Non-geometric fluxes and stable de Sitter in $d = 7$

Jose J. Fernández-Melgarejo[1]

Based on: Fortsch.Phys.(2012): Dibitetto, FM, Marqués, Roest
JHEP 1508 (2015): Cho, FM, Jeon, Park
JHEP 1511 (2015): Dibitetto, FM, Marqués

[1]Physics Department - Harvard University

Iberian Strings - IFT Madrid
January 2016

Gauged SUGRAs

- All possible deformations of SUGRAs by keeping
  - Field content
  - Supersymmetry
- Gauge a subgroup of the global symmetry group
- Embedding tensor formalism: systematic method
- Stückelberg couplings
- Compactifications automatically deform the theory
  - Flux compactifications
  - Scherk-Schwarz (SS) compactifications
  - Coset reductions

Embedding tensor formalism $\vartheta_I{}^A$

Cordaro, deWit, Nicolai, Samtleben,...

- Systematic study of the most general gaugings
- Promote subgroup $G_X \subset G$ to be local
- Embedding tensor (ET) $\vartheta_I{}^A$: $n_v \times \dim\ G$ matrix such that

$$X_I = \vartheta_I{}^A t_A \ , \qquad I = 1 \dots n_v \ , \qquad A = 1 \dots \dim\ G$$

- Constraints
  - quadratic (gauge) $\vartheta_K{}^A X_{IJ}{}^K - \vartheta_J{}^B X_{IB}{}^A = 0$
  - linear (SUSY) $\mathfrak{g} \otimes V = \theta_1 \oplus \theta_2 \oplus \dots \oplus \theta_n$
- Deformation consequences
  - Deformed field strengths
  - Deformed gauge transformations
  - Coupled Bianchi identities
  - Scalar potential (vacua and moduli stabilisation)

$N = 2$ $d = 9$ supergravity

- Global symmetry: $SL(2, \mathbb{R}) \times (\mathbb{R}^+)^2$ FM, Ortín, Torrente-Luján'12
- Field content
  - Vielbein: $e_\mu{}^a$
  - Scalar fields: φ, $\tau \equiv \chi + ie^{-\phi}$
  - p-form potentials: $A_\mu{}^0, A_\mu{}^1, A_\mu{}^2, B_{\mu\nu}{}^1, B_{\mu\nu}{}^2, C_{\mu\nu\rho}$
  - fermions: $\psi_\mu, \tilde{\lambda}, \lambda$
- $\vartheta_I{}^A$, $\quad I = 1, 2, 3, \quad A = 1, \ldots, 5$
- Covariant derivatives: $\mathfrak{D} = d + X_I A^I = d + \vartheta_I{}^A t_A\, A^I$

Gauged quantities: field strengths

- Ungauged

$$\begin{aligned} F^I &= dA^I \\ H^i &= dB^i + \tfrac{1}{2}\delta^i{}_{\mathbf{i}}(A^0 \wedge F^{\mathbf{i}} + A^{\mathbf{i}} \wedge F^0) \\ G &= d[C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}] - \varepsilon_{\mathbf{i}j}F^{\mathbf{i}} \wedge \left(B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{0\mathbf{j}}\right) \end{aligned}$$

- Gauged

$$\begin{aligned} F^I &= dA^I + \tfrac{1}{2}X_{JK}{}^I A^J \wedge A^K + Z^I{}_i B^i \\ H^i &= \mathfrak{D}B^i + \tfrac{1}{2}\delta^i{}_{\mathbf{i}}(A^0 \wedge F^{\mathbf{i}} + A^{\mathbf{i}} \wedge F^0) + (XA^{012}) + Z^i C \\ G &= \mathfrak{D}[C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}}A^{0\mathbf{ij}}] - \varepsilon_{\mathbf{i}j}F^{\mathbf{i}} \wedge \left(B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}}A^{0\mathbf{j}}\right) + Z_{\mathbf{i}j}B^{\mathbf{i}j} + Z\tilde{C} \\ \tilde{G} &= \mathfrak{D}\tilde{C} + (OLD) + Z^0{}_j B^j \wedge C + (X_{Jij}A^J \wedge B^{ij}) + Z^i \tilde{H}_i \end{aligned}$$

Gauged quantities: gauge transformations

- Ungauged

$$\delta_\Lambda A^I = -d\Lambda^I$$

$$\delta_\Lambda B^i = -d\Lambda^i + \delta^i{}_{\mathbf{i}} \left[\Lambda^{\mathbf{i}} F^0 + \Lambda^0 F^{\mathbf{i}} + \tfrac{1}{2}\left(A^0 \delta_\Lambda A^{\mathbf{i}} + A^{\mathbf{i}} \delta_\Lambda A^0\right)\right]$$

$$\delta_\Lambda [C - \tfrac{1}{6}\varepsilon_{\mathbf{ij}} A^{0\mathbf{ij}}] = -d\Lambda - \varepsilon_{\mathbf{i}j}\left(F^{\mathbf{i}}\Lambda^j + \Lambda^{\mathbf{i}} H^j - \delta_\Lambda A^{\mathbf{i}} B^j + \tfrac{1}{2}\delta^j{}_{\mathbf{j}} A^{0\mathbf{i}} \delta_\Lambda A^{\mathbf{j}}\right)$$

- Gauged

$$\delta_\Lambda A^I = -\mathfrak{D}\Lambda^I + Z^I{}_i \Lambda^i$$

$$\delta_\Lambda B^i = -\mathfrak{D}\Lambda^i + \Lambda^i F^0 + \Lambda^0 F^i + \tfrac{1}{2}\left(A^0 \delta_\Lambda A^i + A^i \delta_\Lambda A^0\right) + Z^i \Lambda$$

$$\delta_\Lambda [C - \tfrac{1}{6}\varepsilon_{ij} A^{0ij}] = -\mathfrak{D}\Lambda - \varepsilon_{ij}\left(F^i\Lambda^j + \Lambda^i H^j - \delta_\Lambda A^i \wedge B^j + \tfrac{1}{2} A^{0i} \delta_\Lambda A^j\right) + Z\tilde{\Lambda}$$

$$\delta_\Lambda \tilde{C} = -\mathfrak{D}\tilde{\Lambda} + (OLD) + Z^i \tilde{\Lambda}_i$$

Gauged quantities: Bianchi identities

Ungauged	Gauged
$dF^I = 0$	$\mathfrak{D}F^I = Z^I{}_i H^i$
$dH^i + F^0 F^i = 0$	$\mathfrak{D}H^i + F^0 F^i = Z^i G$
$dG - F^i H_i = 0$	$\mathfrak{D}G - F^i H_i = Z\tilde{G}$
$d\tilde{G} + F^0 G + \frac{1}{2}\epsilon_{ij} H^i H^j = 0$	$\mathfrak{D}\tilde{G} + F^0 G + \frac{1}{2} H^i H_i = Z^i H_i$
$d\tilde{H}_i + F_i \tilde{G} - H_i G = 0$	$\mathfrak{D}\tilde{H}_i + F_i \tilde{G} - H_i G = Z_i{}^I \tilde{F}_I$
$d\tilde{F}_0 + F^j \tilde{H}_j - \frac{1}{2} GG = 0$	$\mathfrak{D}\tilde{F}_0 + F^i \tilde{H}_i - \frac{1}{2} GG = \vartheta_0{}^A J_A$
$d\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} = 0$	$\mathfrak{D}\tilde{F}_i + F^0 \tilde{H}_i - H_i \tilde{G} = \vartheta_i{}^A J_A$
$\vdots$	$\vdots$

Solving the constraints

- SUSY constraints require
  - $Z = Z(\vartheta)$: Stückelberg couplings depend on the ET
  - 6 independent ET components: $\{\theta^i \,,\, \kappa^{jk}\}$ $\qquad \mathbf{2}_{(+3)} \oplus \mathbf{4}_{(-1)}$
- Quadratic constraints

$$\epsilon_{ij}\,\theta^i\,\kappa^{jk} = 0 \,, \qquad \mathbf{2}_{(-1)}$$

$$\theta^{(i}\,\kappa^{jk)} = 0 \,. \qquad \mathbf{4}_{(-1)}$$

ID	θ^i	κ^{ij}	gauging
1		diag$(1,1)$	SO(2)
2	$(0,0)$	diag$(1,-1)$	SO(1,1)
3		diag$(1,0)$	$\mathbb{R}^+_\gamma$
4	$(1,0)$	diag$(0,0)$	$\mathbb{R}^+_\beta$

Solving the constraints

- SUSY constraints require
  - $Z = Z(\vartheta)$: Stückelberg couplings depend on the ET
  - 6 independent ET components: $\{\theta^i \, , \, \kappa^{jk}\}$ $\qquad \mathbf{2}_{(+3)} \oplus \mathbf{4}_{(-1)}$
- Quadratic constraints

$$\epsilon_{ij}\,\theta^i\,\kappa^{jk} = 0 \; , \qquad \mathbf{2}_{(-1)}$$

$$\theta^{(i}\,\kappa^{jk)} = 0 \; . \qquad \mathbf{4}_{(-1)}$$

ID	θ^i	κ^{ij}	gauging
1		diag(1, 1)	SO(2)
2	(0, 0)	diag(1, −1)	SO(1, 1)
3		diag(1, 0)	$\mathbb{R}^+_\gamma$
4	(1, 0)	diag(0, 0)	$\mathbb{R}^+_\beta$

Half-maximal $d = 8$

- $\mathbb{R}^+ \times \underbrace{SL(2) \times SL(2)}_{O(2,2)}$

- Embedding tensor

$$\left[\begin{array}{l} a_{\alpha i} \\ b_{\alpha i} \end{array} \right.$$

- Quadratic constraints

$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,a_{\beta j} - b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,b_{\beta j} + b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{ij}\,a_{(\alpha i}\,b_{\beta)j} = 0$$
$$\epsilon^{\alpha\beta}\,a_{\alpha(i}\,b_{\beta j)} = 0$$

ID	$a_{\alpha i}$	$b_{\alpha i}$	gauging
1	$\text{diag}(\cos\alpha, 0)$	$\text{diag}(\sin\alpha, 0)$	$\text{Solv}_2 \times \text{SO}(1,1)$
2	$\text{diag}(1,1)$	$\text{diag}(-1,-1)$	$\text{SL}(2) \times \text{SO}(1,1)$
3	$\text{diag}(1,-1)$	$\text{diag}(-1,1)$	

Half-maximal $d = 8$

- $\mathbb{R}^+ \times \underbrace{SL(2) \times SL(2)}_{O(2,2)}$

- Embedding tensor

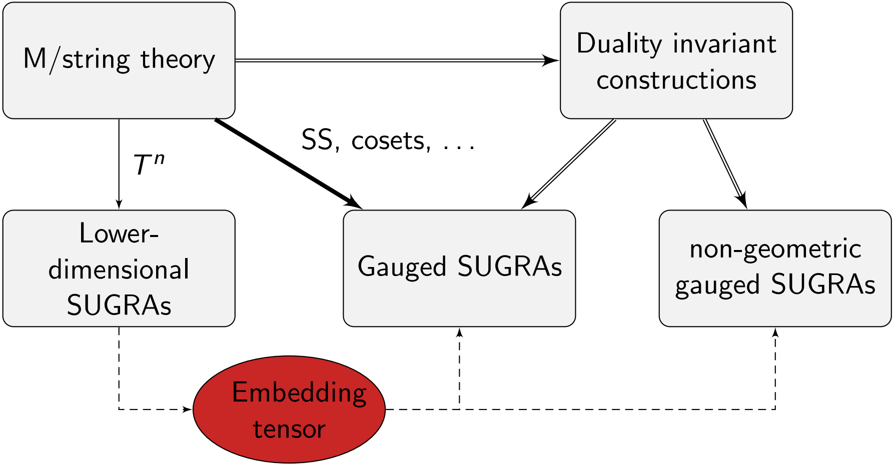
$$\left[\begin{array}{l} a_{\alpha i} \\ b_{\alpha i} \end{array}\right.$$

- Quadratic constraints

$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,a_{\beta j} - b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,b_{\beta j} + b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{ij}\,a_{(\alpha i}\,b_{\beta) j} = 0$$
$$\epsilon^{\alpha\beta}\,a_{\alpha (i}\,b_{\beta j)} = 0$$

ID	$a_{\alpha i}$	$b_{\alpha i}$	gauging
1	$\mathrm{diag}(\cos\alpha, 0)$	$\mathrm{diag}(\sin\alpha, 0)$	$\mathrm{Solv}_2 \times \mathrm{SO}(1,1)$
2	$\mathrm{diag}(1,1)$	$\mathrm{diag}(-1,-1)$	$\mathrm{SL}(2) \times \mathrm{SO}(1,1)$
3	$\mathrm{diag}(1,-1)$	$\mathrm{diag}(-1,1)$	

Higher-dimensional origin of gaugings



Goals

1. Double field theory
   - The basics
   - Strong constraint relaxation and consistency conditions
   - Action fixed by symmetries? [Cho, FM, Jeon, Park]
2. Orbits of non-geometric fluxes [Dibitetto, FM, Marqués, Roest]
   - Scherk-Schwarz compactification of DFT
   - Classification of deformations in $d = 9, 8, 7$
   - Compactification scheme (twist matrices)
3. Stable de Sitter in half-maximal $d = 7$ [Dibitetto, FM, Marqués]
   - Full classification of deformations
   - Structure of vacua and moduli stabilisation
   - First stable dS in half-maximal
4. Conclusions and outlook

Goals

1. Double field theory
   - The basics
   - Strong constraint relaxation and consistency conditions
   - Action fixed by symmetries? [Cho, FM, Jeon, Park]
2. Orbits of non-geometric fluxes [Dibitetto, FM, Marqués, Roest]
   - Scherk-Schwarz compactification of DFT
   - Classification of deformations in $d = 9, 8, 7$
   - Compactification scheme (twist matrices)
3. Stable de Sitter in half-maximal $d = 7$ [Dibitetto, FM, Marqués]
   - Full classification of deformations
   - Structure of vacua and moduli stabilisation
   - First stable dS in half-maximal
4. Conclusions and outlook

Double field theory

Siegel'93 — Hull, Zwiebach'09 — Hohm, Hull, Zwiebach'10

- T-duality invariant effective field theory of string theories

- T-duality group O(D,D): $\eta_{MN} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\quad M, N = 1...2D$

- $X^M = \{\tilde{x}_i,\ x^i\}, \qquad i = 1, \ldots, D$

- NSNS sector

$$\left.\begin{matrix} g_{\mu\nu} \\ B_{\mu\nu} \end{matrix}\right] \quad \rightarrow \qquad \mathcal{H} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix} \in O(D,D)$$

$$e^{-2\phi} \quad \rightarrow \qquad e^{-2d} = \sqrt{g}\, e^{-2\phi}$$

The action

- The action $S_{\mathrm{DFT}} = \int d^{2D}x\, e^{-2d}\mathcal{R}(\mathcal{H}, d)$

$$\mathcal{R}(\mathcal{H}, d) = \tfrac{1}{8}\mathcal{H}^{MN}\partial_M\mathcal{H}^{PQ}\partial_N\mathcal{H}_{PQ} - \tfrac{1}{2}\mathcal{H}^{MN}\partial_N\mathcal{H}^{PQ}\partial_Q\mathcal{H}_{MP} - 2\partial_M d\partial_N\mathcal{H}^{MN} + 4\mathcal{H}^{MN}\partial_M d\partial_N d$$

- Strong constraint (SC)

$$\eta^{MN}\partial_M\partial_N(AB) = 0$$

- For $\tilde{\partial}^i = 0$

$$S_{\mathrm{DFT}}\big|_{\tilde{\partial}^i=0} \propto S_{\mathrm{NSNS}} \propto \int d^Dx\sqrt{g}e^{-2\phi}\left[R - 4(\partial\phi)^2 + \frac{1}{2\cdot 3!}H_{\mu\nu\rho}H^{\mu\nu\rho}\right]$$

Gauge transformations and generalised Lie derivative

- Generalised Lie derivative

$$\hat{\mathcal{L}}_\zeta V^M = L_\zeta V^M + Y^M{}_P{}^N{}_Q \partial_N \zeta^Q V^P$$

where $Y^M{}_P{}^N{}_Q = \delta_Q{}^M \delta_P{}^N - \alpha P^M{}_P{}^N{}_Q + \lambda \delta_P{}^M \delta_Q{}^N$

- For O(D,D) $\quad Y^M{}_P{}^N{}_Q = \eta^{MN}\eta_{PQ}$

- Gauge transformations

$$\delta_\zeta \mathcal{H}_{MN} = \zeta^P \partial_P \mathcal{H}_{MN} + (\partial_M \zeta^P - \partial^P \zeta_M)\mathcal{H}_{PN} + (\partial_N \zeta^P - \partial^P \zeta_N)\mathcal{H}_{MP}$$

$$\delta_\zeta d = \zeta^M \partial_M d - \tfrac{1}{2}\partial_M \zeta^M$$

- Closure of the algebra $\quad [\delta_{\zeta_1}, \delta_{\zeta_2}] = -\delta_{[\zeta_1,\zeta_2]_c}$

$$[\zeta_1, \zeta_2]_c^M \equiv \zeta_1^N \partial_N \zeta_2^M - \frac{1}{2}\,\zeta_{1N}\partial^M \zeta_2^N - (1 \leftrightarrow 2) \qquad C\text{-bracket}$$

- Closure and invariant action upon the SC!!

DFT and gaugings

Geissbühler'11 — Aldazábal *et al.*'11

- Dimensional reduction of DFT: Scherk-Schwarz compactification

$$\mathcal{H}_{MN}(X) = \widehat{\mathcal{H}}_{IJ}(\mathbb{X})\, U^I{}_M(\mathbb{Y}) U^J{}_N(\mathbb{Y})\,, \qquad d(X) = \widehat{d}(\mathbb{X}) + \lambda(\mathbb{Y})$$

- Can we reproduce all the ET configurations upon these reductions? NO
- Problem? Strong constraint is too strong? $\partial^M \partial_M = 0$
  - SS reduction of the SC vs. QC of ET
  - Essential for gauge invariance
  - Essential for closure
  - Consistent relaxation?

Relaxation of the SC

- DFT aimed to find SUGRAs (fermions): $\mathcal{H}_{MN} \to E^{\bar{A}}{}_M$

$$E^{\bar{A}}{}_M = \begin{pmatrix} e_{\bar{a}}{}^i & e_{\bar{a}}{}^j b_{ji} \\ 0 & e^{\bar{a}}{}_i \end{pmatrix} \,, \qquad \mathcal{H}_{MN} = E^{\bar{A}}{}_M S_{\bar{A}\bar{B}} E^{\bar{B}}{}_N$$

$$\mathcal{L}_\xi E^{\bar{A}}{}_M = \xi^P \partial_P E^{\bar{A}}{}_M + \left(\partial_M \xi^P - \partial^P \xi_M\right) E^{\bar{A}}{}_P$$

- Consistency conditions
  - Closure of the algebra
  - Gauge invariant action
  - Jacobi identities

Consistency conditions without SC

Grana,Marqués'12 — Geissbühler *et al.*'13

- Closure and Jacobi $\Leftrightarrow \quad {\Delta_{123}}^M = 0$

$$ {\Delta_{123}}^M = Y^P{}_R{}^Q{}_S \left(2\partial_P \xi^R_{[1}\ \partial_Q \xi^M_{2]}\ \xi^S_3 - \partial_P \xi^R_1\ \xi^S_2\ \partial_Q \xi^M_3 \right) = 0 $$

- Invariant action $S_{\text{rDFT}} = \int d^{2D}x\ e^{-2d} \mathcal{R}_{\text{rDFT}}$

$$ \mathcal{R}_{\text{rDFT}} = \mathcal{F}_{\bar{A}\bar{B}\bar{C}}\ \mathcal{F}_{\bar{D}\bar{E}\bar{F}} \left[\frac{1}{4} S^{\bar{A}\bar{D}} \eta^{\bar{B}\bar{E}} \eta^{\bar{C}\bar{F}} - \frac{1}{12} S^{\bar{A}\bar{D}} S^{\bar{B}\bar{E}} S^{\bar{C}\bar{F}} - \frac{1}{6} \eta^{\bar{A}\bar{D}} \eta^{\bar{B}\bar{E}} \eta^{\bar{C}\bar{F}} \right] + \mathcal{F}_{\bar{A}} \mathcal{F}_{\bar{B}} \left[\eta^{\bar{A}\bar{B}} - S^{\bar{A}\bar{B}} \right] $$

where $\quad \mathcal{F}_{\bar{A}\bar{B}\bar{C}} = E_{\bar{C}M} \mathcal{L}_{E_{\bar{A}}} {E_{\bar{B}}}^M \qquad \mathcal{F}_{\bar{A}} = -e^{2d} \mathcal{L}_{E_{\bar{A}}} e^{-2d}$

Consistency constraints without SC

- Solve ${\Delta_{123}}^M = 0$

- Unique solution found up to now: Scherk-Schwarz-like

$$A^M(X) = \hat{A}^I(\mathbb{X}) {W_I}^M(\mathbb{Y}) \qquad \partial_M \partial^M \hat{A}^I = 0 \qquad \partial_M \hat{A}^I \partial^M \hat{B}^J = 0$$

- Effective fields do not depend on dual coordinates but twist matrices do!!

- Can we reproduce all the ET configurations upon the SS reduction of S_{rDFT}? Next section

- Can we obtain S_{rDFT} from first principles (symmetries)?

- Uniqueness of S_{rDFT}?

Action and symmetries

Cho, FM, Jeon, Park'15

- DFT + SC: action fixed by
  - Generalised diffeo's
  - O(D,D)
  - Double Lorentz
- Relaxed DFT
  - Generalised diffeo's
  - O(D,D)
  - Double Lorentz
  - ...? SUSY?
- Supersymmetric DFT
  - $N = 1$: SUSY invariance completely fixes the action
  - $N = 2$: RR gauge invariance makes SUSY invariance transparent

Goals

1. Double field theory
   - The basics
   - Strong constraint relaxation and consistency conditions
   - Action fixed by symmetries? [Cho, FM, Jeon, Park]

2. Orbits of non-geometric fluxes [Dibitetto, FM, Marqués, Roest]
   - Scherk-Schwarz compactification of DFT
   - Classification of deformations in $d = 9, 8, 7$
   - Compactification scheme (twist matrices)

3. Stable de Sitter in half-maximal $d = 7$ [Dibitetto, FM, Marqués]
   - Full classification of deformations
   - Structure of vacua and moduli stabilisation
   - First stable dS in half-maximal

4. Conclusions and outlook

Relaxed DFT

- SS reduction of rDFT

$$E^{\hat{A}}{}_M(X) = \widehat{E}^{\hat{A}}{}_B(\mathbb{X})\ U^B{}_M(\mathbb{Y}) \ , \qquad d(X) = \widehat{d}(\mathbb{X}) + \lambda(\mathbb{Y})$$

- Structure constants in effective theory

$$f_{ABC} = 3\tilde{\Omega}_{[ABC]} \ , \qquad \tilde{\Omega}_{ABC} = U_A{}^M \partial_M U_B{}^N U_{CN} \ ,$$
$$f_A = \tilde{\Omega}^B{}_{BA} + 2U_A{}^M \partial_M \lambda$$

- Uplift condition (from half-maximal to maximal)

Dibitetto, Guarino, Roest'11

$$f_{ABC} f^{ABC} = 0$$

- (non-) upliftable $\Longleftrightarrow$ (non-) geometric

$$f_{ABC} f^{ABC} = -3\partial_M U^A{}_P \partial^M (U^{-1})^P{}_A - 24\partial_M \lambda \partial^M \lambda + 24 \partial_M \partial^M \lambda = 0$$

Maximal $d = 9$

- $\mathbb{R}^+ \times SL(2)$
- ET

$$\left\{ \theta^i \, , \, \kappa^{jk} \right\} \qquad\qquad \mathbf{2}_{(+3)} \oplus \mathbf{4}_{(-1)}$$

- QC

$$\epsilon_{ij}\, \theta^i\, \kappa^{jk} = 0 \ , \qquad\qquad \mathbf{2}_{(-1)}$$

$$\theta^{(i}\, \kappa^{jk)} = 0 \ . \qquad\qquad \mathbf{4}_{(-1)}$$

ID	θ^i	κ^{ij}	gauging
1		diag(1, 1)	SO(2)
2	(0, 0)	diag(1, −1)	SO(1, 1)
3		diag(1, 0)	$\mathbb{R}^+_\gamma$
4	(1, 0)	diag(0, 0)	$\mathbb{R}^+_\beta$

Maximal $d = 9$

- $\mathbb{R}^+ \times SL(2)$
- ET

$$\left\{\theta^i \,,\, \kappa^{jk}\right\} \qquad\qquad \mathbf{2}_{(+3)} \oplus \mathbf{4}_{(-1)}$$

- QC

$$\epsilon_{ij}\,\theta^i\,\kappa^{jk} = 0 \ , \qquad\qquad \mathbf{2}_{(-1)}$$
$$\theta^{(i}\,\kappa^{jk)} = 0 \ . \qquad\qquad \mathbf{4}_{(-1)}$$

ID	θ^i	κ^{ij}	gauging
1		diag(1, 1)	SO(2)
2	(0, 0)	diag(1, −1)	SO(1, 1)
3		diag(1, 0)	$\mathbb{R}^+_\gamma$
4	(1, 0)	diag(0, 0)	$\mathbb{R}^+_\beta$

Maximal $d = 8$

- $SL(2) \times SL(3)$

- ET

$$\left\{ f_\alpha{}^{(mn)} \, , \, \xi_{\alpha m} \right\} \qquad\qquad (\mathbf{2}, \mathbf{6}') \oplus (\mathbf{2}, \mathbf{3})$$

- QC

$$\begin{aligned} \epsilon^{\alpha\beta} \, \xi_{\alpha p} \xi_{\beta q} &= 0 \, , & & \big(\mathbf{1}, \mathbf{3}'\big) \\ f_{(\alpha}{}^{np} \xi_{\beta) p} &= 0 \, , & & \big(\mathbf{3}, \mathbf{3}'\big) \\ \epsilon^{\alpha\beta} \left(\epsilon_{mqr} f_\alpha{}^{qn} f_\beta{}^{rp} + f_\alpha{}^{np} \xi_{\beta m} \right) &= 0 \, . & & \big(\mathbf{1}, \mathbf{3}'\big) \oplus \big(\mathbf{1}, \mathbf{15}\big) \end{aligned}$$

Maximal $d = 8$

ID	$f_+{}^{mn}$	$f_-{}^{mn}$	ξ_{+m}	ξ_{-m}	gauging
1	$\mathrm{diag}(1,1,1)$				$SO(3)$
2	$\mathrm{diag}(1,1,-1)$				$SO(2,1)$
3	$\mathrm{diag}(1,1,0)$	$\mathrm{diag}(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$ISO(2)$
4	$\mathrm{diag}(1,-1,0)$				$ISO(1,1)$
5	$\mathrm{diag}(1,0,0)$				$CSO(1,0,2)$
6	$\mathrm{diag}(0,0,0)$	$\mathrm{diag}(0,0,0)$	$(1,0,0)$	$(0,0,0)$	$Solv_2 \times Solv_3$
7	$\mathrm{diag}(1,1,0)$				
8	$\mathrm{diag}(1,-1,0)$	$\mathrm{diag}(0,0,0)$	$(0,0,1)$	$(0,0,0)$	$Solv_2 \times Solv_3$
9	$\mathrm{diag}(1,0,0)$				
10	$\mathrm{diag}(1,-1,0)$	$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\frac{2}{9}(0,0,1)$	$(0,0,0)$	$Solv_2 \times SO(2) \ltimes Nil_3(2)$

Maximal $d = 8$

ID	$f_+{}^{mn}$	$f_-{}^{mn}$	ξ_{+m}	ξ_{-m}	gauging
1	$\mathrm{diag}(1,1,1)$	$\mathrm{diag}(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$SO(3)$
2	$\mathrm{diag}(1,1,-1)$				$SO(2,1)$
3	$\mathrm{diag}(1,1,0)$				$ISO(2)$
4	$\mathrm{diag}(1,-1,0)$				$ISO(1,1)$
5	$\mathrm{diag}(1,0,0)$				$CSO(1,0,2)$
6	$\mathrm{diag}(0,0,0)$	$\mathrm{diag}(0,0,0)$	$(1,0,0)$	$(0,0,0)$	$\mathrm{Solv}_2 \times \mathrm{Solv}_3$
7	$\mathrm{diag}(1,1,0)$	$\mathrm{diag}(0,0,0)$	$(0,0,1)$	$(0,0,0)$	$\mathrm{Solv}_2 \times \mathrm{Solv}_3$
8	$\mathrm{diag}(1,-1,0)$				
9	$\mathrm{diag}(1,0,0)$				
10	$\mathrm{diag}(1,-1,0)$	$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\frac{2}{9}(0,0,1)$	$(0,0,0)$	$\mathrm{Solv}_2 \times SO(2) \ltimes \mathrm{Nil}_3(2)$

Half-maximal $d = 10$ and $d = 9$

- $d = 10$ without vector multiplets
  - No freedom to deform

- $d = 9$
  - $\mathbb{R}^+ \times SO(1,1)$
  - Abelian gauging
  - Fully geometric

Half-maximal $d = 8$

- $\mathbb{R}^+ \times \underbrace{SL(2) \times SL(2)}_{O(2,2)}$

- ET

$$a_{\alpha i}$$
$$b_{\alpha i}$$

- QC

$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,a_{\beta j} - b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,b_{\beta j} + b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{ij}\,a_{(\alpha i}\,b_{\beta)j} = 0$$
$$\epsilon^{\alpha\beta}\,a_{\alpha(i}\,b_{\beta j)} = 0$$

ID	$a_{\alpha i}$	$b_{\alpha i}$	gauging
1	$\mathrm{diag}(\cos\alpha, 0)$	$\mathrm{diag}(\sin\alpha, 0)$	$\mathrm{Solv}_2 \times \mathrm{SO}(1,1)$
2	$\mathrm{diag}(1, 1)$	$\mathrm{diag}(-1, -1)$	$\mathrm{SL}(2) \times \mathrm{SO}(1,1)$
3	$\mathrm{diag}(1, -1)$	$\mathrm{diag}(-1, 1)$	

Half-maximal $d = 8$

- $\mathbb{R}^+ \times \underbrace{SL(2) \times SL(2)}_{O(2,2)}$

- ET

$$a_{\alpha i}$$
$$b_{\alpha i}$$

- QC

$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,a_{\beta j} - b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,b_{\beta j} + b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{ij}\,a_{(\alpha i}\,b_{\beta)j} = 0$$
$$\epsilon^{\alpha\beta}\,a_{\alpha(i}\,b_{\beta j)} = 0$$

ID	$a_{\alpha i}$	$b_{\alpha i}$	gauging
1	$\text{diag}(\cos\alpha, 0)$	$\text{diag}(\sin\alpha, 0)$	$\text{Solv}_2 \times \text{SO}(1,1)$
2	$\text{diag}(1,1)$	$\text{diag}(-1,-1)$	$\text{SL}(2) \times \text{SO}(1,1)$
3	$\text{diag}(1,-1)$	$\text{diag}(-1,1)$	

Half-maximal $d = 8$

- $\mathbb{R}^+ \times \underbrace{SL(2) \times SL(2)}_{O(2,2)}$

- ET

$$a_{\alpha i}$$
$$b_{\alpha i}$$

- QC

$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,a_{\beta j} - b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{\alpha\beta}\,\epsilon^{ij}\,(a_{\alpha i}\,b_{\beta j} + b_{\alpha i}\,b_{\beta j}) = 0$$
$$\epsilon^{ij}\,a_{(\alpha i}\,b_{\beta)j} = 0$$
$$\epsilon^{\alpha\beta}\,a_{\alpha(i}\,b_{\beta j)} = 0$$

ID	$a_{\alpha i}$	$b_{\alpha i}$	gauging
1	$\text{diag}(\cos\alpha, 0)$	$\text{diag}(\sin\alpha, 0)$	$\text{Solv}_2 \times \text{SO}(1,1)$
2	$\text{diag}(1,1)$	$\text{diag}(-1,-1)$	$\text{SL}(2) \times \text{SO}(1,1)$
3	$\text{diag}(1,-1)$	$\text{diag}(-1,1)$	

Half-maximal $d = 8$

- Twist matrix

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cosh(m\, y^1 + n\, \tilde{y}_1) & 0 & \sinh(m\, y^1 + n\, \tilde{y}_1) \\ 0 & 0 & 1 & 0 \\ 0 & \sinh(m\, y^1 + n\, \tilde{y}_1) & 0 & \cosh(m\, y^1 + n\, \tilde{y}_1) \end{pmatrix}$$

- ET & fluxes: $f_{ABC} = (X_{\alpha i})_{\beta j}{}^{\gamma k} (G_A)^{\alpha i} (G_B)^{\beta j} (G_C)_{\gamma k}$

$$a_{\alpha i} = -b_{\alpha i} = \text{diag}\left(-\frac{m+n}{2\sqrt{2}}, \ \frac{m-n}{2\sqrt{2}}\right)$$

  - Orbit 2: $m = 0, \ n = -2\sqrt{2}$
  - Orbit 3: $m = -2\sqrt{2}, \ n = 0$

- SC $(m+n)(m-n) = 0$
- $f_{ABC} f^{ABC} \neq 0 \quad \Rightarrow \quad$ non upliftable to maximal!

Half-maximal $d = 8$

- Twist matrix

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cosh(m\, y^1 + n\, \tilde{y}_1) & 0 & \sinh(m\, y^1 + n\, \tilde{y}_1) \\ 0 & 0 & 1 & 0 \\ 0 & \sinh(m\, y^1 + n\, \tilde{y}_1) & 0 & \cosh(m\, y^1 + n\, \tilde{y}_1) \end{pmatrix}$$

- ET & fluxes: $f_{ABC} = (X_{\alpha i})_{\beta j}{}^{\gamma k} (G_A)^{\alpha i} (G_B)^{\beta j} (G_C)_{\gamma k}$

$$a_{\alpha i} = -b_{\alpha i} = \mathrm{diag}\left(-\frac{m+n}{2\sqrt{2}},\ \frac{m-n}{2\sqrt{2}}\right)$$

  - Orbit 2: $m = 0,\ n = -2\sqrt{2}$
  - Orbit 3: $m = -2\sqrt{2},\ n = 0$
- SC $\qquad \cancel{(m+n)(m-n) = 0}$
- $f_{ABC} f^{ABC} \neq 0 \quad \Rightarrow \quad$ non upliftable to maximal!

Half-maximal $d = 7$

- $\mathbb{R}^+ \times \underbrace{SL(4)}_{SO(3,3)}$
- ET truncated

$$\left[\begin{array}{c} Z^{MN,P} \\ Y_{(MN)} \end{array}\right. \longrightarrow \begin{array}{ccccccc} \theta\,, & & \xi_{mn}, & & Q_{mn}, & & \tilde{Q}^{mn} \\ \mathbf{1} & \oplus & \mathbf{6} & \oplus & \mathbf{10} & \oplus & \mathbf{10'} \end{array}$$

- QC

$$\theta \xi_{mn} = 0 \quad (\mathbf{6})$$

$$\left(\tilde{Q}^{mp} + \xi^{mp}\right) Q_{pq} - \frac{1}{4}\left(\tilde{Q}^{np} Q_{np}\right) \delta^m_q = 0 \quad (\mathbf{15})$$

$$Q_{mp}\xi^{pn} + \xi_{mp}\tilde{Q}^{pn} = 0 \quad (\mathbf{15})$$

$$\epsilon^{mnpq}\xi_{mn}\xi_{pq} = 0 \quad (\mathbf{1})$$

Half-maximal $d = 7$

- Truncation $\quad \cancel{\theta}\ ,\ \cancel{\xi_{[mn]}}\ ,\ Q_{(mn)}\ ,\ \tilde{Q}^{(mn)} \qquad \mathbf{10} \oplus \mathbf{10}'$
- $\{f_{abc},\, f_{ab}{}^{c},\, f_{a}{}^{bc},\, f^{abc}\} \equiv \{H_{abc},\, \omega_{ab}{}^{c},\, Q_{a}{}^{bc},\, R^{abc}\}$
- $\mathbf{10}_{\text{SD}} \oplus \mathbf{10}_{\text{ASD}}$ of $SO(3,3) \quad \longleftrightarrow \quad \mathbf{10} \oplus \mathbf{10}'$ of $SL(4)$
- Fluxes & ET: $\qquad f_{ABC} = (X_{mn})_{pq}{}^{rs} (G_A)^{mn} (G_B)^{pq} (G_C)_{rs}$
- Dictionary

$$\begin{aligned} Q &= \text{diag}\left(H_{123},\, Q_1{}^{23},\, Q_2{}^{31},\, Q_3{}^{12}\right) \\ \tilde{Q} &= \text{diag}\left(R^{123},\, \omega_{23}{}^{1},\, \omega_{31}{}^{2},\, \omega_{12}{}^{3}\right) \end{aligned}$$

- Surviving QC

$$\tilde{Q}^{mp}\, Q_{pn} - \frac{1}{4}\left(\tilde{Q}^{pq}\, Q_{pq}\right)\, \delta^m_n \;=\; 0$$

Half-maximal $d = 7$

ID	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1,1,1,1)	diag(1,1,1,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4) , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(3) , & \alpha = \frac{\pi}{4} \end{cases}$
2	diag(1,1,1,−1)	diag(1,1,1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3,1)
3	diag(1,1,−1,−1)	diag(1,1,−1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(2{,}2) , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(2,1) , & \alpha = \frac{\pi}{4} \end{cases}$
4	diag(1,1,1,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(3)
5	diag(1,1,−1,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(2,1)
6	diag(1,1,0,0)	diag(0,0,1,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(2,0,2) , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_1 \quad (\mathrm{Solv}_6) , & \alpha = \frac{\pi}{4} \end{cases}$
7	diag(1,1,0,0)	diag(0,0,1,−1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\begin{cases} \mathrm{CSO}(2,0,2) , & \lvert\alpha\rvert < \frac{\pi}{4} \\ \mathrm{CSO}(1,1,2) , & \lvert\alpha\rvert > \frac{\pi}{4} \\ \mathfrak{g}_0 \quad (\mathrm{Solv}_6) , & \lvert\alpha\rvert = \frac{\pi}{4} \end{cases}$
8	diag(1,1,0,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_1 \quad (\mathrm{Solv}_6)$
9	diag(1,−1,0,0)	diag(0,0,1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(1,1,2) , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_2 \quad (\mathrm{Solv}_6) , & \alpha = \frac{\pi}{4} \end{cases}$
10	diag(1,−1,0,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_2 \quad (\mathrm{Solv}_6)$
11	diag(1,0,0,0)	diag(0,0,0,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathfrak{l} \quad (\mathrm{Nil}_6(3)) , & \alpha \neq 0 \\ \mathrm{CSO}(1,0,3) , & \alpha = 0 \end{cases}$

Half-maximal $d = 7$

ID	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1, 1, 1, 1)	diag(1, 1, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4) \,, & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(3) \,, & \alpha = \frac{\pi}{4} \end{cases}$
2	diag(1, 1, 1, −1)	diag(1, 1, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3, 1)
3	diag(1, 1, −1, −1)	diag(1, 1, −1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(2,2) \,, & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(2,1) \,, & \alpha = \frac{\pi}{4} \end{cases}$
4	diag(1, 1, 1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(3)
5	diag(1, 1, −1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(2, 1)
6	diag(1, 1, 0, 0)	diag(0, 0, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(2,0,2) \,, & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_1 \quad (\mathrm{Solv}_6) \,, & \alpha = \frac{\pi}{4} \end{cases}$
7	diag(1, 1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\begin{cases} \mathrm{CSO}(2,0,2) \,, & \lvert\alpha\rvert < \frac{\pi}{4} \\ \mathrm{CSO}(1,1,2) \,, & \lvert\alpha\rvert > \frac{\pi}{4} \\ \mathfrak{g}_0 \quad (\mathrm{Solv}_6) \,, & \lvert\alpha\rvert = \frac{\pi}{4} \end{cases}$
8	diag(1, 1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_1 \quad (\mathrm{Solv}_6)$
9	diag(1, −1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(1,1,2) \,, & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_2 \quad (\mathrm{Solv}_6) \,, & \alpha = \frac{\pi}{4} \end{cases}$
10	diag(1, −1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_2 \quad (\mathrm{Solv}_6)$
11	diag(1, 0, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathfrak{l} \quad (\mathrm{Nil}_6(3)) \,, & \alpha \neq 0 \\ \mathrm{CSO}(1,0,3) \,, & \alpha = 0 \end{cases}$

Half-maximal $d = 7$

ID	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1,1,1,1)	diag(1,1,1,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(3)\ , & \alpha = \frac{\pi}{4} \end{cases}$
2	diag(1,1,1,−1)	diag(1,1,1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3,1)
3	diag(1,1,−1,−1)	diag(1,1,−1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(2,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(2,1)\ , & \alpha = \frac{\pi}{4} \end{cases}$
4	diag(1,1,1,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(3)
5	diag(1,1,−1,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(2,1)
6	diag(1,1,0,0)	diag(0,0,1,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_1\ \ (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
7	diag(1,1,0,0)	diag(0,0,1,−1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \lvert\alpha\rvert < \frac{\pi}{4} \\ \mathrm{CSO}(1,1,2)\ , & \lvert\alpha\rvert > \frac{\pi}{4} \\ \mathfrak{g}_0\ \ (\mathrm{Solv}_6)\ , & \lvert\alpha\rvert = \frac{\pi}{4} \end{cases}$
8	diag(1,1,0,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_1\ \ (\mathrm{Solv}_6)$
9	diag(1,−1,0,0)	diag(0,0,1,−1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(1,1,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_2\ \ (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
10	diag(1,−1,0,0)	diag(0,0,0,1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_2\ \ (\mathrm{Solv}_6)$
11	diag(1,0,0,0)	diag(0,0,0,1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathfrak{l}\ \ (\mathrm{Nil}_6(3))\ , & \alpha \neq 0 \\ \mathrm{CSO}(1,0,3)\ , & \alpha = 0 \end{cases}$

Half-maximal $d = 7$

ID	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1, 1, 1, 1)	diag(1, 1, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(3)\ , & \alpha = \frac{\pi}{4} \end{cases}$
2	diag(1, 1, 1, −1)	diag(1, 1, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3, 1)
3	diag(1, 1, −1, −1)	diag(1, 1, −1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(2{,}2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(2,1)\ , & \alpha = \frac{\pi}{4} \end{cases}$
4	diag(1, 1, 1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(3)
5	diag(1, 1, −1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(2, 1)
6	diag(1, 1, 0, 0)	diag(0, 0, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_1 \quad (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
7	diag(1, 1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \lvert\alpha\rvert < \frac{\pi}{4} \\ \mathrm{CSO}(1,1,2)\ , & \lvert\alpha\rvert > \frac{\pi}{4} \\ \mathfrak{g}_0 \quad (\mathrm{Solv}_6)\ , & \lvert\alpha\rvert = \frac{\pi}{4} \end{cases}$
8	diag(1, 1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_1 \quad (\mathrm{Solv}_6)$
9	diag(1, −1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(1,1,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_2 \quad (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
10	diag(1, −1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_2 \quad (\mathrm{Solv}_6)$
11	diag(1, 0, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathfrak{l} \quad (\mathrm{Nil}_6(3))\ , & \alpha \neq 0 \\ \mathrm{CSO}(1,0,3)\ , & \alpha = 0 \end{cases}$

Half-maximal $d = 7$

ID	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	range of α	gauging
1	diag(1, 1, 1, 1)	diag(1, 1, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(4)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(3)\ , & \alpha = \frac{\pi}{4} \end{cases}$
2	diag(1, 1, 1, −1)	diag(1, 1, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	SO(3, 1)
3	diag(1, 1, −1, −1)	diag(1, 1, −1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{SO}(2{,}2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathrm{SO}(2,1)\ , & \alpha = \frac{\pi}{4} \end{cases}$
4	diag(1, 1, 1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(3)
5	diag(1, 1, −1, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	ISO(2, 1)
6	diag(1, 1, 0, 0)	diag(0, 0, 1, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_1\ \ (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
7	diag(1, 1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\begin{cases} \mathrm{CSO}(2,0,2)\ , & \lvert\alpha\rvert < \frac{\pi}{4} \\ \mathrm{CSO}(1,1,2)\ , & \lvert\alpha\rvert > \frac{\pi}{4} \\ \mathfrak{g}_0\ \ (\mathrm{Solv}_6)\ , & \lvert\alpha\rvert = \frac{\pi}{4} \end{cases}$
8	diag(1, 1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_1\ \ (\mathrm{Solv}_6)$
9	diag(1, −1, 0, 0)	diag(0, 0, 1, −1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathrm{CSO}(1,1,2)\ , & \alpha \neq \frac{\pi}{4} \\ \mathfrak{f}_2\ \ (\mathrm{Solv}_6)\ , & \alpha = \frac{\pi}{4} \end{cases}$
10	diag(1, −1, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$	$\mathfrak{h}_2\ \ (\mathrm{Solv}_6)$
11	diag(1, 0, 0, 0)	diag(0, 0, 0, 1)	$-\frac{\pi}{4} < \alpha \leq \frac{\pi}{4}$	$\begin{cases} \mathfrak{l}\ \ (\mathrm{Nil}_6(3))\ , & \alpha \neq 0 \\ \mathrm{CSO}(1,0,3)\ , & \alpha = 0 \end{cases}$

Half-maximal $d = 7$

- Orbits 4, 5, 8, 10, 11: geometric $U(y) \in GL(3)$
- Orbits 1_0, 2_0, 3_0, 7, 9: $U(y) \in O(3,3)$

Salam, Sezgin'83 — Cvetic *et al.*'00 ...

- Orbits 1, 2, 3, 6: $U(y, \tilde{y}) \in O(3,3)$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & 0 & B \\ 0 & 0 & 1 & 0 \\ 0 & C & 0 & D \end{pmatrix} , \quad U_4 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \exp\left(t_{IJ}\phi^{IJ}\right) \in O(2,2)$$

$$[t_{IJ}]_K{}^L = \delta^L_{[I}\eta_{J]K} \qquad\qquad \phi^{IJ} = \alpha^{IJ} y^1 + \beta^{IJ} \tilde{y}^1$$

- SC [✓ 1_0, 2_0, 3_0, 4, 5, 7, 8, 9, 10, 11, / × 1, 2, 3 ($\alpha \neq 0$), 6]
- Uplift condition [✓ 1_0, 2_0, 3_0, 4, 5, 6, 7, 8, 9, 10, 11 / × 1, 2, 3 ($\alpha \neq 0$)]

Half-maximal $d = 7$

- Orbits 4, 5, 8, 10, 11: geometric $U(y) \in GL(3)$
- Orbits 1_0, 2_0, 3_0, 7, 9: $U(y) \in O(3,3)$

Salam, Sezgin'83 — Cvetic *et al.*'00 ...

- Orbits 1, 2, 3, 6: $U(y, \tilde{y}) \in O(3,3)$

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & A & 0 & B \\ 0 & 0 & 1 & 0 \\ 0 & C & 0 & D \end{pmatrix} , \quad U_4 = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \exp\left(t_{IJ}\phi^{IJ}\right) \in O(2,2)$$

$$[t_{IJ}]_K{}^L = \delta^L_{[I}\eta_{J]K} \qquad\qquad \phi^{IJ} = \alpha^{IJ}y^1 + \beta^{IJ}\tilde{y}^1$$

- SC $\left[\begin{array}{l} \checkmark\ 1_0,\ 2_0,\ 3_0,\ 4,\ 5,\ 7,\ 8,\ 9,\ 10,\ 11,\ 6 \\ \times\ 1,\ 2,\ 3\ (\alpha \neq 0),\ 6 \end{array} \right.$
- Uplift condition $\left[\begin{array}{l} \checkmark\ 1_0,\ 2_0,\ 3_0,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11 \\ \times\ 1,\ 2,\ 3\ (\alpha \neq 0) \end{array} \right.$

Summary of results

- U-duality orbits

d	9	8
$\sharp$ orbits	4	10

- T-duality orbits

d	10	9	8	7
$\sharp$ orbits	0	1	$3+2$	$11+3$

- Twist degeneracy in twist matrices
- Upliftability (to maximal) condition $f_{ABC}f^{ABC}=0$
- Locally non-geometric fluxes given by $U(y)$ matrices

Goals

1. Double field theory
   - The basics
   - Strong constraint relaxation and consistency conditions
   - Action fixed by symmetries? [Cho, FM, Jeon, Park]

2. Orbits of non-geometric fluxes [Dibitetto, FM, Marqués, Roest]
   - Scherk-Schwarz compactification of DFT
   - Classification of deformations in $d = 9, 8, 7$
   - Compactification scheme (twist matrices)

3. Stable de Sitter in half-maximal $d = 7$ [Dibitetto, FM, Marqués]
   - Full classification of deformations
   - Structure of vacua and moduli stabilisation
   - First stable dS in half-maximal

4. Conclusions and outlook

Half-maximal $d = 7$ SUGRA: field content

- $\mathbb{R}^+ \times SL(4)$

- Field content: $\{e_\mu{}^a,\ A_\mu{}^{[mn]},\ B_{\mu\nu},\ \Sigma,\ \mathcal{V}_m{}^{\underline{m}},\ \psi_{\mu\alpha},\ \chi_\alpha,\ \lambda^{\alpha\hat{\alpha}\hat{\beta}}\}$

- Scalar coset

$$M_{mn} = \mathcal{V}_m{}^{\underline{m}} \mathcal{V}_n{}^{\underline{n}} \delta_{\underline{mn}} \in \frac{\mathrm{SL}(4)}{\mathrm{SO}(4)}$$

$$\mathcal{V}_m{}^{\underline{m}} = \begin{pmatrix} e^{\phi_1/2} & \chi_1\, e^{\phi_2/2} & \chi_2\, e^{\phi_3/2} & \chi_4\, e^{-(\phi_1+\phi_2+\phi_3)/2} \\ 0 & e^{\phi_2/2} & \chi_3\, e^{\phi_3/2} & \chi_5\, e^{-(\phi_1+\phi_2+\phi_3)/2} \\ 0 & 0 & e^{\phi_3/2} & \chi_6\, e^{-(\phi_1+\phi_2+\phi_3)/2} \\ 0 & 0 & 0 & e^{-(\phi_1+\phi_2+\phi_3)/2} \end{pmatrix}$$

Half-maximal $d = 7$ SUGRA: scalar potential

- Embedding tensor

$$\Theta \;\in\; \underbrace{\mathbf{1}_{(-4)}}_{\theta} \;\oplus\; \underbrace{\mathbf{10}'_{(+1)}}_{Q_{(mn)}} \;\oplus\; \underbrace{\mathbf{10}_{(+1)}}_{\tilde{Q}^{(mn)}} \;\oplus\; \underbrace{\mathbf{6}_{(+1)}}_{\xi_{[mn]}}$$

- Gauge generators

$$(X_{mn})_{pq}{}^{rs} \;=\; \frac{1}{2}\,\delta^{[r}_{[m}\,Q_{n][p}\,\delta^{s]}_{q]} \;+\; \frac{1}{4}\,\epsilon_{tmn[p}\,(\tilde{Q}+\xi)^{t[r}\,\delta^{s]}_{q]}$$

- Previous work: $\mathbf{10} \oplus \mathbf{10}'$ Dibitetto,FM,Marqués,Roest'12
- Quadratic constraints

$$\begin{aligned}
\left(\tilde{Q}^{mp} + \xi^{mp}\right) Q_{pn} - \tfrac{1}{4}\left(\tilde{Q}^{pq}\,Q_{pq}\right)\delta^{m}_{n} &= 0\\
Q_{mp}\,\xi^{pn} + \xi_{mp}\,\tilde{Q}^{pn} &= 0\\
\xi_{mn}\,\xi^{mn} &= 0\\
\theta\,\xi_{mn} &= 0
\end{aligned}$$

Half-maximal $d = 7$ SUGRA: scalar potential

- Scalar potential

$$V = \frac{1}{4} Q_{mn} Q_{pq} \Sigma^{-2} \left(2 M^{mp} M^{nq} - M^{mn} M^{pq}\right) + \frac{1}{4} \tilde{Q}^{mn} \tilde{Q}^{pq} \Sigma^{-2} \left(2 M_{mp} M_{nq} - M_{mn} M_{pq}\right) + Q_{mn} \tilde{Q}^{mn} \Sigma^{-2} + \theta^2 \Sigma^8 - \theta \left(Q_{mn} M^{mn} - \tilde{Q}^{mn} M_{mn}\right) \Sigma^3 + \frac{3}{2} \xi_{mn} \xi_{pq} \Sigma^{-2} M^{mp} M^{nq}$$

- In terms of positive quantities

$$V = g^2 \left(- \frac{3}{10} |A_1|^2 + \frac{4}{5} |A_2|^2 + \frac{1}{2} |A_3|^2 \right)$$

where

$$e^{-1} \mathcal{L}_{\text{f. mass}} \supset g \left({A_1}^{\alpha\beta} \bar{\psi}_{\mu\alpha} \gamma^{\mu\nu} \psi_{\nu\beta} \oplus {A_2}^{\alpha\beta} \bar{\psi}_{\mu\alpha} \gamma^{\mu} \chi_{\beta} \oplus {A_3}_{\hat{\alpha}\hat{\beta}\beta}{}^{\alpha} \bar{\psi}_{\mu\alpha} \gamma^{\mu} \lambda^{\beta\hat{\alpha}\hat{\beta}} \right)$$

Orbit classification of deformations ($\theta\,\xi_{mn} = 0$)

- branch 1 ($\theta = 0$): $\mathbf{6} \oplus \mathbf{10} \oplus \mathbf{10}'$ Dibitetto,FM,Marqués'15

ID	ξ_{mn}	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	gauging
1	0_4	$\mathbb{1}_4$	$\mathbb{1}_4$	$\mathrm{SO}(4)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathrm{SO}(3)\,,\quad \alpha = \frac{\pi}{4}$
2		$\mathrm{diag}(1,1,1,-1)$	$\mathrm{diag}(1,1,1,-1)$	$\mathrm{SO}(3,1)$
3		$\mathrm{diag}(1,1,-1,-1)$	$\mathrm{diag}(1,1,-1,-1)$	$\mathrm{SO}(2,2)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathrm{SO}(2,1)\,,\quad \alpha = \frac{\pi}{4}$
4	0_4	$\mathrm{diag}(1,1,1,0)$	$\mathrm{diag}(0,0,0,1)$	$\mathrm{CSO}(3,0,1)$
5		$\mathrm{diag}(1,1,-1,0)$		$\mathrm{CSO}(2,1,1)$
6	$\xi_0 \begin{pmatrix} \epsilon_2 & \\ & 0_2 \end{pmatrix}$	$\mathrm{diag}(1,1,0,0)$	$\mathrm{diag}(0,0,1,1)$	$\mathrm{CSO}(2,0,2)\,,\quad \lvert\xi_0\rvert < 1$ $\mathfrak{f}_1\quad (\mathrm{Solv}_6)^*\,,\quad \lvert\xi_0\rvert = 1$
7			$\mathrm{diag}(0,0,1,-1)$	$\mathrm{CSO}(2,0,2)\,,\quad \lvert\xi_0\rvert < \sqrt{\cos(2\alpha)}$ $\mathrm{CSO}(1,1,2)\,,\quad \lvert\xi_0\rvert > \sqrt{\cos(2\alpha)}$ $\mathfrak{g}_0\quad (\mathrm{Solv}_6)^*\,,\quad \lvert\xi_0\rvert = \sqrt{\cos(2\alpha)}$
8			$\mathrm{diag}(0,0,0,1)$	$\mathfrak{h}_1\quad (\mathrm{Solv}_6)^*$
9	$\xi_0 \begin{pmatrix} \epsilon_2 & \\ & 0_2 \end{pmatrix}$	$\mathrm{diag}(1,-1,0,0)$	$\mathrm{diag}(0,0,1,1)$	$\mathfrak{f}_2\quad (\mathrm{Solv}_6)^*$
10			$\mathrm{diag}(0,0,1,-1)$	$\mathrm{CSO}(1,1,2)$
11			$\mathrm{diag}(0,0,0,1)$	$\mathfrak{h}_2\quad (\mathrm{Solv}_6)^*$
12	$\xi_0 \begin{pmatrix} \epsilon_2 & \\ & 0_2 \end{pmatrix}$	$\mathrm{diag}(1,0,0,0)$	$\mathrm{diag}(0,0,0,1)$	$\mathfrak{l}\quad (\mathrm{Nil}_6(3))^*\,,\quad \xi_0 \neq 0$ $\mathrm{CSO}(1,0,3)\,,\quad \xi_0 = 0$
13	$\xi_0 \begin{pmatrix} \epsilon_2 & \\ & 0_2 \end{pmatrix}$	0_4	0_4	$\left(\mathbb{R}^+ \ltimes (\mathbb{R}^+)^3\right) \times \mathrm{U}(1)^2$

Orbit classification of deformations ($\theta\,\xi_{mn} = 0$)

- branch 2 ($\xi_{mn} = 0$): $\mathbf{1} \oplus \mathbf{10} \oplus \mathbf{10}'$ Dibitetto,FM,Marqués'15

ID	θ	$Q_{mn}/\cos\alpha$	$\tilde{Q}^{mn}/\sin\alpha$	gauging
1	κ	$\mathbb{1}_4$	$\mathbb{1}_4$	$\mathrm{SO}(4)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathrm{SO}(3)\,,\quad \alpha = \frac{\pi}{4}$
2		$\mathrm{diag}(1,1,1,-1)$	$\mathrm{diag}(1,1,1,-1)$	$\mathrm{SO}(3,1)$
3		$\mathrm{diag}(1,1,-1,-1)$	$\mathrm{diag}(1,1,-1,-1)$	$\mathrm{SO}(2,2)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathrm{SO}(2,1)\,,\quad \alpha = \frac{\pi}{4}$
4	κ	$\mathrm{diag}(1,1,1,0)$	$\mathrm{diag}(0,0,0,1)$	$\mathrm{CSO}(3,0,1)$
5		$\mathrm{diag}(1,1,-1,0)$		$\mathrm{CSO}(2,1,1)$
6	κ	$\mathrm{diag}(1,1,0,0)$	$\mathrm{diag}(0,0,1,1)$	$\mathrm{CSO}(2,0,2)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathfrak{f}_1\quad (\mathrm{Solv}_6)^*\,,\quad \alpha = \frac{\pi}{4}$
7	κ		$\mathrm{diag}(0,0,1,-1)$	$\mathrm{CSO}(2,0,2)\,,\quad \lvert\alpha\rvert < \frac{\pi}{4}$ $\mathrm{CSO}(1,1,2)\,,\quad \lvert\alpha\rvert > \frac{\pi}{4}$ $\mathfrak{g}_0\quad (\mathrm{Solv}_6)^*\,,\quad \lvert\alpha\rvert = \frac{\pi}{4}$
8			$\mathrm{diag}(0,0,0,1)$	$\mathfrak{h}_1\quad (\mathrm{Solv}_6)^*$
9	κ	$\mathrm{diag}(1,-1,0,0)$	$\mathrm{diag}(0,0,1,-1)$	$\mathrm{CSO}(1,1,2)\,,\quad \alpha \neq \frac{\pi}{4}$ $\mathfrak{f}_2\quad (\mathrm{Solv}_6)^*\,,\quad \alpha = \frac{\pi}{4}$
10			$\mathrm{diag}(0,0,0,1)$	$\mathfrak{h}_2\quad (\mathrm{Solv}_6)^*$
11	κ	$\mathrm{diag}(1,0,0,0)$	$\mathrm{diag}(0,0,0,1)$	$\mathfrak{l}\quad (\mathrm{Nil}_6(3))^*\,,\quad \alpha \neq 0$ $\mathrm{CSO}(1,0,3)\,,\quad \alpha = 0$

Critical points

- Equations

  - scalar potential

  - eom's for the scalar fields

  - masses

$$\left(m^2\right)_I{}^J \;\equiv\; \frac{1}{|V|}\, K^{JK}\, \partial_K \partial_I V \ , \quad \mathcal{L}_{\rm kin} \;=\; -\frac{1}{2}\, K_{IJ}\, \left(\partial \Phi^I\right)\, \left(\partial \Phi^J\right) \ .$$

- "Going to the origin" method

Dibitetto, Guarino, Roest'11 — Dall'Agata, Inverso'11

$$V(\Theta, \Phi) = V(g \star \Theta, g \star \Phi) \ , \qquad \forall g \in \mathbb{R}^+ \times \mathrm{SL}(4)$$

Critical points: branch 1 ($\theta = 0$)

- No-go argument for $\Lambda \neq 0$

$$V = \Sigma^{-2} V_0(M_{mn})$$

- Run-away direction: $\text{eom}(\Sigma) = 0 \quad \Rightarrow \quad \Lambda = 0$

- Analogous to the SL(2) dilaton for purely electric gaugings in $N = 4$ $D = 4$

Critical points: branch 2 ($\xi_{mn} = 0$)

- Non-semisimple gaugings: no-scale Mkw and AdS
- Semisimple gaugins in the $\mathbf{10}'$ ($\tilde{Q}^{mn} = 0$)
  - $\theta \tilde{Q}^{mn} = 0$ — Samtleben, Weidner'05
  - upliftable to maximal $d = 7$
- Semisimple gaugings in the $\mathbf{10} \oplus \mathbf{10}'$: SO(4)

ID	θ	Q_{mn}	$\tilde{Q}^{mn}$	orbit	V_0	mass spectrum
1	$\frac{1-\lambda}{2}$	$\mathbb{1}_4$	$\lambda\, \mathbb{1}_4$	1	$-\frac{15}{4}(1-\lambda)^2$	$-\frac{8}{15}$ $(\times 10)$
2	$\frac{1-\lambda}{4}$	$\mathrm{diag}(1,1,1,\lambda)$	$\mathrm{diag}(\lambda,\lambda,\lambda,1)$	1 - 4 - 2	$-\frac{15}{16}(1-\lambda)^2$	0 $(\times 3)$ $-\frac{8}{15}$ $(\times 1)$ $\frac{16}{15}$ $(\times 5)$ $\frac{8}{3}$ $(\times 1)$
3	$1-\lambda$	$\mathbb{1}_4$	$\lambda\, \mathbb{1}_4$	1	$-5(1-\lambda)^2$	$-\frac{4}{5}$ $(\times 9)$ $\frac{4}{5}$ $(\times 1)$
4	$\frac{1-\lambda}{2}$	$\mathrm{diag}(1,1,1,\lambda)$	$\mathrm{diag}(\lambda,\lambda,\lambda,1)$	1 - 4 - 2	$-\frac{5}{4}(1-\lambda)^2$	0 $(\times 8)$ $\frac{4}{5}$ $(\times 1)$ $\frac{12}{15}$ $(\times 1)$
5	0	$\mathrm{diag}(1,1,\lambda,\lambda)$	$\mathrm{diag}(\lambda,\lambda,1,1)$	1 - 6 - 3	0	0 $(\times 6)$ $4(1-\lambda)^2$ $(\times 4)$

Semisimple gaugings: SO(3,1)

- Two continuous branches of solutions ($\pm$)

$$Q_{\pm} = \mathrm{diag}(1, \lambda, \lambda, \lambda)\,, \quad \tilde{Q}_{\pm} = f_{\pm}(\lambda)\mathrm{diag}(\lambda, 1, 1, 1)\,, \quad \theta_{\pm} = g_{\pm}(\lambda)$$

where

$$f_{\pm}(\lambda) \equiv \frac{-7+22\lambda-7\lambda^2 \pm (1-\lambda)\sqrt{49-82\lambda+49\lambda^2}}{8\,(2-\lambda)}$$

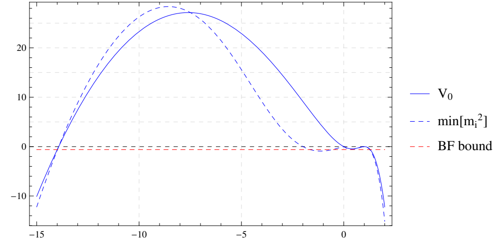
$$g_{\pm}(\lambda) \equiv \left(\frac{1}{1-\lambda} + \frac{15}{8+8\lambda \pm \sqrt{49-82\lambda+49\lambda^2}}\right)^{-1}$$

- Stable dS window

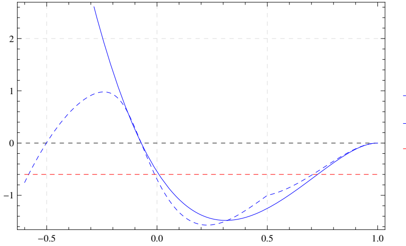
$$-7-4\sqrt{3} < \lambda < \mu_+ \qquad \mu_- < \lambda < -7+4\sqrt{3}$$

where $\mu_{\pm} = \frac{1}{56} - 11 - 3\sqrt{385} \pm \sqrt{450 + 66\sqrt{385}})$

Stable dS in SO(3,1) gauging



$-7 - 4\sqrt{3} < \lambda < \mu_+$



$\mu_- < \lambda < -7 + 4\sqrt{3}$

- Example in branch $(+)$: $\lambda = -3$
  - Cosmological constant $V_0 = \frac{16}{5}(52 - 7\sqrt{46})$
  - Mass spectrum

$$0(\times 3) \quad , \quad \frac{28+\sqrt{46}}{15} \quad (\times 5) \quad , \quad \frac{1}{90}\left(212 - 13\sqrt{46} \pm \sqrt{61310 - 7504\sqrt{46}}\right) \quad (\times 1)$$

SO(3,1) fluxes in $N = 4$ $D = 4$

- SO(3,1) 7-D configuration into 4-D ET: $\{f_{a[\mathbb{MNP}]},\ \ \xi_{a\mathbb{M}}\}$

$$f_{+ABC} = f_{ABC} \qquad f_{-\bar{i}\bar{j}\bar{k}} = \theta \qquad \xi_{+A} = \frac{1}{\sqrt{2}}\,\xi_A$$

where

$$f_{ABC} = (X_{mn})_{pq}{}^{rs}\,[G_A]^{mn}\,[G_B]^{pq}\,[G_C]_{rs}$$

$$\xi_A = \xi_{mn}\,[G_A]^{mn}$$

- de Sitter
- EOMs for scalar fields not satisfied
- Extra fluxes required!

Goals

1. Double field theory
   - The basics
   - Strong constraint relaxation and consistency conditions
   - Action fixed by symmetries? [Cho, FM, Jeon, Park]
2. Orbits of non-geometric fluxes [Dibitetto, FM, Marqués, Roest]
   - Scherk-Schwarz compactification of DFT
   - Classification of deformations in $d = 9, 8, 7$
   - Compactification scheme (twist matrices)
3. Stable de Sitter in half-maximal $d = 7$ [Dibitetto, FM, Marqués]
   - Full classification of deformations
   - Structure of vacua and moduli stabilisation
   - First stable dS in half-maximal
4. Conclusions and outlook

Conclusions

- DFT
  - Basics
  - Relaxed SC formulation (generalised fluxes)
  - $N = 1$ Supersymmetric DFT fixed by symmetries
- Duality orbits of non-geometric fluxes
  - Generalised SS reduction of DFT
  - Gaugings classification of $d = 9, 8, 7$ (half-)maximal SUGRAs
  - $\forall\ X_{MN}{}^{P}\ \exists\ U(\mathbb{Y})\ |\ f_{MN}{}^{P}(U) \cong X_{MN}{}^{P}$
  - Truly non-geometric gaugings $\Leftrightarrow$ non-upliftable $(f_{ABC}f^{ABC} \neq 0)$
- Stable de Sitter in half-maximal $d = 7$
  - Full classification of gaugings
  - Exhaustive classification of non-semisimple vacua
  - SO(3,1) gauging: non-geometric (with massive deformation θ)
  - First examples of stable de Sitter solutions in half-maximal

Outlook

- DFT
  - Global definition (of its relaxed version)
  - Genuinely exotic branes and mixed-symmetry potentials (E_{11})

    Bergshoeff *et al.*'15
  - Truly non-geometric deformations?

    Blumenhagen *et al.*'11 — Condeescu *et al.*'12'13
  - Twist matrices for lower dimensions? Not even guaranteed
- Supergravity
  - DFT as a criterion to identify non-geometric deformations
  - Non-geometric fluxes circumvent no-go theorems (Maldacena-Núñez)
  - Vacua structure of other non-geometric deformations
  - Stable dS in $N = 4$ $D = 4$?
  - Non-geometric deformations from EFT
  - Vacua structure of non-geometric maximal theories
  - AdS spaces of non-geometric deformations?

Thanks