# Asymptotic Symmetries and Gravitational Hair.

with

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## Gauge Invariance:

Is a real Symmetry
or
just redundancy. ?.

The answer to this question depends on:

**Topology** : Large Gauge Transformations (top non trivial)

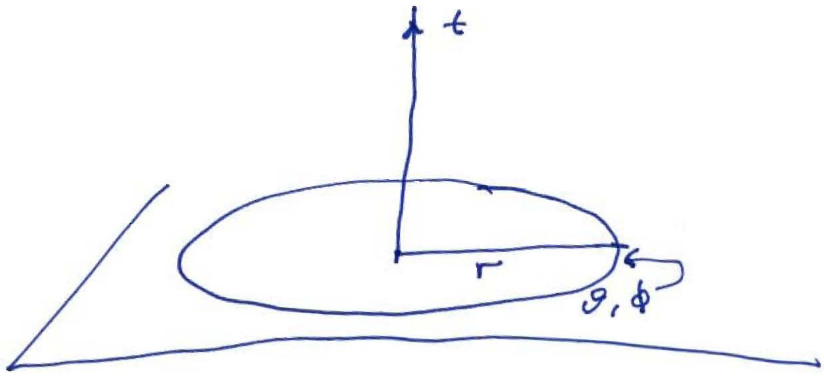
**IR physics** Large Gauge transformations (Those non vanishing at $\infty$)

# QED.

$$A_\mu \to A_\mu + \partial_\mu \varepsilon$$

work in temporal gauge $A^0 = 0$

gauge transformations: $\varepsilon(r, \theta, \phi)$




Large gauge transformations:

$$\varepsilon(\theta \phi) = \lim_{r \to \infty} \varepsilon(r, \theta, \phi) \neq 0$$

Why?

Gauge Invariance $\Rightarrow$ Gauss Law (constraint).

Charged Matter:

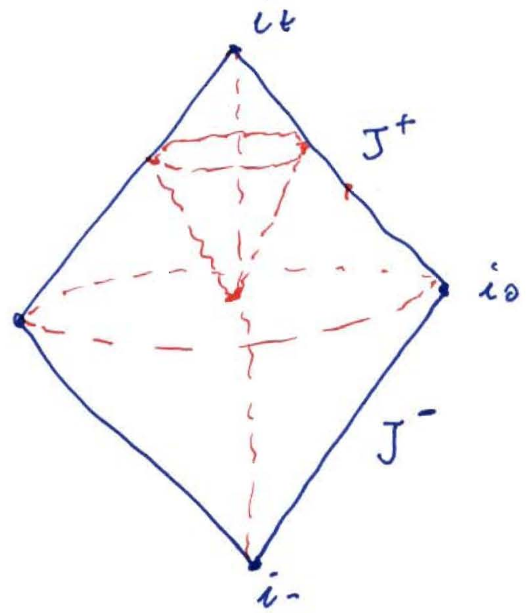
$$\tilde{\psi}(x) = \psi(x) \underbrace{e^{-ie\int_x^\infty A^\mu(\xi)\, d\xi}}_{\text{dressing}}.$$

$A \to A + \partial \varepsilon$ with $\varepsilon(\infty) \neq 0$

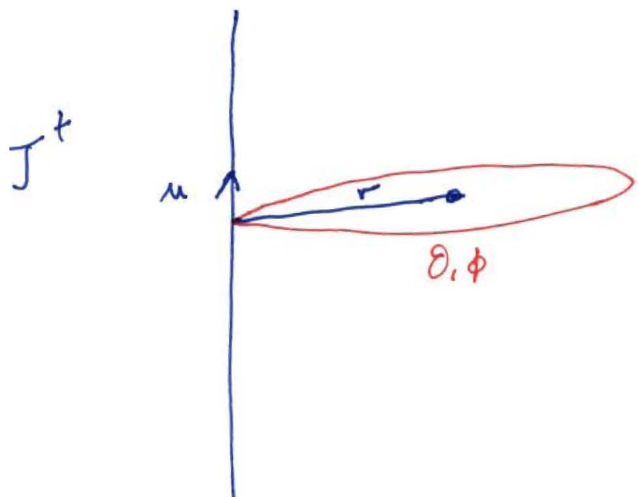
$$\tilde{\psi}(x) \to e^{-ie\varepsilon(\infty)}\, \tilde{\psi}(x)$$

i.e A real symmetry transformation.

Use Penrose's diagram to define $\infty$ in Minkowski space-time.




Coordinates at null infinity $\mathcal{J}^\pm$




$$(r, u, \theta, \phi)$$
$$\Big\downarrow r \to \infty$$
$$\mathcal{J}^+ \quad (u, \theta, \phi)$$
$$\cong$$
$$\mathbb{R} \times S^2$$

"$u$" - time
$r$ - hol coordinate
$S^2$ - hol screen (?)

QED (Asymptotic Symmetries)

gauge connection : $\mathcal{A}(r, u, \theta, \phi)$

$z, \bar{z}$

$A(u, \theta, \phi)$ on $\mathcal{J}^+$

"temporal" gauge $A_u = 0$

Large gauge transformations :

$$A_{\substack{z \\ \bar{z}}} \longrightarrow A_{\substack{z \\ \bar{z}}} + \partial_{\substack{z \\ \bar{z}}} \, \varepsilon(z, \bar{z})$$

arbitrary function on $S^2$

Classical vacua:

"flat" connections

$$F_{z\bar{z}} = \partial_z A_{\bar{z}} - \partial_{\bar{z}} A_z = 0$$

$$\boxed{A_{\substack{z \\ \bar{z}}} = \partial_{\substack{z \\ \bar{z}}} \, \phi(z, \bar{z})}$$

generic function.

Asympt vacua: $\quad | \partial \phi(z\bar{z}) \rangle$

Large Gauge Transformation $\quad \Big\downarrow T_\varepsilon$

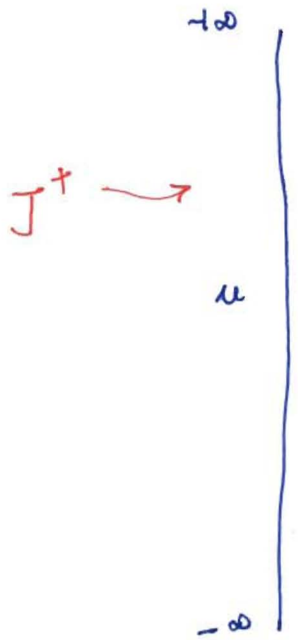
$$| \partial(\phi + \varepsilon) \rangle$$

"Goldstone" picture:

$$\langle \text{Goldstone} | T_\varepsilon | \partial \phi_0 \rangle \neq 0$$

Soft photon.

How to identify the Goldstone?




$$A_\varepsilon(u, z, \bar z) \equiv \begin{cases} -\infty & |\partial\phi(z\bar z)\rangle \\ +\infty & T_\varepsilon |\partial\phi(z\bar z)\rangle = \\ & = |\partial(\phi+\varepsilon)(z\bar z)\rangle \end{cases}$$

Charge:

$$Q_\varepsilon \equiv \int_{-\infty}^{+\infty} du\, d^2z\, \partial_u \left( \partial_z \partial_{\bar z} A_\varepsilon + \leftrightarrow \right)$$

$\approx$ # soft photons "radiated" during The interpolating process.

What about gravity?.

Asymptotic flat:

The analog of
Large Gauge Transformations are

BMS-transformations.

Null-$\infty$ $\mathcal{J}$ $\longrightarrow$ $(\vec{n}, g_{ab})$
$(0++)$

generators:

$\vec{\xi}$ ;

$$\boxed{\begin{aligned} \mathcal{L}_{\vec{\xi}}\, n &= \alpha\, n \\ \mathcal{L}_{\vec{\xi}}\, g_{ab} &= \alpha\, g_{ab} \\ \mathcal{L}_{\vec{n}}\, \alpha &= 0 \end{aligned}}$$

Supertranslations $\vec{\xi}\Big|_{\mathcal{J}} = \tilde{f}\,\vec{n}$ $\quad \tilde{f}(z,\bar{z})$ generic.

So:

$$\vec{\xi} = (f, A, B, C)$$

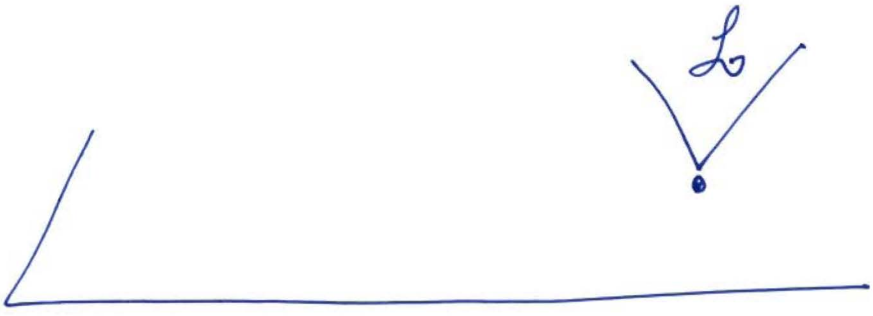
$$f\big|_{\mathcal{J}} = \tilde{f} \qquad \underbrace{A\big|_{\mathcal{J}} = B\big|_{\mathcal{J}} = C\big|_{\mathcal{J}}}_{\to\, O\left(\frac{1}{r}\right)} = 0$$

$$\boxed{\mathcal{B} = \underbrace{\mathcal{S}}_{\text{supertranslations}} \rtimes \underbrace{\mathcal{L}}_{\text{Lorentz}}}$$

very similar to Poincare BUT $\mathcal{S}$ now is $\infty$ - dimensional!

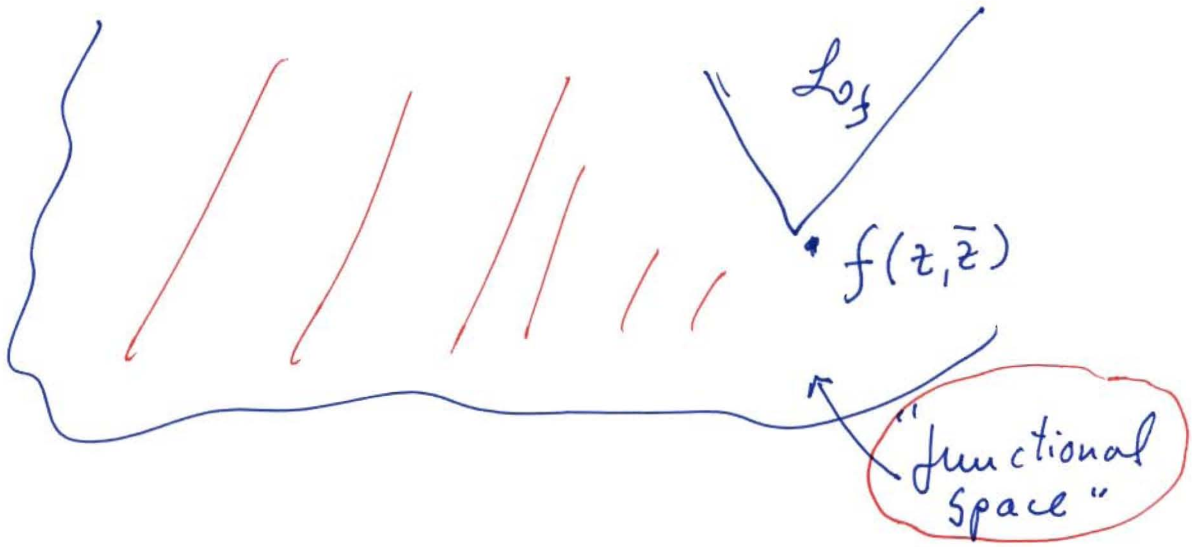
$\Longrightarrow$ $\infty$ - different Lorentz groups!.

$\mathcal{P}$




4-parameter family of Lorentz

$\mathcal{B}$




$\infty$- parameter family of Lorentz

Vacua:

In Q-F.T

$$\boxed{\mathcal{L}\,|0\rangle = 0}$$

But now we have:

$$\boxed{\mathcal{L}_f\,|f\rangle = 0}$$

Each one for each $f$!.

$f \longrightarrow \nabla_f$ flat connection on $\mathcal{J}$

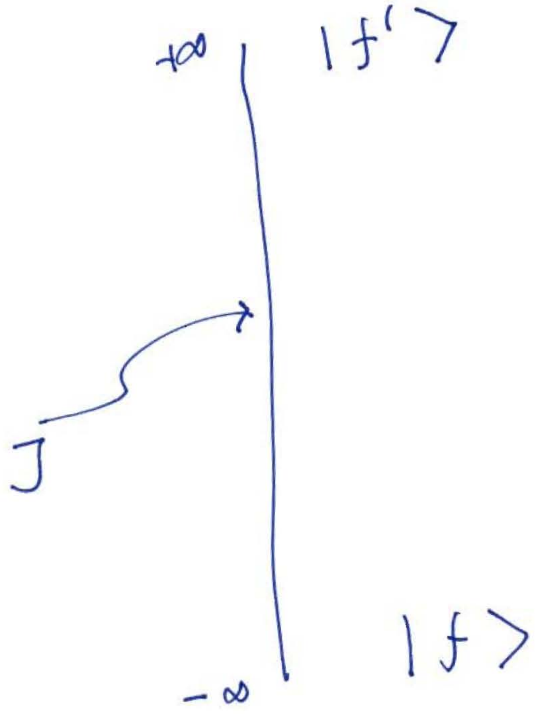
$$C^{abcd}\, n_c n_d \sim 0$$

Weyl tensor.

$$\boxed{T_{f'} \, |f\rangle = |f \circ f'\rangle}$$

$T_{f'}$ → BMS - transformations

Now Goldstone = Soft graviton!




$g_{\mu\nu}(u, \theta, \phi)$
interpolating.

$Q \sim \int du \, \partial_u g$

gravitational radiation!

## Key Questions:

1. What is the meaning of this Vacuum degeneration ?.
   Classically $\rightsquigarrow$ $N = \infty$

2. Can we do the same thing for BH - horizons ?.

3. And for Cosmological horizons ?.
   (cosmological constant)

4. What is the analog of the soft graviton in these cases ?.

5. How to relate "ground state" degeneration in these cases to Entropy ?.

# Asymptotic Symmetries on Horizon.

$$\left(\vec{n}_H, g_{ab}^{(H)}\right) \qquad \vec{\xi}$$

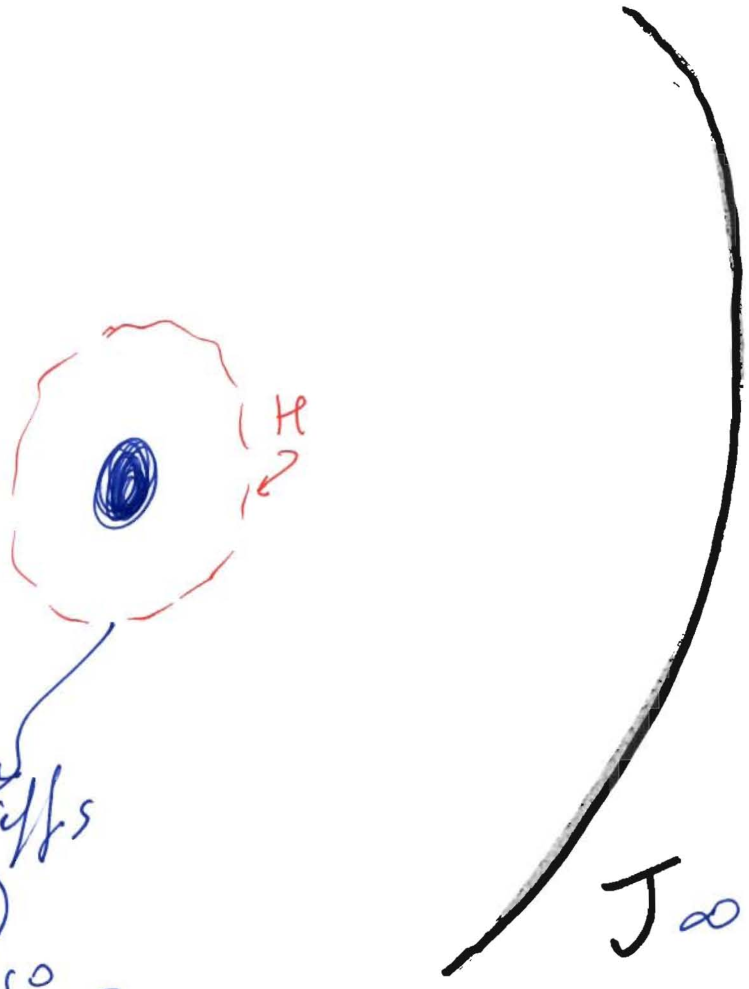
$$\boxed{\begin{aligned} \mathcal{L}_{\bar{\xi}}\, n^H &= \alpha\, n^H \\ \mathcal{L}_{\bar{\xi}}\, g_{ab}^H &= \alpha\, g_{ab}^H \\ \mathcal{L}_{n_H}\, \alpha &= 0 \end{aligned}} \qquad \bar{\xi} = f\, \bar{n}^H$$

$\Rightarrow$ $\infty$ - degenerate

BH "ground states"

$$|BH; f\rangle$$

# Obvious Physics Question:

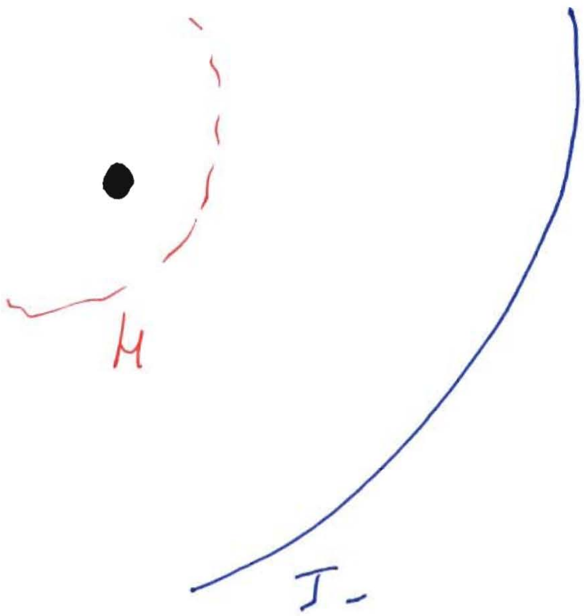


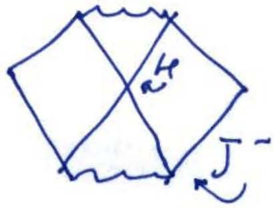

Large Diffs
BMS (H)
are also
symmetries ?

Large Diff's
are
Symmetries
(IR)

Intrinsic to the Horizon:

$$\boxed{\mathcal{A} = BMS(H) \big/ BMS(-)}$$




Quotient BMS(H) by the continuation to H of BMS transformations at $J_-$

Finally:

Are These Asymptotic Symmetries on the Horizon quantum mechanically Anomalous?

$$\langle BH; f' | BH; f \rangle = 0 \quad \text{symmetry}$$

$$\neq 0 \quad \text{anomaly}$$

$$\boxed{\langle BH; f' | BH; f \rangle \sim O\left(\frac{1}{S_{BH}}\right)}$$