

5-brane webs and 5d, 6d SCFTs

Hiroataka Hayashi

(IFT UAM-CSIC)

Based on the collaboration with

- [Sung-Soo Kim, Kimyeong Lee, Masato Taki, Futoshi Yagi](#)
[\[arXiv:1505.04439\]](#) and [\[arXiv:1512.08239\]](#)
- [Sung-Soo Kim, Kimyeong Lee, Futoshi Yagi](#)
[\[arXiv:1509.03300\]](#)

28th of January 2016, Iberian Strings 2016, IFT Madrid

1. Introduction

- Understanding quantum field theories in higher dimensions.

Assumptions: supersymmetry and without gravity

What are UV complete higher dimensional gauge theories?

- Higher dimensional gauge theories are not renormalizable. The gauge coupling becomes infinitely strong at high energies.

- Those theories may make sense if they admit a non-trivial fixed point at UV.
- Superconformal field theories exist when $d \leq 6$. Nahm 78
- Therefore, 5d or 6d gauge theories may be UV complete, but not all of them.
- 6d gauge theories are very restrictive. In particular they always require a tensor multiplet due to the cancellation of anomalies. Seiberg 96

- Recently, it is argued that all the 6d SCFTs may be classified by using F-theory compactifications.

Heckman, Morrison, Vafa 13

Heckman, Morrison, Rudelius, Vafa 15

- What about the classification of UV complete 5d gauge theories?
- They are less restrictive compared to 6d gauge theories due to the absence of the anomalies (there can be a condition for coefficients of cubic terms in the prepotential from the gauge invariance).

- Example: 5d SU(2) gauge theories with N_f flavors

(i) $N_f \leq 7 \rightarrow$ 5d SCFT with E_{N_f+1} flavor symmetry

Seiberg 96

Morrison, Seiberg 96

(ii) $N_f = 8 \rightarrow$ **6d SCFT** called “E-string”

Douglas, Katz, Vafa 96

An M5-brane probing an M9-wall

Ganor, Hanany 96

Klemm, Mayr, Vafa 96

Ganor, Morrison, Seiberg 96

From the 5d viewpoint, instantons become KK modes of S^1 compactification of the 6d theory. In other words, after summing up all the instantons the S^1 direction appears.

- How about the higher rank cases?
- The field theory analysis gives:

Intriligator, Morrison, Seiberg 97

Ex. 5d SU(N) gauge theory with N_f flavors and k CS level

$$N_f + 2|k| \leq 2N$$

- The 5d SU(N) gauge theories that satisfy this condition are supposed to have 5d UV fixed points

- A natural question:

No 5d $SU(N)$ gauge theory admits a 6d UV completion?

- I will revisit the question of UV complete 5d gauge theories by using string theory.

- String theory predicts **new UV complete 5d gauge theories** that admit a 6d UV completion.
- String theory also implies **new 5d “UV dualities”**.

1. Introduction
2. 5d gauge theories from string theory
3. 6d SCFTs from 5-brane webs
4. New 5d UV dualities
5. Conclusion

2. 5d gauge theories from string theory

- We construct a 5d supersymmetric field theory with eight supercharges as the worldvolume theory on a 5-brane web.
- The 5-brane configuration in Type IIB string theory.

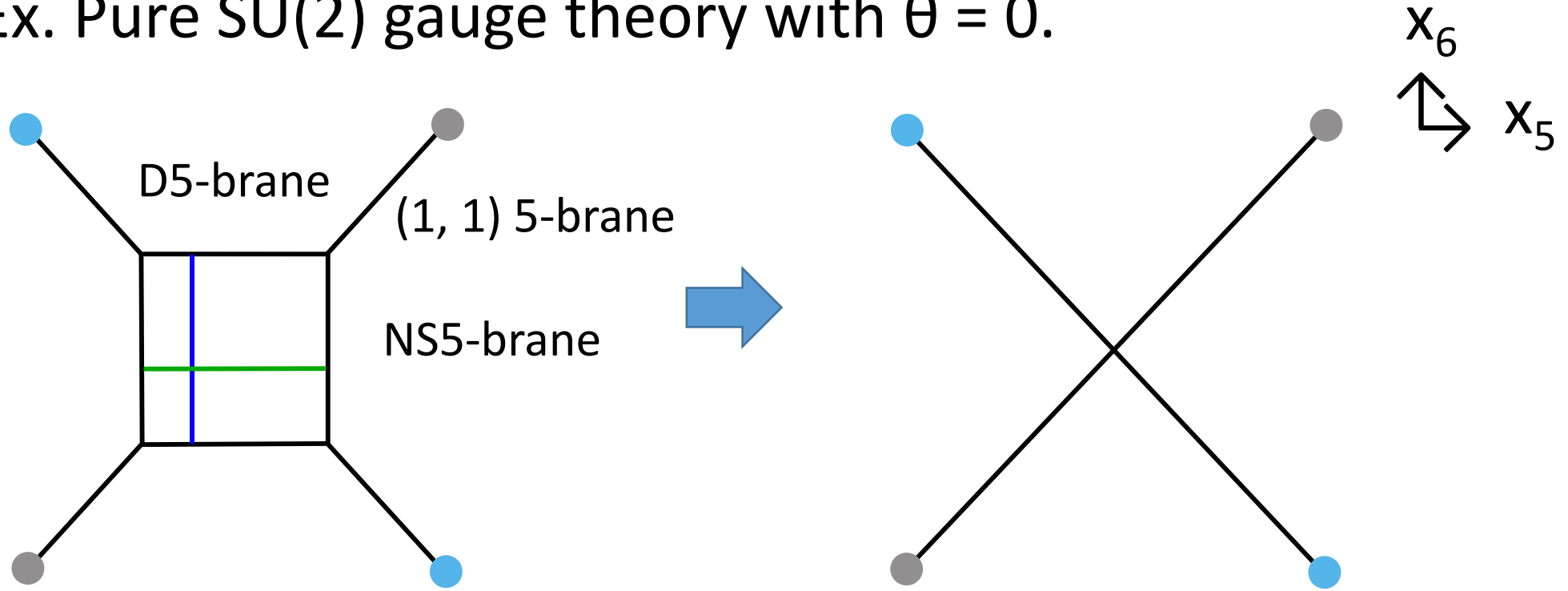
	0	1	2	3	4	5	6	7	8	9
D5-brane	×	×	×	×	×	×				
NS5-brane	×	×	×	×	×		×			
(p, q) 5-brane	×	×	×	×	×	angle				
7-brane	×	×	×	×	×			×	×	×

Aharony, Hanany 97

Aharony, Hanany, Kol 97

DeWolfe, Iqbal, Hanany, Katz 99

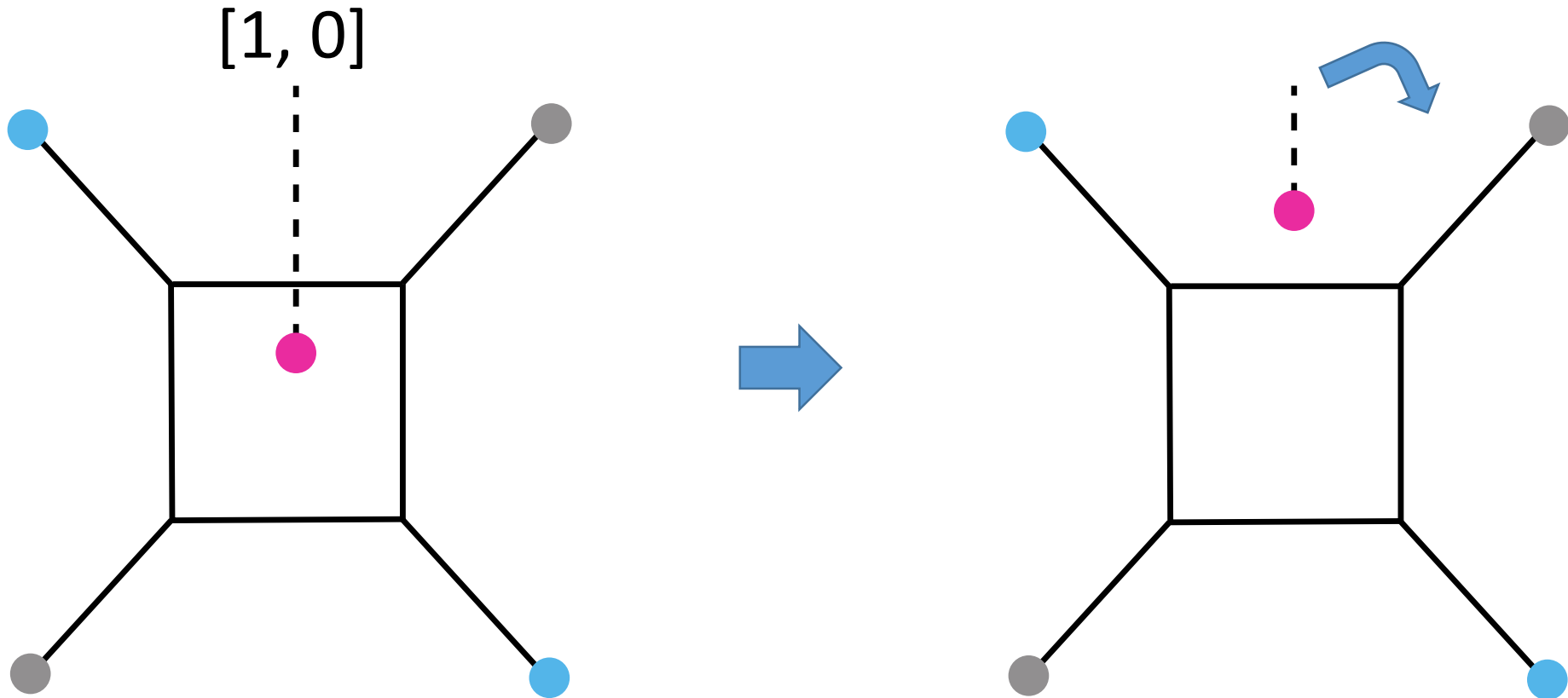
- Ex. Pure $SU(2)$ gauge theory with $\theta = 0$.



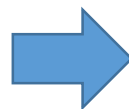
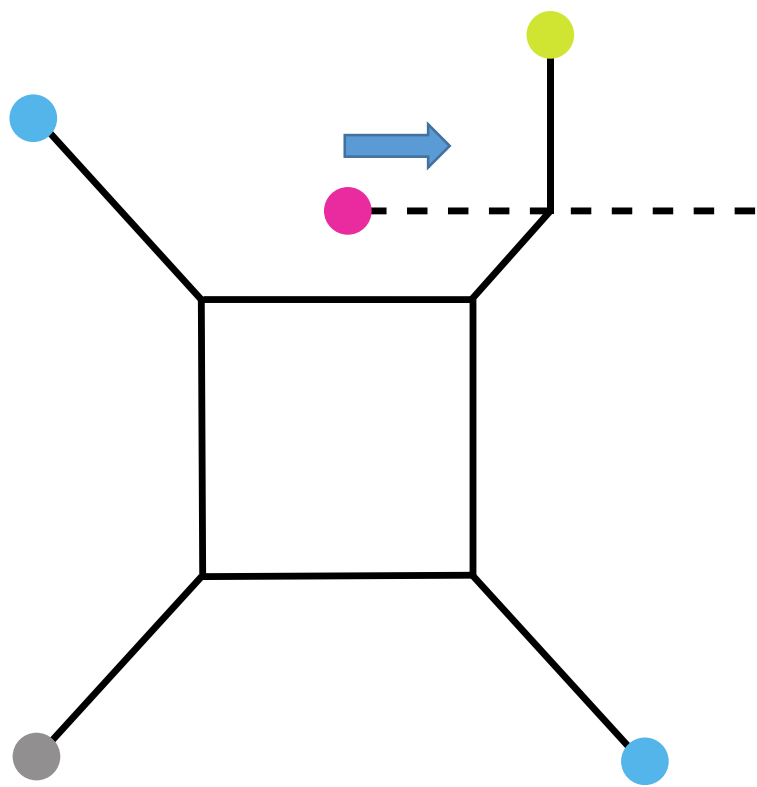
- This theory has a fixed point where the gauge coupling is infinitely strong.
- Enhancement of flavor symmetry: $U(1) \rightarrow SU(2) = E_1$.

Seiberg 96

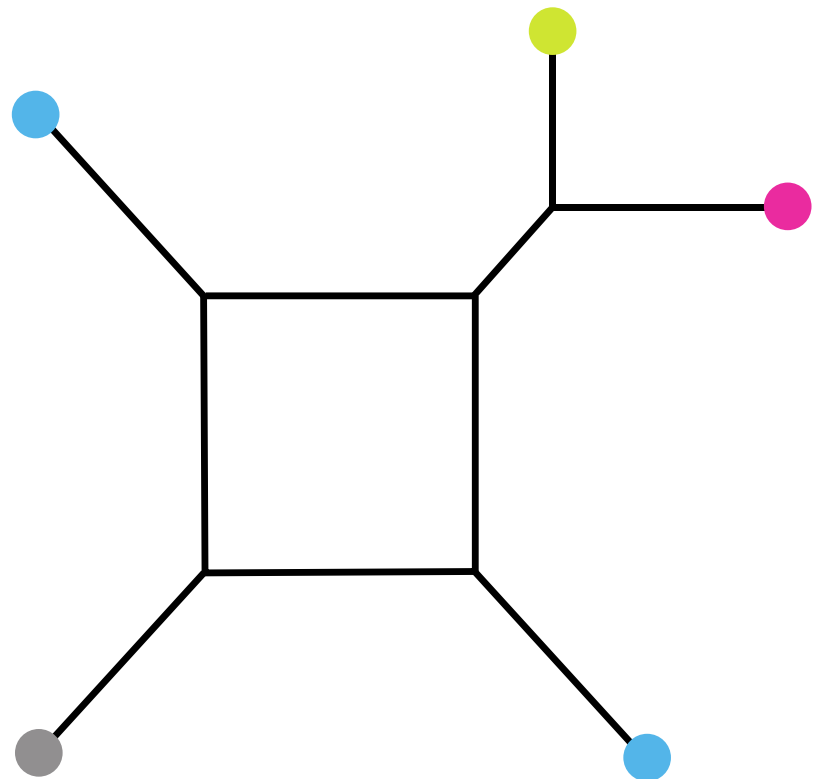
- Ex. 1 flavor : $[U(1) \times U(1) \rightarrow SU(2) \times U(1) = E_2]$



$[0, 1]$

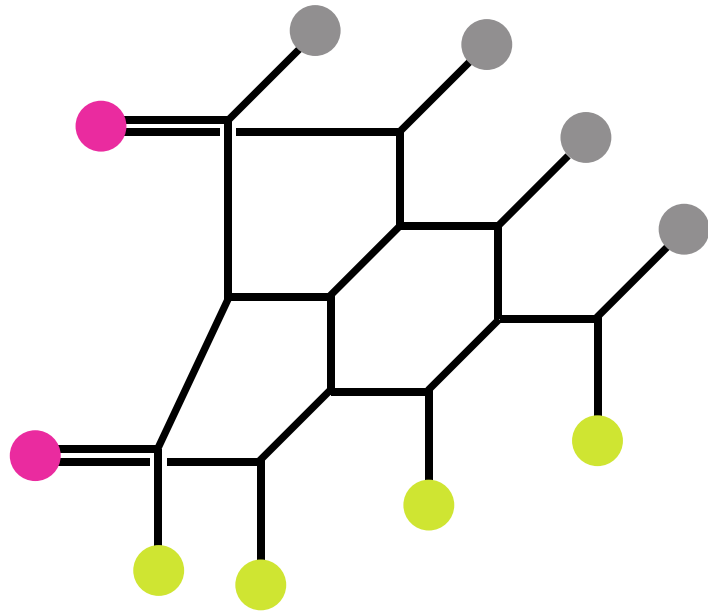


$SU(2)$ with $N_f = 1$

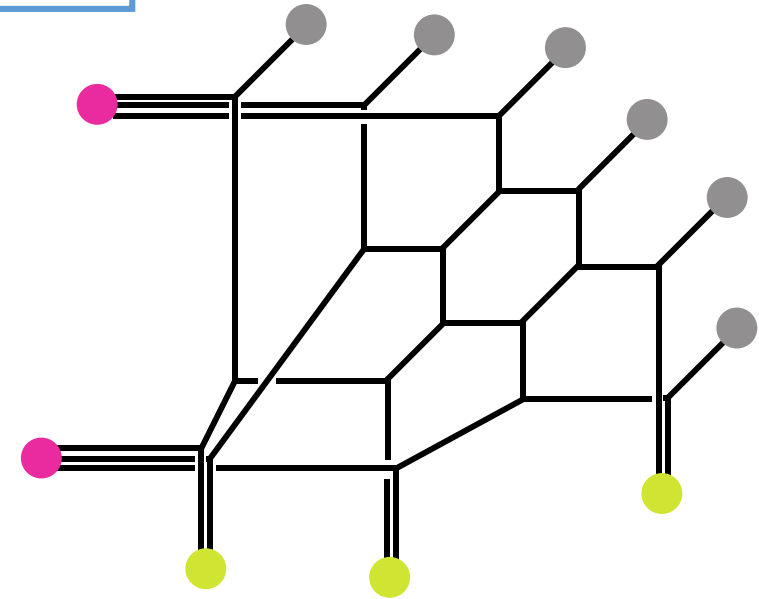


- Further addition of the flavors can be done systematically.

$$N_f = 6 [E_7]$$

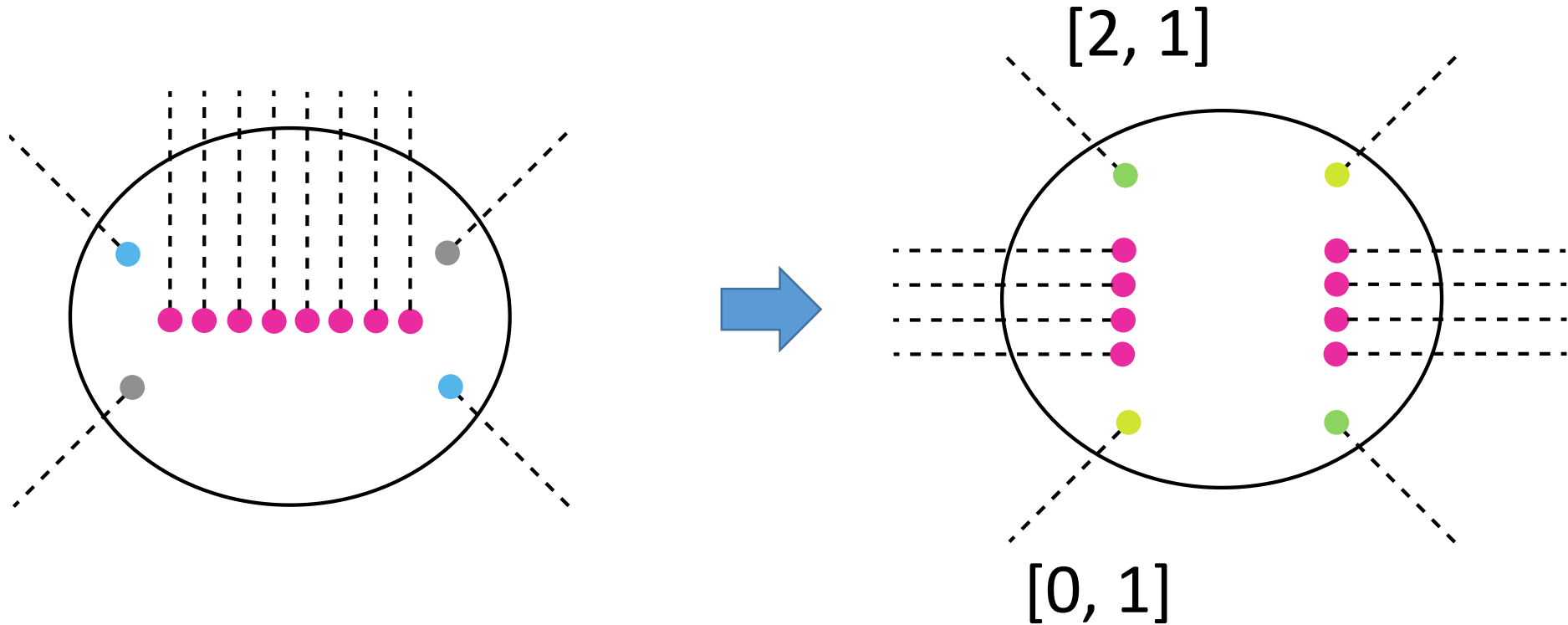


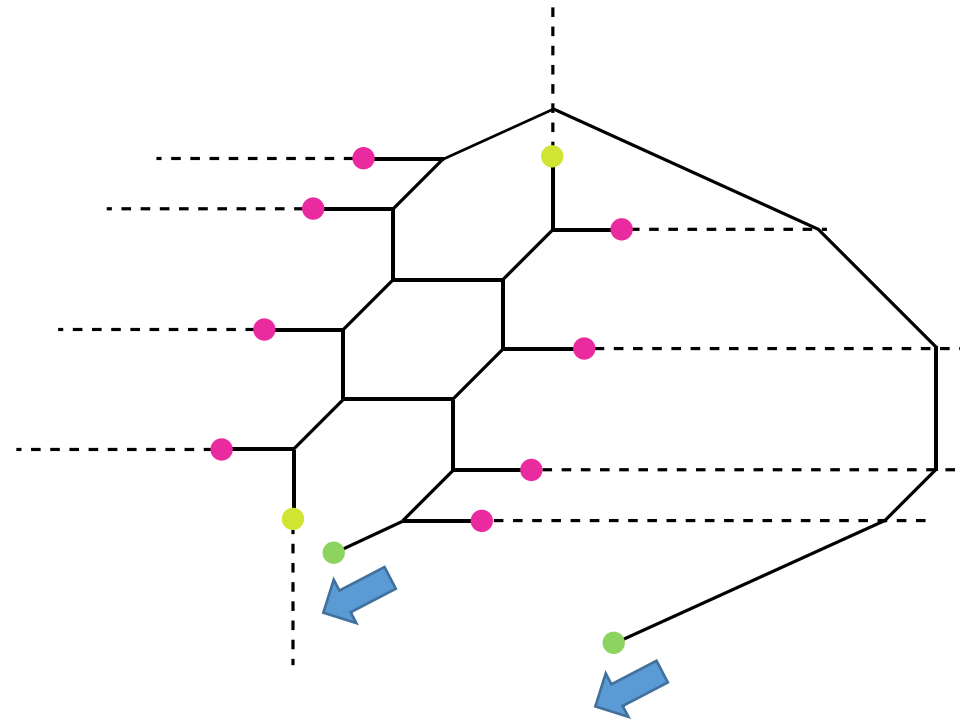
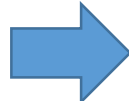
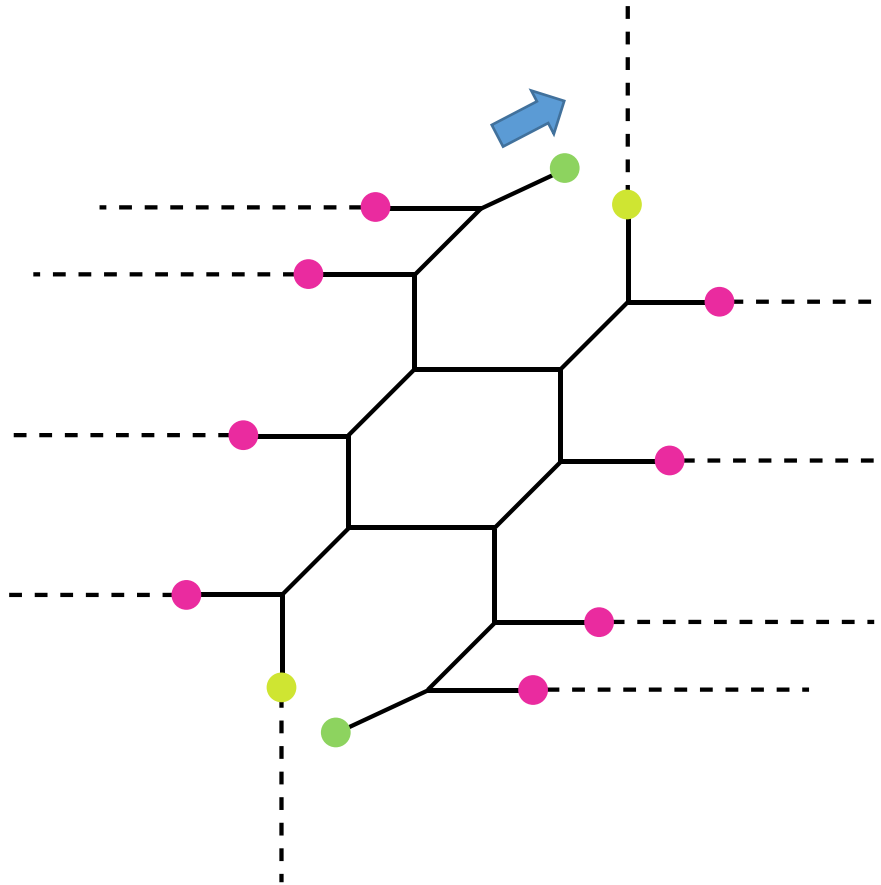
$$N_f = 7 [E_8]$$



Benini, Benvenuti, Tachikawa 09

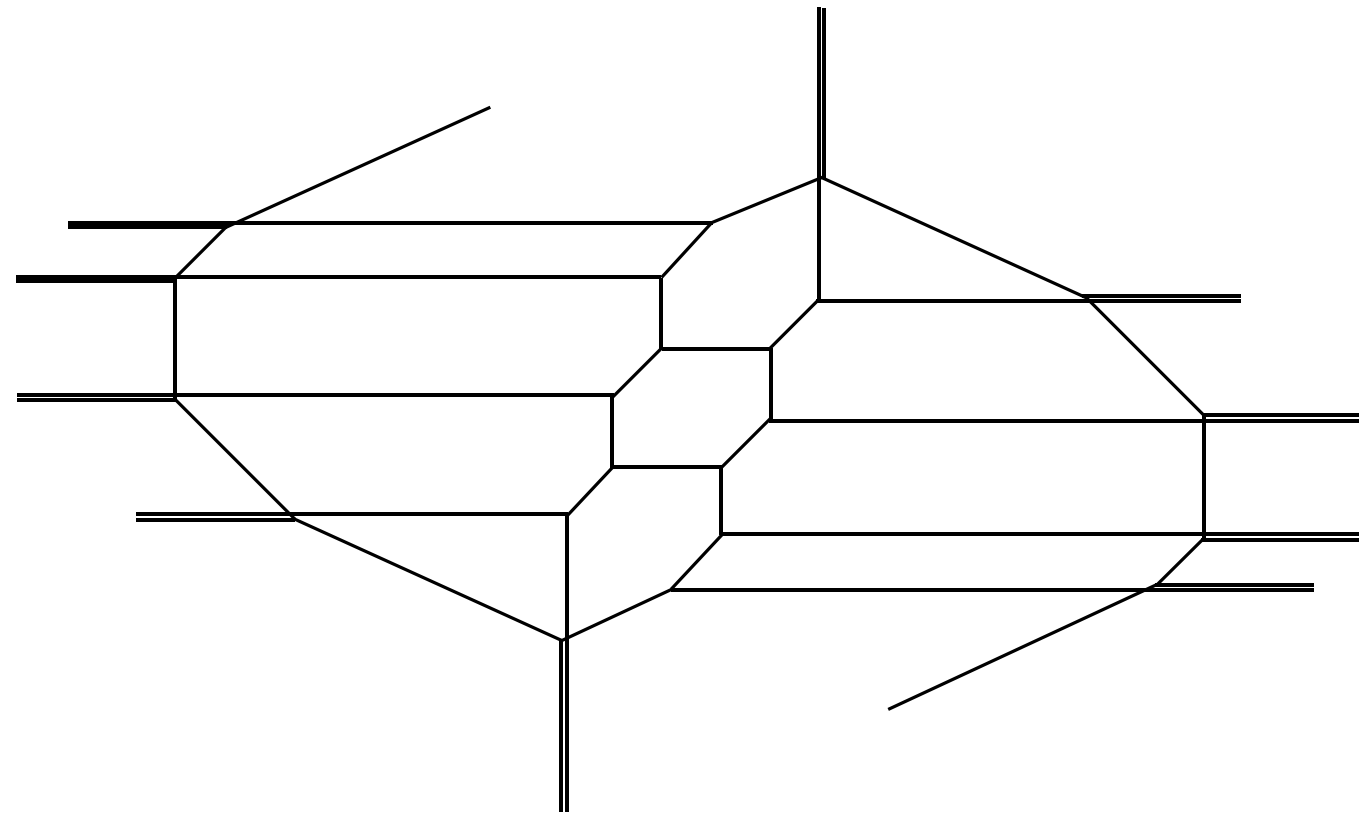
- What happens if we consider the $SU(2)$ gauge theory with 8 flavors? We know that its UV fixed point is the 6d E-string theory.



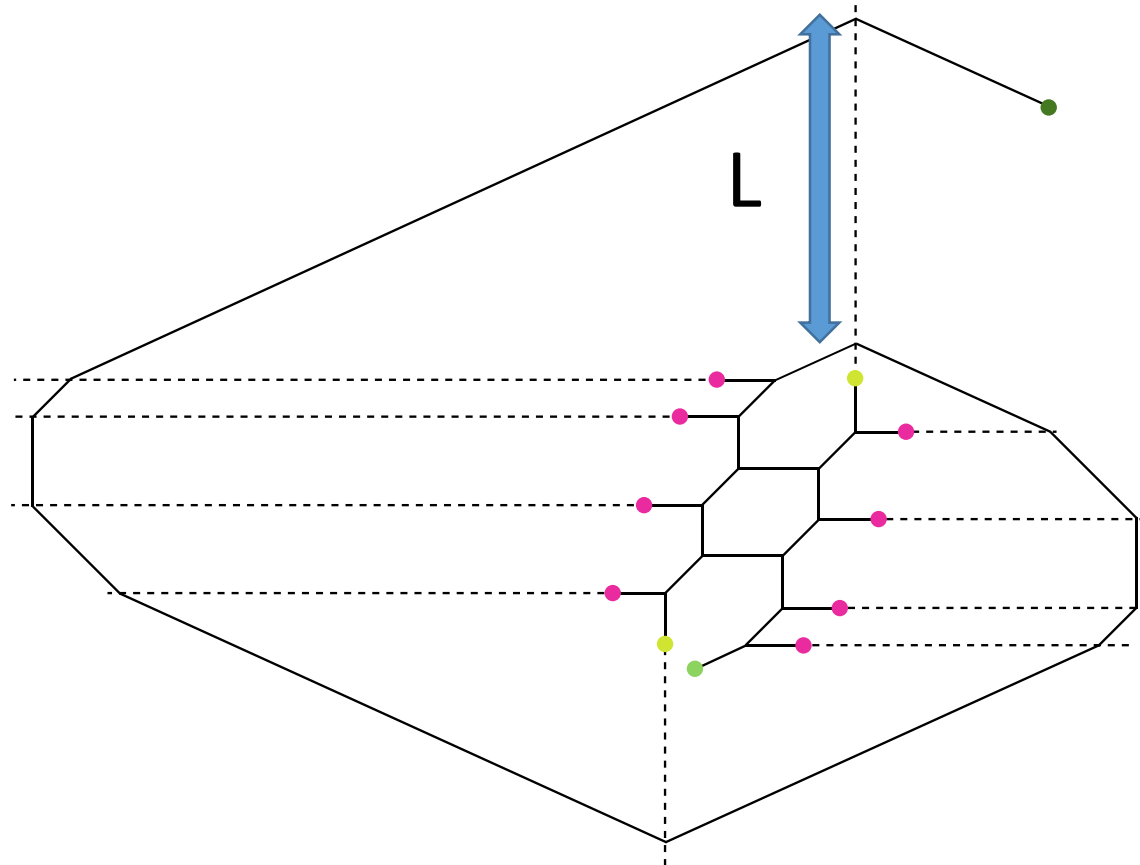


- Then the 5-brane web starts rotating when we pull 7-branes to infinity.

Kim, Taki, Yagi 15



- The “period” of the 5-brane web diagram can be identified with the inverse of the radius of the S^1 .



$$L \approx 1/g^2 \approx R^{-1}$$

- String theory predictions:

- The existence of a standard 5-brane web implies the corresponding 5d theory has a **5d** UV fixed point.

- The existence of an infinitely rotating 5-brane web (or a 5-brane web which has an “ S^1 ” direction) implies the corresponding 5d theory has a **6d** UV fixed point.

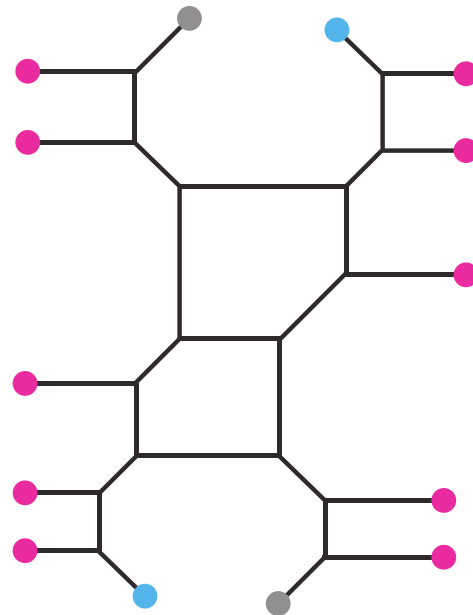
- Then let us think about the generalization by utilizing the infinitely rotating structure of 5-brane webs!

3. 6d SCFTs from 5-brane webs

- The higher rank generalization. The infinitely rotating structure of a 5-brane web predicts the existence of a 5d $SU(N)$ gauge theory with $N_f = 2N+4$.

Kim, Taki, Yagi 15

Ex. 5-brane web for 5d $SU(3)$ with 10 flavors



- Does it corresponds to a 6d SCFT at UV? → Yes!

Hayashi, Kim, Lee, Taki, Yagi 15
Yonekura 15

- Our claim is

6d (D_{n+2}, D_{n+2}) minimal conformal matter theory



S^1 compactification

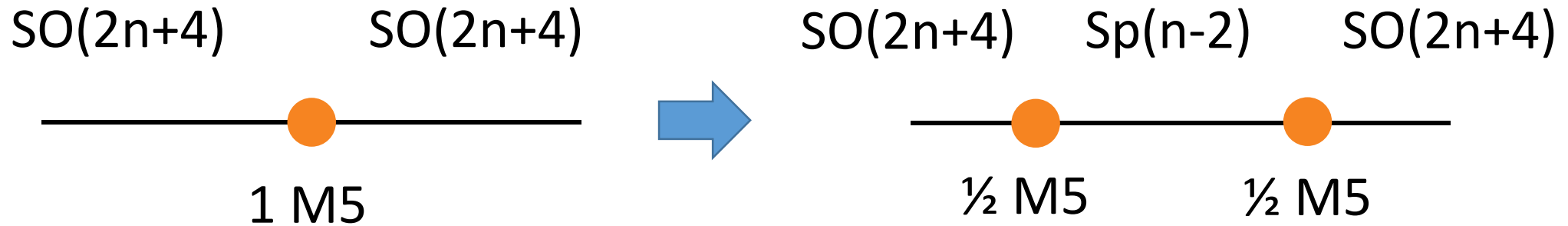
5d $SU(n)$ gauge theory with $N_f = 2n+4$ & CS-level 0

- In fact, we can “prove” this claim by using a brane construction in string theory.

Hayashi, Kim, Lee, Taki, Yagi 15

- A derivation by branes
- The 6d (D_{n+2}, D_{n+2}) minimal conformal matter theory is a 6d SCFT realized on a single M5-brane probing the D_{n+2} type singularity.

Del Zotto, Heckman, Tomasiello, Vafa 14



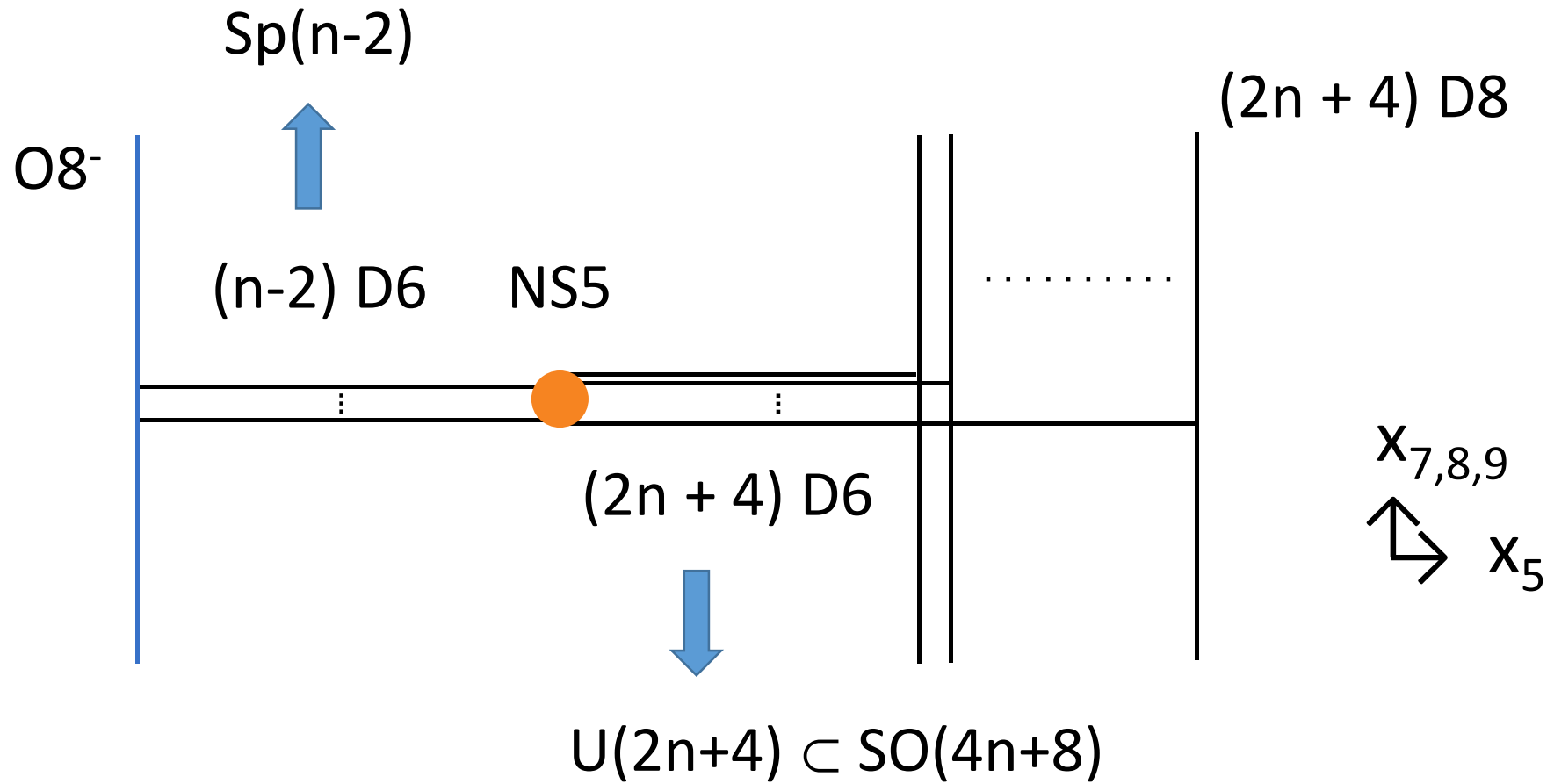
“tensor (Coulomb) branch”

- In the tensor branch, the 6d theory is an $Sp(n-2)$ gauge theory with $2n+4$ fundamental hypermultiplets, coupled to a tensor multiplet.
- The same 6d theory can be realized by a brane configuration in Type IIA string theory.

Brunner, Karch 97
 Hanany, Zaffaroni 97

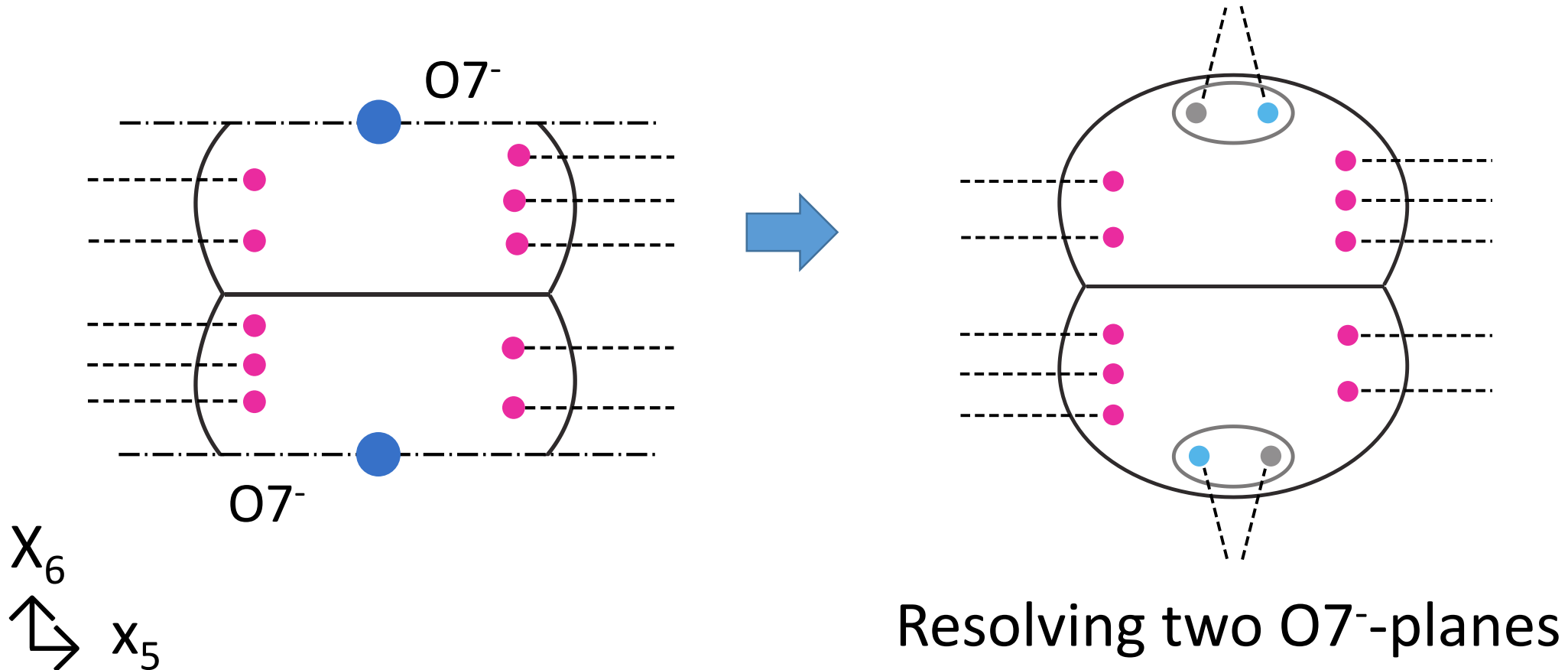
	0	1	2	3	4	5	6	7	8	9
D6-brane	×	×	×	×	×	×	×			
NS5-brane	×	×	×	×	×		×			
D8-brane	×	×	×	×	×		×	×	×	×
O8-plane	×	×	×	×	×		×	×	×	×

- The brane configuration

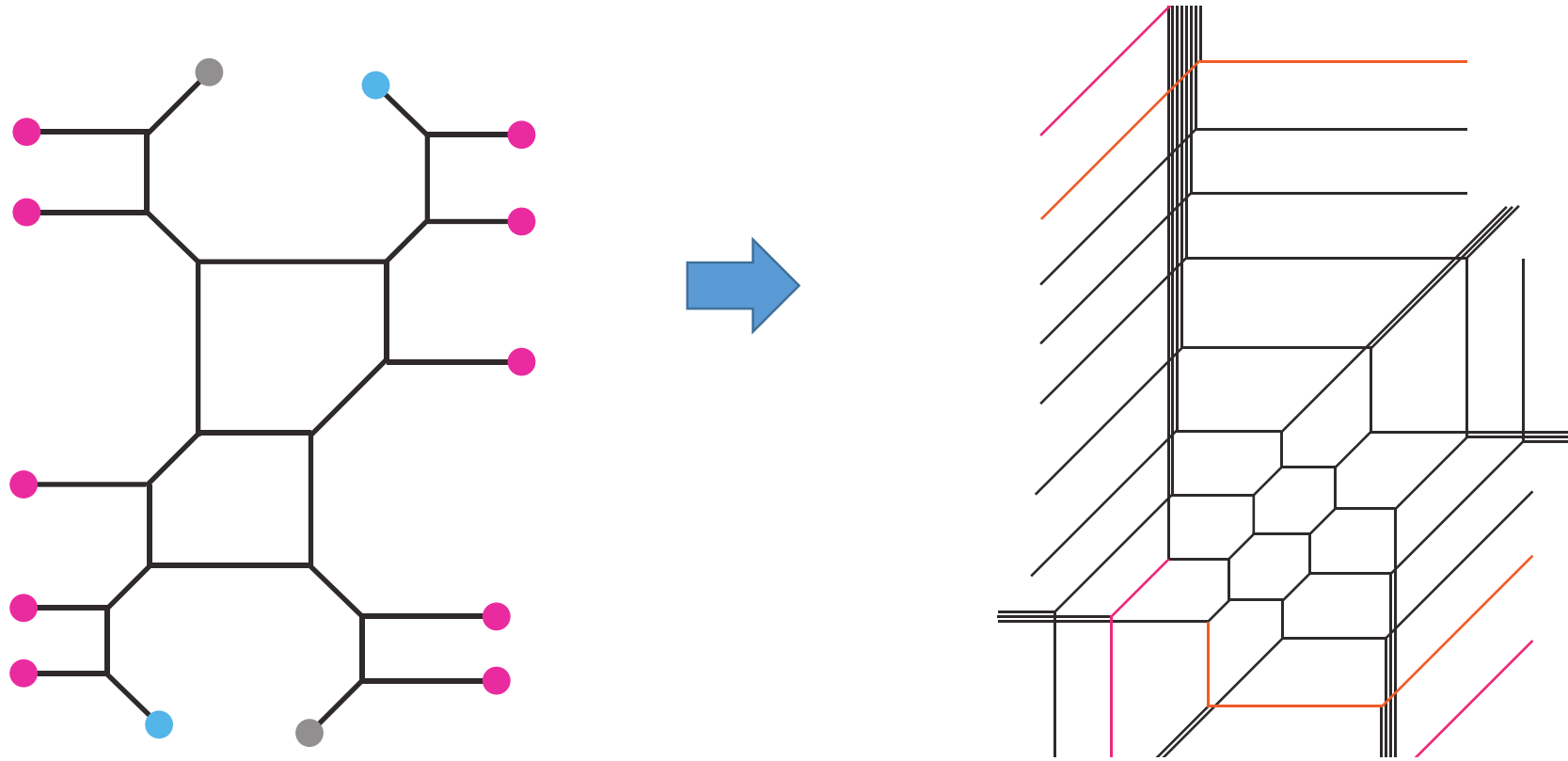


- We then compactify x_6 -direction on S^1 and perform the **T-duality** along the direction.

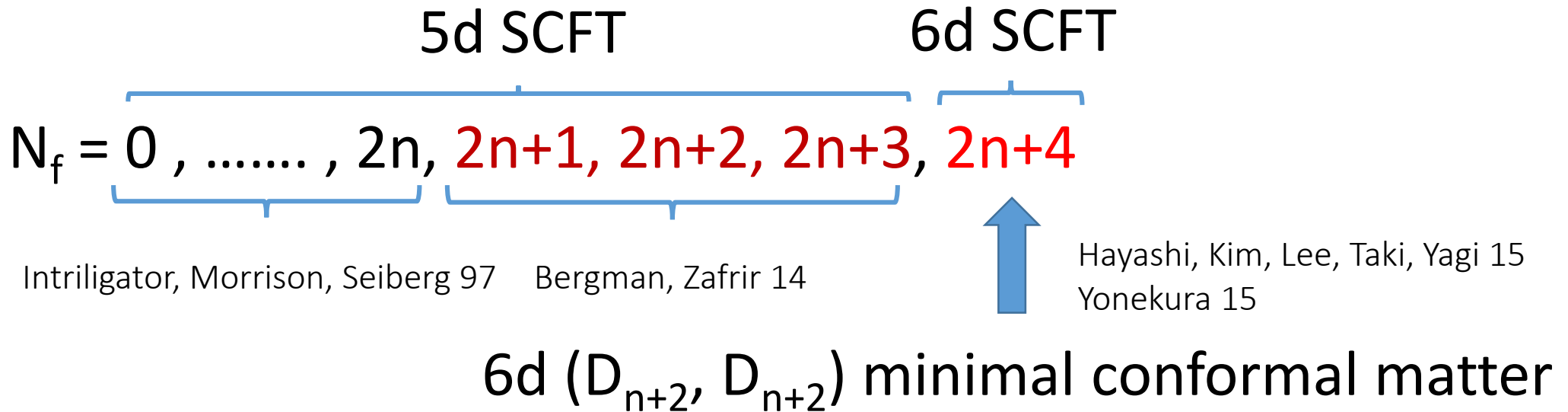
Ex. $n = 3$



- By pulling out the 7-branes outside the 5-brane loop, we obtain the 5-brane web for the $SU(3)$ gauge theory with $N_f = 10$ flavors.



- By decoupling flavors, we obtain 5d theories whose UV completion is 5d SCFTs.



- The new condition

$$N_f + 2|k| \leq 2N + 4$$

- It is possible to generalize the analysis to other 6d SCFTs constructed by type IIA brane configurations.

(i) Including more NS5-branes, which gives **new** 5d A-type quiver gauge theories.

Yonekura 15
Zafir 15,
Hayashi, Kim, Lee, Yagi 15
Ohmori, Shimizu 15

(ii) Including an ON^0 -plane with or without an $O8^-$ -plane, which often gives **new** 5d D-type quiver gauge theories.

Hayashi, Kim, Lee, Taki, Yagi 15

- **Largely expands the landscape of 5d gauge theories!**

Examples:

(i) 6d $SU(2n)$ with $N_f = 2n+8$ and $N_a = 1$



5d $[n+3] - SU(n+1) - SU(n+1) - [n+3]$

(ii) 6d $[8] - SU(2n) - Sp(2n-4) - SO(4n) - Sp(2n-4) - [2n]$



5d $[4] - SU(n+3) \left\{ \begin{array}{l} \backslash \\ / \end{array} \right. SU(2n+2) - SU(2n-2) - SU(2n-6) - [2n-8]$
 $[4] - SU(n+3)$

4. New 5d UV dualities

- The derivation using the brane configuration also implies another interesting 5d dualities.
- 5d $SU(n) \leftrightarrow 5d Sp(n-1)$ duality

Gaiotto, Kim 15

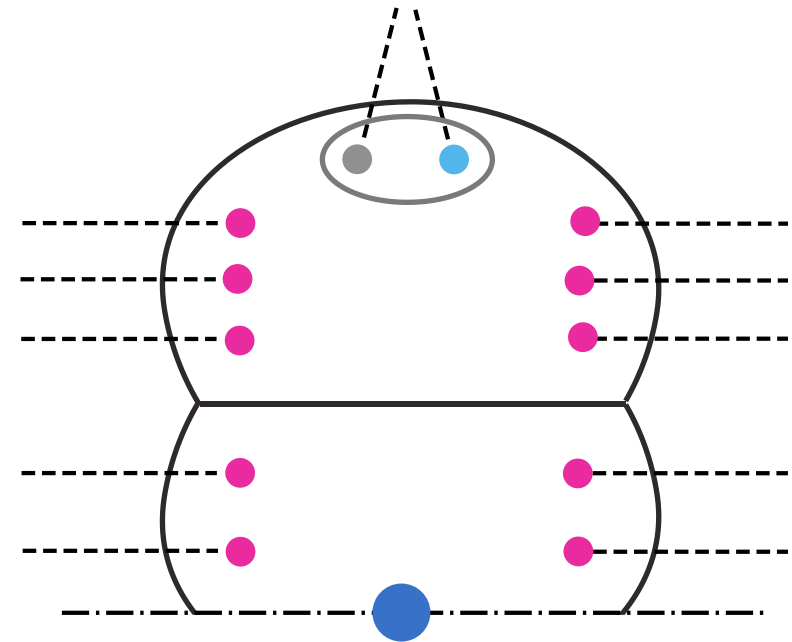
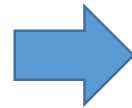
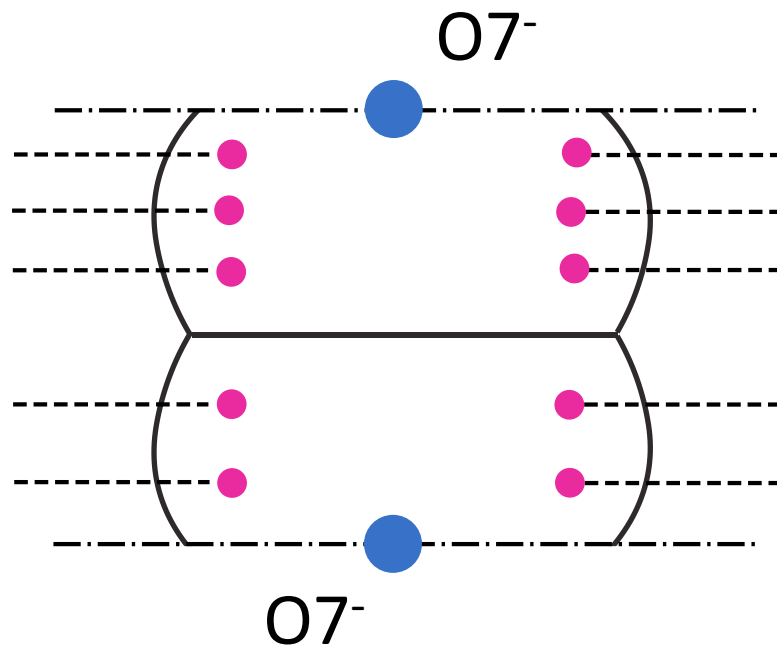
5d $SU(n)$ gauge theory with N_f flavors & $\kappa = \pm(N + 2 - N_f/2)$



5d $Sp(n-1)$ gauge theory with N_f flavors

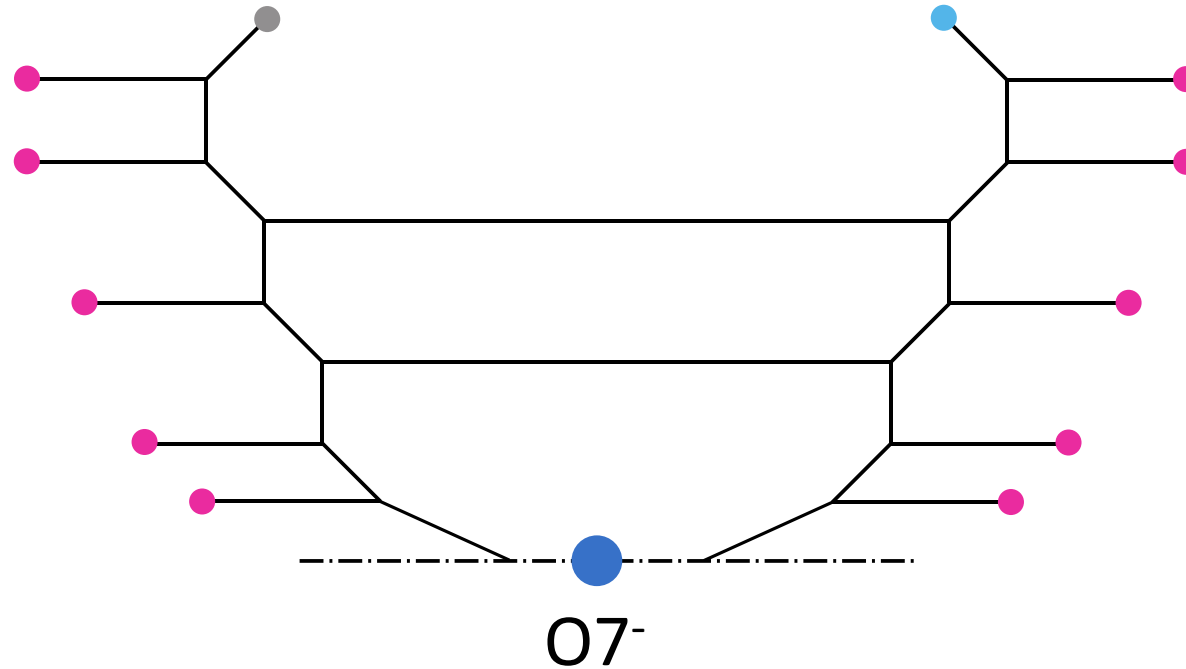
- The derivation again starts from the Type IIA brane configuration for the 6d $Sp(n-2)$ gauge theory with $N_f = 2n+4$ flavors, coupled to a tensor multiplet.

Ex. $n = 3$



Resolving **one** $O7^-$ -plane

- The resulting brane configuration is a 5d $Sp(2)$ gauge theory with $N_f = 10$.



- In general, we obtain a 5d $Sp(n-1)$ gauge theory with $N_f = 2n+4$.

(D_{n+2}, D_{n+2}) minimal conformal matter



$SU(n)$ with $N_f = 2n+4$



$Sp(n-1)$ with $N_f = 2n+4$

- Decoupling flavors on both sides in the same way reproduces the 5d SU – Sp duality.
- The generalization to other 6d SCFTs yields various **new** 5d UV dualities.

Hayashi, Kim, Lee, Yagi 15

Hayashi, Kim, Lee, Taki, Yagi 15

Examples:

(i) $[n+3] - \text{SU}(n+1) - \text{SU}(n+1) - [n+3]$



$\text{SU}(2n+1)$ with $N_f = 2n+7$ and $N_a = 1$

(ii) $[4] - \text{SU}(n+3)$
 $[4] - \text{SU}(n+3) \begin{cases} \diagdown \\ \diagup \end{cases} \text{SU}(2n+2) - \text{SU}(2n-2) - \text{SU}(2n-6) - [2n-8]$



$[8] - \text{SU}(2n+3) - \text{Sp}(2n-1) - \text{SO}(4n-2) - \text{Sp}(2n-5) - [2n-4]$

5. Conclusion

- String theory predicts **new** 5d gauge theories that have a 5d or 6d UV completion.
- The brane configuration and T-duality give a way to directly identify the 6d UV completion of the new 5d gauge theories.
- The method also implies various **new** 5d UV dualities.