Novel Thermal Phases of $AdS_5 \times S^5$

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Based on:

OD, Jorge Santos, Benson Way, 1501.06574

Iberian Strings 2016, IFT — UAM, Madrid January 2016

➞ Recalling the primordial days: AdS_5 / CFT_4

- **Type IIB supergravity:**

$$G_{MN} \equiv R_{MN} - \frac{1}{48} F_{MPQRS} F_N{}^{PQRS} = 0, \qquad \nabla_M F^{MPQRS} = 0\,, \qquad F_{(5)} = \star F_{(5)}$$

- **$AdS_5 x S^5$ is a solution:**

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2 \mathrm{d}\Omega_3^2 + L^2 \mathrm{d}\Omega_5^2\,, \qquad F_{\mu\nu\rho\sigma\tau} = \epsilon_{\mu\nu\rho\sigma\tau}\,, \qquad F_{abcde} = \epsilon_{abcde}$$

$$f(r) = 1 + \frac{r^2}{L^2}$$

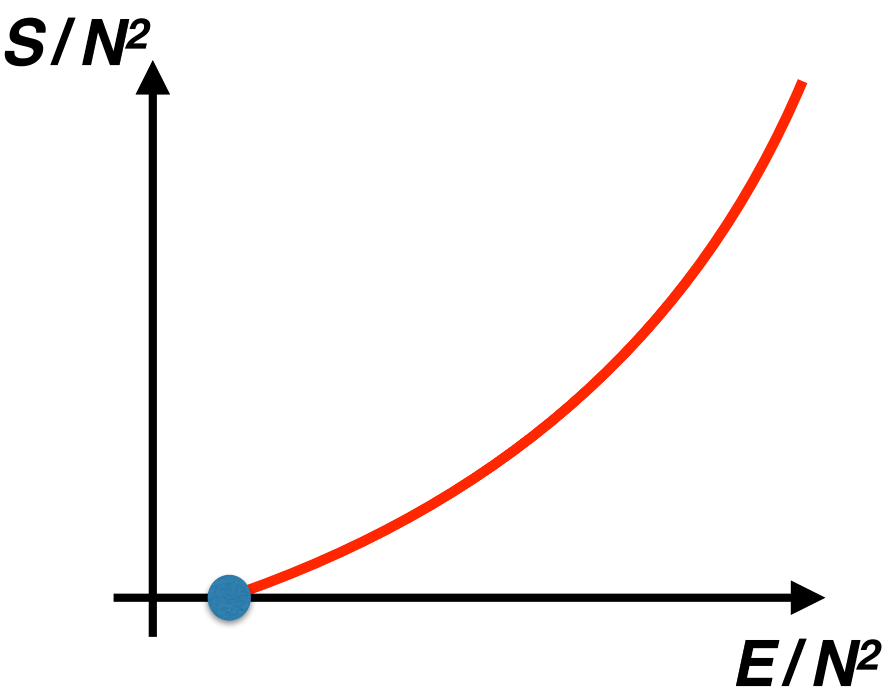
- **Schwarzschild-$AdS_5 x S^5$ is also a solution:**

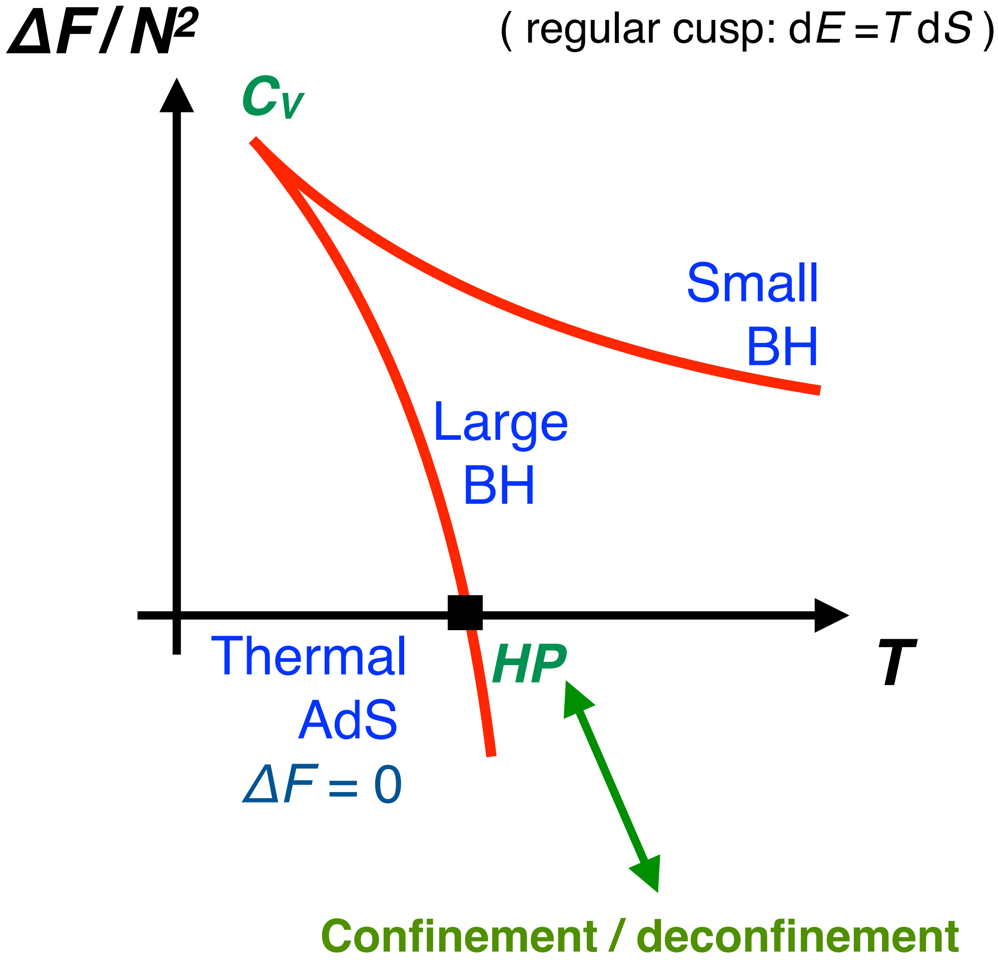
$$f(r) = 1 + \frac{r^2}{L^2} - \frac{r_+^2}{r^2}\left(\frac{r_+^2}{L^2} + 1\right)$$

➞ Thermal Phases of $AdS_5 x S^5$ and their competition

Microcanonical ensemble:
(fixed E)

Canonical ensemble:
(fixed T)



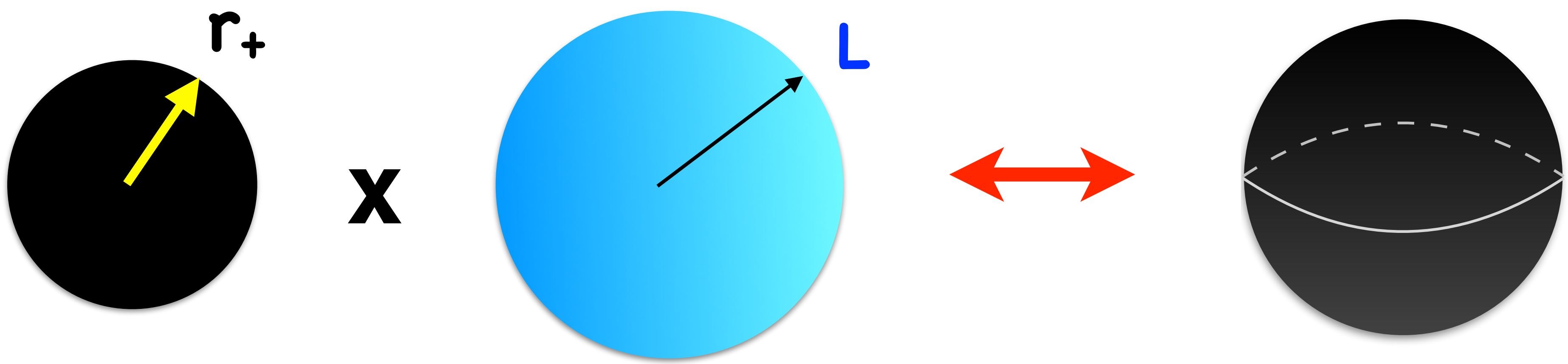


➞ But … we can have hierarchy of scales:

Schwarzschild-AdS$_5$ x S^5:

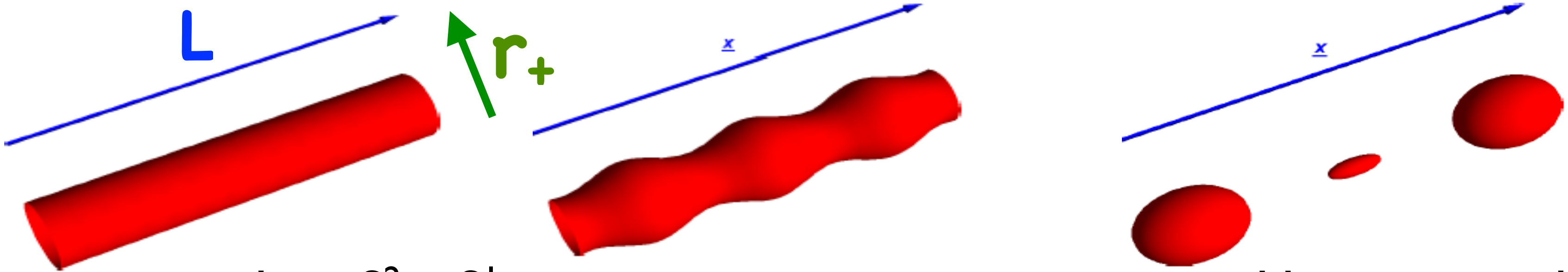
$$f(r) = 1 + \frac{r^2}{L^2} - \frac{r_+^2}{r^2}\left(\frac{r_+^2}{L^2} + 1\right)$$

Horizon topology $S^3 \times S^5$



… **but now we have two scales: horizon radius r_+ and S^5 radius L**

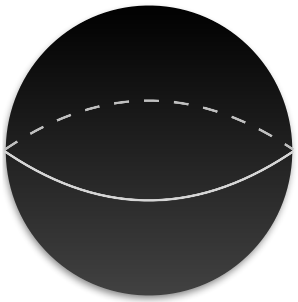
- **Recall *Gregory-Laflamme* instability on a black string Mink $_4$ x S^1 with $r_+ << L$**

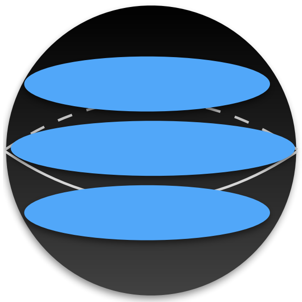


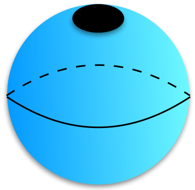
Horizon topology $S^2 \times S^1$

Horizon topology S^3

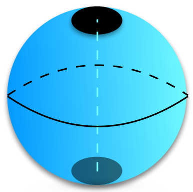
- **And suspect that for $r_+ << L$ Schwarzschild-AdS$_5$ xS^5:**







Horizon topology S^8



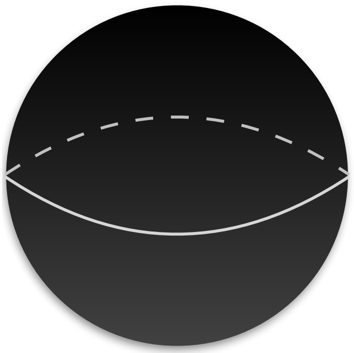
Horizon topology $S^3 \times S^5$

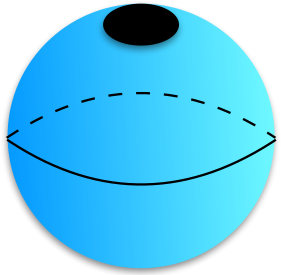
CFT: spontaneous symmetry breaking

$SO(6)$ -> $SO(5)$

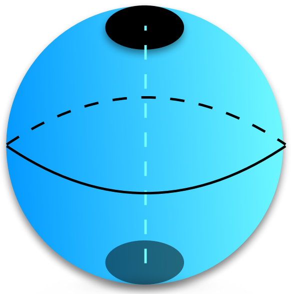
[Banks, Douglas, Horowitz, Martinec, 1998]
[Peet, Ross, 1998]
[Hubeny, Rangamani, 2002]

...yet, we can still be surprised:





Horizon topology S^8

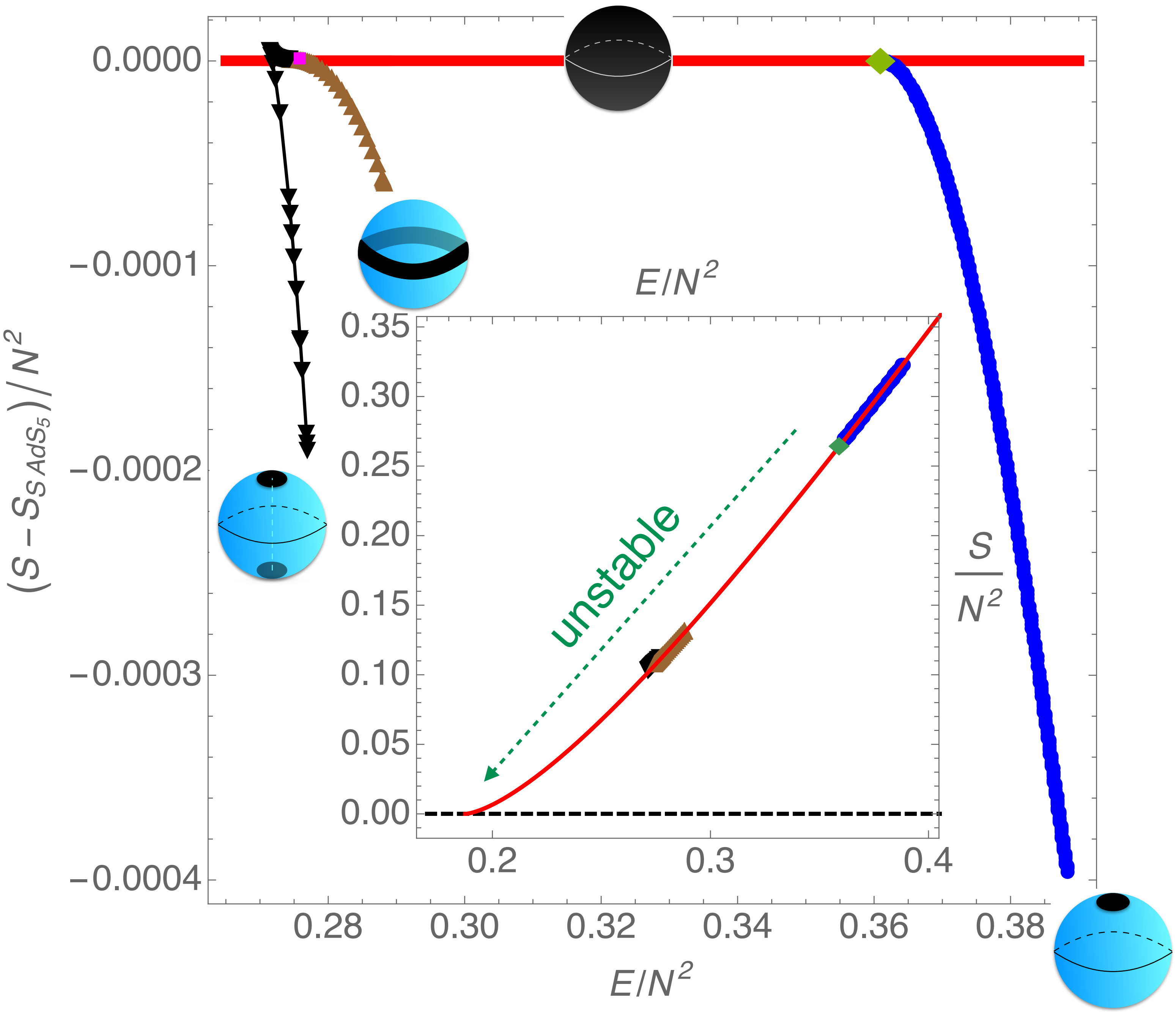


Horizon topology S^8

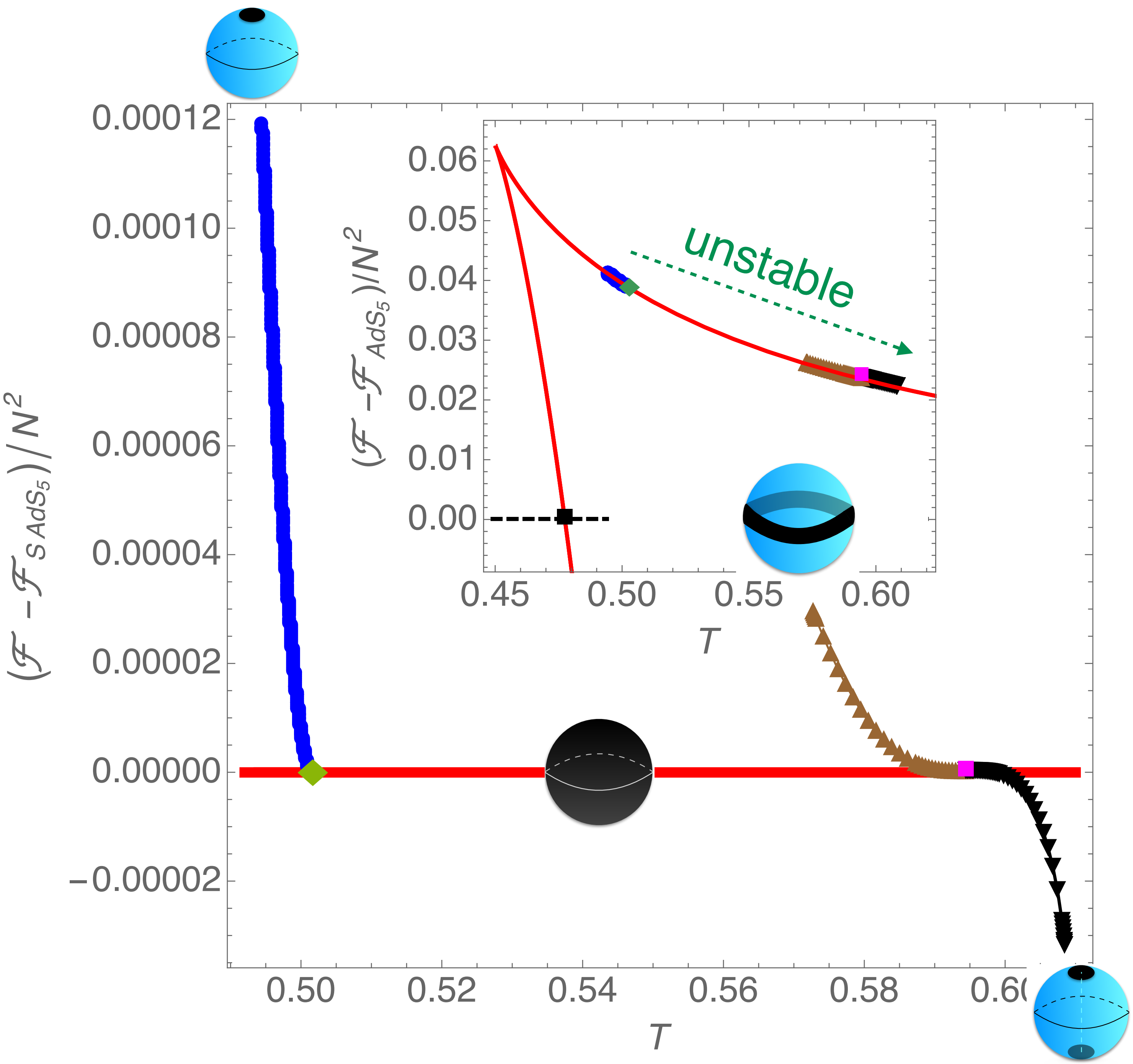
Newcomer: ‘ **black belt** ‘

Horizon topology $s^4 \times S^4$

Microcanonical ensemble:
(fixed E)



Canonical ensemble:
(fixed T)



➙ Numerical technology required to construct these solutions:

Einstein-De Turck method with **spectral discretisation methods** described in:

- **M. Headrick, S. Kitchen, and T. Wiseman, 0905.1822**
- **OD, J. Santos, B. Way, 1510.02804**

  Invited Topical Review for Classical and Quantum Gravity

➞ Technology required to construct these solution:

Kaluza-Klein Holography

- **3 fundamental tasks** to find and describe solution:
  - find asymptotic BCs <—> **turning-off sources** on CFT.
  - **compute E**,
  - **compute VEVs** of dim reduced scalar fields

- Use **Kaluza-Klein Holography:**

1) dim reduction of asymptotic $AdS_5 \times S^5$ ($AdS_p \times X^q$) solns down to solns on AdS_5

2) Apply holographic renormalisation to compute Tab, VEVs on bdry of AdS_5.

[Skenderis, Taylor, hep-th/0603016]

See also Appendix of [OD, Santos, Way 1501.06574]

➞ Kaluza-Klein Holography:

1) dim reduction of asymptotic $AdS_5 \times S^5$ ($AdS_p \times X^q$) solns down to solns on AdS_5

- Write the fields as a **deformation of $AdS_5 \times S^5$**

$$g_{MN} = g^o_{MN} + h_{MN}, \qquad F_{MNPQR} = F^o_{MNPQR} + f_{MNPQR}$$

- Decompose d=10 {h,f} as a **sum** of **spherical harmonics of S^5**, $h = \sum_\ell \psi^\ell(z)\, Y_\ell(x)$,

  that **break SO(6)** but **preserve SO(5):**

$$h_{\mu\nu}(z,X) = \sum_\ell \tilde{h}^\ell_{\mu\nu}(z) Y_\ell(X)\,, \qquad h_{\mu a}(z,X) = \sum_\ell \tilde{B}^\ell_\mu(z) D_a Y_\ell(X)\,,$$

$$h_{(ab)}(z,X) = \sum_\ell \tilde{\phi}^\ell(z) D_{(a} D_{b)} Y_\ell(X))\,, \qquad h^a_a(z,X) = \sum_\ell \tilde{\pi}^\ell(z) Y_\ell(X)$$

- **d=10 Harmonic coefs.** ψ^ℓ (z) — upon dim reduction — give **d=5** fields Ψ^ℓ

  which are interpreted as scalar fields in the reduced AdS_5 theory: **VEVs ~ $\Psi^\ell|_{\partial AdS_5}$**

- d=10 **Harmonic coefs.** $\psi^\ell(z)$ —> dim reduction —> give d=5 fields Ψ^ℓ: **VEVs ~** $\Psi^\ell|_{\partial AdS_5}$

- In general, **highly nonlinear map** between Ψ and ψ.

- For VEVs we only need the field Ψ **up to certain order** in a Fefferman-Graham expansion.

For example, the quadratic expansion for a field Ψ^ℓ takes the form:

$$\Psi^\ell = \psi^\ell + \sum_{k,m} \left(J_{\ell k m}\, \psi^k\, \psi^m + L_{\ell k m}\, D_\mu \psi^k\, D^\mu \psi^m \right) + O([\psi^\ell]^3)\ , \quad \text{const } J,L$$

If Ψ^ℓ is **dual** to an **operator** of **dim** ℓ => FG expand Ψ^ℓ off the boundary to **O(z$^\ell$)**.

But the quadratic terms with $k + m = \ell$ **also contribute** to **O(z$^\ell$)**.

In our case computing VEVs like E only requires **quadratic order**.

- Important to **use gauge invariant** fields to avoid gauge issues

- KK nonlinear map between 5-dim Ψ and 10-dim ψ:

$$\Psi^\ell = \psi^\ell + \sum_{k,m} \left(J_{\ell km}\, \psi^k \psi^m + L_{\ell km}\, D_\mu \psi^k D^\mu \psi^m \right) + O([\psi^\ell]^3)$$

WARNING: this contribution is often missed!

$$G_{\mu\nu} = h^0_{\mu\nu} - \frac{1}{12}\left[\frac{2}{9} D_\mu D^\sigma \hat{\psi}^2 D_\nu D_\sigma \hat{\psi}^2 - \frac{10}{3}\hat{\psi}^2 D_\mu D_\nu \hat{\psi}^2 + \left(\frac{10}{9}(D\hat{\psi}^2)^2 - \frac{32}{9}(\hat{\psi}^2)^2\right) g^o_{\mu\nu}\right].$$

- EOM can be obtained from a **5-dim action**:

$$S_{5d} = \frac{N^2}{2\pi^2}\int d^5x\sqrt{-G}\left[\frac{1}{4}R - 3 + \sum_{\Psi_\ell}\left(\frac{1}{2}G^{\mu\nu}\partial_\mu\Psi_\ell\,\partial_\nu\Psi_\ell + V(\Psi_\ell)\right)\right]$$

- **EOM** in d=5 are:

$$R_{\mu\nu}[G] = 2\left(-2G_{\mu\nu} + T_{\mu\nu} - \frac{1}{3}G_{\mu\nu}T^\sigma_\sigma\right)$$

$$T_{\mu\nu} = \sum_{\Psi_\ell}\left[\partial_\mu\Psi_\ell\,\partial_\nu\Psi_\ell - G_{\mu\nu}\left(\frac{1}{2}(\partial\Psi_\ell)^2 + V(\Psi_\ell)\right)\right]$$

$$\left(\Box - m^2_{\psi_\ell}\right)\Psi_\ell = \lambda_{\Psi\psi\psi}(\psi_\ell)^2$$

$\Box = \Box_G = \Box_{AdS_5}$ up to : $\mathcal{O}(z^4)$

2) Apply holographic renormalisation to compute Tab, VEVs on bdry of AdS5.

- Introduce **5-dim FG coord Z(z)**, &

  Do standard **FG expansion** and **Holographic Renormalisation**:

$$ds_5^2 = \frac{dZ^2}{Z^2} + \frac{1}{Z^2}\left[G_{ij}^{(0)}(X) + Z^2 G_{ij}^{(2)}(X) + Z^4\left(G_{ij}^{(4)}(X) + \log Z^2 H_{ij}^{(4)}(X)\right) + \cdots\right] dX^i dX^j;$$

$$\Phi^2(X,Z) = Z^2\left(\log Z^2 \cancel{\Phi_{(0)}^2}(X) + \tilde{\Phi}_{(0)}^2(X) + \cdots\right), \qquad \text{for} \quad \Delta = \Delta_{BF} = 2;$$

$$\Phi^\Delta(X,Z) = Z^{(4-\Delta)} \cancel{\Phi_{(0)}^\Delta}(X) + \cdots + Z^\Delta \Phi_{(2\Delta-4)}^\Delta(X) + \cdots, \qquad \text{for} \quad \Delta > 2, \qquad \text{(A.4}$$

- **Impose Dirichlet BCs** that **eliminate sources (non-normalizable** modes**):**

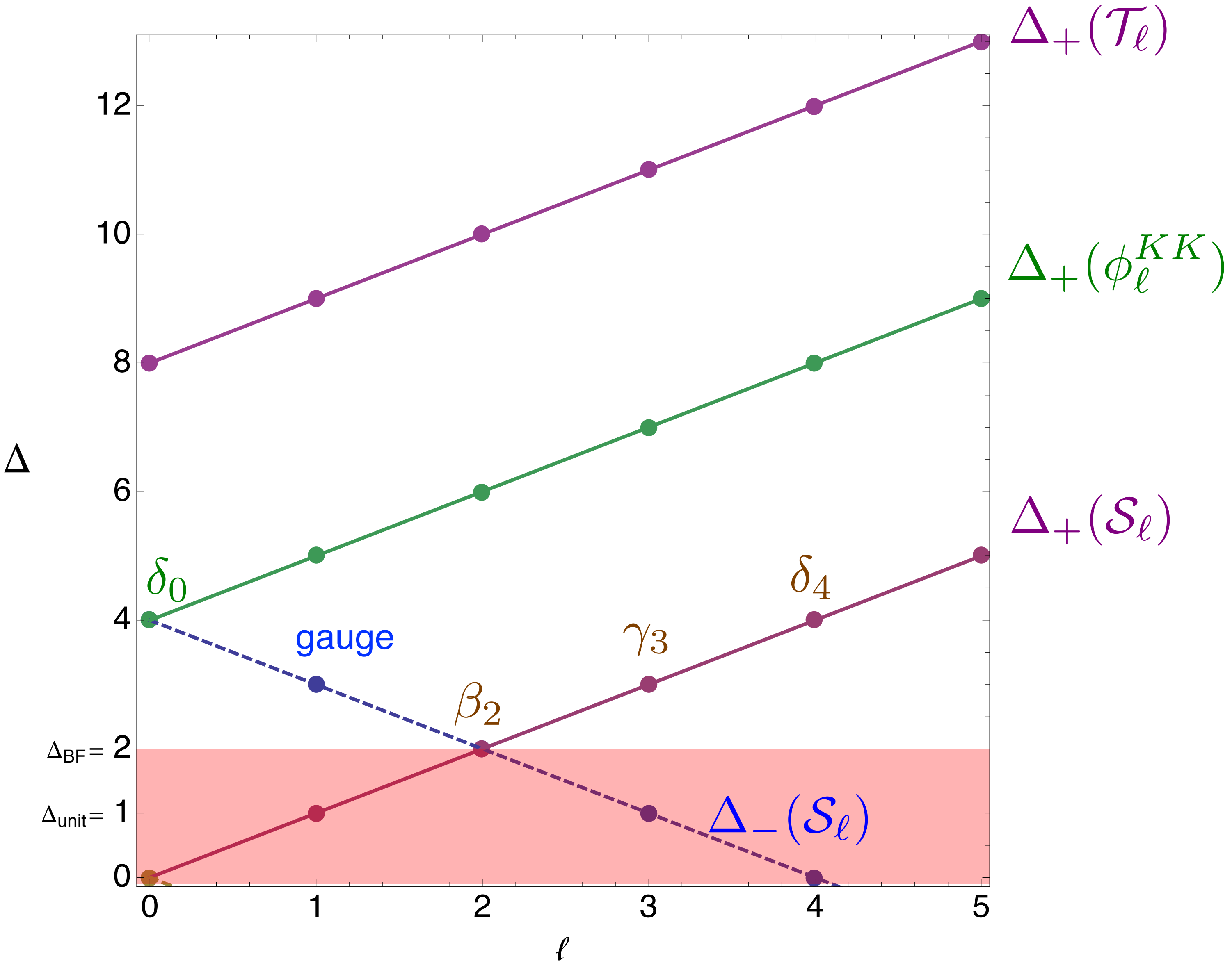
$$G_{ij}^{(0)}(X) = G_{ij}^{(0)}\Big|_{R_t \times S^3}$$

$$\Phi_{(0)}^2(X) = 0, \quad \text{for} \quad \Delta = \Delta_{BF} = 2;$$

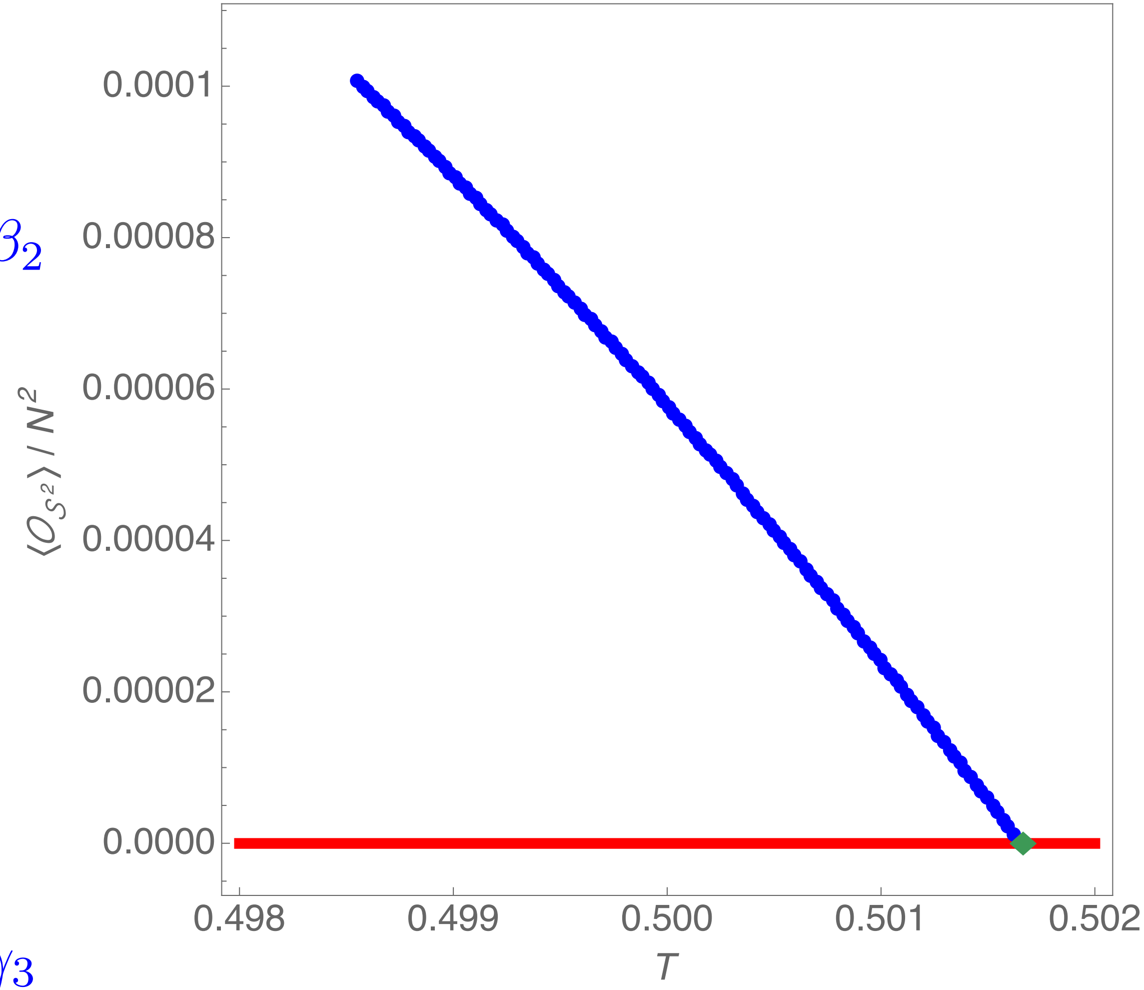
$$\Phi_{(0)}^\Delta(X) = 0, \quad \text{for} \quad \Delta > 2.$$

- **Use standard Holographic Renormalisation on 5-dim spacetime to get VEVs, E**

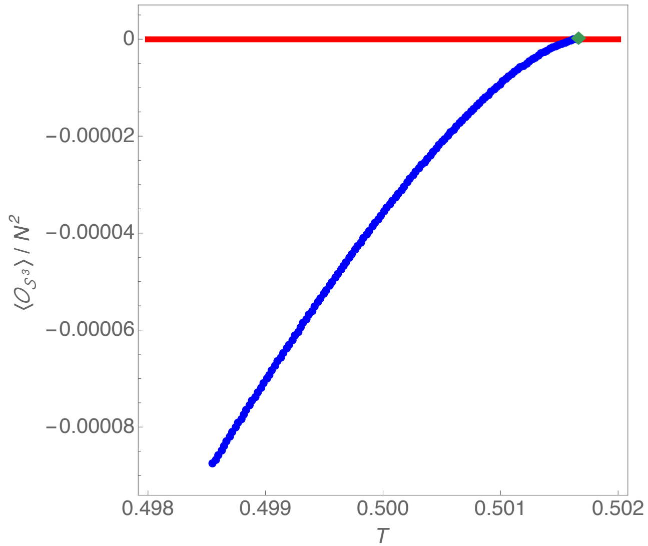
$$\langle T_{ij}\rangle = \frac{N^2}{2\pi^2}\left[\frac{3}{16}+\frac{3}{4}\,y_+^2-\frac{y_+^4}{3072}\left(30\beta_2^2+5\beta_2+12(16\delta_0+\delta_4-192)\right)\right]\,\mathrm{diag}\left\{1,\frac{1}{3}\,\eta_{\hat{i}\hat{j}}\right\}$$



$$\langle \mathcal{O}_{\mathcal{S}^2} \rangle = \frac{N^2}{\pi^2}\,\tilde{\Phi}^2_{(0)} = -\frac{N^2}{\pi^2}\,\frac{1}{8}\sqrt{\frac{5}{3}}\,y_+^2\,\beta_2$$



$$\langle \mathcal{O}_{\mathcal{S}^3} \rangle = \frac{N^2}{\pi^2}\,\Phi^3_{(2)} = \frac{N^2}{\pi^2}\,\frac{1}{8}\sqrt{\frac{5}{6}}\,y_+^3\,\gamma_3$$



→ Thermal Phases of $AdS_5 x S^5$ and their competition

Microcanonical ensemble:
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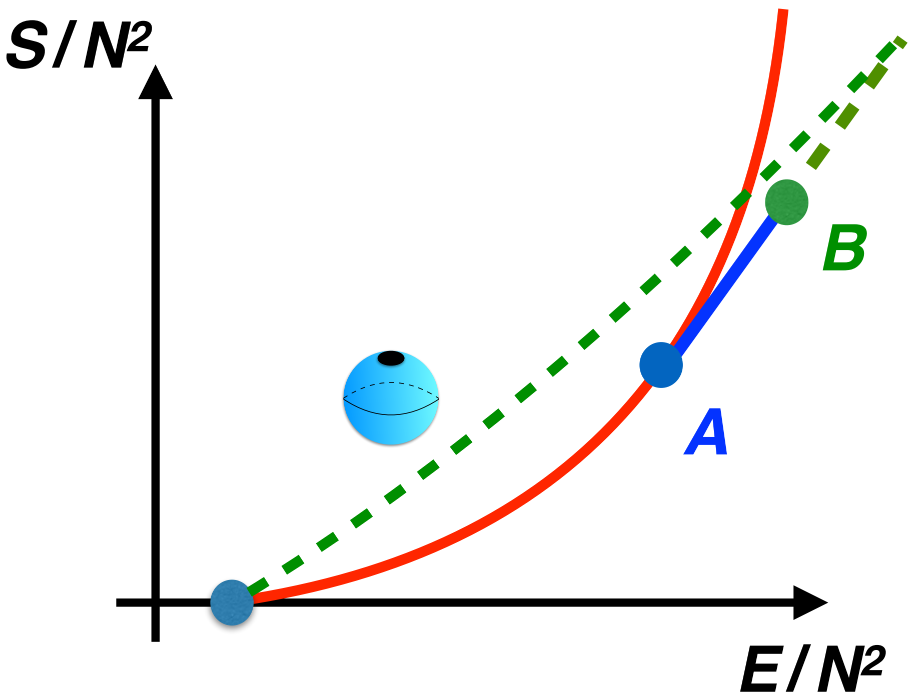

Canonical ensemble:
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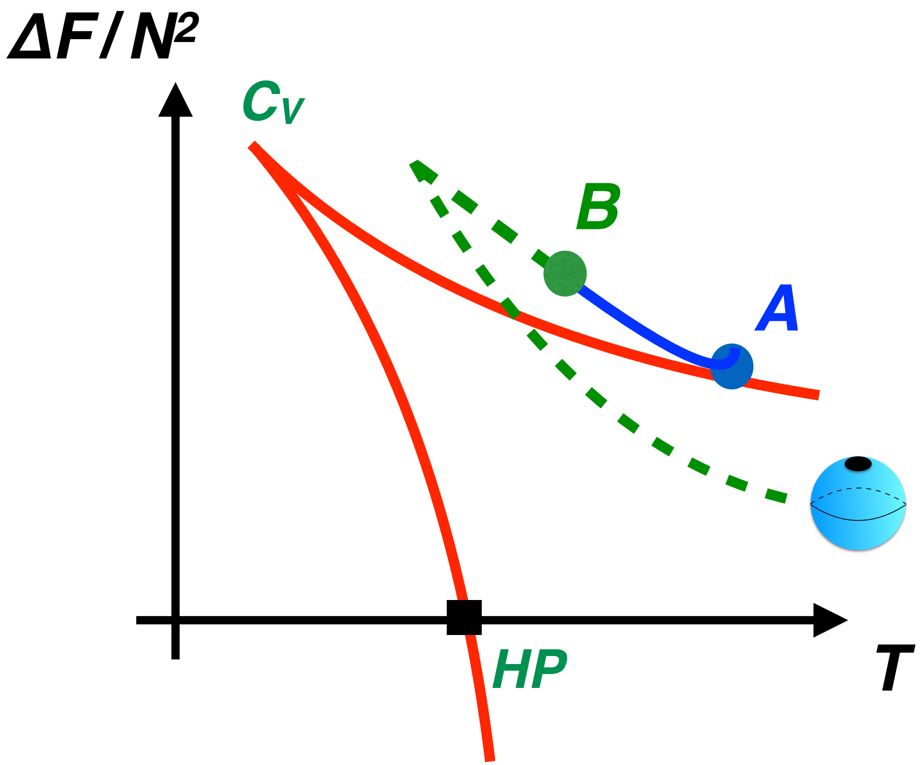
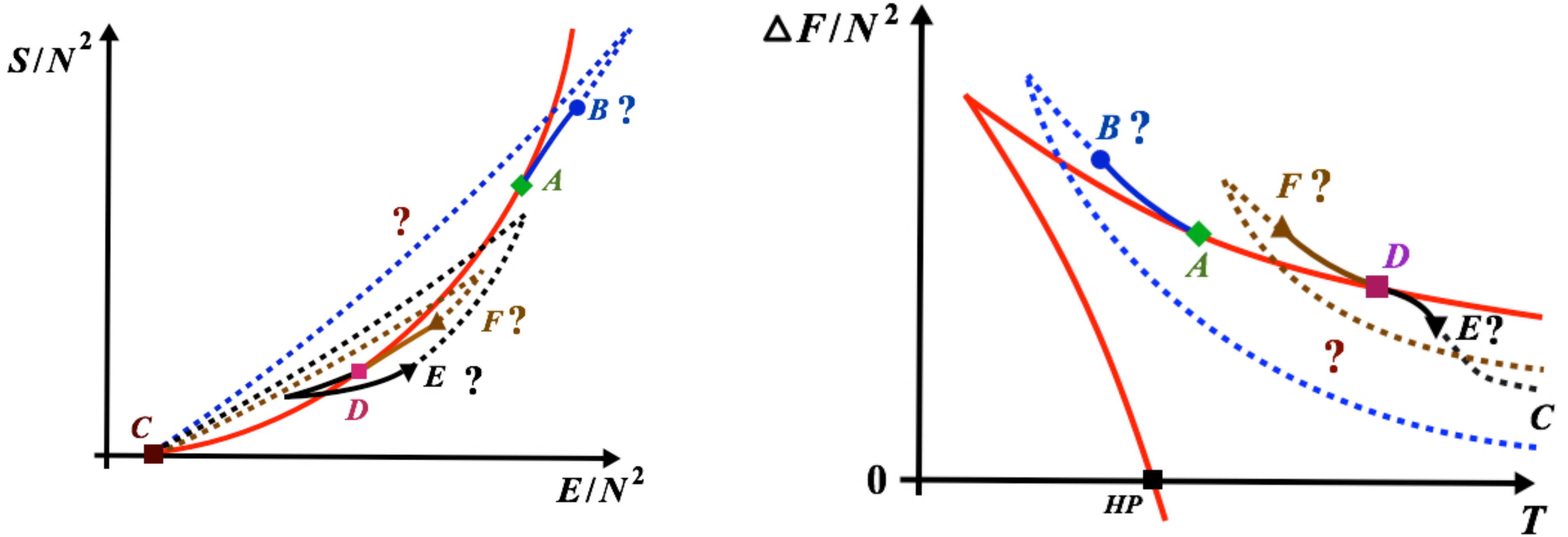


Muchas gracias!

Muito obrigado!

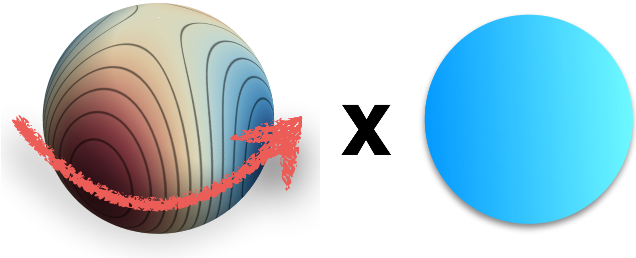


S/N²
B ?
A
?
F ?
E ?
D
C
E/N²
ΔF/N²
B ?
F ?
A
D
E?
?
C
0
HP
T

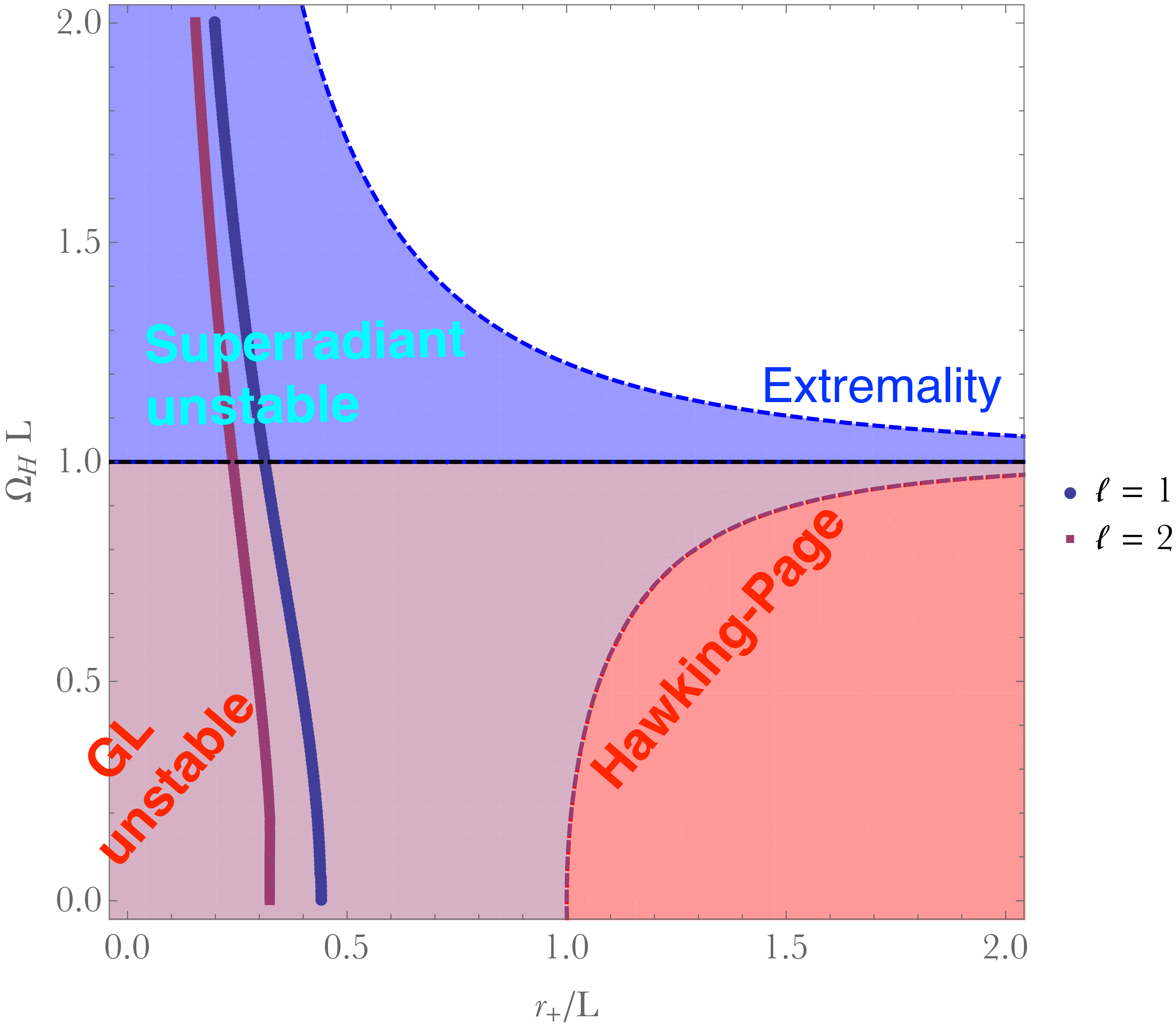
For Kerr-AdS_5xS^5 BHs, could it be that

the GL instability could shield $\Omega_H L > 1$ BHs from the superradiant instability? NO

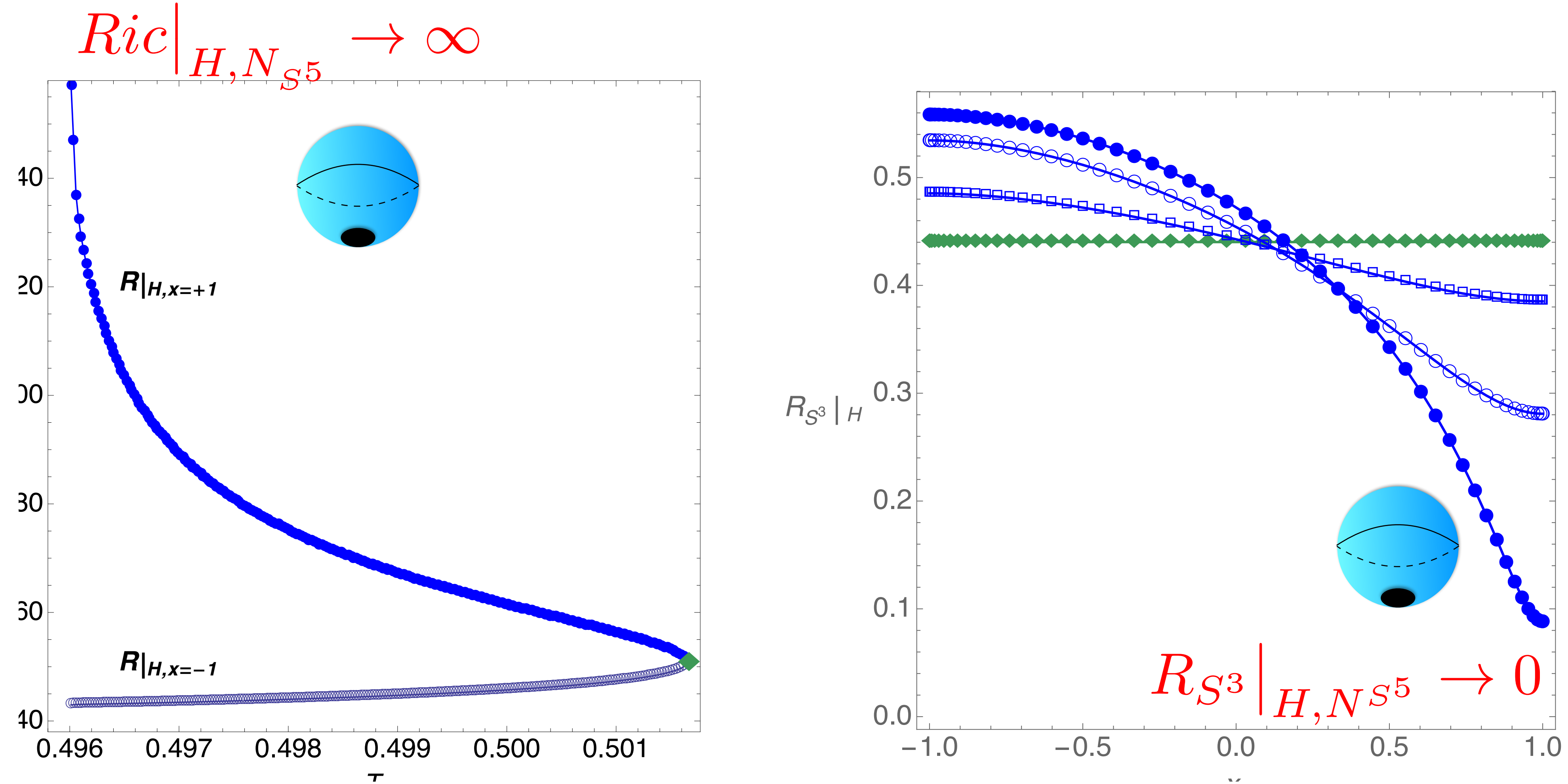
Black resonators



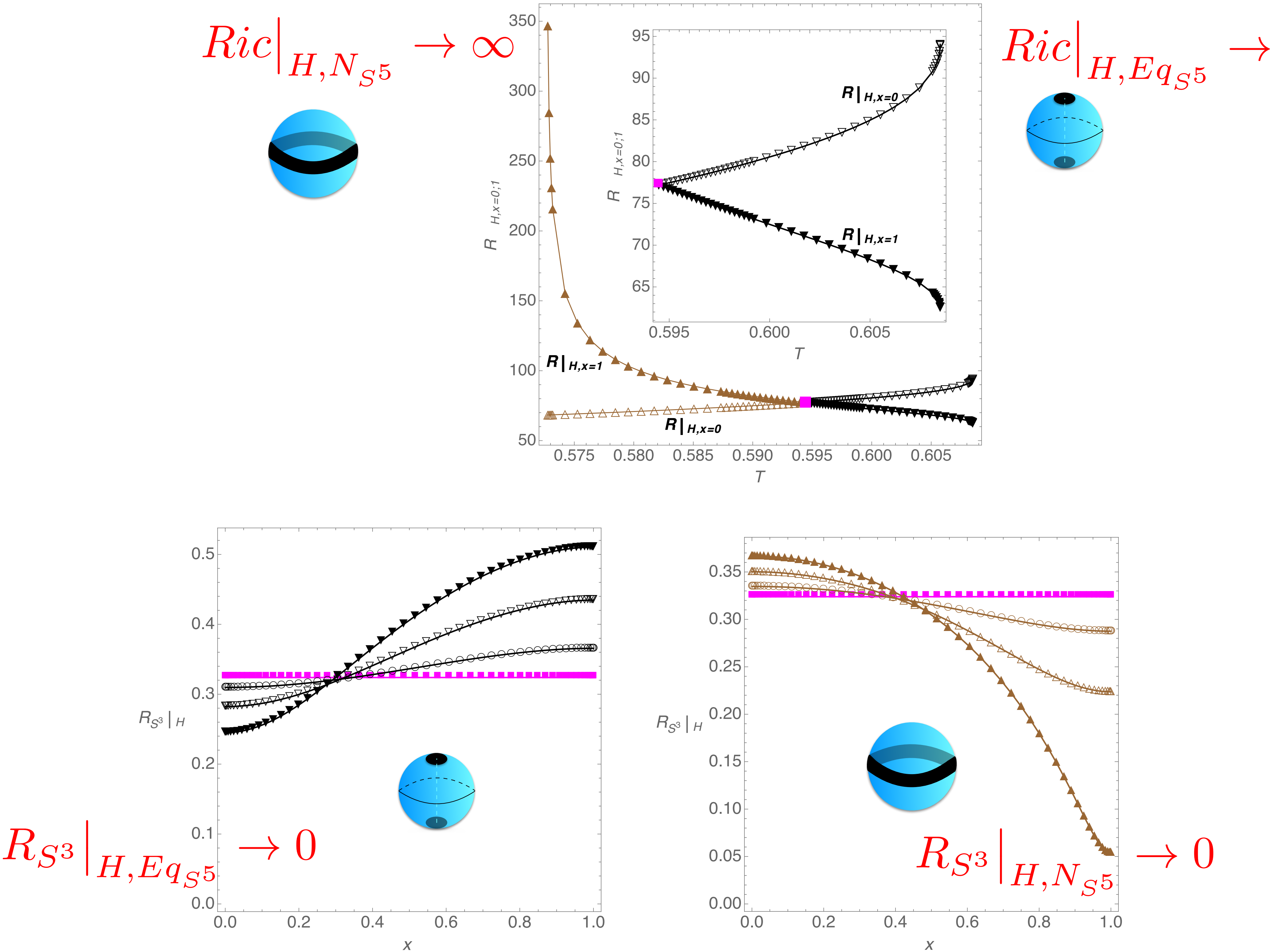
OD, Santos, Way, 1505.04793



OD, Santos, Way, 1510.02804

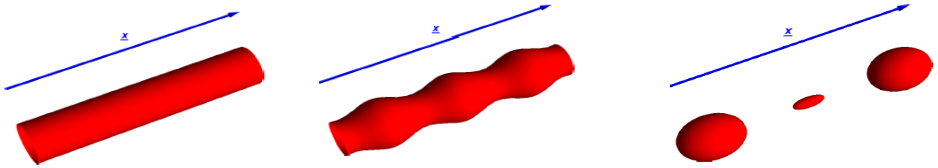


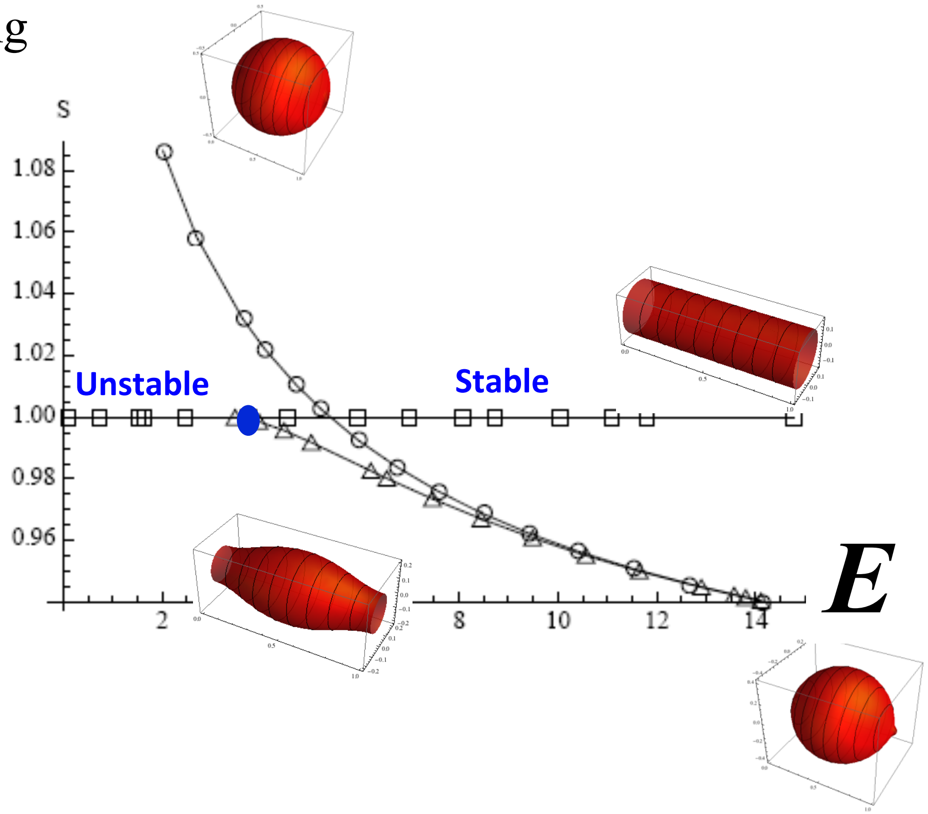
$Ric\big|_{H,N_{S^5}} \to \infty$
$R|_{H,x=+1}$
$R|_{H,x=-1}$
0.496
0.497
0.498
0.499
0.500
0.501
$R_{S^3}\,|\,_H$
0.0
0.1
0.2
0.3
0.4
0.5
−1.0
−0.5
0.0
0.5
1.0
$R_{S^3}\big|_{H,N^{S^5}} \to 0$



$Ric|_{H,N_{S^5}} \to \infty$
$Ric|_{H,Eq_{S^5}} \to$
$R_{H,x=0;1}$
$R|_{H,x=1}$
$R|_{H,x=0}$
T
$R_{S^3}|_H$
x
$R_{S^3}|_{H,Eq_{S^5}} \to 0$
$R_{S^3}|_{H,N_{S^5}} \to 0$

- **Gregory-Laflamme instability** on a black string





- **Ultraspinning instability** on a rotating BH