Black Holes in the 1/D expansion

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Black holes are very important objects in **General Relativity**,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings are very important objects in **Yang-Mills** theories,

but they do not appear in the fundamental formulation of the theory

They're non-linear, extended field configurations with complicated dynamics

Strings *become the basic* objects in the **large N** limit of SU(**N**) YM

In this limit, YM can be reformulated using worldsheet variables

Strings are still extended objects, but their dynamics simplifies drastically

Is there a limit of GR in which Black Hole dynamics simplifies a lot?

Yes, the limit of **large D**

any other parameter?

Is there a limit in which GR can be formulated with black holes as the basic (extended) objects?

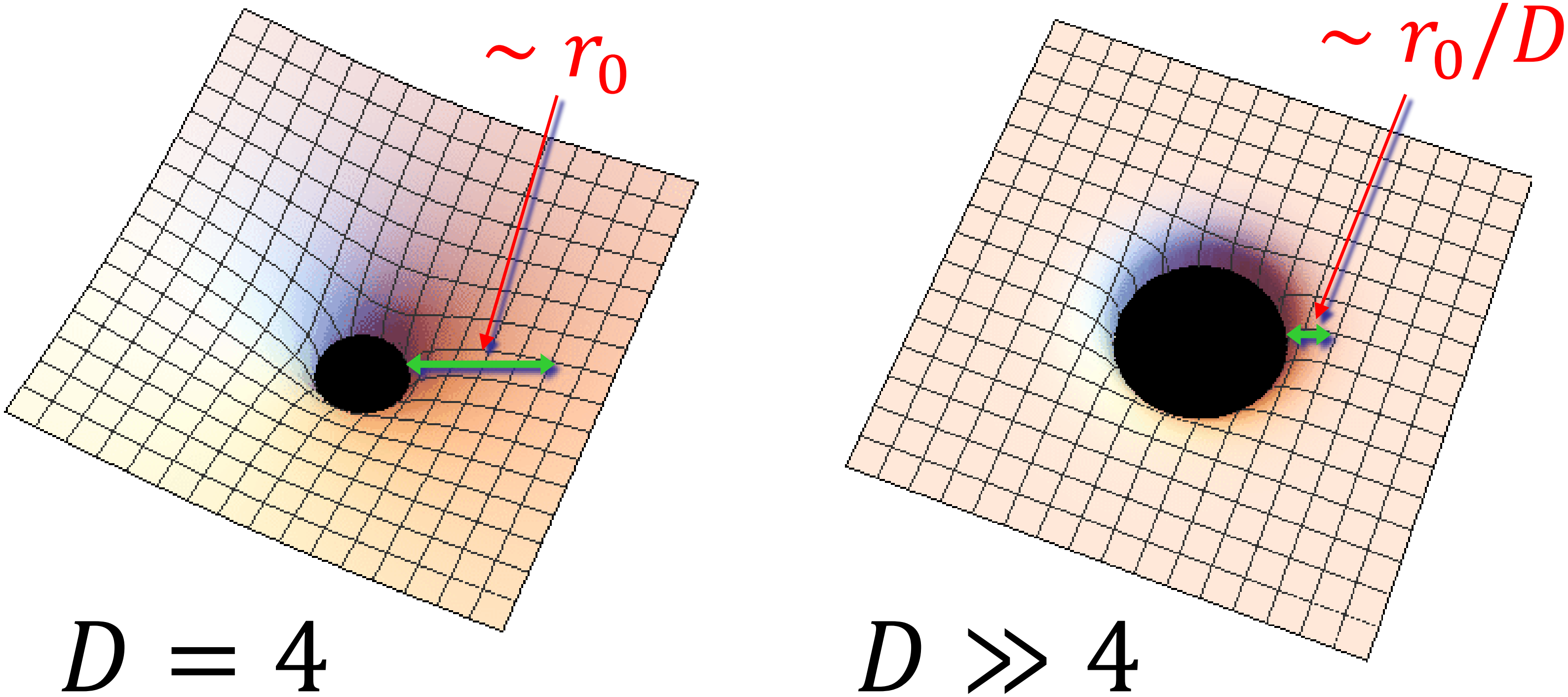
Maybe, the limit of large D

BH in D dimensions

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

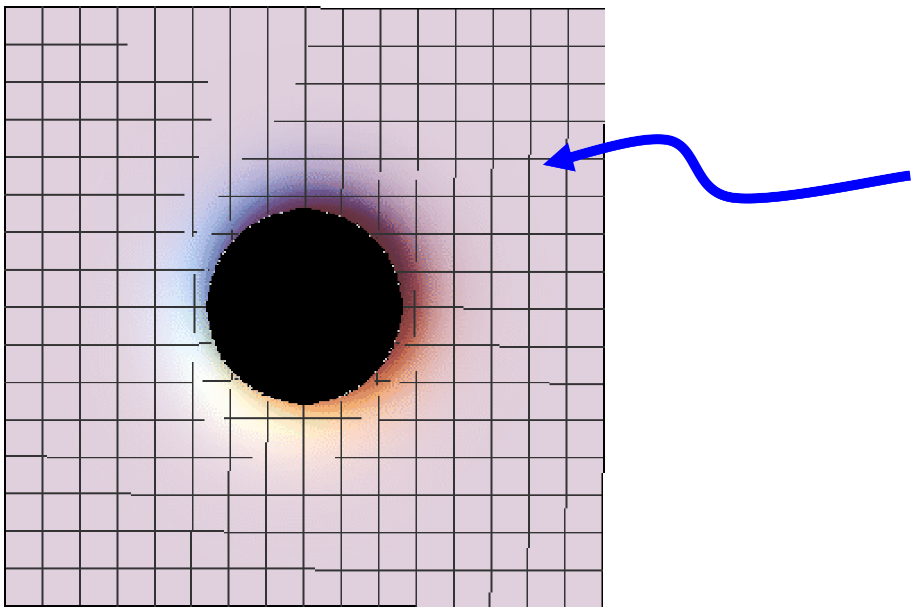
13 Jan 1916 (in $D = 4$)

Large-D ⇒ localization of gravitational field



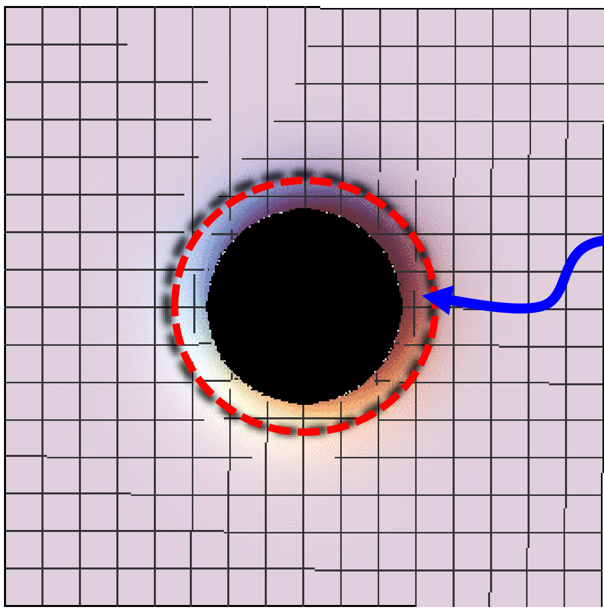
Hierarchy of scales: $\frac{r_0}{D} \ll r_0$

$$r \gg \frac{r_0}{D} \Rightarrow \left(\frac{r_0}{r}\right)^{D-3} \rightarrow 0$$



Flat space

$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \Leftrightarrow r - r_0 \lesssim \frac{r_0}{D}$$



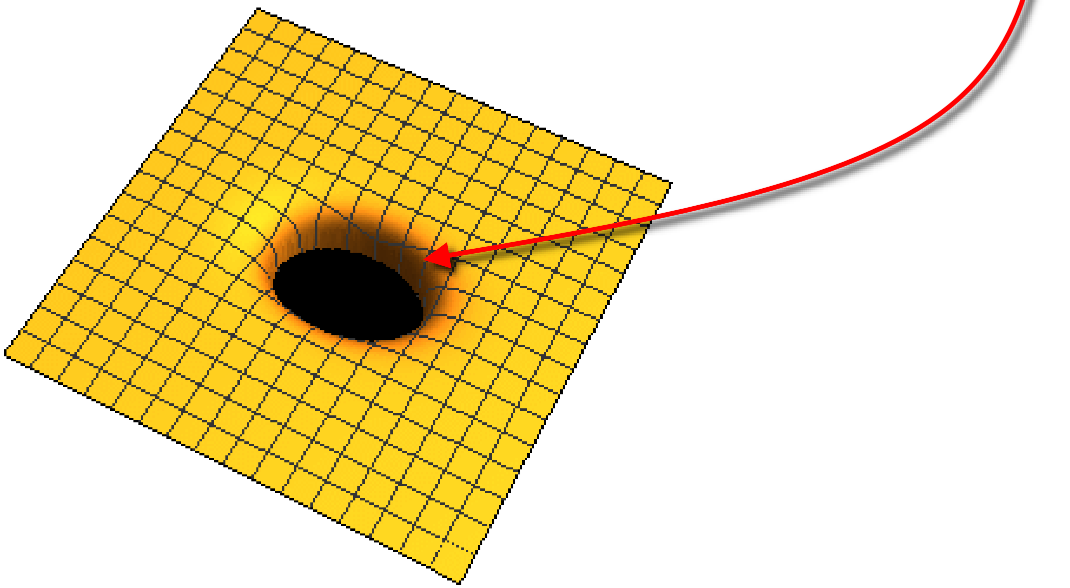
$r - r_0 \sim \frac{r_0}{D}$

non-trivial
gravitational field

Near-horizon region

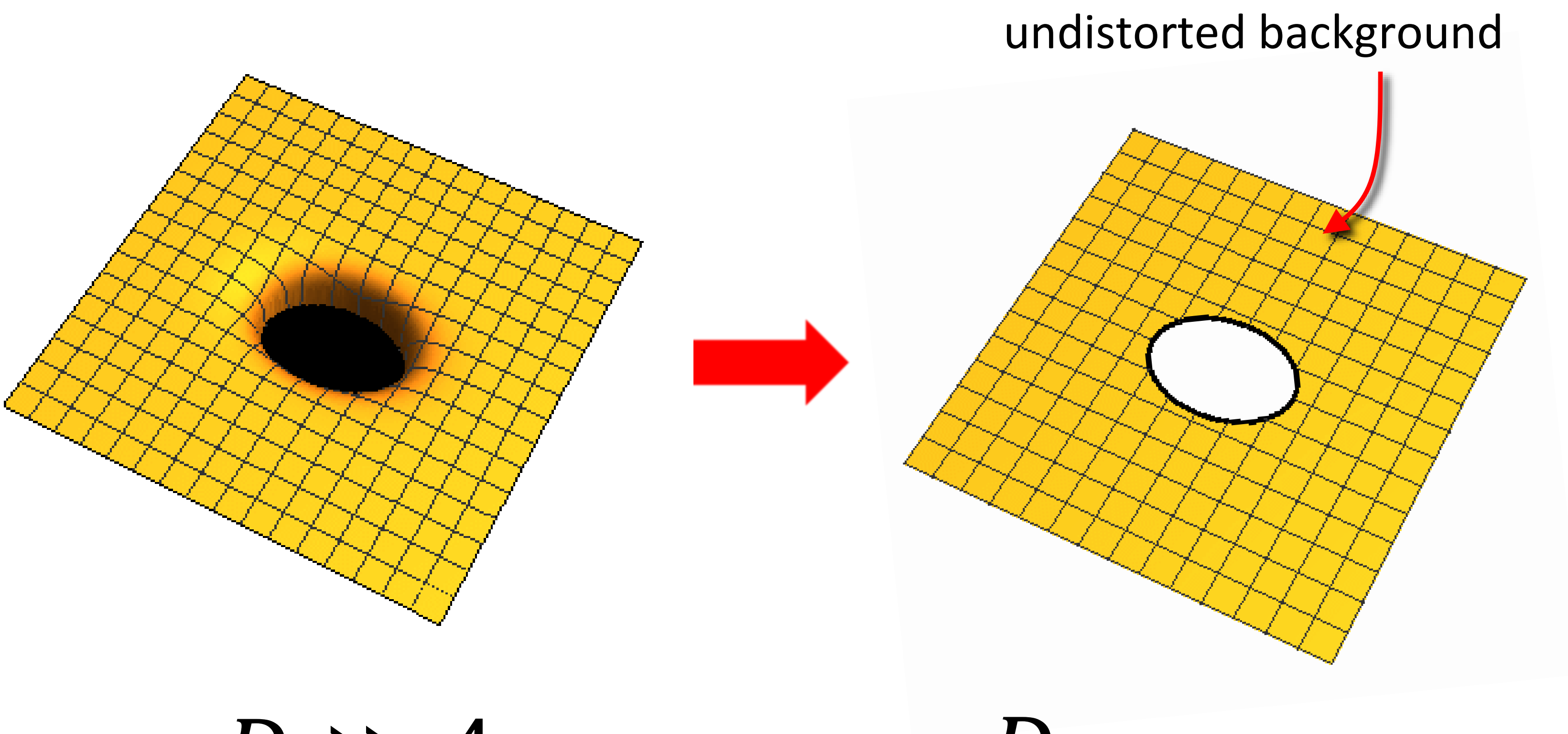
Does this help understand/solve bh dynamics?

All the black hole physics is concentrated here



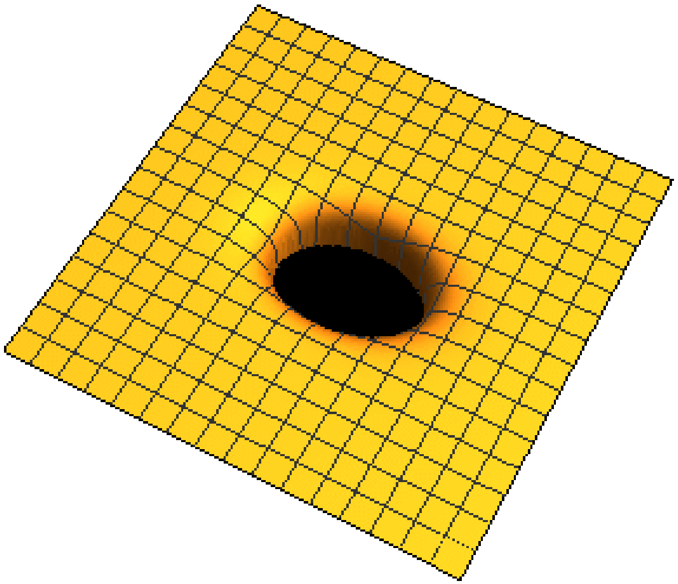
$$D \gg 4$$

Replace bh → Surface ('membrane')

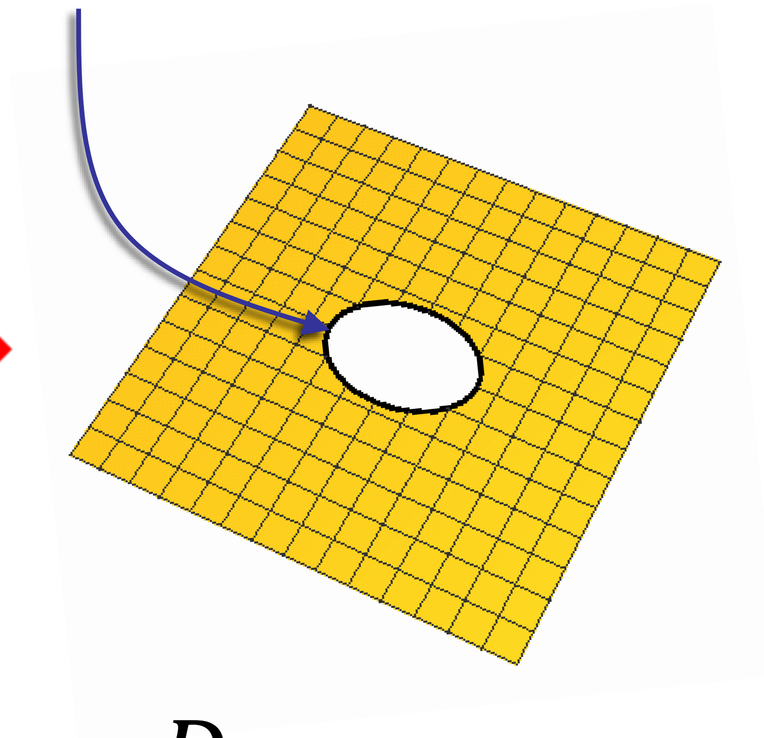


$D \gg 4$ $\qquad$ $D \to \infty$

What's the dynamics of this membrane?



$D \gg 4$



$D \to \infty$

Solve Einstein equations in near-horizon

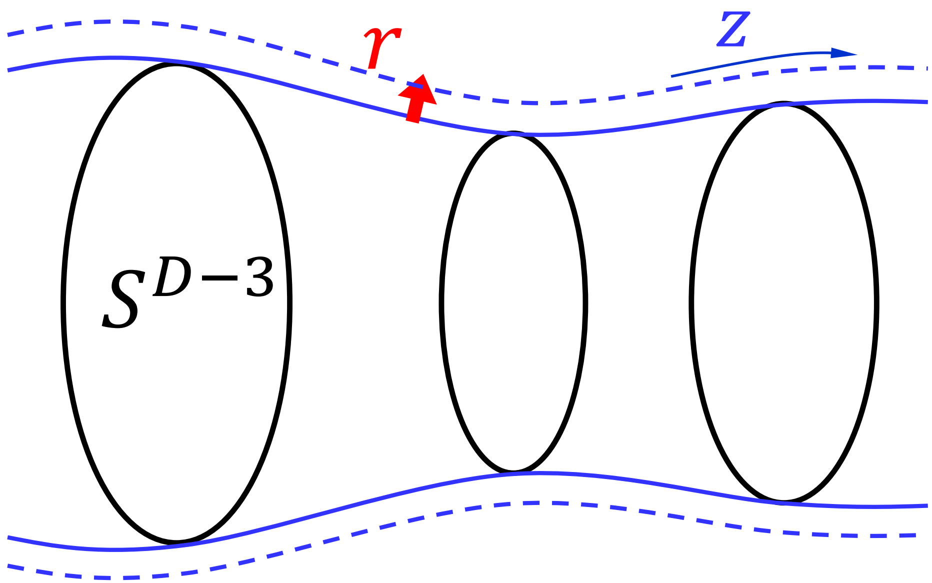
$\rightarrow$ *Effective membrane theory*

captures black hole dynamics in a 1/D expansion

Gradient hierarchy

⊥ Horizon: $\partial_r \sim D$

∥ Horizon: $\partial_z \sim 1$



Solved for stationary configurations

and for some time-dependent systems

RE+Shiromizu+Suzuki+Tanabe+Tanaka

Suzuki+Tanabe

RE+Izumi+Luna+Suzuki+Tanabe

A different (time-dependent) formulation

by

Bhattacharyya + Minwalla et al

Stationary solution

Soap-bubble equation (redshifted)

$$K = 2\gamma\kappa$$

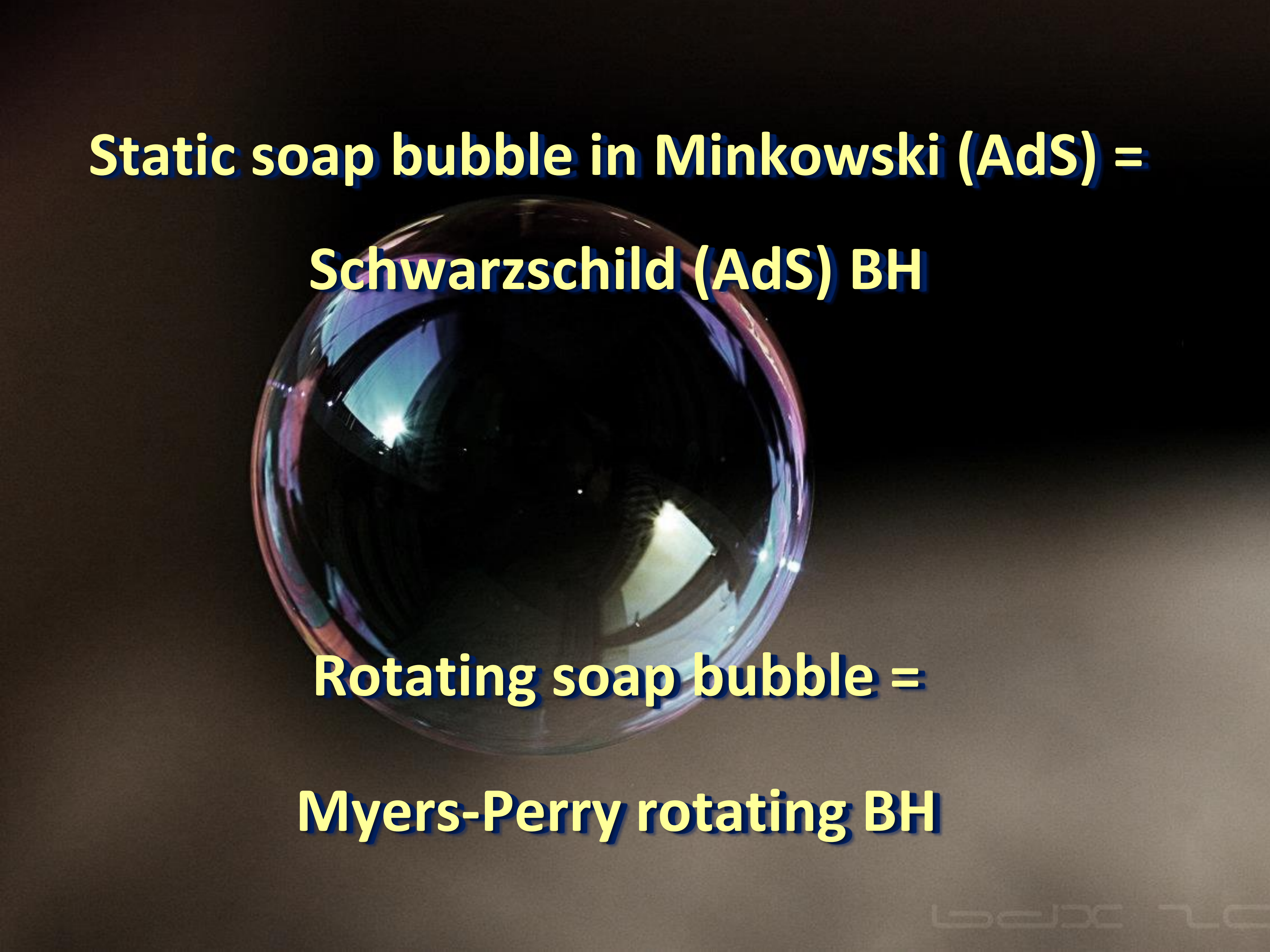
$K =$ **extrinsic curvature** trace

$\gamma =$ **redshift** factor on ‘membrane’

Lorentz boost from rotation

+ gravitational redshift from background

$\kappa =$ **surface gravity**


Static soap bubble in Minkowski (AdS) =
Schwarzschild (AdS) BH
Rotating soap bubble =
Myers-Perry rotating BH

Time-dependence:

Effective theory of black branes

Expect hydrodynamic features

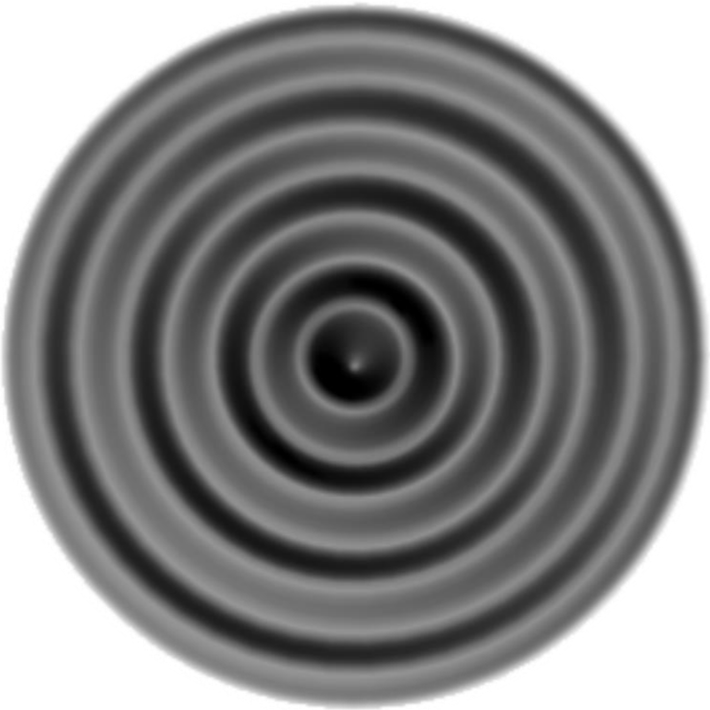
Fluid/gravity correspondences:

black brane dynamics at long wavelengths is fluid dynamics

But:

large-D stationary black branes are elastic soap-bubbles,

not fluids



Pressure waves on a fluid?

or

Wrinkles on a membrane?

Hydro-elastic complementarity

Large-D black branes

evolve dynamically as **fluids**,

but

settle down as **elastic membranes**

(*not* a fluid living on an elastic membrane)

The same variable can play two different (and complementary) roles:

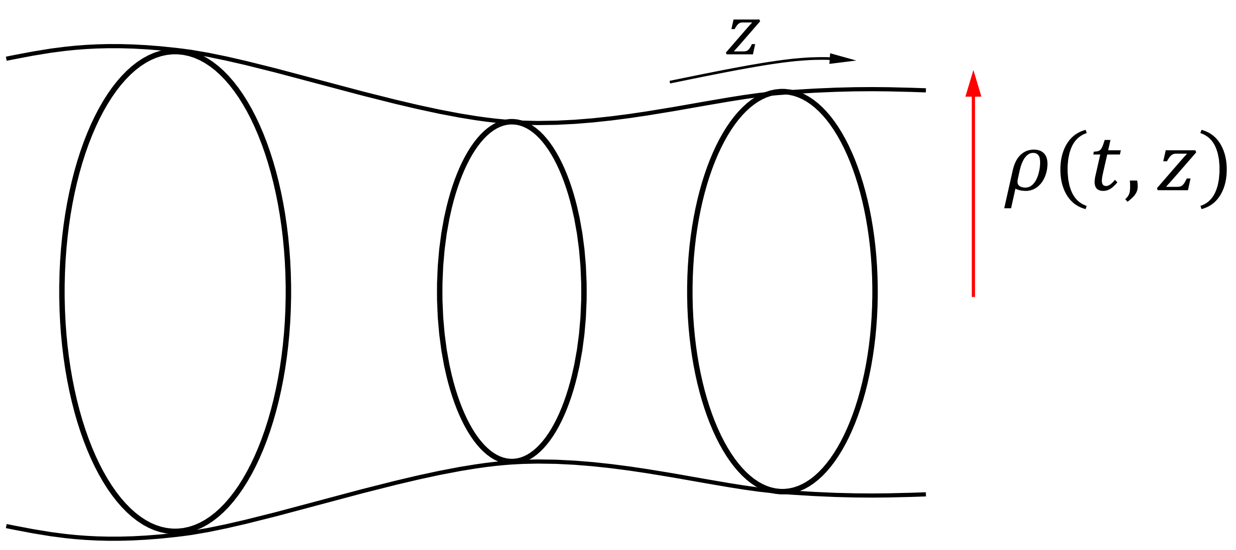
as mass

as horizon radius

$$R_{bh} = 2M$$

as energy density of a fluid

as local radius of a membrane



Solve Einstein equations
for a neutral black brane

$$ds^2 = 2dt\,dr + r^2\left(-\boxed{A}\,dt^2 - \frac{2}{D}C_i dz^i dt + \frac{1}{D}G_{ij}dz^i dz^j\right)$$

$$\boxed{A = 1 - \frac{\rho(t,z)}{r^D}}$$

Horizon at $r^D = \rho(t,z)$

$$C_i = \frac{p_i(t,z)}{r^D}$$

$$p_i = \rho v_i + \partial_i \rho$$

$v_i(t,z)$= velocity along brane

$$G_{ij} = \delta_{ij} + \frac{1}{D}\frac{p_i(t,z)p_j(t,z)}{\rho(t,z)r^D}$$

Effective equations

effective fields $\rho(t,z)$, $v_i(t,z)$

$$\partial_t\rho + \partial_i(\rho v^i) = 0$$

$$\partial_t(\rho v_i) + \partial^j\left(\rho v_i v_j - 2\,\rho\partial_{(i}v_{j)}\right) = -\rho\partial_i(\sqrt{-g_{tt}}K(\rho))$$

$K(\rho)$ = extrinsic curvature with radius ρ

Effective equations

continuity equation (energy conservation)

$$\partial_t \rho + \partial_i (\rho v^i) = 0$$

$$\partial_t (\rho v_i) + \partial^j \left(\rho v_i v_j - 2\, \rho \partial_{(i} v_{j)}\right) = -\rho \partial_i (\sqrt{-g_{tt}} K(\rho))$$

compressible viscous fluid

ρ= energy (mass) density

Effective equations

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

extrinsic curvature with radius ρ

$$\partial_t(\rho v_i) + \partial^j \left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i(\sqrt{-g_{tt}} K(\rho))$$

ρ= local radius of membrane

Effective **static** equations

$$\partial_t \rho + \partial_i(\rho v^i) = 0$$

extrinsic curvature with radius ρ

$$\partial_t(\rho v_i) + \partial^j\left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i\left(\sqrt{-g_{tt}} K(\rho)\right)$$

0

Elastic equation: $\sqrt{-g_{tt}}K$ =constant

can extend to stationary

Hydro-elastic complementarity

$$\partial_t(\rho v_i) + \partial^j \left(\rho v_i v_j - 2\,\rho \partial_{(i} v_{j)}\right) = -\rho \partial_i\left(\sqrt{-g_{tt}}K(\rho)\right)$$

evolve dynamically as <u>fluids</u>,

settle down as <u>elastic membranes</u>

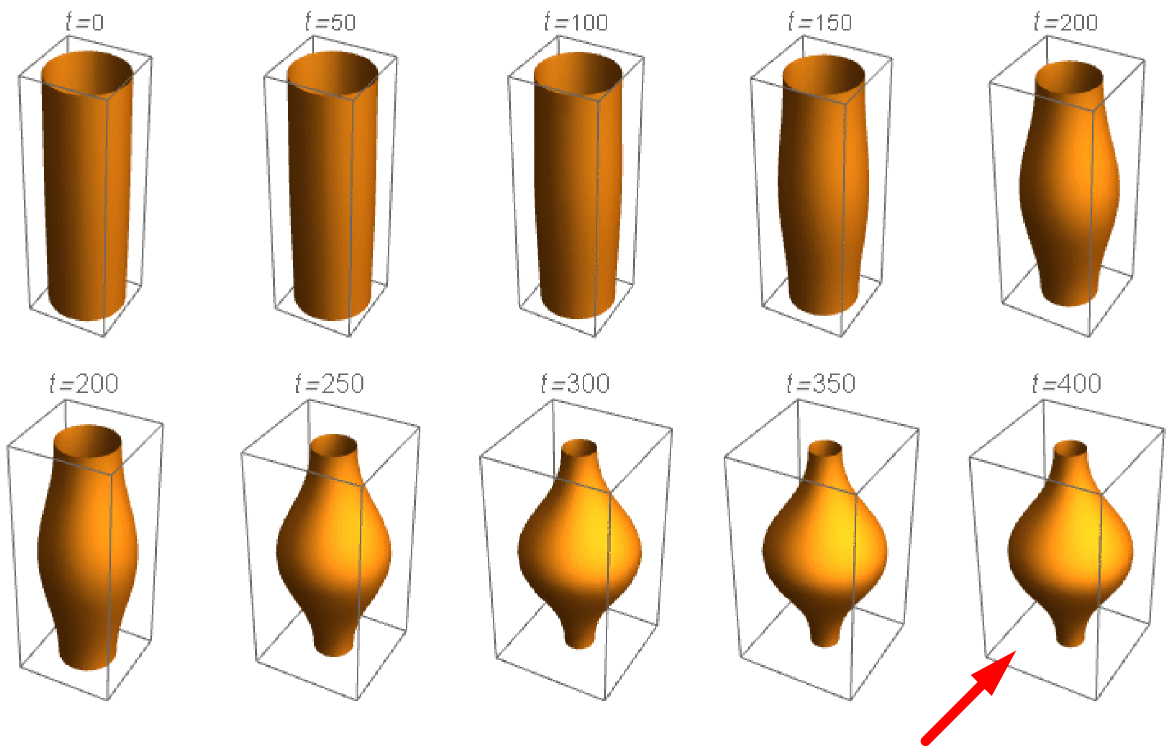
We’ve extended this to

charged black branes

Beyond fluid/gravity @ large D

Instability of black branes:

evolution and endpoint



stable non-uniform black string

Beyond fluid/gravity @ large D

Charged silence in

Reissner-Nordstrom AdS black brane:

quasinormal sound frequency

purely imaginary below a critical wavelength

Conclusion

The Large-D limit efficiently concentrates and extracts black hole physics

$$ds^2 = \frac{N^2(r,x)}{D^2} dr^2 + g_{ab}(r,x) dx^a dx^b$$

Einstein equations (w/ Λ)

$$K^a_b = \frac{D}{N} g^{ac} \partial_r g_{cb}$$ → def: extrinsic curvature

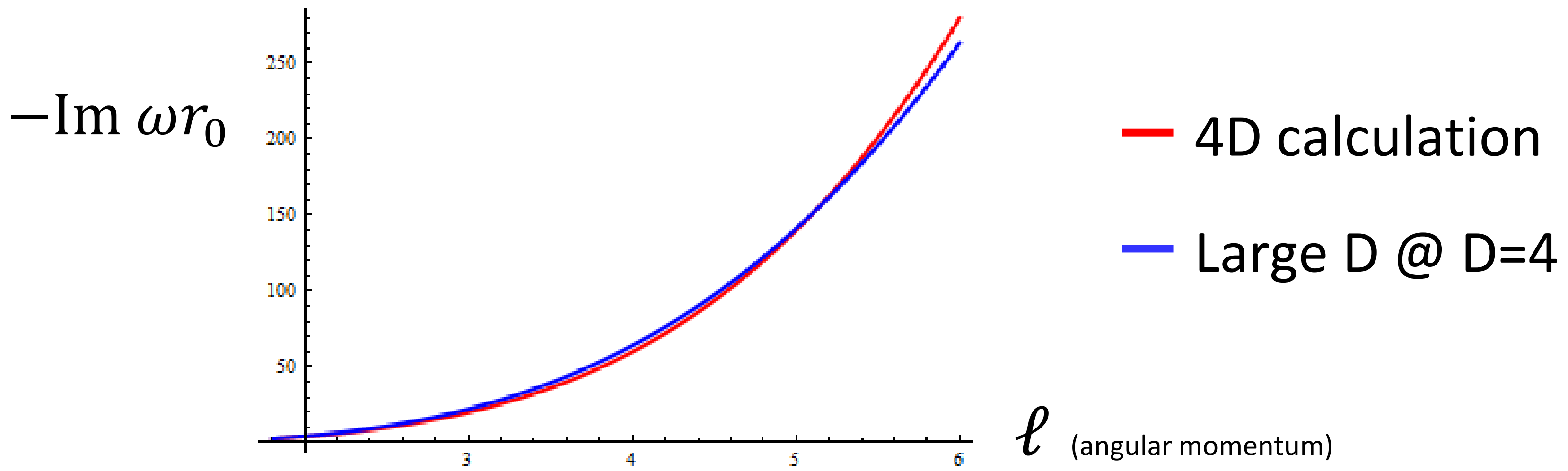
$$\frac{D}{N} \partial_r K^a_b + K\, K^a_b = R^a_b + \frac{\delta^a_b}{L^2} - \frac{\nabla^a \nabla_b N}{N}$$ → radial evolution eqn

$$R - K^2 + K^\mu_\nu K^\nu_\mu + \frac{(D-1)(D-2)}{L^2} = 0$$

$$\nabla_\nu K^\nu_\mu - \nabla_\mu K = 0$$

→ r-independent constraints

Quasinormal frequency in $D = 4$ (vector-type)



Calculation up to $\frac{1}{D^3}$: 6% accuracy in $D = 4$