A holographic model of Weyl semi-metal

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Based on

Karl Landsteiner, YL, **1505.04772**

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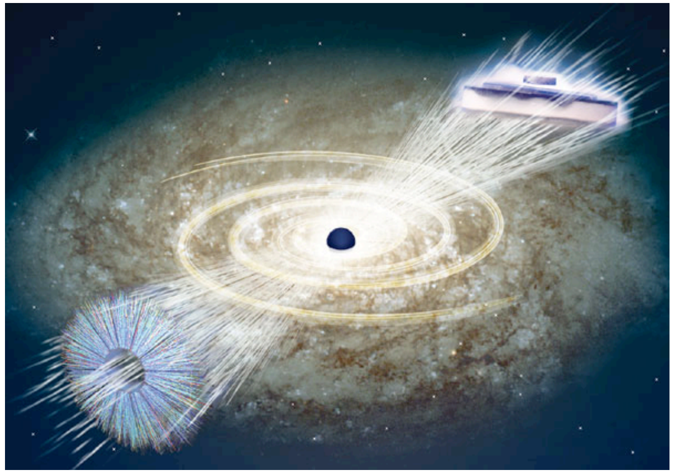
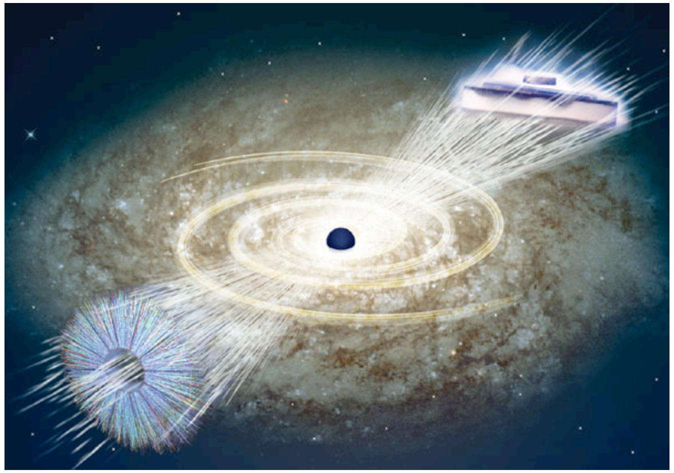
AdS/CMT



- Many phenomena observed in condensed matter systems can be studied by general relativity.
    - Superconducting/superfluid phase transition
    - Fermi surface, non Fermi liquid
    - Lattice/impurity effect …

[Hartnoll, Horowitz, Herzog, H.Liu, McGreevy, Schalm, Zaanen, Tong, et al.]

AdS/CMT



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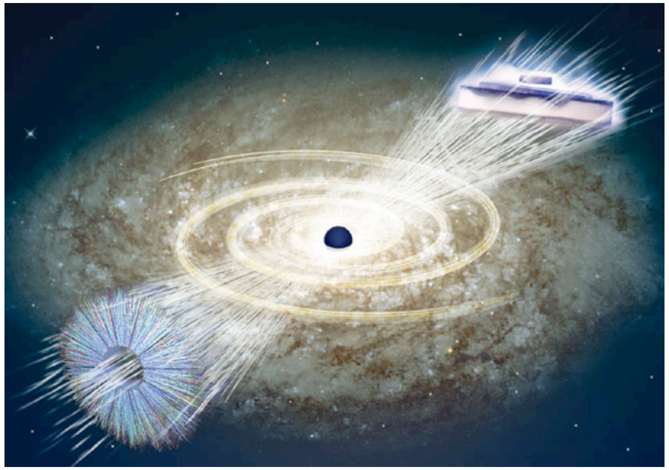
- It is a consequence of **Gauge/gravity duality**

Complicated strongly coupled phenomena ⟷ Simple gravitational dynamics

AdS/CMT



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* AdS/other examples of condensed matter system?

  - Weyl semi-metal (a topological state)

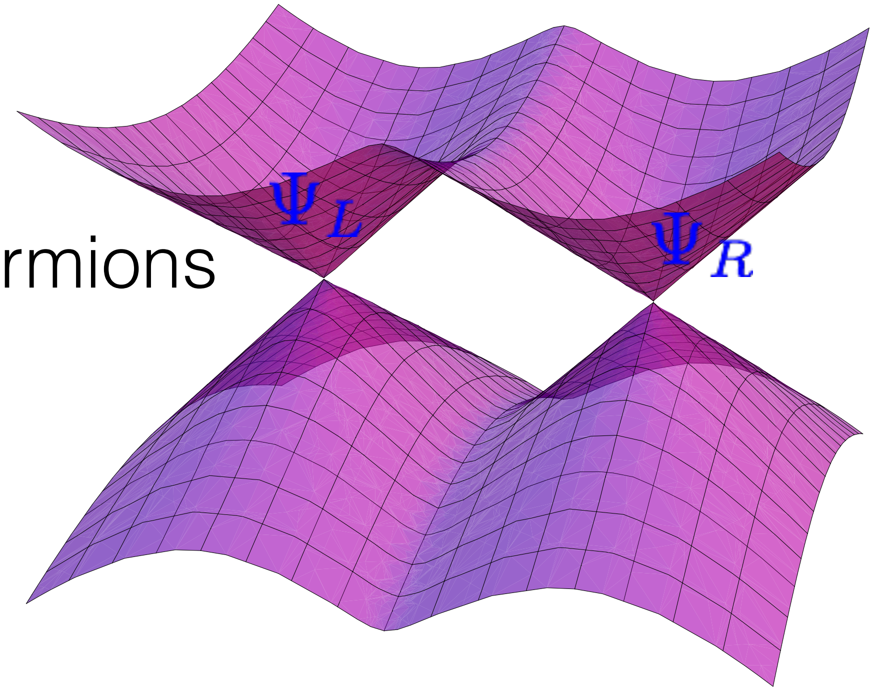
Outline

- Weyl semi-metal (WSM): QFT model

- Holographic model of WSM

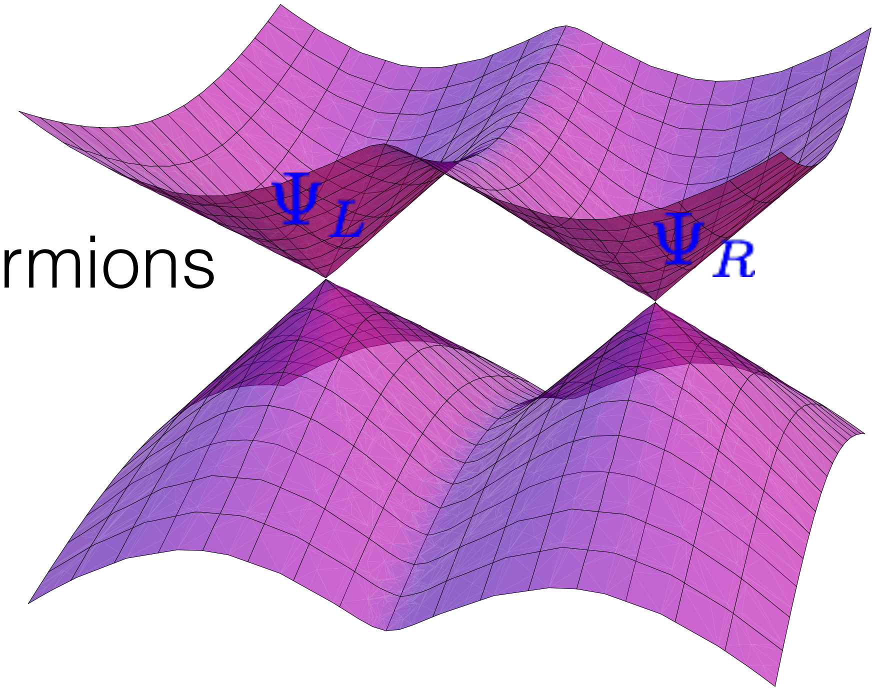
- Summary and open questions

Weyl semi-metal



- Low energy excitation: Weyl fermions
- conduction and valence
  band (linear) touch

Weyl semi-metal

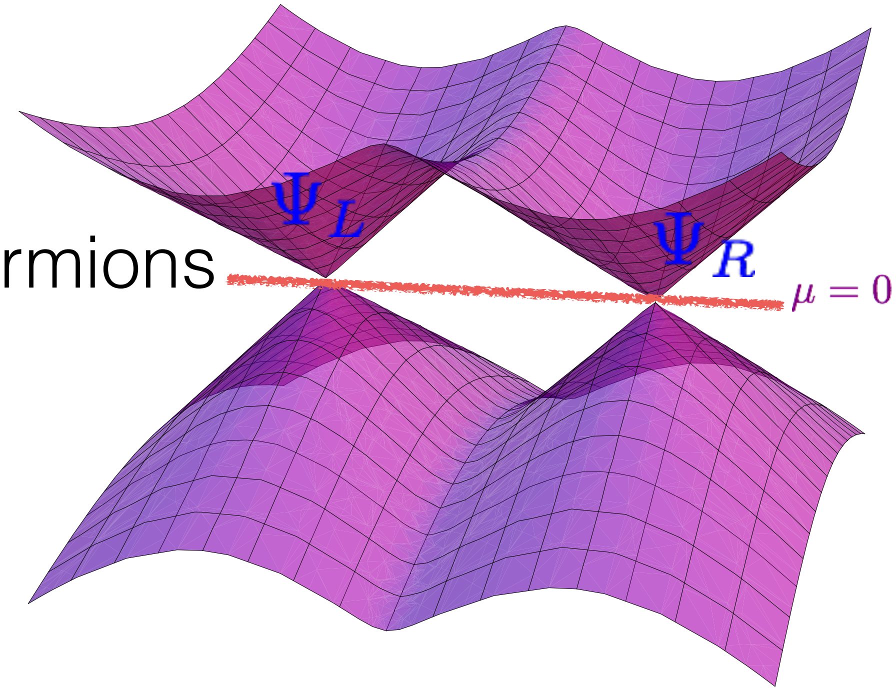


- Low energy excitation: Weyl fermions
- conduction and valence band (linear) touch

- Weyl points:
    - Close to Weyl points, Weyl equation
    - Weyl points appear in +/- pairs with opposite chiralities [Nielsen,Ninomiya]
    - Weyl points are topologically protected in momentum space

- Experimentally realisation: TaAs (2015)…

Weyl semi-metal



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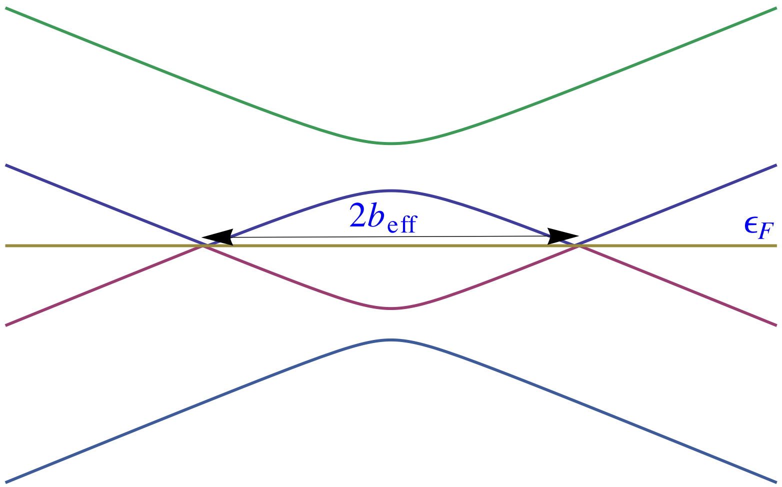
QFT of WSM

$$\boxed{\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu + M - \gamma_5\gamma_z b\right)\Psi\,.}$$

spectrum

$M < b: \quad b_{\text{eff}} = \sqrt{b^2 - M^2}$



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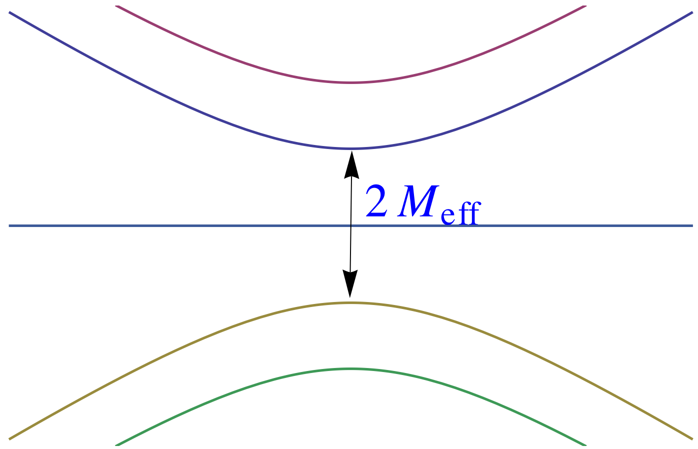


QFT of WSM

- Topological phase

$$M < b: \qquad b_{\text{eff}} = \sqrt{b^2 - M^2} \qquad \mathcal{L}_{\text{eff}} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - \gamma_5 \gamma_z b_{\text{eff}}\right)\psi$$

constant axial gauge field $\quad A_z^5 = b_{\text{eff}}$

axial gauge transformation $\quad \theta_5 = b_{\text{eff}} z$

axial anomaly $\quad W = \int d^4x \theta_5 F \wedge F$

electric current $\quad J^\mu = \dfrac{\delta W}{\delta A_\mu}$

- Anomalous Hall Effect (AHE) [Haldane, 1987]

$$\mathbf{J} = \frac{e^2}{2\pi^2} \mathbf{b}_{\text{eff}} \times \mathbf{E}$$

QFT of WSM

$$M > b: \qquad M_{\text{eff}} = \sqrt{M^2 - b^2} \qquad \mathcal{L}_{\text{eff}} = \bar{\psi}\left(i\gamma^\mu \partial_\mu + M_{\text{eff}}\right)\psi$$

- gapped phase with vanishing AHE.

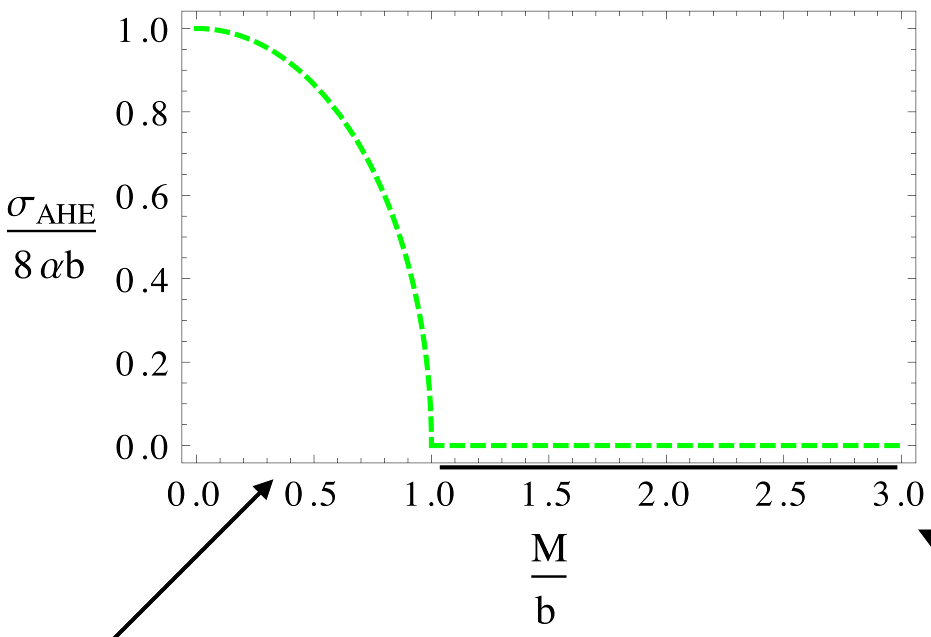
- More generally, more Dirac cones can be present *topologically trivial semi-metal*

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu \partial_\mu + M - \gamma_5 \gamma_z b\right)\Psi + \sum_{j=1}^{N} \bar{\psi}(i\gamma^\mu \partial_\mu)\psi$$

QFT of WSM

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu + M - \gamma_5\gamma_z b\right)\Psi\,.$$

- Topological phase can be characterised by a **nonzero** anomalous Hall conductivity.



topological phase

Trivial phase

Motivation for Holographic WSM

- How does WSM work in strongly coupled case?

without quasiparticle,
no notion of Berry phase (Weyl points)

A holography model for WSM can teach us qualitative lessons!

AdS/CFT: dictionary

[Maldacena; Witten; Gubser, Klebanov, Polyakov]

Bulk AdS	**Boundary field theory**
4+1D gravity weakly coupled	3+1D field theory strongly coupled
metric	stress tensor
gauge field	current
local symmetry	global symmetry
scalar field	scalar operator

$$e^{iS_{\text{bulk}}}\Big|_{\phi\to J} = \langle exp(i\int d^4x\sqrt{-g_0}J\mathcal{O})\rangle$$

Model WSM from gravity

- Weakly coupled theory $\mathcal{L} = \bar{\Psi}\left(i\gamma^\mu\partial_\mu + M - \gamma_5\gamma_z b\right)\Psi\,.$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^\mu_5 = \frac{1}{16\pi^2} 3\mathcal{F}\wedge\mathcal{F} + 2M\bar{\Psi}\gamma_5\Psi$$

- **Ingredients** for AdS gravity model:
  - Gravity with negative cosmological constant
  - One gauge field dual to electric current
  - One gauge field dual to axial current
  - One scalar field charged under axial gauge symmetry

Holographic WSM

- Holographic model

$$\mathcal{L} = \frac{1}{2\kappa^2}\Big(R + \frac{12}{L^2}\Big) - \frac{1}{4}\mathcal{F}^2 - \frac{1}{4}F_5^2$$
$$+ \frac{\alpha}{3}A_5 \wedge \big(F_5 \wedge F_5 + 3\mathcal{F} \wedge \mathcal{F}\big) +$$
$$+|(\partial_\mu - iqA_\mu^5)\Phi|^2 - V(\Phi)$$

- CS structure = anomaly form
- the dimension of the dual scalar operator is chosen to be 3, mass deformation

Holographic WSM

- Currents:

$$J^\mu = \frac{\delta S}{\delta A_\mu(r=\infty)} = \lim_{r\to\infty}\sqrt{-g}\Big(\mathcal{F}^{\mu r} + 4\alpha\epsilon^{r\mu\nu\rho\alpha}A^5_\nu\mathcal{F}_{\rho\alpha}\Big)$$

$$J^\mu_5 = \frac{\delta S}{\delta A^5_\mu(r=\infty)} = \lim_{r\to\infty}\sqrt{-g}\Big(F_5^{\mu r} + 4\alpha\epsilon^{r\mu\nu\rho\alpha}A^5_\nu F^5_{\rho\alpha}\Big)$$

- Wald identity

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu J^\mu_5 = \Big(\frac{\alpha}{3}\Big[F_5\wedge F_5 + 3\mathcal{F}\wedge\mathcal{F}\Big] - iq\sqrt{-g}\big[\Phi(D_r\Phi)^* - \Phi^*(D_r\Phi)\big]\Big)\Big|_{r\to\infty}$$

Holographic WSM

- **Ansatz (T=0)**

$$ds^2 = u(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u} + h dz^2 \,,$$

$$\Phi = \phi \,,$$

$$A^5 = A^5_z dz \,.$$

- **Near UV**

Metric: $$ds^2|_{r\to\infty} = \frac{dr^2}{r^2} + r^2(-dt^2 + d\vec{x}^2)$$

vector gauge field: $$A_\mu = 0$$

axial gauge field: $$A^5_\mu|_{r\to\infty} = b\delta^z_\mu$$

scalar field: $$r\Phi_\mu|_{r\to\infty} = M$$

Holographic WSM

At zero temperature: 3 distinct classes of solutions (leading order solution @ IR)

- **M/b<0.744** (Topological phase) $A_z^5(0) = b_{\text{eff}}, \quad \Phi(0) = 0$
- **M/b=0.744** (Critical point) $A_z^5 = r^\beta, \quad \Phi(0) = \phi_0$

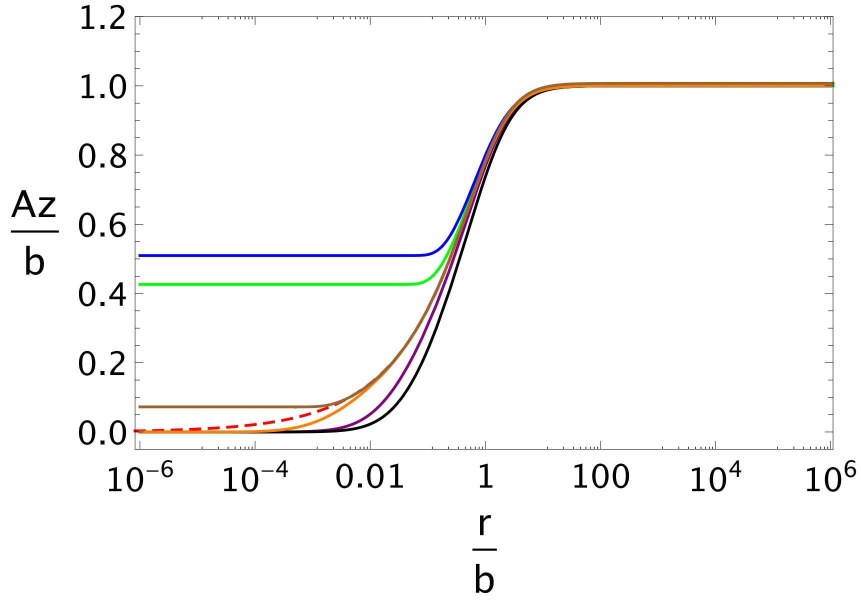
$$ds^2 = u_0 r^2(-dt^2 + dx^2 + dy^2) + \frac{dr^2}{u_0 r^2} + h_1 r^{2\beta} dz^2,$$

- **M/b>0.744** (Trivial phase) $A_z^5(0) = 0, \quad \Phi(0) = \phi_{\text{min}}$

$$\frac{dV}{d\phi}(\phi_{\text{min}}) = 0$$

Holographic WSM

Bulk profile of axial gauge field



$M/b = 0.695$

$M/b = 0.712$

$M/b = 0.743$

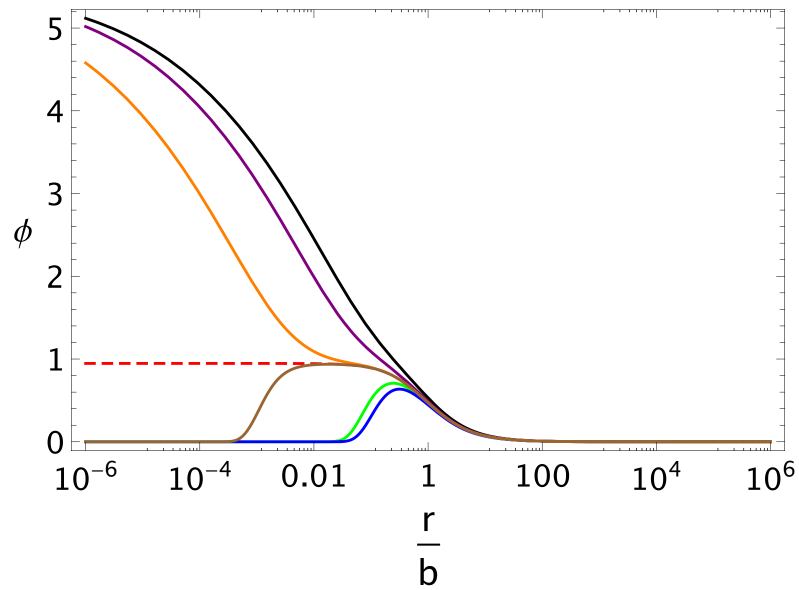
$M/b = 0.744$

$M/b = 0.745$

$M/b = 0.778$

$M/b = 0.856$

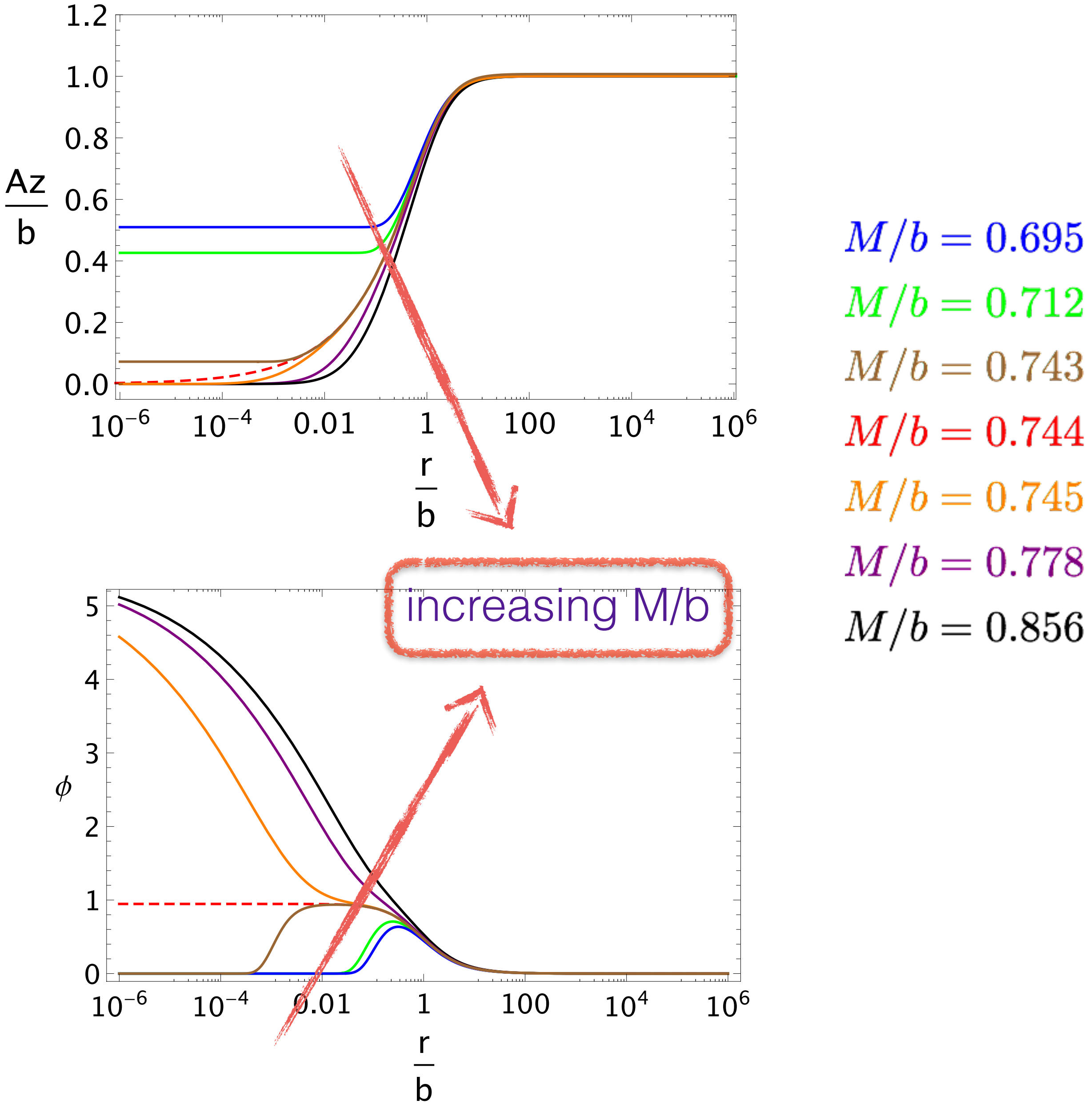
Bulk profile of scalar field



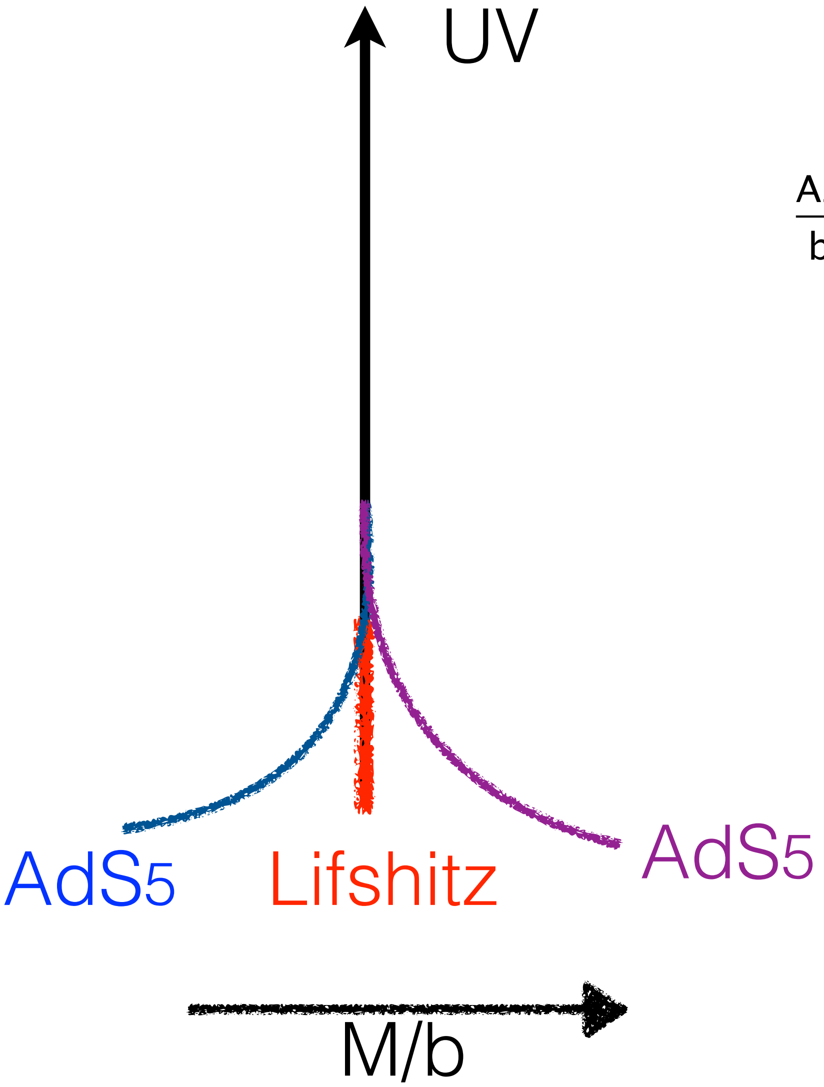
Holographic WSM

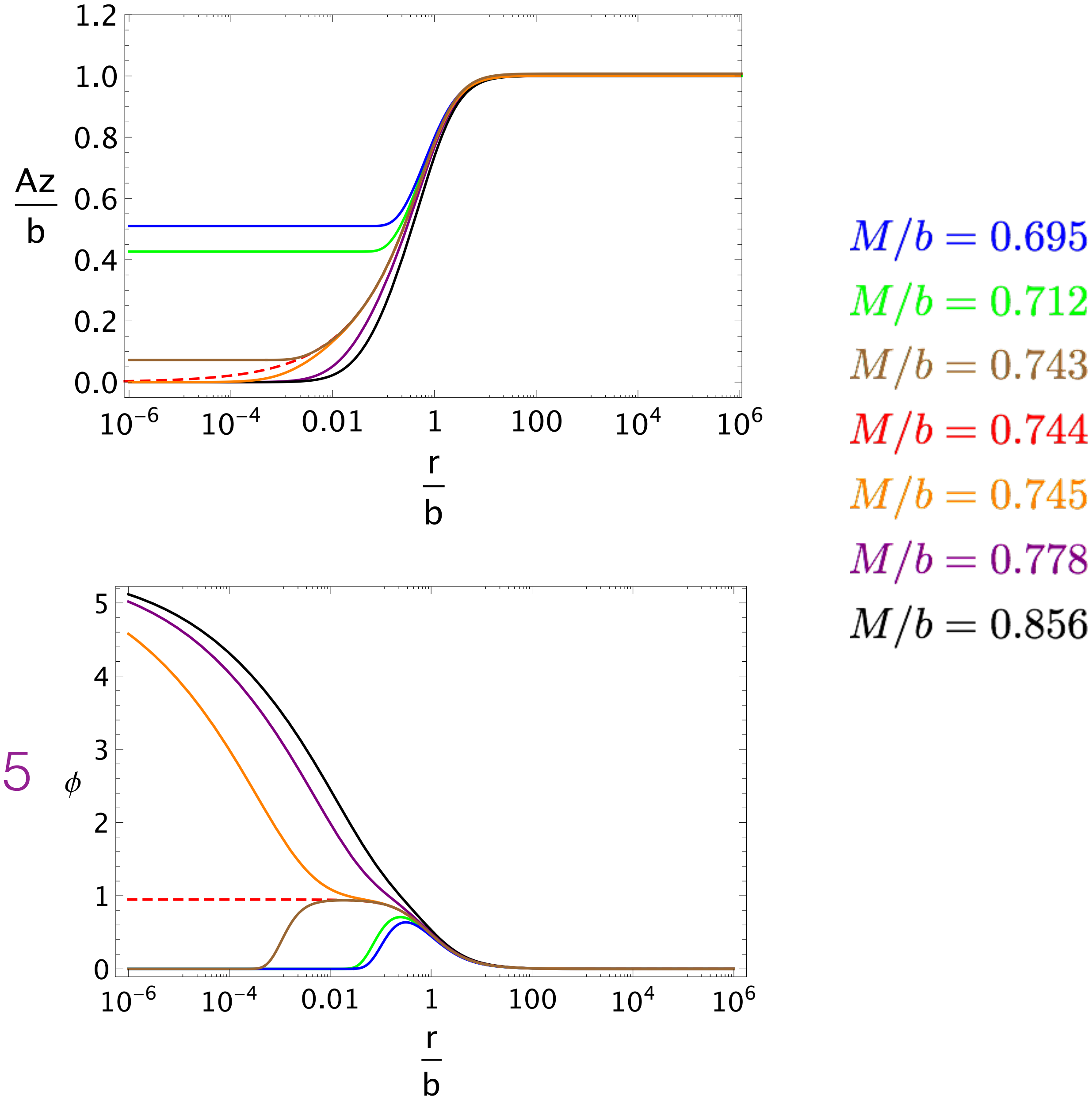
Bulk profile of axial gauge field

Bulk profile of scalar field

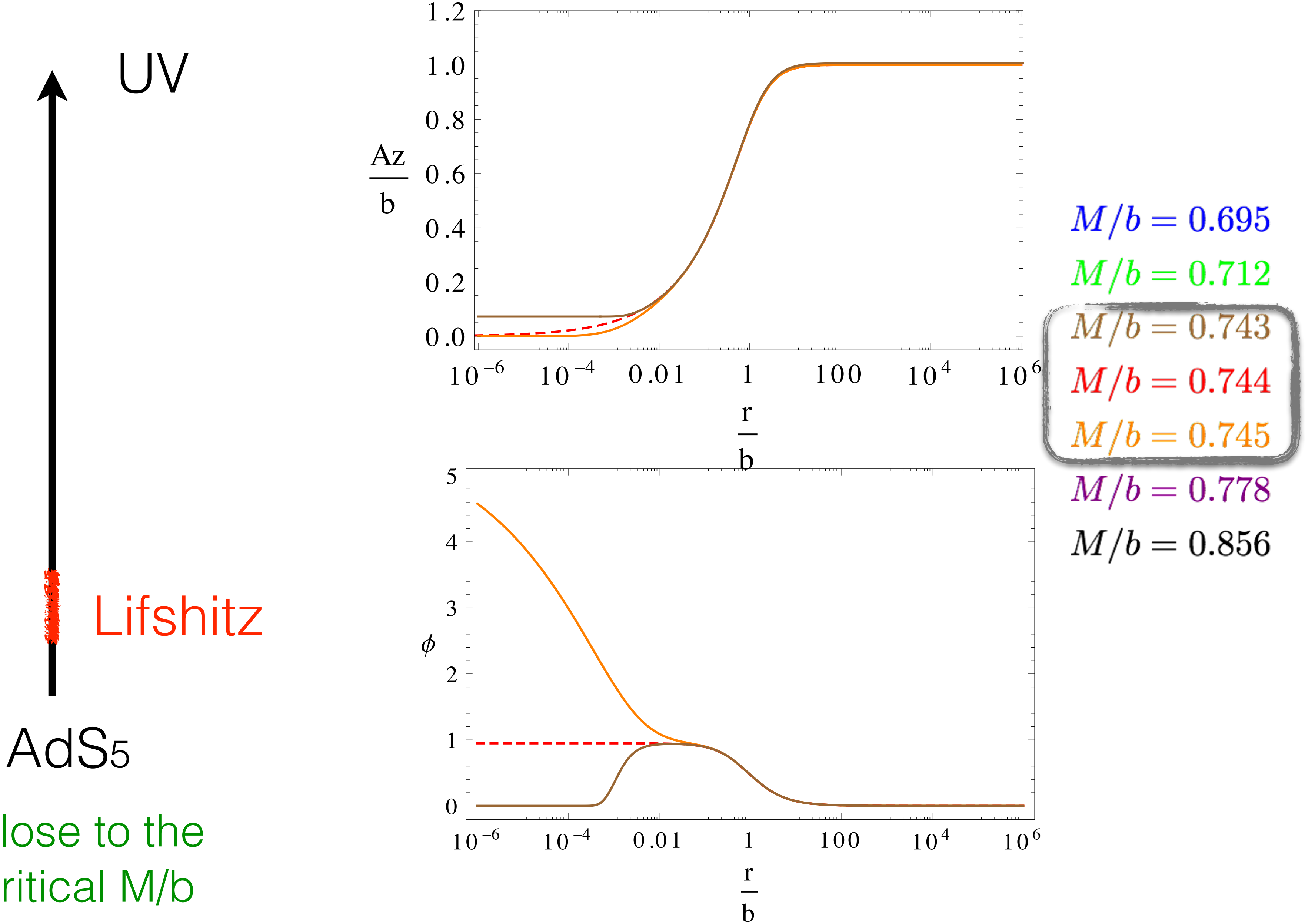


Holographic WSM





Holographic WSM

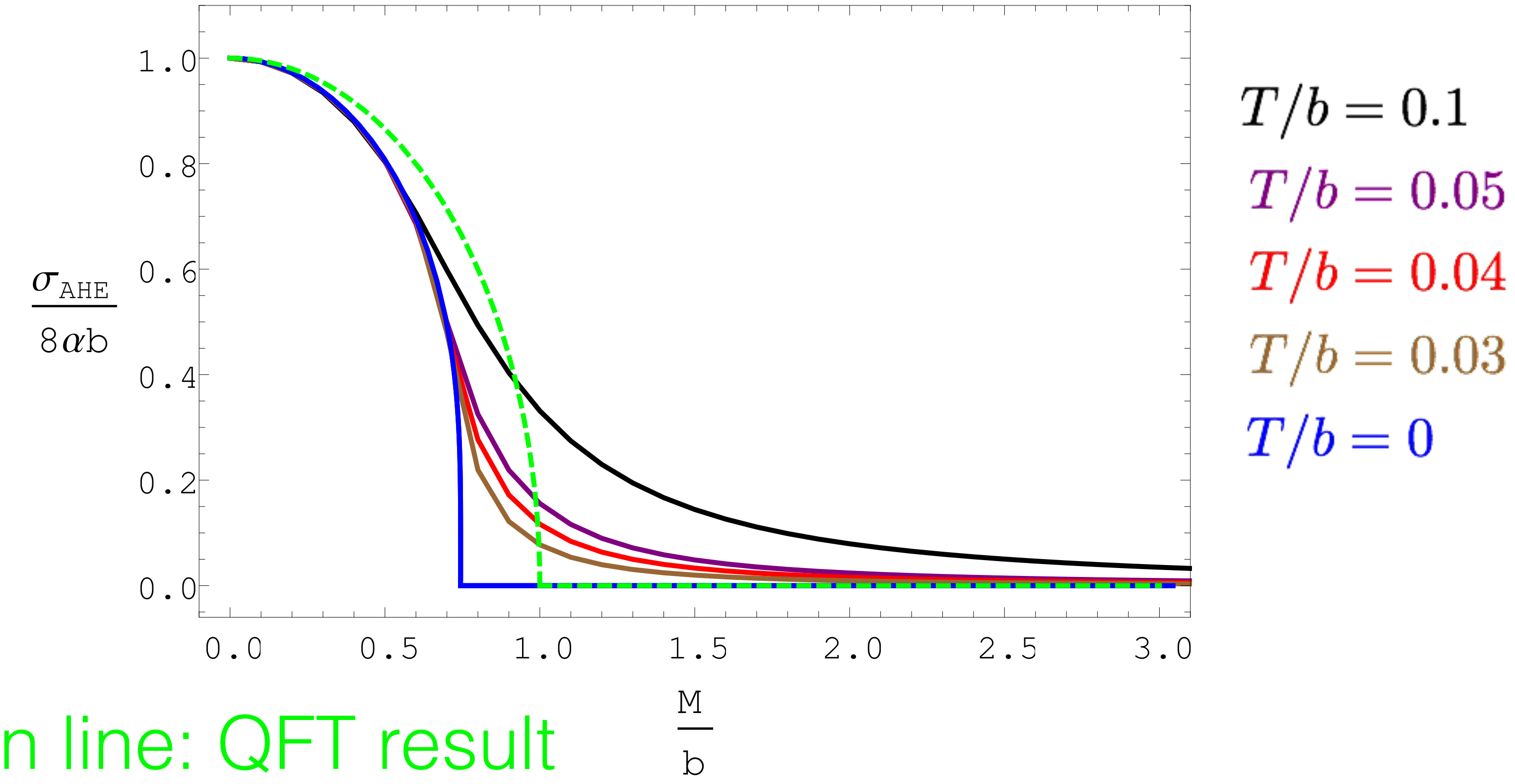


Close to the critical M/b

Holographic WSM

- Order parameter of topological state of matter: AHE

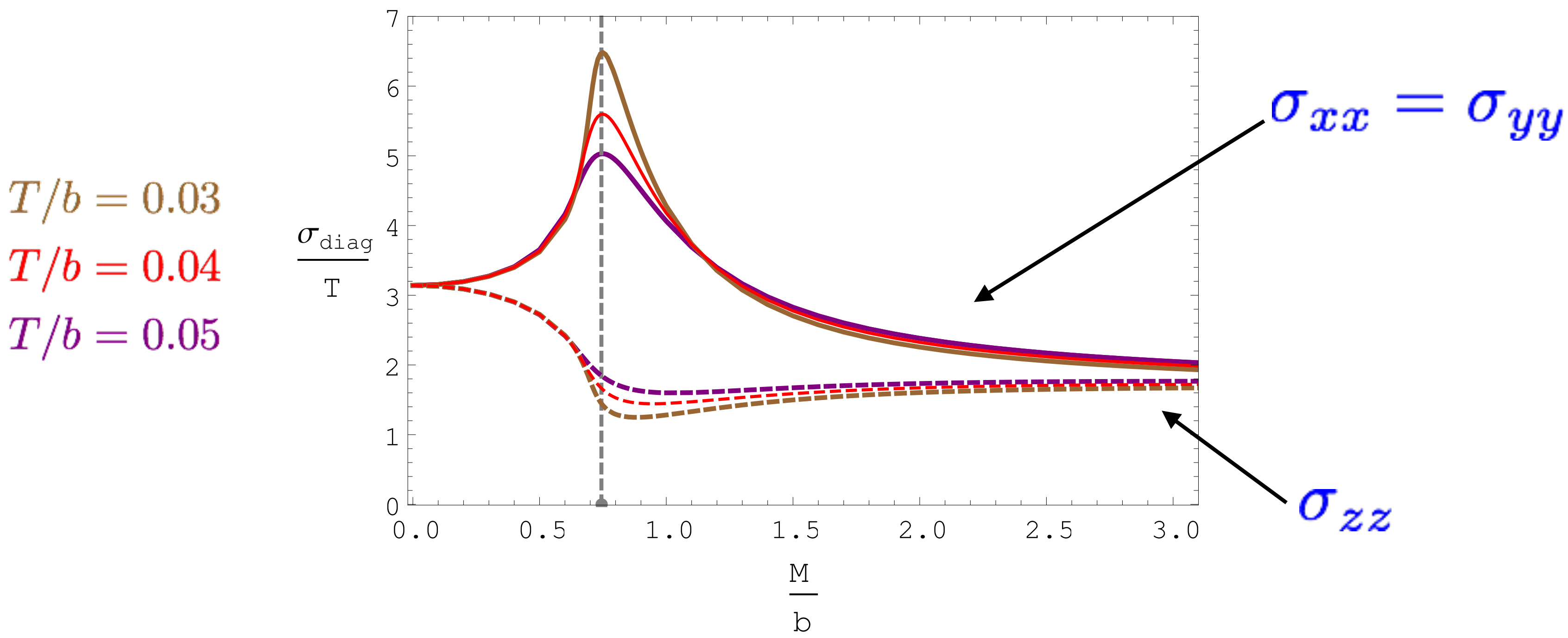
$$\sigma_{\text{AHE}} = 8\alpha A_z^5(0)$$



- green line: QFT result

Holographic WSM

- Diagonal conductivities at T=0: $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0$
- Diagonal conductivities at T>0:



Summary

- A holographic model for Weyl semi-metal using gauge/gravity duality.
- Order parameter = AHE
- Varying M/b, a quantum phase transition between topological non trivial state and topological trivial state.
- Diagonal conductivities (peak/dip behaviour)

Open questions

- Holographic realisation of phase transition between topological non-trivial state and insulator?
- Holographic surface state (Fermi arc)?
- Effective field theory description for the topological phase transition from holographic WSM?
- ...

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Thank You!

Free energy