Black Holes
And Democratic Systems

Javier M. Magán

ArXiv:1601.04663
ArXiv:1508.05339 (Phys. Rev. Lett)

Delta Institute for Theoretical Physics
ITP
Universiteit Utrecht

What I would like to communicate today:

A question:
What are the dynamics of Information in
Black holes and democratic systems?

A toy model:
Random particles

An answer:
Large-N factorization as
Extensivity of entanglement evolution

What I would like to communicate today:

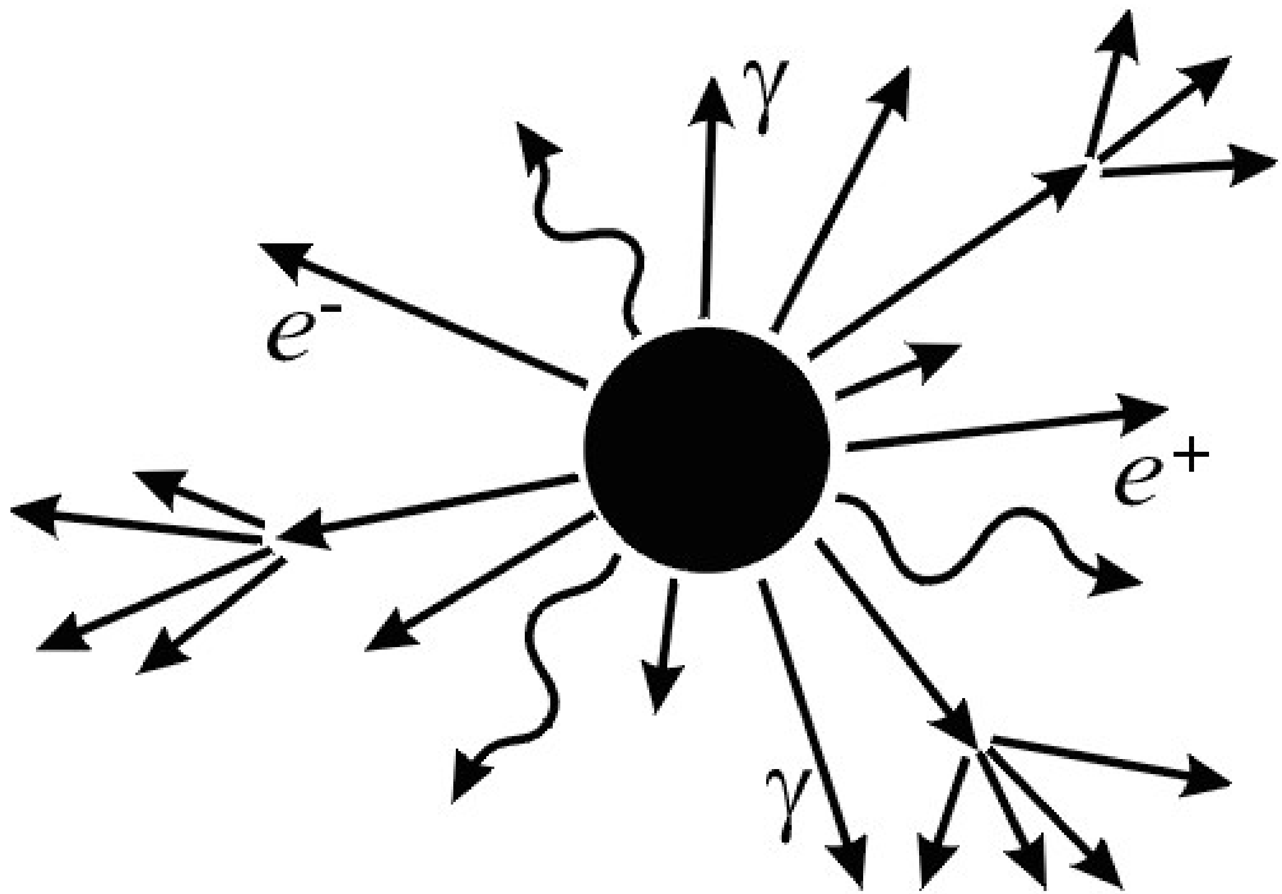
A question:
What are the dynamics of Information in
Black holes and democratic systems?

A toy model:
Random particles

An answer:
Large-N factorization as
Extensivity of entanglement evolution

Do not be scared!! I do not assume familiarity with these concepts!!!

$$S_{\mathrm{BH}} = \frac{A}{4\,G} \qquad T_{\mathrm{H}} = (\frac{\partial\, S}{\partial M})^{-1}$$



A Question:

What are the Dynamics of Information ?

Why?

Defines the interaction structure of the microscopic model.
No need of conserved charges!
Information transport always appear.

↓

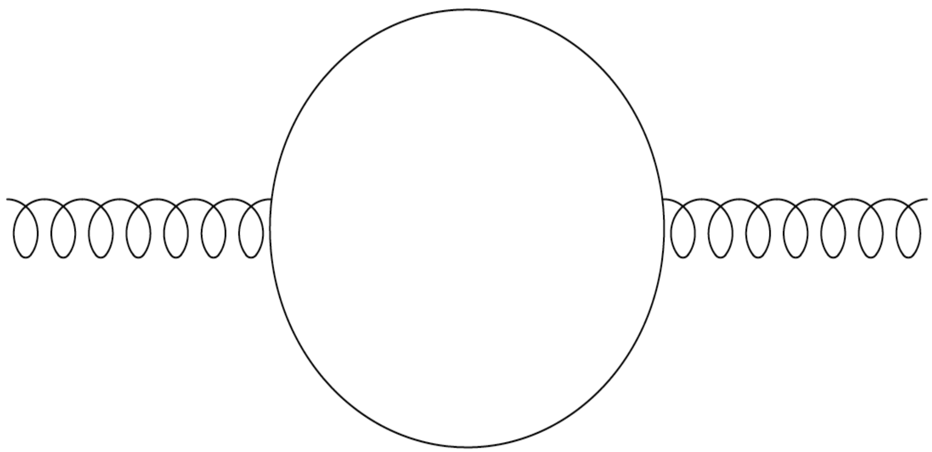
Potential implications for:

Information paradox (Hawking)
Fast Scrambling Conjecture (Hayden and Preskill, Sekino and Susskind)
Event Horizon locality (Many authors).
Entanglement and geometry (Many authors).
Microscopic models of Black Holes(AdS/CFT, Sachdev, Gomez/Dvali...)

String Theory Hamiltonians:

Non-perturbatively described by
Large-N Matrix Models (Gauge Theories)

Banks,Fischler,Shenker,Susskind (1997)
Maldacena's AdS/CFT (1998)



$$H \supset \sum_{ij}^{N} \pi_i A_{ij} \pi_j$$

Black Holes → High energy states
Very difficult problem, even at infinite N.
We want finite N!!

Gauge invariant entanglement entropy?

Sadchev-Ye-Kitaev model:

Fermions with random quartic couplings

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;lm}\, c_i^\dagger\, c_j^\dagger\, c_l\, c_m$$

$J_{ij;lm}$ → Random numbers

Black Holes → High energy states

Very difficult problem, even at infinite N.

Sadchev-Ye-Kitaev model:

Fermions with random quartic couplings

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;lm}\, c_i^\dagger\, c_j^\dagger\, c_l\, c_m$$

$J_{ij;lm}$ → Random numbers

Black Holes → High energy states

Very difficult problem, even at infinite N.

Too difficult problems at finite N!!!

A Toy Model?

First requirement:

Solvability

Second requirement:

Quantum thermalization
(two different approaches: correlators and entanglement entropies)

Third requirement:

Democracy

Second requirement:

Quantum Many Body system
displaying Quantum Thermalization

$$\langle \psi(t) | \mathcal{O}_{\mathbf{A}} | \psi(t) \rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}} \mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Second requirement:

Quantum Many Body system displaying Quantum Thermalization

$$\langle \psi(t) | \mathcal{O}_{\mathbf{A}} | \psi(t) \rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}} \mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Eigenstate Thermalization Hypothesis (ETH)

$$\langle E | \, \mathcal{O}_{\mathbf{A}} \, | E \rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}} \, \mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Srednicki (1994)
Jensen and Shankar (1985)
Gogolin,Eisert (2015)

Second requirement:

Quantum Many Body system displaying Quantum Thermalization

$$\langle\psi(t)|\mathcal{O}_{\mathbf{A}}|\psi(t)\rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}}\mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Eigenstate Thermalization Hypothesis (ETH)

$$\langle E|\,\mathcal{O}_{\mathbf{A}}\,|E\rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}}\,\mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Srednicki (1994)
Jensen and Shankar (1985)
Gogolin,Eisert (2015)

Information theory and entanglement thermalization

$$\rho_{\mathrm{A}}$$

⬇

$$S_{\mathrm{A}}$$

Second requirement:

Quantum Many Body system displaying Quantum Thermalization

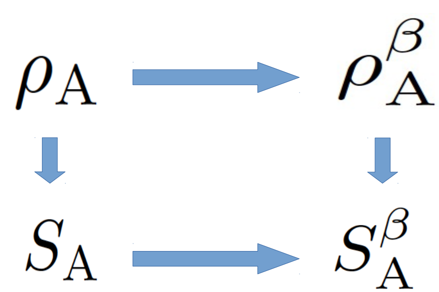
$$\langle\psi(t)|\mathcal{O}_{\mathbf{A}}|\psi(t)\rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}}\mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Eigenstate Thermalization Hypothesis (ETH)

$$\langle E|\,\mathcal{O}_{\mathbf{A}}\,|E\rangle = \mathrm{Tr}(\rho^{\mathrm{Gibbs}}\,\mathcal{O}_{\mathbf{A}}) \pm \mathrm{error}$$

Srednicki (1994)
Jensen and Shankar (1985)
Gogolin,Eisert (2015)

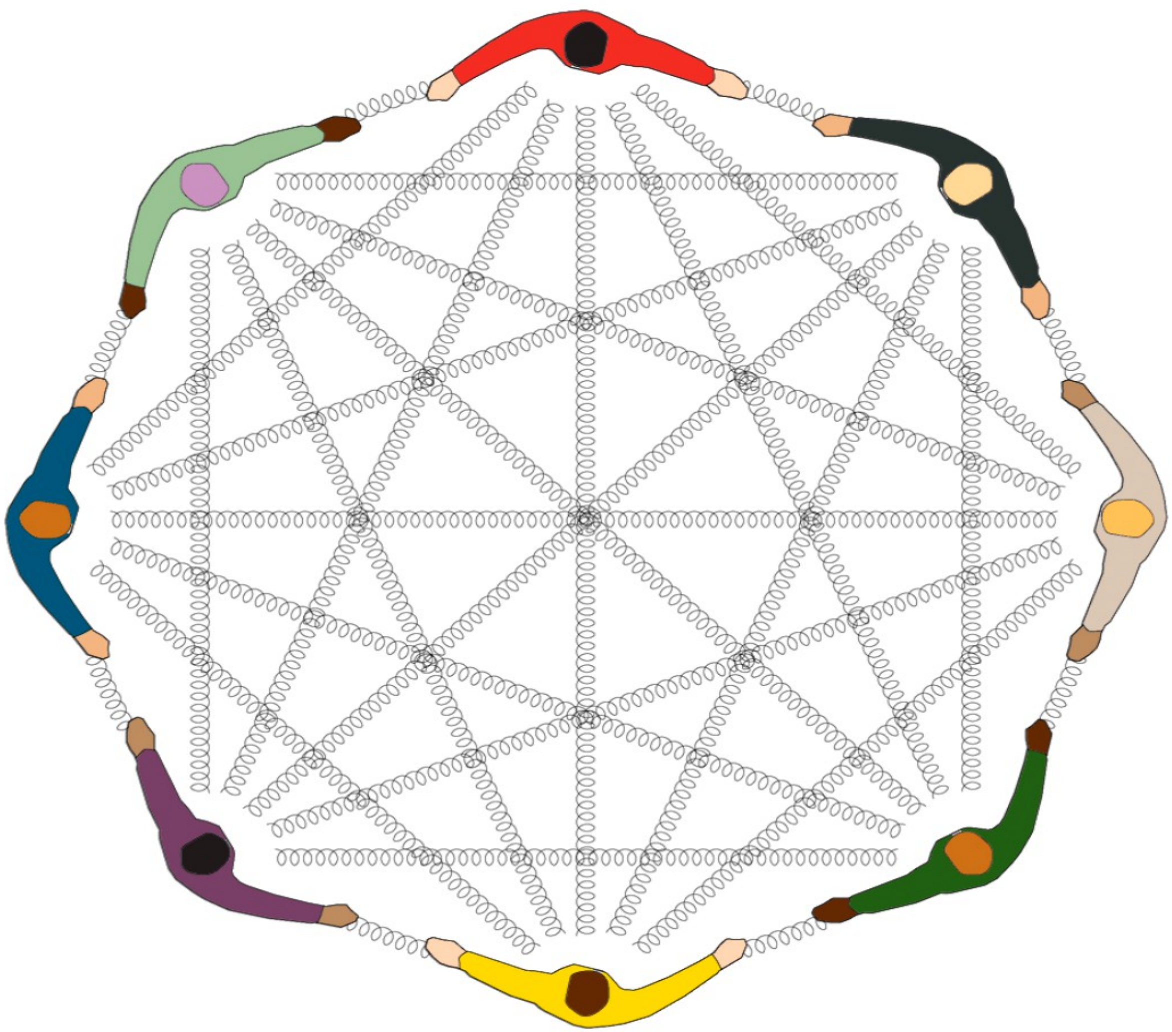
Information theory and entanglement thermalization



Third requirement:

"Democratic" system.

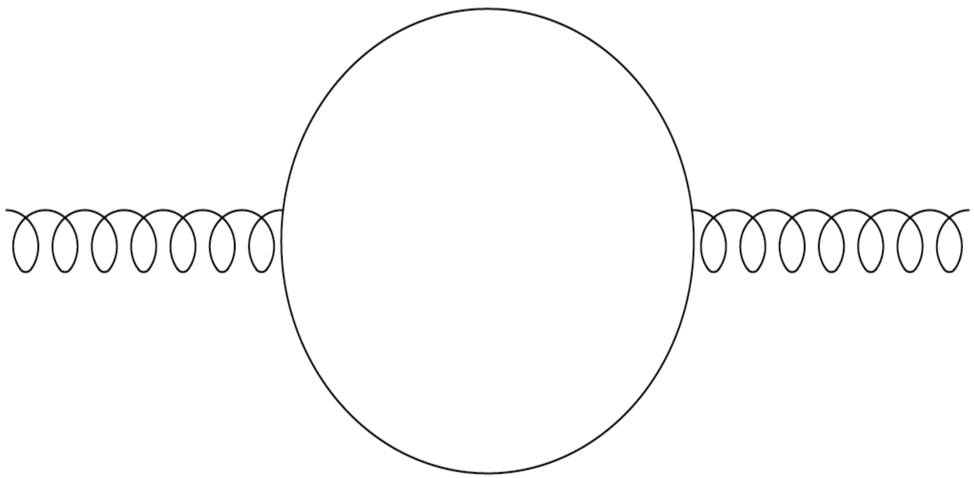
(As non-local as it can be.
There is no locality structure whatsoever)



Previous examples are democratic!

String Theory Hamiltonians:

Large-N Matrix Models



$$H \supset \sum_{ij}^{N} \pi_i A_{ij} \pi_j$$

Sadchev-Ye-Kitaev model:

Fermions with random quartic couplings

$$H = \frac{1}{(2N)^{3/2}} \sum_{i,j,k,l=1}^{N} J_{ij;lm}\, c_i^\dagger\, c_j^\dagger\, c_l\, c_m$$

$J_{ij;lm}$ → Random numbers

What are the common features?

First requirement:

Solvability

Second requirement:

Quantum thermalization
(two different approaches: correlators and entanglement entropies)

Third requirement:

Democracy

The Toy Model:
Black Holes as “random particles”

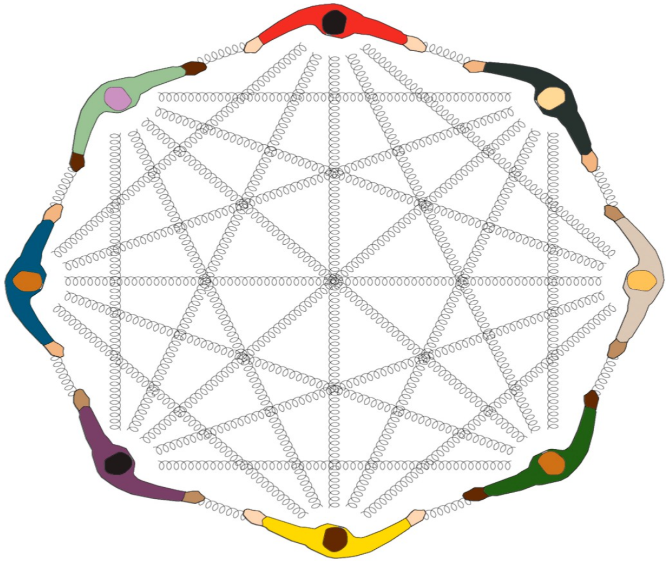
J.M.M (arXiv:1601.04663)

Complexity comes to the rescue. Take the simplest (gaussian/free) but most complex Hamiltonian

$$H = \alpha \sum_{i=1}^{N} c_i^\dagger \, c_i + \eta \sum_{i,j=1}^{N} c_i^\dagger \, V_{ij} \, c_j$$

A system of free particles with Gaussian random couplings:

A collection of “random particles”



Information/Entanglement structure of Eigenstates

J.M.M (arXiv:1508.05339, Phys. Rev. Lett)

Within time independent hamiltonians,
random particles constitute the first analytical example of:

Information/Entanglement structure of Eigenstates

J.M.M (arXiv:1508.05339, Phys. Rev. Lett)

Within time independent hamiltonians,
random particles constitute the first analytical example of:

Eigenstate Thermalization Hypothesis

$$C_{ij}^{E_{N_p}} = \langle\, E_{N_p} |\, c_i^\dagger c_j \,|\, E_{N_p} \rangle = \frac{N_p}{N} + \mathrm{error} = C_{ij}^{\beta} + \mathrm{error}$$

$$\mathrm{errors} \sim \mathcal{O}(1/\sqrt{N})$$

Information/Entanglement structure of Eigenstates

J.M.M (arXiv:1508.05339, Phys. Rev. Lett)

Within time independent hamiltonians,
random particles constitute the first analytical example of:

Eigenstate Thermalization Hypothesis

$$C_{ij}^{E_{N_p}} = \langle E_{N_p} | \, c_i^\dagger c_j \, | E_{N_p} \rangle = \frac{N_p}{N} + \text{error} = C_{ij}^{\beta} + \text{error}$$

$$\text{errors} \sim \mathcal{O}(1/\sqrt{N})$$

Entanglement Thermalization

$$S_{\mathrm{A}}^{\Psi^{N_p}} = |\mathrm{A}| \, S_1^{N_p} + \text{error} = S_{\mathrm{A}}^{\beta} + \text{error}$$

$$S_1^{N_p} = -\frac{N_p}{N} \log \frac{N_p}{N} - (1 - \frac{N_p}{N}) \log (1 - \frac{N_p}{N})$$

Thermalization is typical within the space of gaussian systems!!

Information/Entanglement Dynamics in Democratic systems

"Throwing in" an unentangled particle

$$\rho_{\text{in}} = \rho_1 \otimes \rho_\beta \qquad \mathcal{C}_{ij}(t) = \langle\, c_i^\dagger(t) c_j(t) \rangle$$

⬇

Thermalization of occupation densities

$$\mathcal{C}_{11}(t) \simeq f + (n - f)\left(\frac{4(J_1(Rt))^2}{t^2} \pm \mathcal{O}(1/N)\right)$$

Information/Entanglement Dynamics in Democratic systems

"Throwing in" an unentangled particle

$$\rho_{\text{in}} = \rho_1 \otimes \rho_\beta \qquad \mathcal{C}_{ij}(t) = \langle\, c_i^\dagger(t) c_j(t) \rangle$$

$$\Downarrow$$

$$\mathcal{C}_{i\neq j}(t) = \mathcal{O}(1/N)$$

Information transport is instantaneous and structureless.
Analytical example of large-N factorization
in high energy/entropic sectors.

The answer:

J.M.M (arXiv:1601.04663)

Large-N factorization translates into extensive entanglement dynamics

$$S_{\mathrm{A}}(t) = \sum_{i \in \mathrm{A}} S_i(t)$$

The answer:

J.M.M (arXiv:1601.04663)

Large-N factorization translates into extensive entanglement dynamics

$$S_{\mathrm{A}}(t) = \sum_{i \in \mathrm{A}} S_i(t)$$

Where if the degrees of freedom are fermions:

$$S_i^{\text{fermions}}(t) = -\big(n_i(t) \log n_i(t) + (1 - n_i(t)) \log(1 - n_i(t)) \big)$$

While for bosons:

$$S_i^{\text{bosons}}(t) = (n_i(t) + 1) \log(n_i(t) + 1) - n_i(t) \log n_i(t)$$

The answer:

J.M.M (arXiv:1601.04663)

Large-N factorization translates into extensive entanglement dynamics

$$S_{\mathrm{A}}(t) = \sum_{i \in \mathrm{A}} S_i(t)$$

For large-N matrix models this law should apply for "Generalized free field" modes

$$S^{\mathrm{A}}(t) = \sum_{i \in \mathrm{A}} (n_{\mathcal{O}^i_{\omega,k}}(t) + 1) \log(n_{\mathcal{O}^i_{\omega,k}}(t) + 1) - n_{\mathcal{O}^i_{\omega,k}}(t) \log(n_{\mathcal{O}^i_{\omega,k}}(t))$$

Entanglement entropy shows quasinormal ringing

First insights for black hole physics:

The results challenge the "Fast Scrambling Conjecture"

Hayden,Preskill (2007) Sekino,Susskind (2008)

$$t_{\text{scrambling}} \sim \beta \log S$$

While we find

$$t_{\text{scrambling}} \sim \beta$$

First insights for black hole physics:

The results shed new light on the Information Paradox.

$$\mathcal{C}_{ij}(t)^{\infty} = \lim_{N \to \infty} \mathcal{C}_{ij}(t) = f\delta_{ij} + (n - f)\frac{4(J_1(Rt))^2}{t^2}\delta_{11}$$

$$S(\rho) = S_1(t) + S_{BH} = S(t)$$

In Democratic systems,
Unitarity is lost in the thermodynamic limit.
This should be contrasted with the behavior of local systems

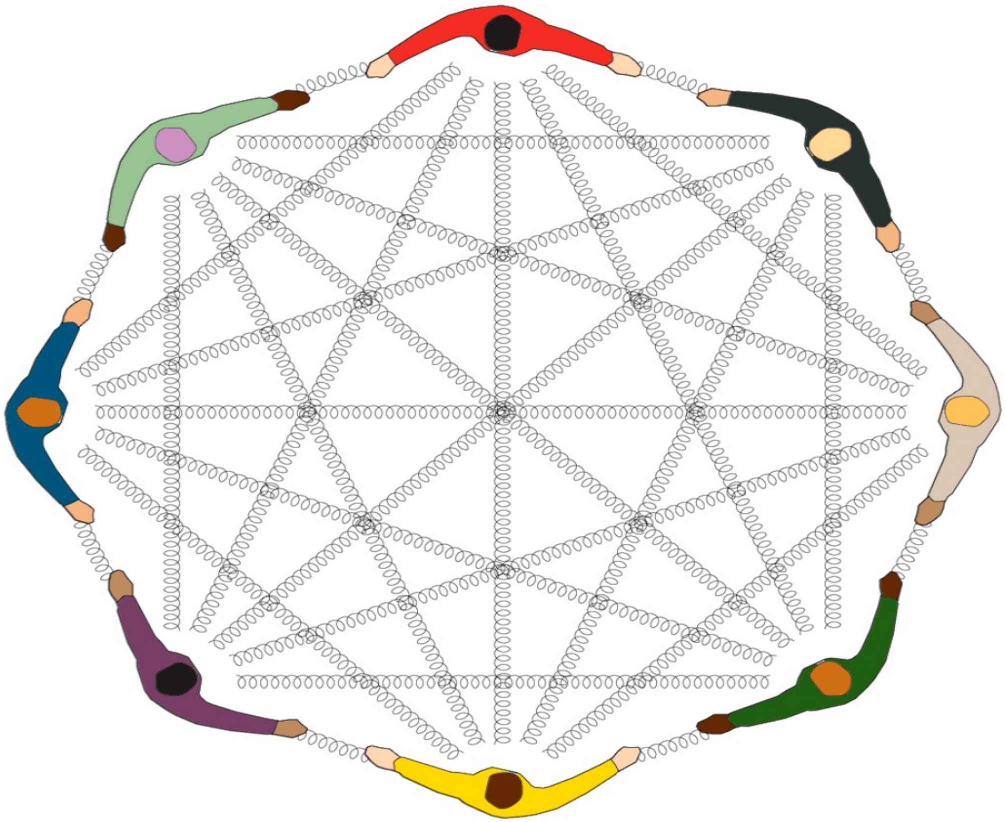
Summary:

Random particles, the most complex gaussian system,
is the simplest example of Democratic systems
displaying Quantum Thermalization.
It is solvable at finite N!!

Key result on information dynamics:
Large-N factorization implies extensivity of
Entanglement entropy in time evolved scenarios.
Quasinormal ringing of entanglement entropy.

First insights for Black Hole physics:
Challenge the Fast Scrambling Conjecture
Non-Unitarity of the large-N limit

THANK YOU



Javier Martínez Magán
j.martinezmagan@uu.nl