Speed limits in holography

Carlos Hoyos
Universidad de Oviedo

w/.
David Rodriguez, N. Jokela, A. Vuorinen,

Iberian Strings
January 28, 2016

Outline

- Basics
- Bound on the speed of sound in holographic models
- ‘Trivial’ breaking of the bound
- A non-trivial counterexample

Basics

Sound from hydrodynamics

Hydrodynamic equations = conservation of the energy-momentum tensor

$$\partial_\mu \langle T^{\mu\nu} \rangle = 0$$

Constitutive relations (relativistic ideal fluid)

$$T^{\mu\nu} = (\varepsilon + p)\, u^\mu u^\nu + p\eta^{\mu\nu}$$

Small deviations from equilibrium

$$u^\mu \simeq (1, v^i), \quad p = p_0 + \delta p, \quad \varepsilon = \varepsilon_0 + \delta\varepsilon$$

$\Rightarrow$ linearized equations

Sound from hydrodynamics

Expand in plane waves

$$\delta p = \int \frac{d\omega d^3k}{(2\pi)^4} e^{-i\omega t + ik\cdot x} \delta\tilde{p}.$$

Sound mode = solution of hydrodynamic equations with linear dispersion relation:

$$\omega^2 = v_s^2 k^2$$

The speed of sound is

$$v_s^2 = \left(\frac{\partial p}{\partial \varepsilon}\right)$$

Sound and equation of state

The equation of state fixes p vs ε

$$p = p(\varepsilon)$$

In a CFT

$$0 = \langle T^\mu_\mu \rangle = -\varepsilon + 3p \;\Rightarrow\; v_s^2 = \frac{1}{3}.$$

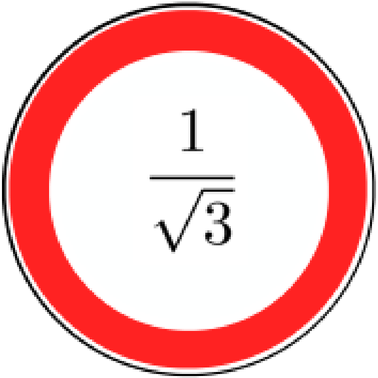
Causality $v_s \leq 1$ restricts the equation of state

$$p \leq \varepsilon$$

Stiffness

- Compress the fluid $\Rightarrow$ increase energy density
- Pressure also increases $\Rightarrow$ opposes compression
- The larger $\frac{\partial p}{\partial \varepsilon}$ is, the less compressible is the fluid
- The EOS is 'stiffer' or 'softer' for larger or smaller speeds of sounds

Speed limits in holographic models



Holographic models ($RG\,flows$)

- First computed in $\mathcal{N} = 4$ SYM [Policastro, Son, Starinets '02]

$$v_s = \frac{1}{\sqrt{3}}$$

- Masses for fermions (m_f) and scalars (m_b) ($\mathcal{N} = 2^*$) [Benincasa, Buchel, Starinets '05]

$$v_s = \frac{1}{\sqrt{3}}\left(1 - \frac{\left[\Gamma\left(\frac{3}{4}\right)\right]^4}{3\pi^4}\left(\frac{m_f}{T}\right)^2 - \frac{1}{18\pi^4}\left(\frac{m_b}{T}\right)^4 + \cdots\right)$$

- Klebanov-Strassler ($\mathcal{N} = 1$): logarithmic running with scale Λ [Aharony, Buchel, Yarom '05]

$$v_s^2 = \frac{1}{3} - \frac{2}{9}\frac{1}{\log\frac{T}{\Lambda}} + \cdots$$

- Exceptions: thermodynamically unstable phases [Buchel, Pagnutti '09]

Holographic models ($RG flows$)

For a class of models:
relevant deformation ($\Delta < 4$) of a CFT$_4$ $=$ minimally coupled scalar field

$$S = \frac{1}{2\kappa^2}\int d^5x \left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

At high temperatures the speed of sound is below the conformal value
[Cherman, Cohen, Nellore '09; Hohler, Stephanov '09]

$$v_s^2 = \frac{1}{3} - C(\Delta)(LT)^{\Delta-4} + \cdots$$

Holds for several scalars [Cherman, Nellore '09]

Conjecture: universal bound

Holographic models ($RG flows$)

For a class of models:
relevant deformation ($\Delta < 4$) of a CFT$_4$ $=$ minimally coupled scalar field

$$S = \frac{1}{2\kappa^2}\int d^5x \left(R - \frac{1}{2}(\partial\phi)^2 - V(\phi) \right)$$

At high temperatures the speed of sound is below the conformal value
[Cherman, Cohen, Nellore '09; Hohler, Stephanov '09]

$$v_s^2 = \frac{1}{3} - C(\Delta)(LT)^{\Delta-4} + \cdots$$

Holds for several scalars [Cherman, Nellore '09]

Conjecture: universal bound

Further evidence: D-brane intersections

D-brane intersections

- $D3/D7$: flavor mass m, condensate c [Mateos, Myers, Thomson '07]

$$v_s^2 - \frac{1}{3} \simeq \frac{\lambda_{YM}}{24\pi^2}\frac{N_f}{N_c}\left(mc + \frac{1}{3}mT\frac{\partial c}{\partial T}\right) < 0.$$

- $D3/D7$ (flavors) at $T = 0$, $\mu \neq 0$
  Massless [Karch, Son, Starinets '08] and massive [Kulaxizi, Parnachev '08]

$$v_s^2 = \frac{\mu^2 - m^2}{3\mu^2 - m^2} \leq \frac{1}{3} \quad (v_s^2 < 1)$$

Field theory counterexample

- QCD at non-zero isospin chemical potential μ_I
- For $\mu_I > m_\pi \Rightarrow$ Pion condensate
- Speed of sound [Son, Stephanov '00]

$$v_s^2 = \frac{\mu_I^2 - m_\pi^2}{\mu_I^2 + 3m_\pi^2}$$

$v_s^2 \to 1$ when $m_\pi/\mu_I \to 0$

The bound in neutron stars

- Neutron stars: largest mass depends on equation of state
- Observations find up to $\sim 2M_{\odot}$
- Needs stiff equation of state
- Bound on the speed of sound strongly disfavored [Bedaque, Steiner '14]

The bound in neutron stars

- Neutron stars: largest mass depends on equation of state
- Observations find up to $\sim 2M_{\odot}$
- Needs stiff equation of state
- Bound on the speed of sound strongly disfavored [Bedaque, Steiner '14]

The bound does not hold for nuclear matter

Is there a bound for deconfined matter?
(with holographic dual)

‘Trivial’ breaking of the bound

‘Trivial’ breaking of the bound

$=$ the UV is not a CFT

D-brane intersections

- Dp/Dq: $(n|p \perp q)$ $n =$ common spatial directions $(q \geq p)$
  [Karch, Kulaxizi, Parnachev '09; Goykham, Parnachev, Zaanen '12; DiNunno, Ihl, Jokela, Pedraza '14; Jokela, Ramallo '15]

$$v_s^2 = \frac{2}{2n + \frac{p-3}{2}(p+q-2n-8)}$$

Dp/Dq	n	$SUSY$	$v_s^2 > 1/n$
$Dp/D(p+4)$	p	✓	$p > 3$
$Dp/D(p+2)$	$p-1$	✓	$p > 3$
Dp/Dp	$p-2$	✓	$p > 3$
$D4/D8/\overline{D8}$	3	✗	✓
$D3/D7'$	2	✗	✗
$D2/D8'$	2	✗	✓

Non-relativistic theories

Non-relativistic scaling

$$t \to \lambda^z t, \quad x^i \to \lambda x^i, i = 1, \ldots, n$$

Equation of state

$$p = \frac{z}{n}\varepsilon$$

Stiffer than a CFT for $z > 1$

Non-relativistic theories

Non-relativistic scaling

$$t \to \lambda^z t, \quad x^i \to \lambda x^i, i = 1, \ldots, n$$

Equation of state

$$p = \frac{z}{n}\varepsilon$$

Stiffer than a CFT for $z > 1$

Sound dispersion relation

[Hoyos, O'Bannon, Wu '10; Lee,Pang, Park'10; Dey, Roy '13]

$$\omega^2 = \frac{z}{n} C_{z,n} \rho^{2(z-1)/n} k^2$$

Valid for $2 > z \geq 1$. For $z > 2$ there is no sound mode.

A non-trivial counterexample

- Hint: neutron stars are high density states

- Hint: neutron stars are high density states

- However: many finite density states satisfy the bound
    - $\mathcal{N} = 4$ with R-charge (single $U(1) \subset SU(4)$) plus small mass deformation ($\mathcal{N} = 2^*$). Bound satisfied on critical line $\mu/T = \pi/\sqrt{2}$ [Buchel '10]
    - 3d SYM+dynamical flavors [Faedo, Kundu, Mateos, Pantelidou, Tarrio '15]
    - Flows between AdS and Lifshitz [Goldstein, Kachru, Prakash, Trivedi '09, Bertoldi, Burrington, Peet, (Zadeh) '10 ('11)]

- Hint: neutron stars are high density states
- However: many finite density states satisfy the bound
  - $\mathcal{N}=4$ with R-charge (single $U(1)\subset SU(4)$) plus small mass deformation ($\mathcal{N}=2^*$). Bound satisfied on critical line $\mu/T=\pi/\sqrt{2}$ [Buchel '10]
  - 3d SYM+dynamical flavors [Faedo, Kundu, Mateos, Pantelidou, Tarrio '15]
  - Flows between AdS and Lifshitz [Goldstein, Kachru, Prakash, Trivedi '09, Bertoldi, Burrington, Peet, (Zadeh) '10 ('11)]

First step: study the bound in a simple model

The model

- $\mathcal{N} = 4$ SYM with R-charge (same in two $U(1) \subset SU(4)$)
- Consistent truncation from 10d to 5d [Gunaydin, Romans, Warner '86]

$$S = \frac{1}{2\kappa^2} \int d^5x \left(R - \frac{1}{4} e^{4\phi} F_{\mu\nu}^2 - 12(\partial_\mu \phi)^2 - V(\phi) \right)$$

$$V(\phi) = -\frac{1}{L^2} \left(8e^{2\phi} + 4e^{-4\phi} \right)$$

The model

- Analytic solution
  [Cvetic, Duff, Hoxha, Liu, Lu, Martinez-Acosta, Pope, Sati, Tran ' 99]

$$ds^2 = \frac{L^2 e^{2B}}{f(r)} dt^2 + \frac{r^2}{L^2} e^{2A} \left(-f(r)dt^2 + d\vec{x} \cdot d\vec{x}\right)$$

$$A = \frac{1}{3} \log\left(1 + Q^2 \frac{L^2}{r^2}\right), \quad B = -\frac{2}{3} \log\left(1 + Q^2 \frac{L^2}{r^2}\right),$$

$$\phi = \frac{1}{6} \log\left(1 + Q^2 \frac{L^2}{r^2}\right), \quad f = 1 - \frac{M^2}{\left(Q^2 + \frac{r^2}{L^2}\right)^2},$$

$$A_t = \sqrt{2} QM \left(\frac{1}{Q^2 + \frac{r^2}{L^2}} - \frac{1}{Q^2 + \frac{r_H^2}{L^2}}\right).$$

- Asymptotic expansion of the scalar $r \to \infty$

$$\phi = \frac{Q^2 L^2}{6r^2} + \cdots$$

- Normalizable mode $\leftrightarrow$ vev

$$v_0 = \frac{Q^2}{6L^2} = \frac{\mu^2}{12}$$

- Non-normalizable mode $\leftrightarrow$ relevant deformation

$$\phi = \frac{L^4 J}{r^2} \log \frac{r}{L} + \frac{L^4 v_0}{r^2} + \cdots$$

$J \sim (\text{mass})^2$

Thermodynamics $J = 0$

Vev of scalar operator

$$\langle \mathcal{O} \rangle = -\frac{3N^2}{8\pi^2} v_0$$

Conformal equation of state

$$\left\langle T^\mu_\mu \right\rangle = 0$$

$$\varepsilon_0 = 3p_0 = \frac{3\pi^2 N^2}{64} \left(T^2 + \frac{\mu^2}{2\pi^2} \right)^2$$

Speed of sound

$$v_s^2 = \frac{1}{3}$$

Thermodynamics $J \neq 0$

Vev of scalar operator (κ = finite counterterm)

$$\langle \mathcal{O} \rangle = \frac{N^2}{32\pi^2} \left(\kappa J - 12 v \right)$$

Non-conformal equation of state

$$\langle T^\mu_\mu \rangle + 2J \langle \mathcal{O} \rangle = \mathcal{A}, \quad \mathcal{A} = \frac{3N^2}{16\pi^2} J^2$$

Energy density and pressure

$$\varepsilon = \varepsilon_0 + \sigma \quad p = p_0 - \sigma$$

$$\sigma = \frac{N^2}{64\pi^2} J \left((\kappa - 3) J - 12 v \right)$$

Perturbative expansion

$$J \ll T^2 + \frac{\mu^2}{2\pi^2}$$

Deviation from conformal invariance $v = v_0 + \delta v$

$$\sigma \simeq -\frac{3N^2}{16\pi^2} J v_0 = -\frac{N^2}{64\pi^2} J \mu^2$$

Speed of sound

$$v_s^2 \simeq \frac{1}{3} + \frac{T}{\varepsilon_0}\left[\frac{\partial\sigma}{\partial T} - \left(\frac{\mu}{2T} + \frac{3\pi^2 T}{\mu}\right)\frac{\partial\sigma}{\partial\mu}\right]$$

$$v_s^2 \simeq \frac{1}{3} + \frac{J}{3\pi^2}\frac{6T^2 + \frac{\mu^2}{\pi^2}}{\left(T^2 + \frac{\mu^2}{2\pi^2}\right)^2} > \frac{1}{3} \quad (J > 0)$$

Perturbative expansion

$$J \ll T^2 + \frac{\mu^2}{2\pi^2}$$

Scalar field solution

$$\phi = \phi^{(0)} + \delta\phi$$

Other fields: metric and gauge field

$$A = A^{(0)} + \delta A \quad f = f^{(0)} + \delta f \quad A_t = A_t^{(0)} + \delta A_t$$

Gauge fixing

$$\delta B = 0$$

Solve a system of linear ODEs

Perturbative expansion

Asymptotic expansion of scalar

$$\delta\phi = \frac{L^4 J}{r^2} \log \frac{r}{L} + \frac{L^4 \delta v}{r^2} + \cdots$$

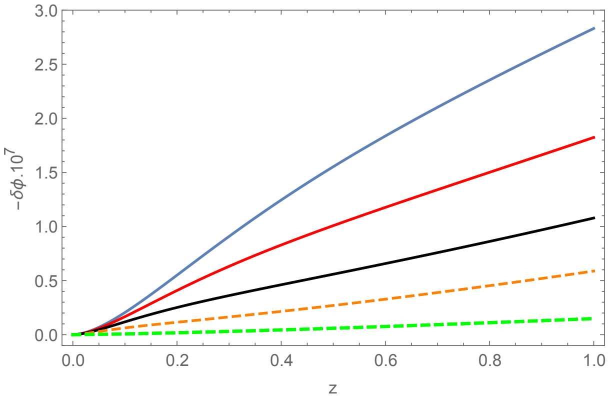
Fix boundary metric and chemical potential

$$\lim_{r\to\infty} \delta A, \ \delta f, \ \delta A_t = 0$$

Impose regularity at the horizon $\Rightarrow$ solution completely determined by J

Perturbative expansion

Numerical solutions $z = r_H/r$



Curve goes down as μ/T increases

Conclusions and outlook

- No bound on v_s in finite density systems
- How large can v_s be beyond perturbative approximation?
- What are the general conditions for $v_s^2 > 1/3$?
- Can there be strongly coupled quark matter in the core of neutron stars?