Adventures with Stringy Axions

Wieland Staessens
based on (1503.01015, 1503.02965 [hep-th]) + work in progress
with G. Shiu (& F. Ye)

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Inflation and Axions

- BICEP2 (2014) $\Rightarrow$ illusion of large field inflation
  Lyth (1996): tensor pert. with $r > 0.01 \overset{\text{single field}}{\Leftrightarrow} \Delta\phi\big|_{\text{inf}} > M_{\text{Pl}}$
  - ⋆ Shift symmetry of axions $\rightarrow$ ~~dim 6 operators~~ $\rightsquigarrow$ slow roll: $|\eta| \ll 1$ ✓
  - ⋆ Nonperturbative effects $\rightsquigarrow V_{inf} \sim \Lambda^4 \left[1 - \cos\frac{a}{f}\right]$

  $\Rightarrow$ natural inflation Freese-Frieman-Olinto (1990) with $f > M_{Pl}$ for slow roll

- Entering String Theory $\sim$ UV complete theory with quantum gravity
  $\Rightarrow$ realising $f > M_{Pl}$ upon dimensional reduction to 4d
  - ⋆ plethora of axions + local origin of shift symmetries

    Banks-Dixon (1988), Kallosh-Linde-Linde-Susskind (1995), Banks-Seiberg (2010), ...
  - ⋆ But $\exists$ no-go theorems and arguments forbidding $f > M_{Pl}$ (1 axion)

    Banks-Dine-Fox-Gorbatov ('03), Svrček-Witten ('06), Arkhani-Hamed-Motl-Nicolis-Vafa ('06), Conlon-Krippendorf ('16)

    $\rightsquigarrow$ multiple axions: N-flation (2005), aligned natural inflation (2004)
  - ⋆ $\exists$ no-go theorems for multiple axions?? (Generalized WGC)

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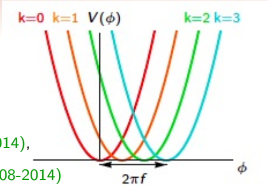

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- For N axions & $M(\geq N)$ instantons $\rightsquigarrow$ definition of f_{eff} ambiguous $\rightsquigarrow$ consider the diameter of axion moduli space

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- Presence of other (heavy) fields $\rightsquigarrow$ affects inflation process $\Rightarrow 0.8 M_{\mathrm{Pl}} < f_{\mathrm{eff}} < M_{\mathrm{Pl}}$ consistent with data

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$\Rightarrow$ Understanding of axion EFT indispensable

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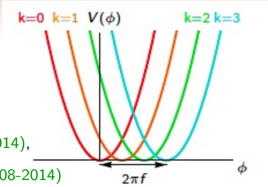

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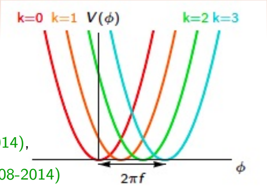

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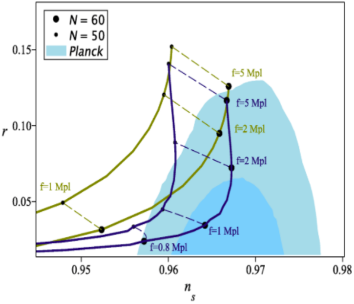

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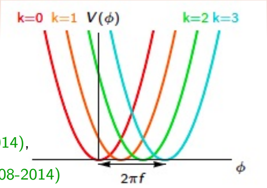

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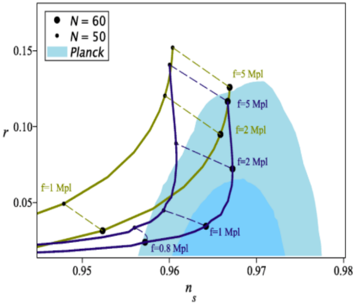

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Axions & String Theory

reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011), Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)

- Closed string axions a^i from dim. red. of p-forms $C_{(p)}$ on $\mathcal{M}_{1,3} \times \mathcal{X}_6$ ($C_{(p)} \in$ RR-forms + NS 2-form in Type II)

$$a^i \equiv (2\pi)^{-1} \int_{\Sigma^i} C_{(p)}, \qquad p - \text{cycle } \Sigma^i \subset \mathcal{X}_6, \qquad i \in \{1, \ldots, \begin{matrix} h_{11} \\ h_{21}+1 \end{matrix} \}$$

  Kinetic terms for p-forms $C_{(p)} \in \rightsquigarrow$ kinetic terms for a^i

- Including D-branes wrapping p-cycle Σ^i:
  - ⋆ D-brane Chern-Simons terms $\rightsquigarrow$ anomal. coupling "$a^i\, Tr(G \wedge G)$"
  - ⋆ Eucl. D-branes $\rightsquigarrow$ additional non-pert. effects (D-brane instantons)

- For a single D-brane wrapping p-cycle with $\Omega\mathcal{R}(\Sigma^i) \neq \Sigma^i$
  $\rightsquigarrow$ a^i acquires Stückelberg charges under $U(1)$

Axions & String Theory: Example

reviews: Blumenhagen-Cvetič-Langacker-Shiu ('05); Blumenhagen-Körs-Lüst-Stieberger ('06); Ibañez-Uranga ('12)

e.g. Type IIA D6-branes on $CY_3/\Omega\mathcal{R} \rightsquigarrow \Sigma^i = \Sigma^i_+ + \Sigma^i_-$

$$\int_{\Sigma^i_-} C_{(5)} \wedge F \neq 0 \qquad \rightsquigarrow \qquad \text{Stückelberg coupling for } a^i$$

$T^6/\Omega\mathcal{R}$ with 4 $\Omega\mathcal{R}$-even 3-cycles $\underbrace{\Sigma_+^{i=0,1,2,3}}_{4 \text{ axions } a^i}$ and 4 $\Omega\mathcal{R}$-odd 3-cycles $\Sigma_-^{i=0,1,2,3}$

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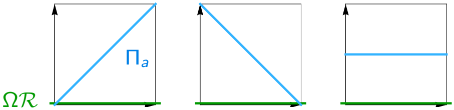

$$\Pi_a = \underbrace{\Sigma^0_+ - \Sigma^3_+}_{} + \underbrace{\Sigma^1_- - \Sigma^2_-}_{}$$

$$\begin{array}{cc} \Downarrow & \Downarrow \\ a^0 F_a \wedge F_a & (da^1 - A_a)^2 \\ -a^3 F_a \wedge F_a & (da^2 + A_a)^2 \end{array}$$

An Effective Action...

w/ Shiu-Ye 1503.01015, 1503.02965 [hep-th]

- String Theory compactifications $\rightsquigarrow$ 4d EFT with mixing axions

$$\mathcal{S}^{\text{eff}}_{axion} = \int \left[\frac{1}{2} \sum_{i,j=1}^{N} \mathcal{G}_{ij} (\mathrm{d}a^i - k^i A) \wedge \star_4 (\mathrm{d}a^j - k^j A) - \frac{1}{8\pi^2} \left(\sum_{i=1}^{N} r_i a^i \right) \mathrm{Tr}(G \wedge G) + \mathcal{L}_{gauge} \right]$$

- 2 types of kinetic mixing
  - (1) metric mixing: $\mathcal{G}_{ij}$ is not diagonal
  - (2) $U(1)$ mixing: $k^i \neq 0$ for some $i \in \{1, \ldots, N\}$
    gauged axions: $a^i \to a^i + k^i \chi, \quad A \to A + d\chi$
- $\mathrm{Tr}(G \wedge G)$-term associated to non-Abelian gauge group
  $\rightsquigarrow$ collective periodicity: $\sum_{i=1}^{N} r_i a^i \simeq \sum_{i=1}^{N} r_i a^i + 2\pi$
- axions couple to D-brane instantons
  $\rightsquigarrow$ individual periodicity: $a^i \to a^i + 2\pi\nu^i$, $\nu^i \in \mathbb{Z}$
  note: effective contributions of D-brane instantons to $\mathcal{S}^{\text{eff}}_{axion}$ is model-dependent
  see e.g. Ibáñez-Uranga (2007, 2012), Blumenhagen-Cvetič-Kachru-Weigand (2009)

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...for Mixing Axions

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To determine axion decay constants
To figure out axions eaten by $A_{U(1)}$ } $\Longrightarrow$ Diagonalise kinetic terms

eigenbasis for kinetic terms $\neq$ eigenbasis for potentials

$\Downarrow$

axionic directions with large f_a?

Note: different from N-flation Dimopoulos-Kachru-McGreevy-Wacker (2005),
KNP-alignment with large N Choi-Kim-Yung (2014),
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2 Mixing Axions

- minimal set-up: 2 axions + 1 $U(1)$ + 1 Non-Abelian gauge group
- 1 axion eaten by $U(1)$ gauge boson, $\perp$ axion ξ with decay constant:

$$f_\xi = \frac{\sqrt{\lambda_+\lambda_-}M_{st}}{\cos\frac{\theta}{2}\left(\lambda_+k^+r_2+\lambda_-k^-r_1\right)+\sin\frac{\theta}{2}\left(\lambda_-k^-r_2-\lambda_+k^+r_1\right)}$$

with $\lambda_\pm$ eigenvalues of $\mathcal{G}_{ij}$ and $M_{st} \equiv \sqrt{\lambda_+(k^+)^2+\lambda_-(k^-)^2}$

$$\cos\theta = \frac{\mathcal{G}_{11}-\mathcal{G}_{22}}{\lambda_+-\lambda_-}, \quad \sin\theta = \frac{2\mathcal{G}_{12}}{\lambda_+-\lambda_-}, \qquad \begin{pmatrix} k^+ \\ k^- \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} k^1 \\ k^2 \end{pmatrix}$$

- Contour plot representation of f_ξ (in units $\sqrt{\mathcal{G}_{11}}$)

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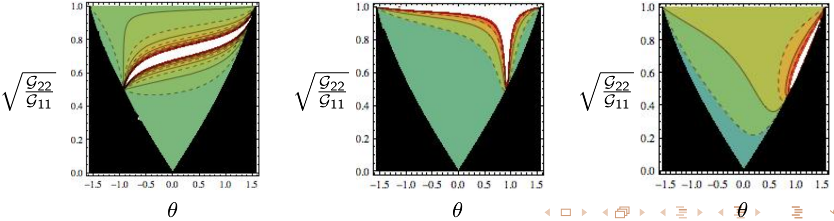
- Contour plot representation of f_ξ (in units $\sqrt{\mathcal{G}_{11}}$)

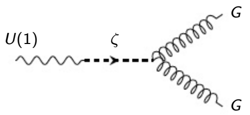
$2k^1 = k^2 = 2r_1 = 2r_2$ $\qquad$ $k^1 = 2k^2 = r_1 = 2r_2$ $\qquad$ $k^1 = -2k^2 = r_1 = 2r_2$



Consistency Conditions

- $U(1)$ gauge invariance: $A \to A + \mathrm{d}\chi$, $\zeta \to \zeta + \tilde{k}\chi$

$$\mathcal{S}_{sub} = \int \left[\frac{f_2^2}{2} \left(\mathrm{d}\zeta - \tilde{k}A\right) \wedge \star_4 \left(\mathrm{d}\zeta - \tilde{k}A\right) - \frac{1}{g_1^2} F \wedge \star_4 F - \underbrace{\frac{1}{8\pi^2} \zeta \, \mathrm{Tr}(G \wedge G)}_{\text{not } U(1) \text{ invariant}} \right]$$



- Integrating out massive $U(1)$ boson
  $\rightsquigarrow$ 1 axion ξ + 1 non-Abelian gauge group + chiral fermions

$$\mathcal{S} = \int \frac{1}{2} \mathrm{d}\xi \wedge \star_4 \mathrm{d}\xi - \frac{1}{8\pi^2} \frac{\xi}{f_\xi} \mathrm{Tr}(G \wedge G) - \frac{\mathcal{C}}{f_2^2} \underbrace{\mathcal{J}_\psi \wedge \star_4 \mathcal{J}_\psi}_{4-fermion} + \mathcal{L}_\psi$$

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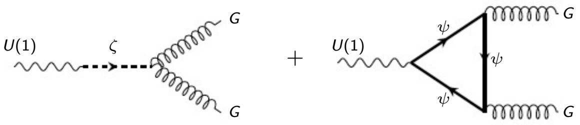
$+$ chiral fermions ψ



"reversed" GS mechanism

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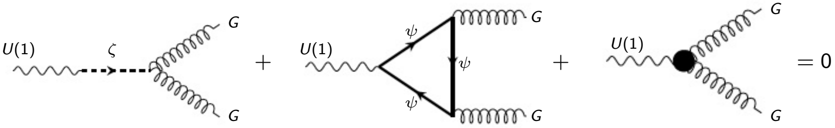
+ chiral fermions ψ + generalised CS-terms



"reversed" GS mechanism for non-symm $\mathcal{A}^{\text{anomaly}}$

Aldazabel-Ibáñez-Uranga ('03), Anastasopoulos et al ('06)
De Rydt-Rosseel-Schmidt-Van Proeyen-Zagerman ('07)

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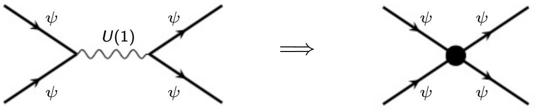

+ global $U(1)_{\mathrm{anom}}$

Inflationary potential

@ strong coupling: fermion condensate $\langle\overline{\psi}\psi\rangle$ + gluon condensate $\langle G\tilde{G}\rangle$

- ⋆ global $U(1)_{\rm anom} \rightsquigarrow \partial_\mu J^\mu_{U(1)} = \underbrace{\frac{1}{16\pi^2}\mathrm{Tr}(G_{\mu\nu}\tilde{G}^{\mu\nu})}_{\text{gluon condensate}} \to$ mass-term for phase of $\overline{\psi}\psi$ $\sim \eta^{(\prime)}$-meson
- ⋆ 4-Fermion coupling $\rightsquigarrow$ mass-terms for axion ξ
  see e.g. reviews 't Hooft (1986), Peccei (2006), Svrček-Witten (2006)

⇒ Effective potential

$$V_{eff}(\xi,\eta) = V_{\rm fermion}(\xi) + V_{\rm gluon}(\eta,\xi)$$

In vacuum of $V_{eff} \rightsquigarrow \frac{m_\eta}{m_\xi} \sim \frac{f_\xi}{f_\eta} \gg 1$

Upon integrating out η-meson

$$V_{eff}(\xi) = \Lambda^4 \cos\left(\frac{\xi}{f_\xi} - \pi\right) \qquad \Lambda^4 \sim \left(\frac{\langle\overline{\psi}\psi\rangle}{f_2}\right)^2$$

Conclusions

- Enhanced f_ξ for mixing axions charged under $U(1)$
  if $\exists$ isotropy relations among contin. moduli λ_- & λ_+ (white regions)
- $V_{\rm inf}$: from non-perturbative effects of 1 Non-Abelian gauge group
- Realisations in String Theory: $\begin{cases} \text{IIA w/ inters. D6-branes} \\ \text{IIB w/ inters. magnetised D7-branes} \end{cases}$
  $\longrightarrow$ more suitable examples are desired

Open issues

☞ Validity of eff. description requires moduli stabilisation??
Conlon (2006), Choi-Jeong (2006), Hristov (2008), Higaki-Kobayashi (2011),
Cicoli-Dutta-Maharana (2014)

☞ No-go theorems for multiple axions??
Brown-Cottrell-Shiu-Soler(2015), Montero-Uranga-Valenzuela (2015), Junghans (2015),
Rudelius (2014/15), Hebecker-Mangat-Rompineve-Witkowski (2015), Bachlechner-Long-McAllister (2015)

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Obrigado  Muchas gracias

Eskerrik asko

Moltes gràcies

Moitas grazas

WGC in a Perpetuum Mobile

- WGC (weak form) states that $\exists$ state with $\left(\frac{M}{Q}\right) \leq M_{Pl}$

  $\rightsquigarrow$ conjectured generalisation for 0-forms with $\mathcal{S}_{inst} \leq \frac{M_{Pl}}{f}$

  Arkhani-Hamed-Motl-Nicolis-Vafa (2006)

- Consistent compactifications for

$$\begin{array}{ccc} \text{M-theory} & \overset{S,T}{\longleftrightarrow} & \text{Type IIB} \\ \mathcal{M}_{1,2} \times S^1_M \times \tilde{S}^1 \times X_6 & & \mathcal{M}_{1,2} \times S^1 \times X_6 \end{array}$$

  $\rightsquigarrow$ Constraints on effective axion decay constant by applying WGC on 5dim BH in M-theory

  Brown-Cottrell-Shiu-Soler (2015)

- Generalisations of WGC $\rightarrow$ Lattice WGC (stronger form!)

  Heidenreich-Reece-Rudelius (2014/15)

- **BUT** in our simple model: 2 axions + 1 $U(1)$ + 1 gauge instanton

  Does it satisfy Lattice WGC?

  Are higher harmonics are still troublesome for $\perp$ axion?

  Montero-Uranga-Valenzuela (2015)

Inflation and Axions

- Single field models discriminated by Lyth-bound: Lyth (1996)

$$\frac{\Delta\phi}{M_{Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2} \qquad \begin{array}{ll} r < 0.01 & \text{small field inflation } (\Delta\phi < M_{Pl}) \\ r > 0.01 & \text{large field inflation } (\Delta\phi > M_{Pl}) \end{array}$$

$\rightsquigarrow$ measurable tensor perb. $\Delta_T^2(k)$ for $r \equiv \frac{\Delta_T^2(k)}{\Delta_S^2(k)} > 0.01$

- η-problem: sensitivity to dim 6 operators $\Rightarrow |\Delta\eta| \gg \mathcal{O}(1)$ ↯ slow roll
- Shift symmetry of axions forbids such corrections, while inflaton potential follows from non-perturb. effects

$$V(a) \sim \Lambda^4 \left[1 - \cos\frac{a}{f}\right]$$

$\Rightarrow$ natural inflation Freese-Frieman-Olinto (1990) with $f > M_{Pl}$ for slow roll

- String Theory: reviews: Baumann (2009), Baumann-McAllister (2009,2014), Westphal (2014), ...
  - ⋆ UV complete theory with plethora of axions
  - ⋆ local origin of shift symmetries Banks-Dixon (1988), Kallosh-Linde-Linde-Susskind (1995), Banks-Seiberg (2010), ...

But $\exists$ no-go theorems and arguments forbidding $f > M_{Pl}$ (1 axion)

Banks-Dine-Fox-Gorbatov (2003), Svrček-Witten (2006), Arkhani-Hamed-Motl-Nicolis-Vafa (2006)

Main question

Apart from 3 traditional multiple axion scenarios

- N-flation Dimopoulos-Kachru-McGreevy-Wacker (2005)
- aligned natural inflation (KNP) Kim-Nilles-Peloso (2004), Kappl-Krippendorf-Nilles (2014), Choi-Kim-Yun (2014), Ben-Dayan-Pedro-Westphal (2014), Long-McAllister-McGuirk (2014)
- axion monodromy Silverstein-Westphal (2008), McAllister-Silverstein-Westphal (2008), Kaloper-(Lawrence)-(Sorbo) (2008/11/14), Marchesano-Shiu-Uranga (2014), Franco-Galloni-Retolaza-Uranga (2014), McAllister-Silverstein-Westphal-Wrase (2014), Hebecker-Kraus-Witkowski (2014), Blumenhagen-Herschmann-Plauschinn (2014), Retolaza-Uranga-Westphal (2015)

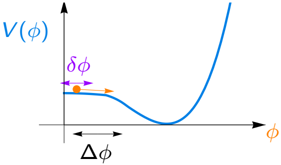
$\exists$ other scenarios to obtain $f > M_{Pl}$ in string theory?

Some aspects of Inflation

reviews: Baumann (2009); Baumann-McAllister (2009,2014); Westphal (2014); ...

- Inflation = cure for horizon problem and flatness problem
- Inflation = explanation for fluctuations in nearly scale-invariant, nearly Gaussian CMB
- typical model: slow-roll single scalar field with potential V satisfying

$$\epsilon \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1, \qquad |\eta| \equiv \left|M_{Pl}^2 \frac{V''}{V}\right| \ll 1 \qquad \text{during inflation}$$



$\Delta\phi$: distance traveled during inflation

- QM fluctuations $\delta\phi$ during inflation

$$\Rightarrow \begin{cases} \text{scalar pert. } \Delta_S^2(k) \sim 10^{-9} \text{ (WMAP+PLANCK)} \\ \text{tensor pert. } \Delta_T^2(k) \end{cases}$$

Some aspects of Inflation

- spectral index n_s: deviation from scale-invariance

$$\Delta_S(k) = A_S \left(\frac{k}{k_\star}\right)^{n_s - 1} \qquad k_\star : \text{reference scale}$$

- tensor-to-scalar ratio r: $r \equiv \frac{\Delta_T^2(k)}{\Delta_S^2(k)}$ sets the inflation scale

$$V^{1/4} \sim \left(\frac{r}{0.01}\right)^{1/4} 10^{16}\,\text{GeV}$$

- Lyth bound: relates field displacements $\Delta\phi$ to r during inflation

$$\frac{\Delta\phi}{M_{Pl}} = \mathcal{O}(1)\left(\frac{r}{0.01}\right)^{1/2}$$

$r < 0.01$ small field inflation ($\Delta\phi < M_{Pl}$)
$r > 0.01$ large field inflation ($\Delta\phi > M_{Pl}$)

- (n_s, r) are related to slow-roll parameters (ϵ, η):

$$n_s - 1 = 2\eta - 6\epsilon \qquad r = 16\epsilon$$

$\Rightarrow$ (n_s, r) measurements give direct info about potential V

Some Considerations about Axions

- axions = CP-odd real scalars with a continuous shift symmetry:

$$a \to a + \varepsilon, \qquad \varepsilon \in \mathbb{R}$$

- shift symmetry is broken to a discrete symmetry by non-perturbative effects (gauge instantons, D-brane instantons, etc.):

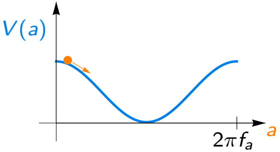
$$a \to a + 2\pi n, \qquad n \in \mathbb{Z}$$

☞ symmetry constrains axionic couplings:

$$\mathcal{S} = \int \frac{f_a^2}{2} \mathrm{d}a \wedge \star_4 \mathrm{d}a - \Lambda^4 \left[1 \pm \cos(a)\right] \star_4 \mathbf{1}$$

f_a: axion decay constants (coupling strength of axion to other matter)

- natural inflation: axion as inflaton candidate Freese-Frieman-Olinto (1990)



$$\epsilon = \frac{M_{Pl}^2}{2f_a^2}\left(\frac{\sin(a/f_a)}{1 \pm \cos(a/f_a)}\right)^2 \ll 1$$

$$\eta = \frac{M_{Pl}^2}{f_a^2}\left|\frac{\cos(a/f_a)}{1 \pm \cos(a/f_a)}\right| \ll 1$$

"Lifting" the flat direction

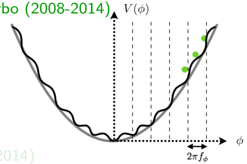
(1) Monodromy effects: shift symmetry is softly broken

- F-term (torsional cycles, fluxes) $\longrightarrow V(\xi) \sim \xi^{p\geq 2}$

  Marchesano-Shiu-Uranga (2014), McAllister-Silverstein-Westphal-Wrase (2014), Blumenhagen-Herschmann-Plauschinn (2014), Kaloper-Lawrence-Sorbo (2008-2014)

- D-term (D-branes) $\longrightarrow V(\xi) = \sqrt{L^4 + \xi^2} \sim \xi$

  Silverstein-Westphal (2008), McAllister-Silverstein-Westphal (2008)



(2) Alignment effects: 2 non-Abelian gauge groups

Kim-Nilles-Peloso (2004), Kappl-Krippendorf-Nilles (2014); Choi-Kim-Yun (2014)

$$V_{axion}^{\text{eff}} = \Lambda_1^4 \left[1 - \cos\left(\frac{a^-}{f_1} + \frac{a^+}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{a^-}{f_2} + \frac{a^+}{g_2}\right)\right]$$

see also BenDayan-Pedro-Westphal (2014), Long-McAllister-McGuirk (2014)

(3) $U(1)$ gauge symmetry:

- 1 axionic direction eaten by $U(1)$ boson (Stückelberg mechanism)
- orthogonal direction acquires mass due to non-perturbative effect

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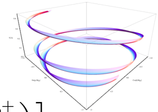

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(3) $U(1)$ gauge symmetry:

- 1 axionic direction eaten by $U(1)$ boson (Stückelberg mechanism)
- orthogonal direction acquires mass due to non-perturbative effect

Aligned natural inflation

- Consider 2 strongly coupled non-Abelian gauge groups:

$$V_{axion}^{\text{eff}} = \Lambda_1^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_1} + \frac{\hat{a}^+}{g_1}\right)\right] + \Lambda_2^4 \left[1 - \cos\left(\frac{\hat{a}^-}{f_2} + \frac{\hat{a}^+}{g_2}\right)\right]$$

with decay constants

$$f_1 = \frac{\sqrt{\lambda_-}}{|r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}|} \qquad g_1 = \frac{\sqrt{\lambda_+}}{|r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}|} \qquad f_2 = \frac{\sqrt{\lambda_-}}{|s_1 \sin\frac{\theta}{2} - s_2 \cos\frac{\theta}{2}|} \qquad g_2 = \frac{\sqrt{\lambda_+}}{|s_1 \cos\frac{\theta}{2} + s_2 \sin\frac{\theta}{2}|}$$

- Perfect alignment:

$$\frac{f_1}{g_1} = \frac{f_2}{g_2} \qquad \Rightarrow \qquad \left|\frac{r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}}{r_1 \sin\frac{\theta}{2} - r_2 \cos\frac{\theta}{2}}\right| = \left|\frac{s_1 \cos\frac{\theta}{2} + s_2 \sin\frac{\theta}{2}}{s_1 \sin\frac{\theta}{2} - s_2 \cos\frac{\theta}{2}}\right|$$

- Deviation from perfect alignment

$$\alpha_{dev} \equiv g_2 - \frac{f_2}{f_1} g_1 = \frac{\sqrt{\lambda_+}\,(s_1 r_2 - r_1 s_2)}{\left(\frac{s_1^2 - s_2^2}{2} \sin\theta - s_1 s_2 \cos\theta\right)\left(r_1 \cos\frac{\theta}{2} + r_2 \sin\frac{\theta}{2}\right)}$$

- Continuous parameter θ allows for $\alpha_{\text{dev}} \approx 0.009\sqrt{\lambda_+}$ with $r_i, s_i \sim O(1-10)$

$U(1)$ gauge invariance

- $U(1)$ gauge invariance: $A \to A + \mathrm{d}\chi$, $a'^2 \to a'^2 + \tilde{k}\chi$

$$\mathcal{S}_{sub} = \int \left[\frac{f_2^2}{2} \left(\mathrm{d}a'^2 - \tilde{k}A \right) \wedge \star_4 \left(\mathrm{d}a'^2 - \tilde{k}A \right) - \frac{1}{g_1^2} F \wedge \star_4 F - \underbrace{\frac{1}{8\pi^2} a'^2 \,\mathrm{Tr}(\, G \wedge G)}_{\text{not } U(1) \text{ invariant}} \right]$$

$\rightsquigarrow$ requires presence of chiral fermions

"reversed" GS mechanism $\Rightarrow \delta\mathcal{S}^{mix}_{anom} = - \int \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{mix}} \chi \,\mathrm{Tr}(\, G \wedge G)$

- If anomaly also contains non-symmetric contributions

$\rightsquigarrow$ Generalised Chern-Simons terms Aldazabel-Ibáñez-Uranga (2003), Anastasopoulos et al (2006) De Rydt-Rosseel-Schmidt-Van Proeyen-Zagerman (2007)

$$\mathcal{S}^{\mathrm{GCS}}_{sub} = \int \frac{1}{8\pi^2} \mathcal{A}^{\mathrm{GCS}} A \wedge \Omega,$$

- Full $U(1)$ gauge invariance: $\boxed{\tilde{k} + \mathcal{A}^{mix} + \mathcal{A}^{\mathrm{GCS}} = 0}$

- $+$ 3 additional constraints from non-Abelian and mixed Abelian/non-Abelian anomaly cancelation

Anomalies

- Spectrum of chiral fermions

	$SU(N)$	$U(1)$
ψ_L^i	R_1^i	q_L^i
ψ_R^i	R_2^i	q_R^i

- mixed anomaly coefficient:
$\mathcal{A}^{\rm mix} = \sum_i \left[\mathrm{Tr}(q_L^i \{ T_a^{R_1^i}, T_b^{R_1^i} \}) - \mathrm{Tr}(q_R^i \{ T_a^{\overline{R}_2^i}, T_b^{\overline{R}_2^i} \}) \right]$

- mixed anomaly: $\mathcal{A}^{\rm GCS} - \mathcal{A}^{\rm mix} = 0$

- cubic $U(1)$ anomaly: $\mathcal{A}^{U(1)^3} = \sum_i \left[(q_L^i)^3 - (q_R^i)^3 \right] = 0$

- cubic $SU(N)$ anomaly:
$\mathcal{A}^{SU(N)^3} = \sum_i \left[\mathrm{Tr}(T_a^{R_1^i} \{ T_b^{R_1^i}, T_c^{R_1^i} \}) - \mathrm{Tr}(T_a^{\overline{R}_2^i} \{ T_b^{\overline{R}_2^i}, T_c^{\overline{R}_2^i} \}) \right] = 0$

Integrating out

- Potential for a'^1? $\rightarrow$ integrating out A ($+\ a'^2$)

Step 1: e.o.m for massive A in unitary gauge

$$-\frac{1}{g_1^2}d(\star_4 dA) + (f_{\tilde{a}_2}\tilde{k}^2)^2 \star_4 A = -\frac{\mathcal{A}^{\text{GCS}}}{8\pi^2}\Omega - \star_4\mathcal{J}_\psi$$

Step 2: Deduce Lorenz-gauge condition

$$(f_{\tilde{a}^2}\tilde{k}^2)^2 d(\star_4 A) = -\frac{\mathcal{A}^{\text{GCS}} + \mathcal{A}^{\text{mix}}}{\mathcal{A}^{\text{mix}}} d(\star_4\mathcal{J}_\psi)$$

Step 3: Re-insert relation between A and $\mathcal{J}_\psi$

$$\mathcal{S} = \int \frac{f_1^2}{2}\mathrm{d}a'^1 \wedge \star_4 \mathrm{d}a'^1 - \frac{1}{8\pi^2}a'^1 \text{Tr}(G \wedge G) - \frac{\mathcal{C}}{f_2^2}\underbrace{\mathcal{J}_\psi \wedge \star_4\mathcal{J}_\psi}_{4-fermion}$$

- Potential for a'^1? $\rightarrow$ integrating out non-Abelian gauge group + chiral fermions

$$\mathcal{S} = \int \frac{f_1^2}{2}\mathrm{d}a'^1 \wedge \star_4 \mathrm{d}a'^1 - \Lambda^4\left[1 - \cos\left(a'^1\right)\right] \star_4 \mathbf{1}$$

Axions from String Theory

Dimensional reduction of String Theory on $\mathcal{M}_{1,3} \times \mathcal{X}_6$

$\Rightarrow$ plethora of axions see e.g. Witten (1984), Banks-Dine-Fox-Gorbatov (2003), Svrček-Witten (2006)

(1) Closed String Axions

- 1 Model-independent: NS 2-form B_2 along $\mathcal{M}_{1,3}$

- $h_{11} + h_{21}$ Model-dependent: $\left\{ \begin{array}{l} \text{NS 2-form } B_2 \\ \text{RR-forms } C_p \end{array} \right.$ along $\mathcal{X}_6$

(2) Open String Axions

- Wilson-line: reduction of (D-brane) gauge field

  see e.g. ArkaniHamed-Cheng-Creminelli-Randal, Marchesano-Shiu-Uranga (2014)

- Field-type: phase of $\mathbb{C}$ scalar field in $\mathcal{N} = 1$ chiral multiplet @ intersection of 2 D-branes

String Theory embedding I

Grimm-Louis (2005), Grimm-Lopes (2011), Kerstan-Weigand (2011)

- Type IIA on $\mathcal{M}_{1,3} \times \mathcal{X}_6$ with D6-branes
  $\mathcal{M}_{1,3}$ maximally symmetric 4-dim spacetime
  $\mathcal{X}_6$ admits a symplectic basis (α_i, β^j) for $H^3(\mathcal{X}_6)$: $\int_{\mathcal{X}_6} \alpha_i \wedge \beta^j = \ell_s^6 \delta_i{}^j$
  e.g. $\mathcal{X}_6 = CY_3/\Omega\mathcal{R}$
- axions emerge from reduction of RR-form C_3
  & charges under (D-brane) $U(1)$ from reduction of RR-form C_5:

$$C_3 = \frac{1}{2\pi}\sum_{i=1}^{b_3} \xi^i(x)\,\alpha_i(y) + \dots \qquad C_5 = \frac{1}{2\pi}\sum_{i=1}^{b_3} D_{(2)i} \wedge \beta^i + \dots$$

- Reduction of bulk action $\longrightarrow$ kinetic terms

$$\mathcal{S}_R^{\text{bulk}} \ni -\frac{\pi}{2\ell_s^8}\int \mathrm{d}C_i \wedge \star_4 \mathrm{d}C_{i=3,5} \longrightarrow \mathcal{S}_{kin} = -\frac{1}{4\ell_s^2}\int d\xi^i \wedge \star_4 d\xi^j \mathcal{K}_{ij} + dD_{(2)i} \wedge \star_4 dD_{(2)_j} \mathcal{K}^{ij}$$

with $\mathcal{K}_{ij} = \frac{1}{2\pi\ell_s^6}\int_{\mathcal{X}_6} \alpha_i \wedge \star_6 \alpha_j$ and $\mathcal{K}^{ij} = \mathcal{K}_{ij}^{-1}$

String Theory embedding II

- D6-brane wraps $\mathcal{M}_{1,3} \times \Delta_3$, w.r.t. de Rahm-dual basis (γ_i, δ^j)

$$\Delta_3 = \sum_{i=1}^{b_3/2} (r^i \gamma_i + p_i \delta^i) \qquad \int_{\gamma_j} \alpha_i = \ell_s^3 \delta_i{}^j = \int_{\delta^i} \beta^j$$

- Reduction of D-brane action $\longrightarrow$ anomalous coupling + $U(1)$ charges

$$\mathcal{S}_{CS}^{D6} \ni \int_{D6} \frac{1}{4\pi\ell_s^3} C_3 \wedge F^2 + \frac{1}{\ell_s^5} C_5 \wedge F \longrightarrow \frac{1}{8\pi^2} \sum_{i=1}^{b_3/2} r^i \int_{\mathcal{M}_{1,3}} \xi^i F \wedge F + \frac{1}{2\pi\ell_s^2} \sum_{j=1}^{b_3/2} p_j \int_{\mathcal{M}_{1,3}} D_{(2)j} \wedge \mathrm{d}A$$

- $D_{(2)i}$ is Hodge-dual to $\xi^i \rightsquigarrow$ dualisation in favour of ξ^i:

$$\boxed{\mathcal{S}_{axion} = -\frac{1}{2\ell_s^2} \int_{\mathcal{M}_{1,3}} \left[\frac{1}{2} \left(d\xi^i - \frac{p_i}{\pi} A \right) \wedge \star_4 \left(d\xi^j - \frac{p_j}{\pi} A \right) \mathcal{K}_{ij} \right] + \frac{1}{8\pi^2} \sum_i r^i \int_{\mathcal{M}_{1,3}} \xi^i F \wedge F}$$

- similar reasoning for C_4-axions in type IIB with D7-branes

  Grimm-Louis (2004), Jockers-Louis (2005), Haack-Krefl-Lust-Van Proeyen-Zagermann (2006)