

# Brane induced supersymmetry breaking and de Sitter supergravity

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based on: [arXiv:1511.03024 \[hep-th\]](https://arxiv.org/abs/1511.03024)=JHEP (in press) with Luca Martucci, Dima Sorokin and Mario Tonin,.

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## de Sitter type spacetime and supergravity

- A generic supersymmetric (SUSY) solutions of supergravity (=SUGRA=SG) usually describe Minkowski or AdS spacetime.
- [See however recent work on gauged SUGRA, in particular talk by Fernandez Melgarejo]
- But an existence of a meta-stable, non-SUSY dS-type solution of supergravity/string theory equations is not excluded.
- **In string theory:**
- **KKLT:** Kachru, Kallosh, Linde, Trivedi, PRD2003= hep-th/0301240.
- **Lagre volume scenario (LVS):** Balasubramanian, Berglund, Conlon, and Quevedo, JHEP 2005= [hep-th/0502058].

## Recently a renewed interest to (the effective field theory description of) dS SUGRA

- as a SUGRA interaction with Volkov-Akulov (VA) goldstino  $\vartheta(\xi)$ :
- Dudas, Ferrara, Kehagias and Sagnotti, JHEP 2015=1507.07842.
- Bergshoeff, Freedman, Kallosh, Van Proeyen (BFKvP), PRD 2015=[1507.08264]
- Wrace ..., Kuzenko, ..., Quevedo, Valandra, ..., Uranga, ..., Lüst, ..., Dall'Agata, Zwimmer, Farakos, ... García– Etxeberria, Retolaza, ...
- Usually VA goldstino is described by nilpotent superfield technique [Rocek 78, Lindström, Rocek 79; Casalbuoni, De Curtis, Domicini, Feruglio, Gatto 1989; Komargorodski, Seiberg 2010]
- $X^2 = 0$  for  $X = \phi + i\theta^\alpha\psi_\alpha + 1/2\theta\theta F$ , in an assumption that  $F \neq 0$  implies:
  - $\phi \propto \psi\psi/F$  (no moduli stabilization problem)
  - $\delta_{susy}\psi = \epsilon F + \dots$  so that  $\psi =$  goldstino (= Volkov-Akulov goldstone fermion, 1972) and susy is spontaneously broken.
  - $\psi(x) = \vartheta(x) + \dots$  – a non-linear field redefinition [Kuzenko 2010]

## Brane induced supersymmetry breaking and VA fermion coupled to supergravity

- But in string theory a metastable dS vacuum may arise from KKLT construction involving anti-D3-brane ( $\overline{D3}$ ), in which VA goldstino  $\vartheta(\xi) =$  fermionic coordinate functions of  $\overline{D3}$  in target 10D superspace.
  - $\overline{D3}$  are just D3-branes but with 'opposite' RR charges so that they completely break the SUSY preserved by a compactification.
  - In 10D SUGRA regime, the dynamics of  $\overline{D3}$  is described by a superspace DBI-like action plus WZ term, which is manifestly invariant under local 10D SUSY.
  - This suggests that 4D effective theory should be given by a 4D superspace Green-Schwarz-DBI-like action for spacetime filling 3-brane = original Volkov-Akulov type action (see [Bergshoeff, Kallosh, Van Proeyen, Wrase 2015]) coupled to a superspace SUGRA.
- Such a description, alternative to the nilpotent superfield one and establishing a more direct link with string theory constructions involving  $\overline{D3}$ , is the subject of this talk.

In this talk, following [arXiv:1511.03024](https://arxiv.org/abs/1511.03024),

- We consider a minimal possible realization of such a 4D theory: a space-filling 3-brane carrying the Volkov-Akulov goldstino  $\vartheta(x)$  and coupled to minimal SUGRA (with auxiliary fields).
- To the quadratic order in  $\vartheta(x)$  our action coincides with earlier constructions of SUGRA with nilpotent superfields, while matching the higher-orders will require a non-linear redefinition of fields.
- In the unitary gauge,  $\vartheta(x) = 0$ , the action coincides with that of Volkov and Soroka [1973].
- We also show how a nilpotency constraint on a chiral curvature superfield emerges in this formulation.

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## Superspace action for SUGRA+ spacetime filling VA $\bar{3}$ -brane

has a suggestive geometric form of the sum of three different volumes:

$$S = \frac{3}{4\kappa^2} \int d^8 z \text{Ber } E + \frac{m}{2\kappa^2} \left( \int d^6 \zeta_L \mathcal{E} + \text{c.c.} \right) + f^2 \int d^4 \xi \det \mathbb{E}(z(\xi)) .$$

The first term  $\int d^8 z \text{Ber } E$  is the action of minimal SUGRA

- $\kappa^2$  is the gravitational constant
- $z^M = (x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}})$  are coordinates of the  $N = 1$   $D = 4$  superspace  $\Sigma^{(4|4)}$
- $\text{Ber } E := \text{Ber } E_M^A(Z) \equiv \text{sdet } E_M^A(Z)$ ,  $E^A := dz^M E_M^A(z) = (E^a, E^\alpha, E^{\dot{\alpha}})$  is the supervielbein one-form

The second term,  $\int d^6 \zeta_L \mathcal{E} + \text{c.c.}$ ,

- gives rise to the anti-de-Sitter cosmological term and the corresponding mass term for the gravitino.
- $\mathcal{E}$  is the volume measure of the chiral subspace  $\zeta^{\mathcal{M}} = (x_L^\mu, \theta^\alpha)$

The third term  $\int d^4 \xi \det \mathbb{E}(z(\xi))$  describes the spacetime filling 3-brane.

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The third term  $\int d^4 \xi \det \mathbb{E}(z(\xi))$  describes the spacetime filling 3-brane.

- It couples to supergravity via embedding of its worldvolume  $W^4$  as a surface of maximal bosonic dimension in curved superspace  $\Sigma^{(4|4)}$ :  
 $W^4 : \quad z^M = z^M(\xi) = (x^m(\xi), \theta^\mu(\xi), \bar{\theta}^{\dot{\mu}}(\xi))$
- $\xi^i$  ( $i = 0, 1, 2, 3$ ) are the local coordinates on  $W^4$ .
- $\det \mathbb{E}(z(\xi)) = \det \mathbb{E}_i{}^a(z(\xi))$ , where  $\mathbb{E}_i{}^a(z(\xi)) = \partial_i z^M(\xi) E_M{}^a(z(\xi))$ ;  
 $E^a(z(\xi)) = d\xi^i E_i^a$  is the pull-back of vector supervielbein to  $W^4$ .
- $f^2$  is the brane tension which gives a positive contribution to the cosmological constant:  $\Lambda = f^2 - \frac{2m^2}{\kappa^2}$ .
- [ $f^2 > 0$  to have a correct sign of kinetic term for  $\theta(\xi)$ ]
- $\Lambda = f^2 - \frac{2m^2}{\kappa^2}$  can be positive, hence allowing for de Sitter vacua  
 [following BFKvP 2015 such theories are dubbed 'de Sitter Supergravity']

### Superspace action for SUGRA+ spacetime filling VA $\bar{3}$ -brane

$$S = \frac{3}{4\kappa^2} \int d^8 z \text{Ber } E + \frac{m}{2\kappa^2} \left( \int d^6 \zeta_L \mathcal{E} + \text{c.c.} \right) + f^2 \int d^4 \xi \det \mathbb{E}(z(\xi)) .$$

The third term  $\int d^4 \xi \det \mathbb{E}(z(\xi))$  describes VA goldstino coupled to SUGRA.

### Conceptual difference with nilpotent superfield approach:

- The 3-brane action  $\int d^4 \xi \det \mathbb{E}_i{}^a(z(\xi))$  is invariant under worldvolume diffeomorphisms  $\xi^i \mapsto \xi'^i(\xi)$ .
- $\Rightarrow$  embedding field  $x^m(\xi)$ , which could be regarded as a bosonic superpartner of  $\theta^\mu(\xi)$ , carries only pure gauge d.o.f.'s.  $\Rightarrow$  it can be eliminated by gauge fixing (without any need in nilpotency constraints).
- The connection with Volkov-Akulov theory becomes manifest in the 'static gauge'  $x^m(\xi) = \xi^i \delta_i{}^m$  in which the goldstino  $\theta^\mu(\xi)$  remains the only worldvolume field.

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Fixing WZ and static gauge and integrating first two terms over fermionic coordinates

we arrive at  $S = S_{SG} + S_{VA}$  with

$$S_{SG} = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) \right. \\ \left. - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) + \frac{3}{32} G_a G^a + \frac{3}{8} (4m + R)(4m + \bar{R}) - 6m^2 \right]$$

$$S_{VA} = f^2 \int d^4x \det \mathbb{E}_m^a(x, \theta(x), \bar{\theta}(x)) .$$

$S_{SG}$ = standard pure  $N = 1$  AdS SG action:

- $e = \det e_m^a(x)$ , where  $e_m^a(x)$  is the space-time vielbein,
- $\psi_m^\alpha(x)$  and  $\bar{\psi}_m^{\dot{\alpha}}(x)$  are the Weyl-spinor gravitino,
- $\hat{\nabla} = d - \hat{\omega}$  with conventional spin connection  $\hat{\omega}$  including gravitino bilinears,
- $\mathcal{R}(\hat{\omega})$  is the curvature scalar associated with connection.
- $G^a$ ,  $R$  and  $\bar{R} = (R)^*$  are the old minimal supergravity auxiliary fields. When  $m = 0$ ,  $S_{SG}$  reduces to the old minimal off-shell supergravity action [Stelle, West 78, Ferrara, van Nieuwenhuizen 1978].

Fixing WZ and static gauge and integrating first two terms over fermionic coordinates

we arrive at  $S = S_{SG} + S_{VA}$  with

$$S_{SG} = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnpq} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) \right. \\ \left. - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) + \frac{3}{32} G_a G^a + \frac{3}{8} (4m + R)(4m + \bar{R}) - 6m^2 \right]$$

$$S_{VA} = f^2 \int d^4x \det \mathbb{E}_m^a(x, \theta(x), \bar{\theta}(x)).$$

the coupling of the Volkov-Akulov goldstino to the supergravity fields is encoded in  $S_{VA}$ ,

- in which

$$\mathbb{E}_m^a(x, \theta(x), \bar{\theta}(x)) = E_m^a(x, \theta(x), \bar{\theta}(x)) + \partial_m \vartheta^\alpha(x) E_{\underline{\alpha}}^a(x, \theta(x), \bar{\theta}(x)). \\ = E_m^a(x, \theta(x), \bar{\theta}(x)) + \partial_m \theta^\alpha(x) E_\alpha^a(x, \theta(x), \bar{\theta}(x)) + \\ + \partial_m \bar{\theta}^{\dot{\alpha}}(x) E_{\dot{\alpha}}^a(x, \theta(x), \bar{\theta}(x))$$

- its explicit form will be discussed below.

The action  $S = S_{SG} + S_{VA}$  with

$$S_{SG} = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) \right. \\ \left. - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) + \frac{3}{32} G_a G^a + \frac{3}{8} (4m + R)(4m + \bar{R}) - 6m^2 \right] \\ S_{VA} = f^2 \int d^4x \det (E_m^a(x, \theta(x), \bar{\theta}(x)) + \partial_m \vartheta^\alpha(x) E_\alpha^a(x, \theta(x), \bar{\theta}(x))) .$$

is invariant under the local SUSY

- which acts on SUGRA fields by

$$\delta e_m^a = 2i(\epsilon \sigma^a \bar{\psi}_m - \psi_m \sigma^a \bar{\epsilon}) , \\ \delta \psi_m = \hat{\nabla} \epsilon + \frac{i}{8} \bar{R}(\bar{\epsilon} \tilde{\sigma}_m)^\alpha + \frac{i}{16} (3G_m \epsilon^\alpha - (\epsilon \sigma_a \tilde{\sigma}_m)^\alpha G^a) , \\ \delta R = -\frac{16}{3} \hat{\nabla}_m \psi_n \sigma^{mn} \epsilon - 2i\epsilon \sigma_a \bar{\psi}^a R - i\epsilon \psi_a G^a , \\ \delta G_a = -\frac{40i}{3e} e_{ma} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\epsilon} - \epsilon \sigma_l \hat{\nabla}_n \bar{\psi}_k) - \frac{32}{3} e_a^{[m} (\hat{\nabla}_m \psi_n \sigma^n] \bar{\epsilon} + \epsilon \sigma^n] \hat{\nabla}_m \bar{\psi}_n) \\ + iG^a (\psi_b \sigma^b \bar{\epsilon} - \epsilon \sigma^b \bar{\psi}_b) + \frac{i}{2} \epsilon_{abcd} (\psi^b \sigma^c \bar{\epsilon} + \epsilon \sigma^b \bar{\psi}^c) G^d - 2iR \bar{\psi}_a \bar{\epsilon} + 2i\bar{R} \epsilon \psi_a ,$$

- and on the VA goldstino by

$$\delta \vartheta^\alpha(x) = -\epsilon^\alpha(x, \vartheta(x)) + \epsilon^m(x, \vartheta(x)) \partial_m \vartheta^\alpha(x) .$$

$$\delta\theta^\alpha(x) = -\epsilon^\alpha(x, \theta(x), \bar{\theta}(x)) + \epsilon^m(x, \theta(x), \bar{\theta}(x))\partial_m\theta^\alpha(x)$$

- Here  $\epsilon^A(x, \theta(x), \bar{\theta}(x)) = (\epsilon^m(x, \theta(x), \bar{\theta}(x)), \epsilon^\alpha(x, \theta(x), \bar{\theta}(x)))$  parametrizes the superspace (super)diffeomorphisms which reproduce the SUSY transformations of the spacetime component fields.
- the term with  $\partial_m\theta^\alpha(x)$  appears because of a worldvolume diffeomorphism is required to preserve the static gauge  $x^m(\xi) = \delta_i^m\xi^i$ .
- The explicit form of  $\epsilon^A(x, \theta(x), \bar{\theta}(x))$  is determined from the requirement of the preservation of the Wess-Zumino gauge.
- To the second order in  $\theta, \bar{\theta}$  we have obtained

$$\begin{aligned} \delta\theta^\alpha = & -\epsilon^\alpha - i(\theta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\theta})(\psi_m^\alpha + \nabla_m\theta^\alpha) - \\ & -(\theta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\theta})(\theta\sigma^n\bar{\psi}_m - \psi_m\sigma^n\bar{\theta})(\psi_n^\alpha + \nabla_m\theta^\alpha) + \\ & + \frac{1}{16}(\theta\sigma^a\bar{\epsilon} - \epsilon\sigma^a\bar{\theta})\left[2\theta^\alpha G_a + (\theta\sigma_{ab})^\alpha G^b + 2(\bar{\theta}\bar{\sigma}_a)^\alpha R\right] + \dots \end{aligned}$$

- This explicitly shows that  $\theta^\alpha(x)$  is a goldstino field
- the presence of which implies the spontaneous SUSY breaking.



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Let us discuss in more detail

the derivation and the structure of this action  $S = S_{SG} + S_{VA}$

$$\mathcal{N} = 1, D = 4 \text{ 'AdS SUGRA' action } S = \frac{3}{4\kappa^2} \int d^8z \text{Ber } E + \frac{m}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} + \text{c.c.} \right)$$

- to express it in terms of the component fields of the minimal off-shell SUGRA one must **use the supergravity constraints**,
- and **impose the Wess-Zumino gauge**, which fixes part of the superDiffs so that what remains is local SUSY and 4d diffs.

## The superspace constraints

- are imposed on the supervielbein  $E^A = dz^M E_M^A(z)$  and spin connection  $\Omega^{ab} = dz^M \Omega_M^{ab}(z)$  of curved superspace.
- To be precise, they are imposed on the covariant field strengths of  $E^A = dz^M E_M^A(z)$  and  $\Omega^{ab} = dz^M \Omega_M^{ab}(z)$ , superspace torsion and curvature, and determine the form of these

•

$$\begin{aligned} T^a &:= DE^a = dE^a - E^b \wedge \omega_b^a = \\ &= -2i\sigma_{\alpha\dot{\alpha}}^a E^\alpha \wedge \bar{E}^{\dot{\alpha}} - \frac{1}{8} E^b \wedge E^c \varepsilon^a{}_{bcd} G^d, \end{aligned}$$

$$\begin{aligned} T^\alpha &:= DE^\alpha = dE^\alpha - E^\beta \wedge \omega_\beta^\alpha = \\ &= \frac{i}{8} E^c \wedge E^\beta (\sigma_c \tilde{\sigma}_d)_\beta{}^\alpha G^d - \frac{i}{8} E^c \wedge \bar{E}^{\dot{\beta}} \varepsilon^{\alpha\beta} \sigma_{c\beta\dot{\beta}} R + \frac{1}{2} E^c \wedge E^b T_{bc}{}^\alpha, \end{aligned}$$

$$T^{\dot{\alpha}} = \dots = (T^\alpha)^*,$$

$$R^{ab} = d\Omega^{ab} - \Omega^{ac} \wedge \Omega_c^b = -\frac{1}{4} E^\alpha \wedge E^\beta \sigma_{ab\alpha\beta} \bar{R} + \text{c.c.} + \dots$$

The above expressions for superspace torsion and curvature

$$T^a = -2i\sigma_{\alpha\dot{\alpha}}^a E^\alpha \wedge \bar{E}^{\dot{\alpha}} - \frac{1}{8} E^b \wedge E^c \varepsilon^a{}_{bcd} G^d,$$

$$T^\alpha = \frac{i}{8} E^c \wedge E^\beta (\sigma_c \tilde{\sigma}_d)_{\beta}{}^\alpha G^d - \frac{i}{8} E^c \wedge \bar{E}^{\dot{\beta}} \varepsilon^{\alpha\beta} \sigma_{c\beta\dot{\beta}} R + \frac{1}{2} E^c \wedge E^b T_{bc}{}^\alpha,$$

- involve main off-shell superfields of  $N = 1$   $D = 4$  SUGRA,  $G_a(z)$  and  $R(z) = (\bar{R})^*$ , which obey

$$\mathcal{D}_\alpha \bar{R} = 0, \quad \bar{\mathcal{D}}_{\dot{\alpha}} R = 0, \quad \bar{\mathcal{D}}^{\dot{\alpha}} G_a \sigma_{\alpha\dot{\alpha}}^a = -\mathcal{D}_\alpha R, \quad \text{and c.c. .}$$

- Their lowest  $\vartheta = 0$  components are the auxiliary fields of the minimal SUGRA:  $G_a(x) \equiv G_a(z)|_{\vartheta=0}$  and  $R(x) \equiv R(z)|_{\vartheta=0}$ .
- The complete set of the main superfields also includes  $W^{\alpha\beta\gamma} := 4\tilde{\sigma}^{ab(\alpha\beta} T_{ab}{}^{\gamma)}$  and its c.c.  $\bar{W}^{\dot{\alpha}\dot{\beta}\dot{\gamma}} = -4\tilde{\sigma}^{ab(\dot{\alpha}\dot{\beta}} T_{ab}{}^{\dot{\gamma})}$ ,
- which are chiral,  $\bar{\mathcal{D}}_{\dot{\alpha}} W^{\alpha\beta\gamma} = 0$ , and obey  $\mathcal{D}_\gamma W^{\alpha\beta\gamma} = \bar{\mathcal{D}}_{\dot{\gamma}} \mathcal{D}^{(\alpha} G^{\beta)\dot{\gamma}}$
- while  $\mathcal{D}_{(\alpha} W_{\beta\gamma\delta)} = -\sigma_{ab(\alpha\beta} \sigma_{\gamma\delta)}^{cd} \mathcal{R}_{cd}{}^{ab}$  gives the superfield generalization of the irreducible (spin-tensor) components of the Weyl tensor.

### Wess-Zumino (WZ) gauge

- the conditions of the WZ gauge, which fixes part of the sDiffs so that what remains is local SUSY and 4d diffs, can be written in the form

$$\iota_{\vartheta} E^A(z) := \vartheta^{\hat{\mu}} E_{\hat{\mu}}{}^A(z) = \vartheta^{\hat{\mu}} \delta_{\hat{\mu}}{}^A, \quad \iota_{\vartheta} \Omega^{ab}(z) := \vartheta^{\hat{\mu}} \Omega_{\hat{\mu}}{}^{ab}(z) = 0$$

- In it all the component  $E^A = dz^M E_M^A(z)$  and  $\Omega^{ab} = dz^M \Omega_M^{ab}(z)$  on  $\vartheta^{\hat{\mu}} = (\theta^{\mu}, \bar{\theta}^{\dot{\mu}})$  are expressed in terms of auxiliary fields

$$R(x) = R(x, 0), \quad \bar{R}(x) = \bar{R}(x, 0), \quad G^a(x) = G^a(x, 0).$$

of the field strengths of the physical fields

$$e_m^a(x) = E_m^a(x, 0), \quad \psi_m^{\alpha}(x) = E_m^{\alpha}(x, 0), \quad \bar{\psi}_m^{\dot{\alpha}}(x) = \bar{E}_m^{\dot{\alpha}}(x, 0),$$

and of the spacetime covariant derivatives of these.

We should also take into account that:

- Due to our choice of the torsion constraint, the conventional SUGRA connection  $\hat{\omega}^{ab}(x)$  is related to the lowest component of  $\Omega_m^{ab}$  as follows

$$\hat{\omega}_m^{ab} := \omega_m^{ab} + 2i(\psi^{[a}\sigma^{b]}\bar{\psi}_m + \psi_m\sigma^{[a}\bar{\psi}^{b]} + \psi^{[a}\sigma_m\bar{\psi}^{b]}) = \Omega_m^{ab}|_0 + \frac{1}{8}e_{m\ c}\varepsilon^{abcd}G_d|_0,$$

where  $\omega_m^{ab}(x)$  is torsion-less spin connection expressed in terms of the vielbein  $e_m^a(x)$  and  $|_0 := |_{\theta=0}$ .

- In the Wess-Zumino gauge the chiral measure  $\mathcal{E}$  has the following form

$$\mathcal{E} = e \left[ 1 + 2i\Theta^\alpha (\sigma_a \bar{\psi}^a)_\alpha + \Theta\Theta \left( \frac{3}{4}\bar{R} - 2\bar{\psi}^a \sigma_{ab} \bar{\psi}^b \right) \right],$$

where  $\Theta$  is a 'new Grassmann coordinate' (see e.g. [Bagger+Wess 82])

- by the requirement that covariantly chiral superfield ( $\bar{D}_\alpha \Phi = 0$ ) can be written as  $\Phi = \Phi|_0 + \Theta^\alpha (\mathcal{D}\Phi|_0) + \frac{1}{2}\Theta\Theta(-\frac{1}{2}\mathcal{D}^\alpha \mathcal{D}_\alpha \Phi|_0)$ .
- Then  $\int d^6\zeta_{L\dots} = \int d^4x \frac{\partial}{\partial\Theta^1} \frac{\partial}{\partial\Theta^2} = \frac{1}{2} \int d^4x \epsilon^{\alpha\beta} \frac{\partial}{\partial\Theta^\alpha} \frac{\partial}{\partial\Theta^\beta}$

Thus in the WZ gauge  $S_{SG} = \frac{3}{4\kappa^2} \int d^8z \text{Ber } E + \frac{m}{2\kappa^2} \left( \int d^6\zeta_L \mathcal{E} + \text{c.c.} \right)$  is equal to

$$S_{SG} = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) + \frac{3}{32} G_a G^a + \frac{3}{8} (4m + R)(4m + \bar{R}) - 6m^2 \right].$$

- If this were the complete action, one could integrate-out the auxiliary fields by substituting into  $S_{SG}$  their equations of motion

$$\boxed{G^a = 0, \quad R = \bar{R} = -4m} \quad \text{and thus produce the } N = 1 \text{ SUGRA}$$

- with  $\Lambda = -\frac{3m^2}{\kappa^2} < 0$  and gravitino mass  $m \neq 0$  [Townsend:1977qa]

$$S_{\text{AdS}} = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) - 6m^2 \right].$$

- But in our case we have to account also for interaction of SUGRA with non-BPS VA 3-brane.
- The 3-brane backreacts and change eqs. of motion. In particular we cannot use  $G_a = 0, R = -4m$ .

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### Coupling of the 3-brane to minimal SG

- After imposing the static gauge  $x^m(\xi) = \delta_i^m \xi^i$ , the interaction of the VA goldstino with SG is encoded in  $S_{VA} = f^2 \int d^4x \det \mathbb{E}_m^a(x, \theta(x), \bar{\theta}(x))$  via a complicated dependence of  $E^a(x, \theta, \bar{\theta})$  on the SG component fields.
- To get the explicit form of this dependence we should expand  $E^a(x, \theta, \bar{\theta})$  in powers of  $\theta$  and  $\bar{\theta}$ .
- The series stops at the fifth order (including  $d\theta^\alpha$  and  $d\bar{\theta}^{\dot{\alpha}}$ ).

### To compute the $\theta$ -expansion of $E^A(x, \theta, \bar{\theta})$ in WZ gauge

- we use the following well known procedure:
- rescale  $\vartheta \rightarrow t\vartheta$  and define the  $t$ -rescaled supervielbein

$$E^A(t) := E^A(x, t\vartheta) = dx^m E_m^A(x, t\vartheta) + td\vartheta^{\hat{\alpha}} E_{\hat{\alpha}}^A(x, t\vartheta) \\ = E_{(0)}^A + tE_{(1)}^A + t^2 E_{(2)}^A + t^3 E_{(3)}^A + t^4 E_{(4)}^A + t^5 E_{(5)}^A,$$

- and observe that in the Wess-Zumino gauge the  $t$ -rescaled superforms obey ...

To compute the  $\theta$ -expansion of  $E^A(x, \theta, \bar{\theta})$  in WZ gauge

- ...
- and observe that in the WZ gauge the  $t$ -rescaled superforms obey

$$\begin{aligned}\frac{d}{dt} E^A(t) &= \mathcal{D}_{\iota_\vartheta} E^A(t) + \iota_\vartheta T^A(t), \\ \frac{d}{dt} \Omega^{ab}(t) &= \iota_\vartheta \mathcal{R}^{ab}(t).\end{aligned}$$

- Taking into account the SUGRA constraints we obtain

$$\begin{aligned}\frac{d}{dt} E^a &= \iota_\theta T^A = 2i\theta\sigma^a \bar{E} - 2iE\sigma^a \bar{\theta}, \\ \frac{d}{dt} E^\alpha &= \mathcal{D}\theta^\alpha + \iota_\theta T^\alpha = \mathcal{D}\theta^\alpha + \frac{i}{8} E^c [(\theta\sigma_c \tilde{\sigma}_d)^\alpha G^d + \bar{\theta}_{\dot{\alpha}\beta} \tilde{\sigma}_c^{\dot{\alpha}\beta} R], \\ \frac{d}{dt} \Omega^{\alpha\beta} &= -E^{(\alpha}\theta^{\beta)} \bar{R} - \frac{i}{8} E^c \left( \tilde{\sigma}_c^{\dot{\gamma}(\alpha}\theta^{\beta)} \bar{\mathcal{D}}_{\dot{\gamma}} \bar{R} - \right. \\ &\quad \left. (\theta\sigma_c \tilde{\sigma}_d)^{(\alpha}\mathcal{D}^{\beta)} G^d + (\sigma_c \bar{\theta})_\gamma W^{\alpha\beta\gamma} \right).\end{aligned}$$

- These set of equations can be solved order by order in  $t$
- with the 'initial conditions'

$$E_{(0)}^a = e^a, \quad E_{(0)}^\alpha = \psi^\alpha(x), \quad \Omega_{(0)}^{ab} = \hat{\omega}^{ab} + \dots$$

The decomposition of bosonic supervielbein  $E^a(x, \theta, \bar{\theta})$  we are interested in

- can be expressed in terms of the  $\vartheta$ -expansion of the fermionic  $E^\alpha, \bar{E}^{\dot{\alpha}}$ ,

$$\begin{aligned}
 E^a &= e^a + E_{(1)}^a + E_{(2)}^a + E_{(3)}^a + E_{(4)}^a + E_{(5)}^a \\
 &= dx^m e_m^a - 2 \left[ i \left( \psi + \frac{1}{2} E_{(1)} + \frac{1}{3} E_{(2)} + \frac{1}{4} E_{(3)} + \frac{1}{5} E_{(4)} \right) \sigma^a \bar{\theta} + \text{c.c.} \right].
 \end{aligned}$$

- Then e.g. the first term in the  $\vartheta$ -expansion of the spinorial supervielbein:

$$E^\alpha = \psi^\alpha + \hat{\nabla} \theta^\alpha + \frac{i}{16} e^b [2\theta^\alpha G_b + (\theta \sigma_{[b} \tilde{\sigma}_{c]})^\alpha G^c] + \frac{i}{8} e^b (\bar{\theta} \tilde{\sigma}_b)^\alpha R + \mathcal{O}(\vartheta^2)$$

allows us to find  $E^a$  up to second order in  $\vartheta$

$$\begin{aligned}
 E^a &= e^a + 2i\theta\sigma^a\bar{\psi} - 2i\psi\sigma^a\bar{\theta} + i\theta\sigma^a\hat{\nabla}\bar{\theta} - i\hat{\nabla}\theta\sigma^a\bar{\theta} \\
 &+ \frac{1}{4} e^b G_b \theta \sigma^a \bar{\theta} - \frac{1}{4} e^{[a} G^{b]} \theta \sigma_b \bar{\theta} + \frac{1}{8} e^a (\theta \bar{R} + \bar{\theta} \bar{R}) + \mathcal{O}(\vartheta^3).
 \end{aligned}$$

- Note that when the auxiliary fields are put to zero, the terms entering the above second order expansions coincide with the supervielbeins first constructed in 1973 by Volkov and Soroka [Volkov:1973jd].

One can then iterate the above procedure to identify the higher order terms

- At the third order we obtain

$$E_{(3)}^a = e^{b\bar{\theta}\bar{\theta}} \left[ -\frac{i}{6}\theta\sigma^a\bar{\psi}_b R - \frac{i}{6}\theta\psi_b G^a - \frac{i}{6}\theta\sigma^a\tilde{\sigma}_c\psi_b G^c + \frac{i}{3}\theta\sigma^a\bar{T}_b + \frac{i}{6}\theta\sigma_b\bar{T}^a - \frac{i}{12}\theta\sigma^a{}_b\sigma_c\bar{T}^c - \frac{2}{9}\eta^{ad}(\theta\sigma_{[d}{}^c T_{b]c}) - \frac{2}{9}(\eta^{ac}\theta T_{cb} + \frac{i}{2}\eta_{be}\varepsilon^{abcd}\theta T_{cd}) \right] + \text{c.c.},$$

- where

$$\begin{aligned} \psi_{ab}^\alpha &:= 2e_a^m e_b^n \hat{\nabla}_{[m}\psi_{n]}^\alpha \\ T_{ab}{}^\alpha &:= T_{ab}{}^\alpha|_0 = \psi_{ab}^\alpha - \frac{i}{8}(\psi_{[a}\sigma_{b]}\tilde{\sigma}^c)^\alpha G_c - \frac{i}{8}\psi_{[a}{}^\alpha G_{b]} - \frac{i}{4}(\bar{\psi}_{[a}\tilde{\sigma}_{b]})^\alpha R \\ \bar{T}_{\dot{\alpha}}^a &:= \varepsilon^{abcd} T_{bc}{}^\alpha \sigma_{d\alpha\dot{\alpha}} = \varepsilon^{abcd} \psi_{bc}{}^\alpha \sigma_{d\alpha\dot{\alpha}} + \frac{1}{2}(\bar{\psi}_b\tilde{\sigma}^{ab})_{\dot{\alpha}} R \\ &\quad + \frac{i}{8}\varepsilon^{abcd}(\psi_b\sigma_c)_{\dot{\alpha}} G_d + \frac{1}{2}(\psi_c\sigma^{[c})_{\dot{\alpha}} G^a]. \end{aligned}$$

One can then iterate the above procedure to identify the higher order terms

- The fourth order is quite complicated

$$\begin{aligned}
 E_{(4)}^a &= -\frac{i}{2} E_{(3)} \sigma^a \bar{\theta} + \frac{i}{2} \theta \sigma^a \bar{E}_{(3)} = \\
 &= \frac{i}{24} \theta \theta \hat{\nabla} \theta \sigma^a \bar{\theta} \bar{R} - \frac{i}{24} \bar{\theta} \bar{\theta} \left( \hat{\nabla} \theta^\gamma \theta_\gamma G^a + \hat{\nabla} \theta \sigma^c \tilde{\sigma}^a \theta G_c \right) \\
 &\quad - \frac{i}{24} \bar{\theta} \bar{\theta} \theta \sigma^a \hat{\nabla} \bar{\theta} R + \frac{i}{24} \theta \theta \left( \hat{\nabla} \bar{\theta}_\gamma \bar{\theta}^\gamma G^a + \hat{\nabla} \bar{\theta} \tilde{\sigma}^c \sigma^a \bar{\theta} G_c \right) \\
 &\quad + \theta \theta \bar{\theta} \bar{\theta} e^a \left[ \frac{1}{384} (\mathcal{D} \mathcal{D} R + \bar{\mathcal{D}} \bar{\mathcal{D}} \bar{R})|_0 + \frac{1}{192} G_c G^c - \frac{1}{96} R \bar{R} \right] \\
 &\quad + \theta \theta \bar{\theta} \bar{\theta} e^b \left[ \frac{1}{768} (4 \eta^{ac} \eta_{bd} + 6 \delta_{[b}^a \delta_{d]}^c) (\tilde{\sigma}_c)^{\dot{\alpha}\alpha} [\bar{\mathcal{D}}_{\dot{\alpha}}, \mathcal{D}_\alpha] G^d|_0 \right. \\
 &\quad \left. + \frac{1}{64} \varepsilon^a{}_{bcd} \mathcal{D}^c G^d|_0 - \frac{5}{192} G_b G^a + \frac{i}{12} \psi^\alpha \mathcal{D}_\alpha G^a|_0 - \frac{i}{12} \bar{\psi}_b \dot{\alpha} \bar{\mathcal{D}}_{\dot{\alpha}} G^a|_0 \right. \\
 &\quad \left. - \frac{i}{64} (\bar{\psi}_b \tilde{\sigma}^a \sigma_c)_{\dot{\alpha}} \bar{\mathcal{D}}^{\dot{\alpha}} G^c|_0 - \frac{i}{64} (\psi_b \sigma^a \tilde{\sigma}_c)^\alpha \mathcal{D}_\alpha G^c|_0 \right],
 \end{aligned}$$

- where one still have to substitute expressions for  $\mathcal{D}G|_0$ ,  $\mathcal{D}\mathcal{D}G|_0$ ,  $\mathcal{D}\mathcal{D}R|_0$  etc., in terms of space-time fields (see our arXiv:1511.03024).
- In contrast, the fifth order is quite simple

$$E_{(5)}^a = \frac{i}{6} \theta \theta \bar{\theta} \bar{\theta} \left[ (\mathcal{T}^{ab} + \frac{i}{2} \varepsilon^{abcd} \mathcal{T}_{cd}) \sigma_b d\bar{\theta} - \text{c.c.} \right].$$

What is really important in this long formulae for higher order terms:

- The auxiliary fields enter  $E^a$  starting from the second order in  $\vartheta$ .
- The contribution of the auxiliary fields to  $E_{(2)}^a$  is **linear**, while in higher order terms of  $E^a$  they appear **at most quadratically**.
- There are **only linear terms in space-time derivatives of the auxiliary fields** and they appear only at the quartic order.
- This means that in  $S_{VA} = f^2 \int d^4x \det \mathbb{E}_m^a(x, \theta(x), \bar{\theta}(x))$  the auxiliary fields appear only linearly or quadratically and without derivatives (modulo integration by parts).

As such, when the 3-brane action is coupled to the supergravity in  $S_{SG} + S_{VA}$

one can still explicitly solve the equations of motion of the auxiliary fields modified by the presence of the goldstino fields.

Let us focus on the terms of this action which are linear and quadratic in the goldstino:

$$S_{VA} = f^2 \int d^4x e \left[ 1 + 2i(\theta\sigma^a\bar{\psi}_a - \psi_a\sigma^a\bar{\theta}) + i(\theta\sigma^a\hat{\nabla}_a\bar{\theta} - \hat{\nabla}_a\theta\sigma^a\bar{\theta}) - \frac{1}{8}G_a\theta\sigma^a\bar{\theta} \right. \\ \left. + \frac{1}{2}(\theta\bar{\theta}\bar{R} + \bar{\theta}\bar{\theta}R) + \bar{\theta}\bar{\theta}\psi_a\sigma^{ab}\psi_b + \theta\theta\bar{\psi}_a\tilde{\sigma}^{ab}\bar{\psi}_b - 8i\varepsilon^{abcd}\theta\sigma^a\bar{\theta}\psi_b\sigma_c\bar{\psi}_d \right] + \dots$$

- varying  $S_{SG} + S_{VA}$  with respect to  $\bar{R}$  and  $G^a$  we find :

$$R = -4m - \frac{8\kappa^2 f^2}{3}\theta^2 + \dots, \quad G^a = \frac{4\kappa^2 f^2}{3}\theta\sigma^a\bar{\theta} + \dots$$

- Substituting this in the action we arrive at

$$S = \frac{1}{2\kappa^2} \int d^4x e \left[ \mathcal{R}(\hat{\omega}) - 4 \left\{ e^{-1}\varepsilon^{mnlk}\hat{\nabla}_n\psi_k\sigma_l\bar{\psi}_m + m\psi^a\sigma_{ab}\psi^b + c.c. \right\} \right. \\ \left. + \int d^4x e \left\{ \left( f^2 - \frac{3m^2}{\kappa^2} \right) + f^2 \left[ 2i(\theta\sigma^a\bar{\psi}_a - \psi_a\sigma^a\bar{\theta}) + i(\theta\sigma^a\hat{\nabla}_a\bar{\theta} - \hat{\nabla}_a\theta\sigma^a\bar{\theta}) \right] - \right. \right. \\ \left. \left. - 2mf^2(\theta^2 + \bar{\theta}^2) + f^2 \left[ \bar{\theta}\bar{\theta}\psi_a\sigma^{ab}\psi_b + \theta\theta\bar{\psi}_a\tilde{\sigma}^{ab}\bar{\psi}_b - 8i\varepsilon^{abcd}\theta\sigma_a\bar{\theta}\psi_b\sigma_c\bar{\psi}_d \right] \right\} + \right. \\ \left. + \dots - \frac{3\kappa^2 f^4}{2} \int d^4x e \theta^2\bar{\theta}^2. \right.$$

- Modulo our conventions this coincides with the action of [BFKvP 2015] truncated to the second order in the goldstino.

But moreover, this action is related with the one of Volkov and Soroka, 1973.

- One can fix the local supersymmetry by imposing the unitary gauge  $\vartheta(x) = 0$ . In it *the complete but gauge fixed action* reads

$$S = \frac{1}{2\kappa^2} \int e \left[ \mathcal{R}(\hat{\omega}) - 4e^{-1} \varepsilon^{mnlk} (\hat{\nabla}_n \psi_k \sigma_l \bar{\psi}_m + \psi_m \sigma_n \hat{\nabla}_k \bar{\psi}_l) - 4m(\bar{\psi}^a \sigma_{ab} \bar{\psi}^b + \psi^a \sigma_{ab} \psi^b) + (2\kappa^2 f^2 - 6m^2) \right]. \quad (1)$$

- and the effect of the Volkov-Akulov 3-brane is reduced to a positive contribution to the value of the cosmological constant  $\Lambda = f^2 - \frac{3m^2}{\kappa^2}$  which can thus be positive and may give rise to a de Sitter vacuum.
- We expect that at higher orders in  $\theta(x)$ ,  $\bar{\theta}(x)$  the form of our action will differ from that of based on the nilpotent goldstino superfield in the same way as the original rigid-supersymmetry Volkov-Akulov action differs from its 'constrained superfield' counterparts.
- To find the precise correspondence between the two formulations one should generalize the non-linear field redefinitions relating different forms of the Volkov-Akulov Lagrangian found in 2010 [Kuzenko et al 2010,11] .
- But what we would like to stress is that the above 'complete but gauge fixed' action coincide with that for the action in [Volkov and Soroka 1973].



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## Emergence of constrained superfields

- The expression for the complex scalar  $R(x)$  to the second order in  $\theta, \bar{\theta}$   
 $R(x) + 4m = -\frac{8\kappa^2 f^2}{3} \theta^2(x) (1 + \dots)$ , implies that  $R(x)$  is nilpotent, i.e.  
 $(R(x) + 4m)^2 = 0$ .
- Since  $R(x)$  is  $R(z)|_{\vartheta=0}$ , this may be regarded as part of the solution to a nilpotency constraint involving the entire superfield  $R(z)$ , similar to the ones derived in [Antoniadis, Dudas, Ferrara, Sagnotti 2014] and [Dudas, Ferrara, Kehagias, Sagnotti 2015]=[Dudas:2015eha] from SG+ nilpotent superfield action.
- These constraint is automatically solved by the superfield  $R(z)$  obeying its eqs of motion also in our formulation.

Using the Wess–Zumino type variations of constrained superfields we find

- that our superfield action produces the scalar superfield equations

$$R(z) + 4m = \frac{16\kappa^2 f^2}{3} \mathcal{J}(z), \quad \text{where } \mathcal{J}(z) = (\bar{D}\bar{D} - R(z))\mathcal{P}(z)$$

$$\text{with } \mathcal{P}(z) = \int d^4\xi \frac{\det \mathbb{E}(z(\xi))}{\text{Ber } E(z(\xi))} \delta^8(z - z(\xi)),$$

- $\delta^8(z) := \frac{1}{4} \theta^2 \bar{\theta}^2 \delta^4(x)$  is the superspace  $\delta$  function,  $\int d^8z \delta^8(z) = 1$ ,
- and  $\bar{D}\bar{D} - R(z)$  is the chiral projector, *i.e.*  $\bar{D}_{\dot{\alpha}}(\bar{D}\bar{D} - R(z))\mathcal{P}(z) \equiv 0$ , so that the r.h.s. is chiral, as it should be, because  $R(z)$  is.
- Fixing the WZ and the static gauge one finds  $\mathcal{J}(z) = -\frac{1}{2}(\theta - \theta(x))^2(1 + \dots) \Rightarrow \mathcal{J}(z)^2 = 0$
- so that on shell  $(R(z) + 4m)^2 = 0$  as expected.
- The form of this constraint is the same as in [Kuzenko:2015yx] and is similar to that used in 'nilpotent supergravity' of [Dudas:2015eha],  $(R(z) - \lambda)^2 = 0$ , but their the parameter  $\lambda$  (which triggers SUSY breaking) is (*a priori*) not related to the gravitino "AdS mass"  $m$ .

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## A comment on Volkov and Soroka SUGRA action from 1973.

- In 1973 Volkov and Soroka coupled the VA model to a consisting of the graviton and gravitino, and a vector gauge field (omiting vector gauge field this coincide with simple SUGRA multiplet).
- SUSY was considered to be non-linearly realized and the general approach to nonlinearly realized symm-s [Colemann, Wess, Zumino 1969, CWZ+Callan 1969, Volkov 69, 73] was used.
- They gauged superPoincaré introducing corresponding gauge fields  $e_m^a(x)$ ,  $\psi_m^\alpha(x)$ ,  $\bar{\psi}_m^{\dot{\alpha}}(x)$  and independent  $\tilde{\omega}_m^{ab}(x)$ .
- The local SUSY (as can be extracted from V+S=1973):

$$\delta e_m^a = 2i(\epsilon\sigma^a\bar{\psi}_m - \psi_m\sigma^a\bar{\epsilon}), \quad \delta\psi_m = \tilde{\nabla}_m\epsilon, \quad \delta\bar{\psi}_m = \tilde{\nabla}_m\bar{\epsilon},$$

- **but** with the covariant derivative  $\tilde{\nabla} = d - \tilde{\omega}$  containing the independent connection  $\tilde{\omega}_m(x)$ .

A comment on Volkov and Soroka SUGRA action from 1973. II.

- The Volkov-Soroka procedure of gauging the super-Poincaré group was essentially based on the idea to use the goldstino as a Stueckelberg-like field whose variation  $\delta\vartheta = -\epsilon(x)$  compensates the local supersymmetry transformations of the graviton and the gravitino together with a Stueckelberg-like field  $X^a(x)$  to compensate local Poincaré translations in the tangent space.
- The latter can be gauge-fixed to zero thus reducing the action of the Poincaré group in the tangent space to local Lorentz rotations.
- The SUSY invariant one-forms constructed in this way,  
 $\tilde{\psi} = \psi + \tilde{\nabla}\theta$ ,  $\tilde{e}^a(x) = e^a(x) + 2i\theta\sigma^a\bar{\psi} - 2i\psi\sigma^a\bar{\theta} + i\theta\sigma^a\tilde{\nabla}\bar{\theta} - i\tilde{\nabla}\theta\sigma^a\bar{\theta}$ ,  
 were used to construct an invariant action

$$S_{VS} = \frac{1}{2\kappa^2} \int d^4x \tilde{e} \left[ \mathcal{R}(\tilde{\omega}) - \frac{4c}{\tilde{e}} \varepsilon^{mnlk} (\tilde{\nabla}_n \tilde{\psi}_k \sigma_l \tilde{\psi}_m + \tilde{\psi}_m \sigma_n \tilde{\nabla}_k \tilde{\psi}_l) - 4m(\tilde{\psi}^a \sigma_{ab} \tilde{\psi}^b + \tilde{\psi}^a \sigma_{ab} \tilde{\psi}^b) + \lambda \right].$$

Note that in this action the coefficients  $c$ ,  $m$  and  $\lambda$  are arbitrary.

### A comment on Volkov and Soroka SUGRA action from 1973. III

- Now, we can use the original SUSY to put  $\vartheta(x) = 0$ , absorb the constant  $c$  in the re-scaled gravitino field ( $\psi \rightarrow c^{-\frac{1}{2}}\psi$ ), redefine  $\lambda = 2\kappa^2(f^2 - \frac{3m^2}{\kappa^2})$  and finally substitute the solution of the  $\tilde{\omega}$  field equation back in to the action.
- The result is our gauge-fixed action described above.
- Back to 1973, when constructing their spontaneously broken supergravity action Volkov and Soroka were, probably, too much concentrated on the local supersymmetry breaking associated with the shift of the goldstino and the corresponding super-Higgs effect, so they did not pose the question whether (for a suitable choice of the parameters) their action can still be supersymmetric even when the goldstino field is gauge fixed to zero.

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## Conclusion

- We have derived the minimal model describing the spontaneous breaking of pure  $N = 1$ ,  $D = 4$  supergravity induced by a space-filling 3-brane which carries the Volkov-Akulov goldstino on its worldvolume and provides a tunable constant contribution to the cosmological constant which can be made positive.
- To the quadratic order in  $\vartheta(x)$  our action coincides with earlier constructions of SUGRA with nilpotent superfields, while matching the higher-orders will require a non-linear redefinition of fields.
- In the unitary gauge,  $\vartheta(x) = 0$ , the action coincides with that of Volkov and Soroka [1973].
- We also show how a nilpotency constraint on a chiral curvature superfield emerges in this formulation.

## Outlook

- To study a (straightforward) generalization to the interacting system of Volkov-Akulov goldstino and a more complicated supergravity-matter systems as a low-energy effective field theories for string compactifications with anti-brane induced supersymmetry breaking. (Bottom up approach).
- But the final resolution of doubts about consistency of KKLT might require to study the complete supersymmetric backreacting IIB SUGRA+ D3-brane system. (Top down approach).
  - ? Probably in terms of D=4 superfields [Marcus+Sagnotti+ Siegel=1983]?
  - ? Probably using the 'complete but gauge fixed description' of the supergravity + super-p-brane interacting systems proposed in [Bandos+de Azcárraga+Lukierski+Izquierdo=2001-05]?

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**THANK YOU FOR YOUR ATTENTION!**