local Higher-derivative Gravity

- Non-local means that we consider an infinite series of higher-derivative terms in the action
- The most general covariant action up to $\mathcal{O}(h^2)$ (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101) is

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{2} + R_{\mu_1\nu_1\lambda_1\sigma_1} \mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2} R^{\mu_2\nu_2\lambda_2\sigma_2} \right]$$

- $\mathcal{O}$ is a differential operator containing covariant derivatives and $\eta_{\mu\nu}$
- The quadratic curvature part of the action up to $\mathcal{O}(h^2)$ can be written, after many simplifications, as

$$S_q = \int \mathrm{d}^4x \left[R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\lambda\sigma} \right],$$

since the covariant derivatives take on the Minkowski values

Non-local Higher-derivative Gravity

- As we shall see later, if we choose $\mathcal{F}_3(\Box) = 0$ & $\mathcal{F}_1(\Box) = \frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} = -\frac{\mathcal{F}_2(\Box)}{2}$, we obtain the ghost-free action (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

Linearized Action

- We perturb the metric fluctuations around the Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- We want to obtain the $\mathcal{O}(h^2)$ part of the action
- If we perturb the metric fluctuations around the Minkowski background, we get (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$S_q = -\int \mathrm{d}^4x \left[\frac{1}{2}h_{\mu\nu}\Box a(\Box)h^{\mu\nu} + h_{\mu}^{\sigma}b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} + hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} \right.$$
$$\left. + \frac{1}{2}h\Box d(\Box)h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\right]$$

Linearized Action

- We have the relations (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box,$$

$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box,$$

$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box,$$

$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box,$$

$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$

- If $f(\Box) = 0 \Rightarrow a(\Box) = c(\Box)$, then we observe

$$2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

Propagator in Non-local Higher-derivative Gravity

- As a consequence of the generalized Bianchi identities, we have

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

- The field equations can be written in the form

$$\Pi_{\mu\nu}^{-1\lambda\sigma} h_{\lambda\sigma} = \kappa\tau_{\mu\nu}$$

- $\Pi_{\mu\nu}^{-1\lambda\sigma}$ is the inverse propagator
- The propagator is $\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$
- To recover GR in the IR, we must have $a(0) = c(0) = -b(0) = -d(0) = 1$

As $k^2 \rightarrow 0$,we obtain the physical graviton propagator $\lim_{k^2 \rightarrow 0} \Pi^{\mu\nu}{}_{\lambda\sigma} = \frac{P^2}{k^2} - \frac{P_s^0}{2k^2}$

Ghosts in Non-local Higher-derivative Gravity

- If we apply the assumption $f = 0 \Rightarrow a = c$, then the propagator becomes

$$\Pi^{\mu\nu}{}_{\lambda\sigma} = \frac{1}{k^2 a(-k^2)}\left(P^2 - \frac{1}{2}P_s^0\right) = \frac{1}{a(-k^2)}\Pi_{\mathrm{GR}}$$

- We are left with a single arbitrary function $a(\Box)$ since $a = c = -b = -d$
- Provided $a(\Box)$ has no zeroes, only the graviton propagator is modified and ghosts are avoided (Biswas, Gerwick, Koivisto, Mazumdar, *Phys. Rev. Lett.* **108** (2012) 031101)
- Choosing $a(-k^2)$ to be a suitable entire function, the ultraviolet behavior of the gravitons can be tamed
- One such choice is $a(-k^2) = e^{k^2/M^2}$
- M is a mass scale at which the non-local modifications become important

Symmetries

- Field equations of GR satisfy the global scaling symmetry

$$g_{\mu\nu} \to \lambda g_{\mu\nu}$$

- Quadratic curvature actions of the form

$$S_q = \int \mathrm{d}^4x\sqrt{-g}\left[R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\lambda\sigma}\right],$$

where the $\mathcal{F}_i$'s are analytic functions of $\Box$,

are invariant under the aforementioned symmetry

- When we expand the action around Minkowski space, the symmetry for $h_{\mu\nu}$ becomes, infinitesimally,

$$h_{\mu\nu} \to (1+\epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

- Relates the free and interaction parts of the action (not a fundamental symmetry); it is useful to have a theory with propagators and vertices having opposing momentum dependence, which is a key feature of gauge theories
- We arrive at the shift-scaling symmetry

$$\phi \to (1+\epsilon)\phi + \epsilon$$

- We can formulate scalar toy model whose quantum behavior resembles that of the full gravitational theory

Degree of Divergence in Non-local Gravity

- Our modified superficial degree of divergence counting exponents is $E = V - I$
- Use again the topological relation $L = 1 + I - V$
- We obtain $E = 1 - L$
- For $L > 1$, E is negative, implying superficially convergent loop amplitudes
- Clear contrast with GR

Scalar Field Theory Toy Model Action

- Our scalar field theory toy model action is

$$S = \frac{1}{2}\int \mathrm{d}^4x \; (\phi\Box a(\Box)\phi)$$
$$+ \frac{1}{M_p}\int \mathrm{d}^4x \left(\frac{1}{4}\phi\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}\phi\Box\phi a(\Box)\phi - \frac{1}{4}\phi\partial_\mu\phi a(\Box)\partial^\mu\phi\right)$$

where $a(\Box) = e^{-\Box/M^2}$

- M is a mass scale at which the nonlocal modifications become important
- Every propagator comes with an exponential suppression and every vertex comes with an exponential enhancement
- The superficial degree of divergence argument for non-local theories of gravity also holds true for the scalar field theory toy model

Propagator

- Our propagator in Euclidean space is

$$\Pi(k^2) = \frac{-i}{k^2 e^{k^2/M^2}}$$

- The propagator is exponentially suppressed
- As $k^2 \to 0$, we obtain the k^{-2} momentum dependence of the propagator in GR, as it should be in the IR

Vertex Factors

- We have that

$$V(k_1, k_2, k_3) = iC\left[1 - e^{k_1^2/M^2} - e^{k_2^2/M^2} - e^{k_3^2/M^2}\right],$$

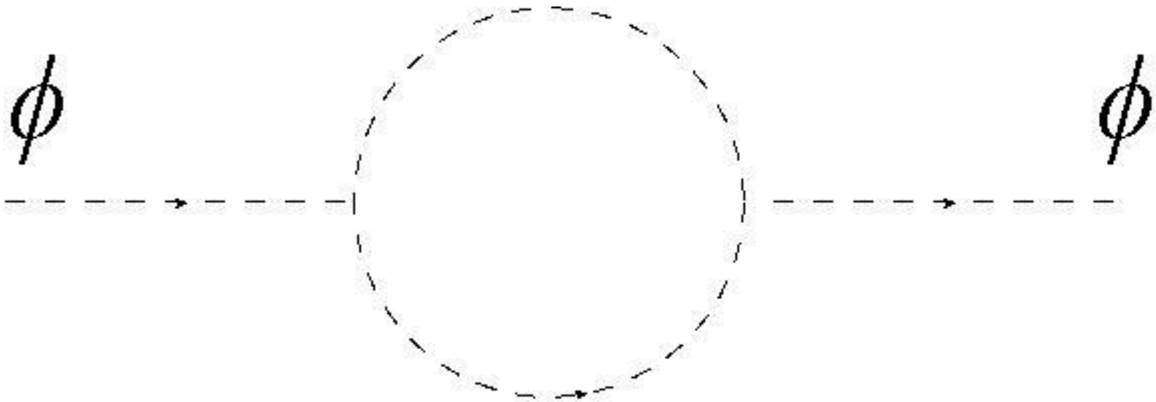
$$\text{where } C = \frac{1}{4}\left(k_1^2 + k_2^2 + k_3^2\right)$$

- The momenta are assumed to be incoming and satisfy the conservation law

$$k_1 + k_2 + k_3 = 0$$

1-loop, 2-point diagram with external momenta

- Here is the 1-loop, 2-point Feynman diagram with external momenta p, -p:



$$\Gamma_{2,1}\left(p^2\right) = \frac{i}{2i^2 M_p^2}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{V^2(-p,\frac{p}{2}+k,\frac{p}{2}-k)}{(\frac{p}{2}+k)^2(\frac{p}{2}-k)^2 e^{(\frac{p}{2}+k)^2/M^2}e^{(\frac{p}{2}-k)^2/M^2}},$$

1-loop, 2-point diagram with external momenta

- There are three types of terms in $\Gamma_{2,1}$:
- i) With no exponential. It leads to a divergence ($\frac{1}{\epsilon}$ pole) when dimensionally regulated.
- ii)With an exponential damping factor. They give rise to convergent results.
- iii)Terms involving $e^{p\cdot k}$. When dimensionally regulated, they give rise to no poles.

1-loop, 2-point diagram with external momenta

- We have that $\Gamma_{2,1}\left(p^2\right) = \frac{iM^4}{M_p^2} f(x),$

$$f(x) = \frac{x^4}{256\pi^2}\left(\frac{2}{\epsilon} - \log\left(\frac{x^2}{4\pi}\right) - \gamma + 2\right)$$
$$+\frac{e^{-x^2}}{512\pi^2 x^2}\Bigg(\left(e^{x^2}-1\right)\Big(-2\left(x^4+3x^2+2\right) - e^{\frac{x^2}{2}}\left(2x^4+5x^2+4\right)$$
$$+e^{x^2}\left(e^{x^2}-1\right)x^6\mathrm{Ei}\left(-\frac{x^2}{2}\right) + e^{\frac{3x^2}{2}}\left(2x^4+5x^2+4\right) + 2e^{x^2}\left(7\left(x^4+x^2\right)+2\right)\Big)$$
$$-2e^{x^2}\left(e^{2x^2}-1\right)x^6\mathrm{Ei}\left(-x^2\right)\Bigg)$$
$$+\frac{1}{128\pi}\int_0^1 \mathrm{d}r\, e^{(1-2r)x^2}\left[p(r,x)Y_0\left(2\sqrt{r-r^2}x^2\right)\right.$$
$$\left.+q(r,x)\sqrt{r-r^2}\,Y_1\left(2\sqrt{r-r^2}x^2\right)\right]\ (\mathrm{DR})$$

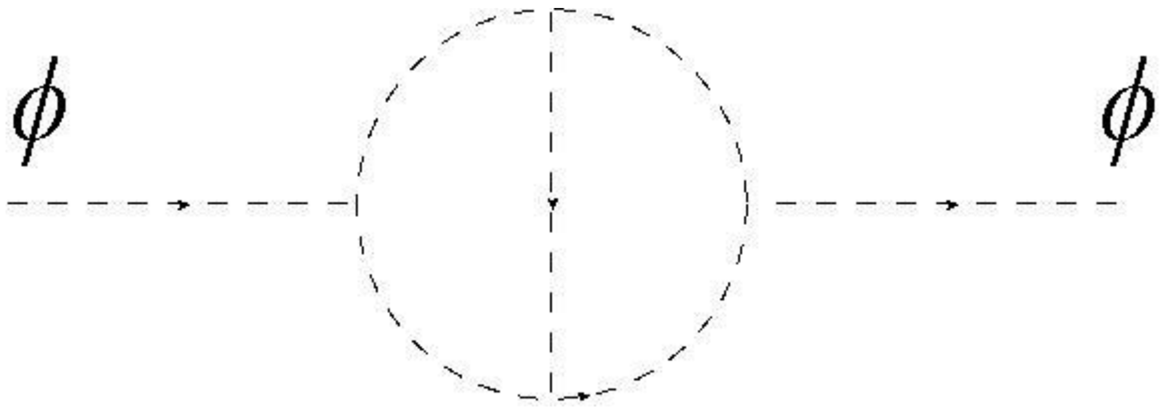
and $x = \frac{p}{M}$ & $p(r,x) = -16x^4r^4 + \left(32x^4+8x^2\right)r^3 - \left(26x^4+12x^2\right)r^2 + \left(10x^4+4x^2\right)r - 2x^4,$

$$q(r,x) = -16x^4r^3 + \left(24x^4+4x^2\right)r^2 - \left(16x^4+4x^2-8\right)r + 4x^4 + 3x^2 - 4$$

The $\frac{1}{\epsilon}$ pole in DR is equivalent to a Λ^4 divergence if we employ a hard cutoff Λ

2-loop, 2-point diagram with zero external momenta

For simplicity, we have set the external momenta equal to zero.



$$\Gamma_{2,2a} = \frac{i^2}{2i^5 M_p^4} \int \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4} \frac{V(k_1,-k_1,0)V(k_2,-k_2,0)V^2(k_1,k_2,k_3)}{k_3^2 k_2^4 k_1^4 e^{k_3^2} e^{2k_2^2} e^{2k_1^2}},$$

where $k_3 = -k_1 - k_2$

2-loop, 2-point diagram with zero external momenta

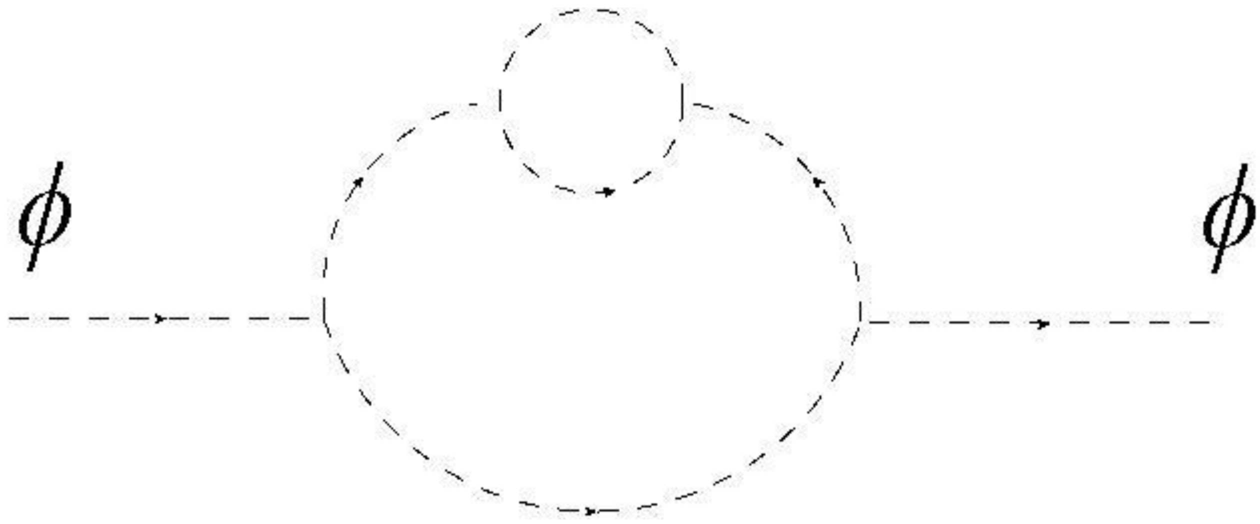
- We can write the exponential part of $\Gamma_{2,2a}$ as

$\sum_i \lambda_i \exp[E_i(k_1, k_2)]$,where the E_i's are quadratic polynomials of k_1, k_2 and λ_i's are constants taking on the values $-2, -1, +1, +2$

- We can write $E_i = \alpha_1 q_1^2 + \alpha_2 q_2^2$ (q_1, q_2 are linear combinations of k_1, k_2)

- If both α_1, α_2 are negative, we get convergent integrals; if one is negative and the other is positive, we get a divergence after analytic continuation; if one is negative and the other is zero, we obtain a divergence

The other 2-loop, 2-point diagram



$$\Gamma_{2,2b} = \frac{i^2}{2i^5 M_p^4} \int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \frac{\mathrm{d}^4 k_2}{(2\pi)^4} \frac{V^2(k_1, -k_1, 0))V^2(k_1, -\frac{k_1}{2} + k_2, -\frac{k_1}{2} - k_2)}{k_1^6 (\frac{k_1}{2} + k_2)^2 (\frac{k_1}{2} - k_2)^2 e^{3k_1^2/M^2} e^{(\frac{k_1}{2}+k_2)^2/M^2} e^{(\frac{k_1}{2}-k_2)^2/M^2}}$$

Upon redefinition of the momenta, the two 2-loop diagrams give exactly the same result.

2-loop, 2-point diagrams with zero external momenta

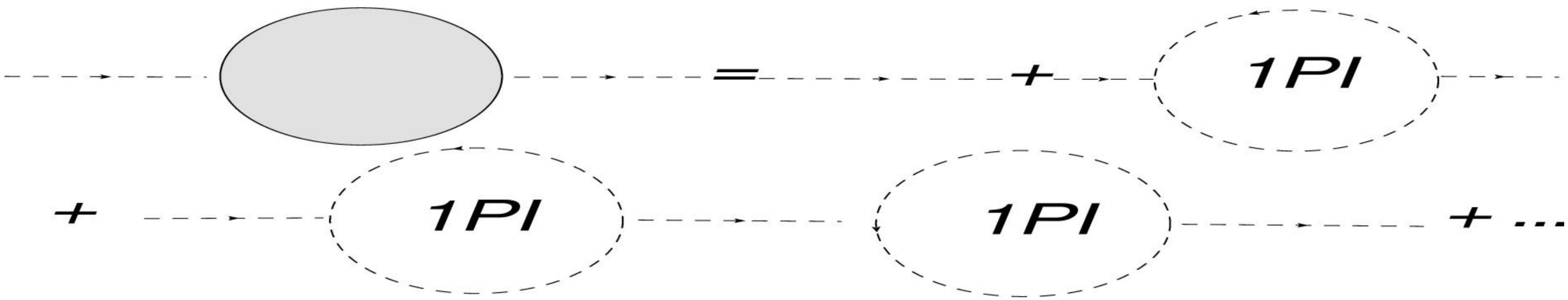
- Using a hard cutoff Λ, we obtain a Λ^4 divergence
- We observe that $\Gamma_{2,1} \sim \Gamma_{2,2} \sim \Lambda^4$
- The degree of divergence stays the same

Summary of Feynman diagram computations

- At 1-loop, the degree of divergence is Λ^4 (hard cutoff)
- At 2-loop, the degree of divergence also stays Λ^4
- Hence, we do not get higher divergences as we proceed from 1-loop to 2-loop
- Gives hope towards renormalizability

Dressed Propagators

- If we sum the infinite geometric series of loop corrections to the propagator, we obtain the dressed propagator



- We have that $\widetilde{\Pi}(p^2) = \dfrac{\Pi(p^2)}{1 - \Pi(p^2)\Gamma_{2,1\text{PI}}(p^2)}$

where $\Gamma_{2,1\text{PI}}(p^2)$ is the renormalized 1-loop, 2-point function

- We have that $\widetilde{\Pi}(p^2) \to \Gamma_{2,1\text{PI}}^{-1}(p^2) \sim e^{-\frac{3p^2}{2M^2}}$ in the UV

Dressed Propagators

- We observe that the dressed propagator is more exponentially suppressed than the bare one
- If we replace the bare propagators with the dressed propagators, convergence of Feynman integrals is improved
- Higher-point 1-loop graphs & 2-loop graphs become finite in the UV
- Only 1-loop, 2-point function diverges
- Once we remove the aforementioned divergence, the theory at the 1-loop level is renormalized
- We believe that higher loops remain finite

Heuristic argument for 2-point & 3-point diagrams

- We consider 2-point & 3-point diagrams which can be constructed out of lower-loop 2-point & 3-point ones
- Since $\widetilde{\Pi}(k^2) \xrightarrow{UV} e^{-\frac{3k^2}{2M^2}}$ & $\Gamma_3 \xrightarrow{UV} \sum_{\alpha,\beta,\gamma} e^{\alpha\frac{k_1^2}{M^2}+\beta\frac{k_2^2}{M^2}+\gamma\frac{k_3^2}{M^2}}$,

where Γ_3 is the 3-point function & $\alpha \geq \beta \geq \gamma,$

we have that the most divergent UV part of the 2-point diagram for zero external momenta is

$$\Gamma_{2,n} \xrightarrow{UV} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{e^{(\alpha_1+\alpha_2+\beta_1+\beta_2)\frac{k^2}{M^2}}}{e^{\frac{3k^2}{M^2}}}$$

Heuristic argument for 2-point & 3-point diagrams

- Similarly, for the 3-point diagram,

$$\Gamma_{3,n} \xrightarrow{UV} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{e^{(\alpha_1+\alpha_2+\alpha_3+\beta_1+\beta_2+\beta_3)\frac{k^2}{M^2}}}{e^{\frac{9k^2}{2M^2}}}$$

- We observe that both the 2- & 3-point diagrams become finite if $\alpha_i + \beta_i < \frac{3}{2}$
- Even when one includes non-zero external momenta, finiteness is assured
- One can recursively check that $\alpha_i + \beta_i < \frac{3}{2}$ for higher loops, which is as would be expected since the exponential suppression coming from the propagators is now stronger than the exponential enhancement originating from the vertices

Conclusions

- Nonlocal gravity possesses many novel features
- Ghosts are avoided
- The degree of divergence stays the same as we proceed from 1-loop to 2-loop
- Dressed propagators improve the convergence at all loop orders
- Once we renormalize the 1-loop graphs, higher-loop graphs do not produce new divergences
- A renormalizable & ghost-free theory of quantum gravity may be within reach