Supergravity (and Gauge Theories) as Double Copies

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Why squaring?

- A recurring theme in attempts to understand the quantum theory of gravity is the idea of "Gravity as the square of Yang-Mills"
- This idea of tensoring left (L) and right (R) multiplets appears in many different (but sometimes overlapping) guises:
  - KLT relations in string theory [Kawai et al:1985]
  - $D = 10$ dimensional Type IIA and IIB supergravity (SG) multiplets from $D = 10$ super Yang-Mills (SYM) multiplets [Green et al:1987]
  - gravity anomalies from gauge anomalies [Antoniadis:1992]
  - (super)gravity scattering amplitudes from those of (super) Yang-Mills in various dimensions (BCJ,etc)

What is the aim?

- Derive off-shell linearised Supergravity local transformations via a convolution of SYM superfields.
- Obtain global U-duality algebra $\mathfrak{g}$ and its maximally compact subalgebra $\mathfrak{h}$ in all dimensions, for all $\mathcal{N}$ (via the division algebras).

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Local symmetries

- We tensor left and right off-shell multiplets with arbitrary non-Abelian gauge groups G_L and G_R
- Focus on the Yang-Mills origin of the *local* gravitational symmetries of general covariance, local Lorentz invariance, local supersymmetry and p-form gauge invariance acting on the classical fields.
- Traditionally:

$$g_{\mu\nu}(x) = A_\mu(x)(L) \otimes A_\nu(x)(R)$$

- This can't reproduce the symmetries correctly! (not in a supergravity sense)

Local symmetries

- Instead we define[Anastasiou, Borsten, Duff, Hughes, SN 2014]:

$$g_{\mu\nu}(x) = [A_\mu{}^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R)](x)$$

  where $\Phi_{ii'}$ is the "spectator" bi-adjoint scalar field introduced by [Hodges 2013] and [Cachazo 2014]

- The convolution is defined as

$$[f \star g](x) = \int d^4y f(y) g(x-y).$$

  and is a consequence of the momentum-space origin of squaring: product in momentum space is convolution in position space!

- Importantly, is **doesn't** obey the Leibnitz rule:

$$\partial_\mu (f \star g) = (\partial_\mu f) \star g = f \star (\partial_\mu g)$$

Example

- Look at $\mathcal{N}=1$ supergravity in $D=4$, obtained by tensoring the $(4+4)$ off-shell $\mathcal{N}_L=1$ Yang-Mills multiplet $(A_\mu(L), \chi(L), D(L))$ with the $(3+0)$ off-shell $\mathcal{N}_R=0$ multiplet $A_\mu(R)$.
- We obtain the new-minimal formulation of $\mathcal{N}=1$ supergravity [Sohnius] with its 12+12 multiplet $(e_\mu^a, \psi_\mu, V_\mu, B_{\mu\nu})$.

Transformation rules

- $\mathcal{N}=1$ supergravity is described by a superfield φ_μ transforming at linearised level separately under local transformations with chiral parameter S_μ and real parameter ϕ and under global super-Poincaré with parameters a, λ, ϵ:

$$\delta\varphi_\mu = S_\mu + \bar{S}_\mu + \partial_\mu\phi + \delta_{(a,\lambda,\epsilon)}\varphi_\mu$$

where $\delta_{(a,\lambda,\epsilon)}F = (aP + \lambda M + \epsilon Q + \bar{\epsilon}\bar{Q})F$

- The left supermultiplet is described by a vector superfield $V^i(L)$ transforming at linearised level separately under local Abelian gauge transformations with parameter $\Lambda^i(L)$, non-Abelian global G_L transformations with parameter $\theta^i(L)$ and global super-Poincaré:

$$\delta V^i(L) = \Lambda^i(L) + \bar{\Lambda}^i(L) + f^i{}_{jk}V^j(L)\theta^k(L) + \delta_{(a,\lambda,\epsilon)}V^i(L).$$

Transformation rules

- The right Yang-Mills field ${A_\nu}^{i'}(R)$ transforms separately under local Abelian gauge transformations with parameter $\sigma^{i'}(R)$, non-Abelian global G_R transformations with parameter $\theta^{i'}(R)$ and global Poincaré:

$$\delta {A_\nu}^{i'}(R) = \partial_\nu \sigma^{i'}(R) + {f^{i'}}_{j'k'} {A_\nu}^{j'}(R)\theta^{k'}(R) + \delta_{(a,\lambda)} {A_\nu}^{i'}(R).$$

- The spectator bi-adjoint scalar field transforms under non-Abelian global $G_L \times G_R$ and global Poincaré:

$$\delta \Phi_{ii'} = -{f^j}_{ik} \Phi_{ji'} \theta^k(L) - {f^{j'}}_{i'k'} \Phi_{ij'} \theta^{k'}(R) + \delta_a \Phi_{ii'}.$$

Dictionary

$$\varphi_\mu = V^i(L) \star \Phi_{ii'} \star A_\mu{}^{i'}(R)$$

$$\phi = V^i(L) \star \Phi_{ii'} \star \sigma^{i'}(R)$$

$$S_\mu = \Lambda^i(L) \star \Phi_{ii'} \star A_\mu{}^{i'}(R)$$

Works without imposing external constraints!

Component Formalism

- Work in Wess-Zumino gauge; field dictionary becomes:

$$\begin{aligned} Z_{\mu\nu} \equiv h_{\mu\nu} + B_{\mu\nu} &= A_\mu{}^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R) \\ \psi_\nu &= \chi^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R) \\ V_\nu &= D^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R), \end{aligned}$$

- Parameter dictionary:

$$\begin{aligned} \alpha_\mu(L) &= A_\mu{}^i(L) \star \Phi_{ii'} \star \sigma^{i'}(R), \\ \alpha_\nu(R) &= \sigma^i(L) \star \Phi_{ii'} \star A_\nu{}^{i'}(R), \\ \eta &= \chi^i(L) \star \Phi_{ii'} \star \sigma^{i'}(R), \\ \Lambda &= D^i(L) \star \Phi_{ii'} \star \sigma^{i'}(R), \end{aligned}$$

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But how do we square now?

- Tensoring two on-shell super Yang-Mills multiplets in dimensions $D \leq 10$ yields an on-shell supergravity multiplet, possibly with additional matter multiplets.
- We take tensor product over little group representations and exterior product over the R-symmetry representations:

$\otimes$	$\tilde{A}_\nu$	$\tilde{\lambda}^{a'}$	$\tilde{\phi}^{i'}$
A_μ	$g_{\mu\nu} + B_{\mu\nu} + \phi$	$\psi_\mu^{a'} + \lambda^{a'}$	$A_\mu^{i'}$
λ^a	$\psi_\nu^a + \lambda^a$	$\phi_{RR}^{aa'} + \cdots$	$\lambda^{ai'}$
ϕ^i	A_ν^i	$\lambda^{ia'}$	$\phi^{ii'}$

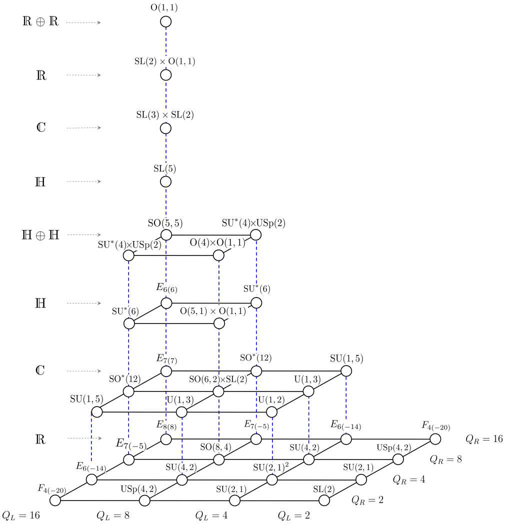

Figure : The U-duality group G in all dimensions. The amount of supersymmetry is determined by the horizontal axis. The spacetime dimension is determined by the division algebra $\mathbb{A}_D$

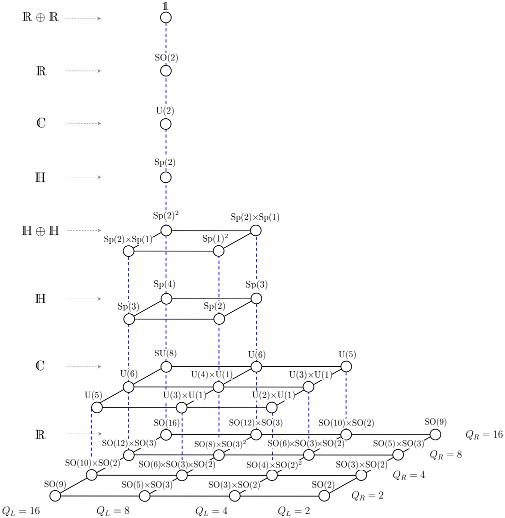

Figure : Pyramid of maximal compact subgroups $H \subset G$. The amount of supersymmetry is determined by the horizontal axis. The spacetime dimension is determined by the division algebra $\mathbb{A}_D$

Conclusions-global symmetries

- One can build the U-duality algebra $\mathfrak{g}$ and its maximally compact subgroup $\mathfrak{h}$ of theories obtained from squaring in any dimensions from knowledge of the multiplet content and global symmetries of the original SYM theories.
- The general prescription [Anastasiou, Borsten, Duff, Hughes, SN 2015]:

$$\mathfrak{g}(\mathcal{N}_L, \mathcal{N}_R, D) = \mathfrak{sym}(\mathcal{N}_L) \oplus \mathfrak{sym}(\mathcal{N}_R) + (Q'_L \otimes Q'_R) + \sum (\Phi_L \otimes \Phi_R)$$

where $\mathfrak{sym}(\mathcal{N}_{L/R})$ is the R-symmetry of the SYM and $Q'_L \otimes Q'_R$ are bosonic transformations on the supergavity multiplet that follow from L/R supersymmetry by suppressing the spacetime indices.

- Associating a division algebra $\mathbb{A}_D$ with every dimension D allows for a unified description of these algebras.

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Applications

- Black hole solutions [Monteiro et al 2015]
- Note that the formula given previously will work for any squaring (e.g. we can obtain supergravity coupled to n vector multiplets)-study supersymmetric solutions
- Ultimately, we want to obtain full non-linear (super)gravity theory from squaring.

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Chiral Squaring

- Can we go further and decompose SYM multiplets?
- We have

$$(\mathcal{N}=4)_{SYM} = (\mathcal{N}=1)_{chiral} \times (\mathcal{N}=1)_{chiral}$$

- One can obtain all the symmetries of $\mathcal{N}=4$ SYM from two copies of the chiral multiplet.[SN 2014]
- We need to introduce new SUSY generators via SO(2) rotation of the bosonic states.

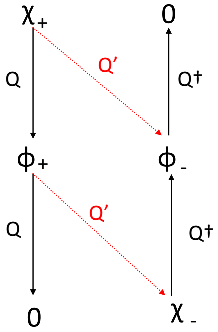

Figure : extra susy generators

- We can write $\mathcal{N} = 8$ as a quadruple copy; all its symmetries can be derived from those of $\mathcal{N} = 1$ chiral multiplet.
- Extends to $D = 6$:

$$[\mathcal{N} = (2,0)]_{tensor} = [\mathcal{N} = (1,0)] \times [\mathcal{N} = (1,0)]$$
$$[\mathcal{N} = (1,1)]_{SYM} = [\mathcal{N} = (1,0)] \times [\mathcal{N} = (0,1)]$$

Applications

- Extension of KLT relations to chiral squaring [Schreiber 2016]
- Fermion double copies as dark matter candidates [de la Cruz et al 2016]
- Off-shell description of double copy theories?

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