$\mathcal{N} = 3$ four dimensional field theories

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Based on
[1512.06434] with I. García-Etxebarria

Motivation and outline

Motivation:

- Original motivation: F-theory at terminal singularities (no supersymmetric smoothing). O3-planes are examples of this.
- In particular, D3-branes probing codimension 4 terminal singularities.
- The simplest unknown cases turn out to lead to $\mathcal{N} = 3$ theories on the worldvolume of the D3s.

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Outline:

- D3s probing an O3 from several perspectives:
  - Worldsheet
  - M/F-theory
  - 4d field theory
- Generalize the O3-plane:
  - M/F-theory
  - 4d field theory ($\mathcal{N} = 3$)

O3s in perturbation theory

- In 2d CFT, O3s are defined as the quotient of 10d Type IIB by $\mathcal{I}(-1)^{F_L}\Omega$

$$\mathcal{I} : (z_1,\, z_2,\, z_3) \to (-z_1, -z_2, -z_3)$$

$(-1)^{F_L}$: left moving spacetime fermion number

Ω : orientation reversal on the worldsheet

$$\left.\begin{matrix} (-1)^{F_L} \\ \Omega \end{matrix}\right\} \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix}$$

- When including N parallel D3s, we need to specify an action on the Chan-Paton factors. [Gimon, Polchinski]

Before the quotient	After the quotient
$4d\ \mathcal{N}=4\ \mathfrak{u}(N)$ →	$4d\ \mathcal{N}=4\ \mathfrak{so}(N)$
	$4d\ \mathcal{N}=4\ \mathfrak{usp}(N) \quad (N \in 2\mathbb{Z})$

- There are different kinds of O3-planes.

O3s in M/F-theory (I)

- 10d Type IIB is given by the F-theory limit of M-theory on a torus.

M-th. on $\mathbb{R}^{1,2} \times \mathbb{C}^3 \times T^2$ $\xrightarrow{T^2 \to 0}$ IIB on $\mathbb{R}^{1,3} \times \mathbb{C}^3$

Complex structure of T^2 $\longrightarrow$ Axio-dilaton ($\tau \supset g_s$)

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- The M-theory lift of the O3 is given by [Hanany, Kol]

M-th. on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$ with $(z_1, z_2, z_3, u) \to (-z_1, -z_2, -z_3, -u)$

$(-1)^{F_L}\Omega$ lifts to: $\mathcal{M}_{(-1)^{F_L}\Omega} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in SL(2,\mathbb{Z})$

This can be seen by looking at the action of the O3 on $\begin{pmatrix} B_2 \\ C_2 \end{pmatrix}$, which comes from reducing C_3 along the one-cycles in the torus.

O3s in M/F-theory (II)

- Four fixed points, which locally look like $\mathbb{C}^4/\mathbb{Z}_2$.

  This is a terminal singularity: no low-energy dynamics associated to O3.

  [Morrison, Stevens; Anno]

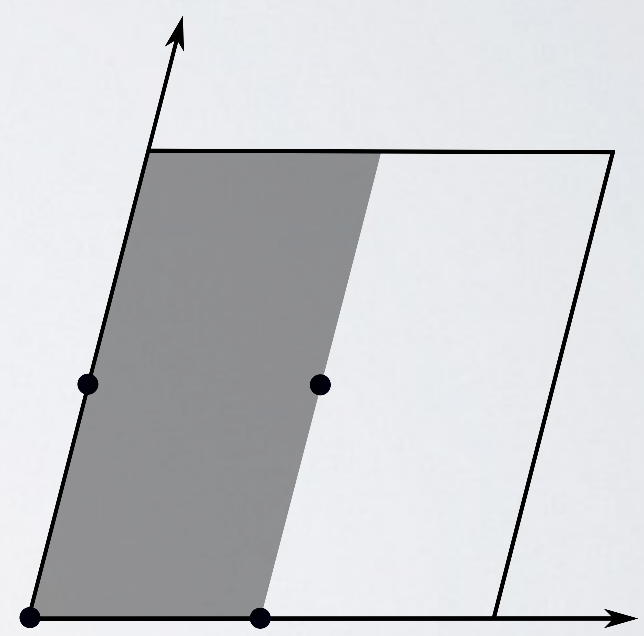
- D3-branes parallel to the O3-plane lift to M2-branes.

- In M-theory, this is precisely ABJM (at level $k = 2$).

  The F-theory limit provides the 4d lift of ABJM.

  $k = 1 : \quad 4d\ \mathcal{N} = 4\ \mathfrak{u}(N)$

  $k = 2 : \quad 4d\ \mathcal{N} = 4\ \mathfrak{so}(N), \mathfrak{usp}(N)$



- Orientifold variants: discrete flux $\longrightarrow$ $O3^-, O3^+, \widetilde{O3}^-, \widetilde{O3}^+$.

  [Hanany, Kol]

O3s in field theory

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- We have seen that $(-1)^{F_L}\Omega$ maps to $\mathbb{Z}_2^S \subset SL(2,\mathbb{Z})$.

$SL(2,\mathbb{Z})$ is a duality, not a symmetry: $\tau \to \dfrac{a\tau+b}{c\tau+d}$

However, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in \mathbb{Z}_2^S$ is a symmetry: $\tau \to \tau$

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  However, $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \in \mathbb{Z}_2^S$ is a symmetry: $\tau \to \tau$
- Therefore, the orientifold corresponds to gauging $\mathbb{Z}_2^{O3} = \mathbb{Z}_2^R \cdot \mathbb{Z}_2^S$.
- Supercharges: $Q_{\alpha a}$ is charged under both $\mathbb{Z}_2^R$ and $\mathbb{Z}_2^S$. [Kapustin, Witten]

  $\mathbb{Z}_2^{O3} : Q_{\alpha a} \to Q_{\alpha a}$ (the O3 does not break SUSY further)

Beyond the O3

Three different ways to look at the O3:

- Worldsheet: quotient by $\mathcal{I}(-1)^{F_L}\Omega$.
- M/F-theory: F-theory limit of $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$.
- 4d gauge theory: quotient by R-symmetry ($\mathbb{Z}_2^R$) and $SL(2,\mathbb{Z})$ ($\mathbb{Z}_2^S$).

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The last two admit a generalization: $\mathbb{Z}_2 \longrightarrow \mathbb{Z}_k$

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We call the associated objects $\mathrm{OF3}_k$-planes. $(\mathrm{OF3}_2 = \mathrm{O3})$

OF3s in M/F-theory (I)

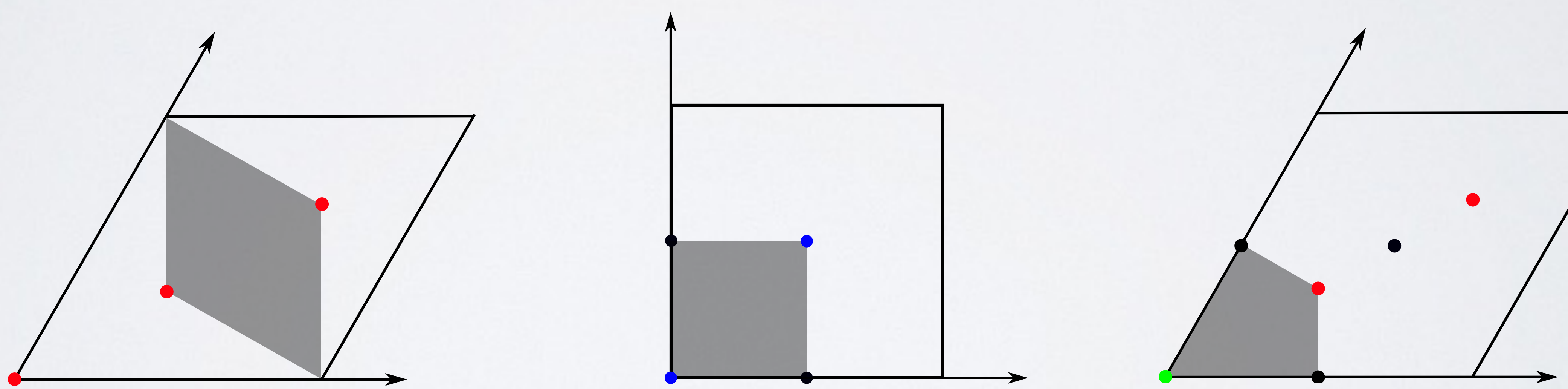
- We want to consider M/F-theory on $\mathbb{R}^{1,2} \times (\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$

$$(z_1, z_2, z_3, u) \to (\zeta_k z_1, \bar{\zeta}_k z_2, \zeta_k z_3, \bar{\zeta}_k u) \quad \text{with} \quad \zeta_k = e^{2\pi i/k} \qquad (k = 2, 3, 4, 6)$$

$\mathrm{OF3}_k$-planes exist only for some values of k.

Only well-defined for special values of the complex structure τ (g_s^{IIB}).

$T^2/\mathbb{Z}_k$:



$k = 3$, $\tau = e^{i\pi/3}$ | $k = 4$, $\tau = i$ | $k = 6$, $\tau = e^{i\pi/3}$

(Different kinds of singularities for a given k)

OF3s in M/F-theory (II)

- Similarly to $k=2$, these do not have supersymmetric resolutions.

[Morrison, Stevens ; Anno]

- Preserve twelve supercharges, $\mathcal{N}_{3d}=6$ or $\mathcal{N}_{4d}=3$. $(k>2)$

- The charge of these objects can be computed from curvature coupling:

$$S_M \supset -\int C_3 \wedge I_8(R) \quad \Longrightarrow \quad Q = \int I_8(R) = \chi/24$$

[Sethi]

- ABJM at level $k>2$ preserves $\mathcal{N}_{3d}=6$. The lift only works for some values of k because there has to be a torus in M-theory.

- M-theory geometry admits discrete flux $\longrightarrow$ Different $\mathrm{OF3}_k$

OF3s in field theory

- The theory on N D3s probing an OF3 should arise as a $\mathbb{Z}_k$ quotient of $4d$ $\mathcal{N}=4$ $\mathfrak{u}(N)$ SYM.

- Just like before, $\mathbb{Z}_k^{\mathrm{OF}} = \mathbb{Z}_k^R \cdot \mathbb{Z}_k^S$ with

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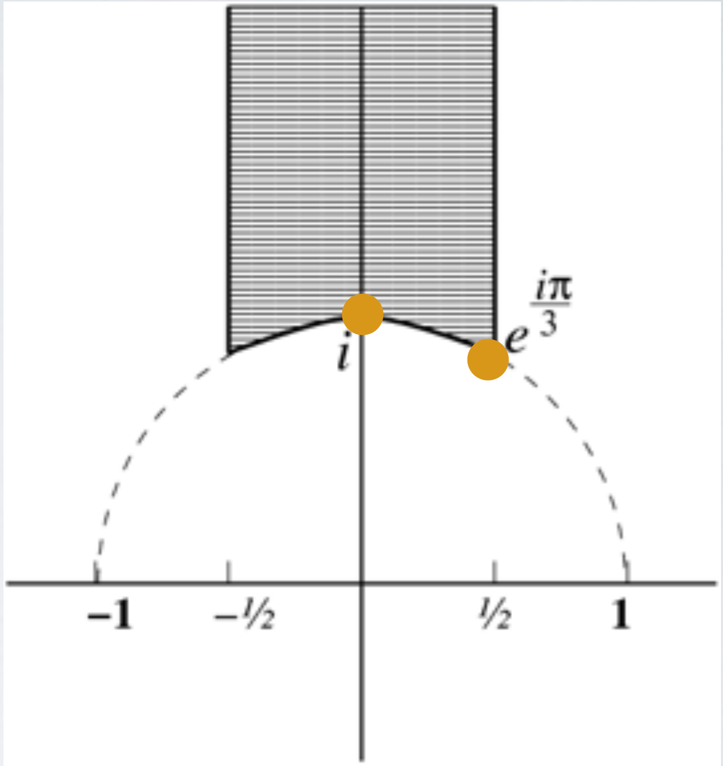

$\tau - \mathrm{plane}$

- Again, $SL(2,\mathbb{Z})$ is not a symmetry, $\tau \to \dfrac{a\tau+b}{c\tau+d}$.

  However, for special values of τ there is a $\mathbb{Z}_k^S$ symmetry (τ invariant). Restriction in both k and τ. The theory is stuck at strong coupling.

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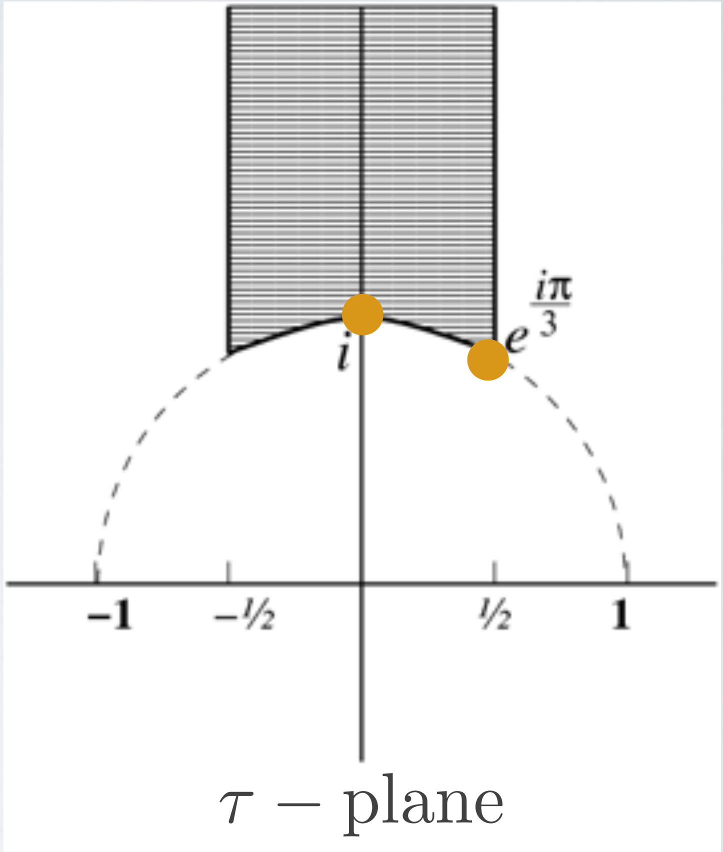

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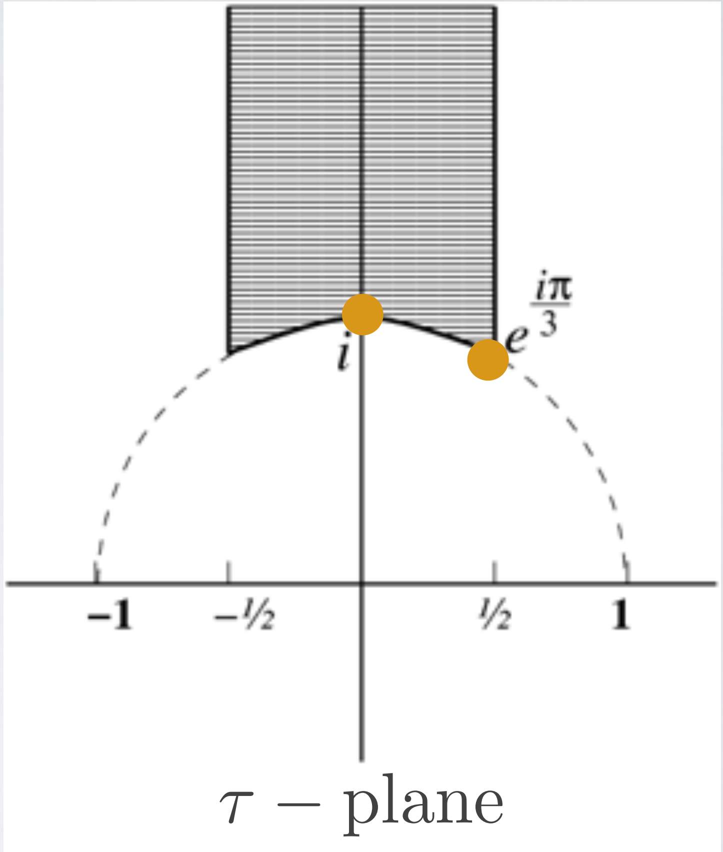

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- The action on the supercharges shows that $\mathcal{N}=3$.

- The quotient does not act just on the elementary fields (mixes E&M). Generalization of the action on Chan-Paton factors is unclear.

$\mathcal{N} = 3$ and CPT invariance

- Having $\mathcal{N} = 3$ in 4d is surprising. There is an argument saying that, in the absence of gravity,

$$\mathcal{N} = 3 + \mathrm{CPT} \quad \Longrightarrow \quad \mathcal{N} = 4$$

[e.g. Weinberg]

Elementary field: vector multiplet.

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- For the case where the parent theory is $4d$ $\mathcal{N}=4$ $U(1)$, which is free, we can check explicitly. Only $\mathcal{N}=3$ is found.

Large N limit

Holographic dual of the $\mathcal{N} = 3$ theories?

- Large number of D3s probing an $\mathrm{OF3}_k$-plane.

- For $k = 1, 2$:

  Without the OF3 ⟶ The usual $AdS_5 \times S^5$ $(k = 1)$

  For an O3 ⟶ IIB orientifold $AdS_5 \times S^5/\mathbb{Z}_2$ $(k = 2)$ [Witten]

- For $k > 2$:

  F-theory/IIB on $AdS_5 \times S^5/\mathbb{Z}_k$ with a $\mathbb{Z}_k$-bundle over $S^5/\mathbb{Z}_k$.

  Axio-dilaton is projected out ⟶ No marginal deformation.

Conclusions and outlook

Conclusions:

- We have built the first examples of $\mathcal{N}=3$ field theories in 4d as quotients of $\mathcal{N}=4$ SYM by particular R-symmetry and $SL(2,\mathbb{Z})$ symmetries.
- Only works for specific values of the coupling. Isolated field theories.
- The worldvolume theory of D3s probing OF3s (generalized orientifolds).
- Can be thought of as the 4d version of ABJM (only for some k).
- Large N limit as a quotient of $AdS_5 \times S^5$ acting on the IIB coupling.

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- Better understanding in M-theory (BPS states).
- Other (less supersymmetric) terminal singularities.

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Thank you!

Extra

- Orientifold variants: discrete torsion.

The transverse space to the singularity is $S^7/\mathbb{Z}_2$.

Discrete G_4 flux in $H^4(S^7/\mathbb{Z}_2,\mathbb{Z}) = \mathbb{Z}_2$.

Many fixed points $\longrightarrow$ many orientifolds (in 3d)

Some become equivalent/trivial in the F-theory limit $\longrightarrow$ different O3s

$$O3^-,\, O3^+,\, \widetilde{O3}^-,\, \widetilde{O3}^+$$

$$\mathbb{Z}_2^{O3} : (A, \lambda, \phi) \to (-A, -\lambda, -\phi)$$

$$\chi = 6(8 + 4h_{0,2} + h_{1,1} + h_{1,3} - h_{1,2})$$

$$\chi(\mathbb{C}^4/\mathbb{Z}_k) = \frac{1}{24}\left(k - \frac{1}{k}\right)$$