Holographic negative magneto-resistivity

Ya-Wen Sun
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Based on work with A. Jimenez-Alba, K.Landsteiner, Yan Liu and Qing Yang
(1410.6399,1504.06566, and work in progress)

Motivation: anomalous magneto-transport in chiral fluid

- *Chiral fluid:* QGP, 3+1-D Dirac/Weyl (semi-) metal

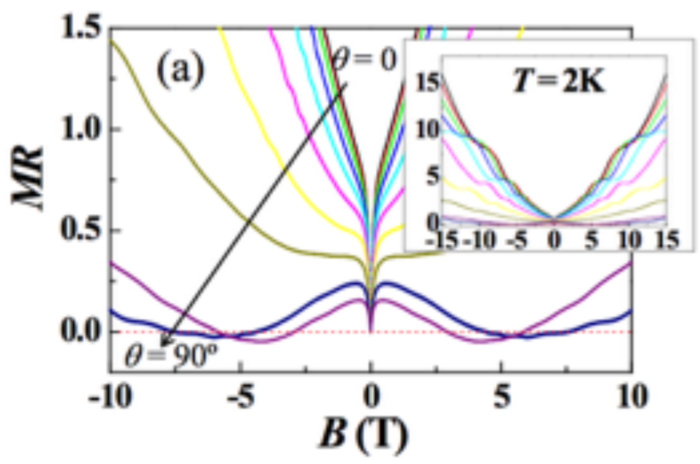
- *Anomalous magneto-transport associated with chiral anomaly:* negative magneto-resistivity, anomalous Hall effect, chiral magnetic effect, etc…

- *Negative Magneto-resistivity:* Longitudinal Magneto-resistivity decreases with increasing magnetic field, Positive magneto-conductivity

- *Longitudinal magneto-resistivity:* electric conductivity with a parallel background magnetic field; Chiral (Adler-Bell-Jackiw, or axial) anomaly: "E.B" effect.

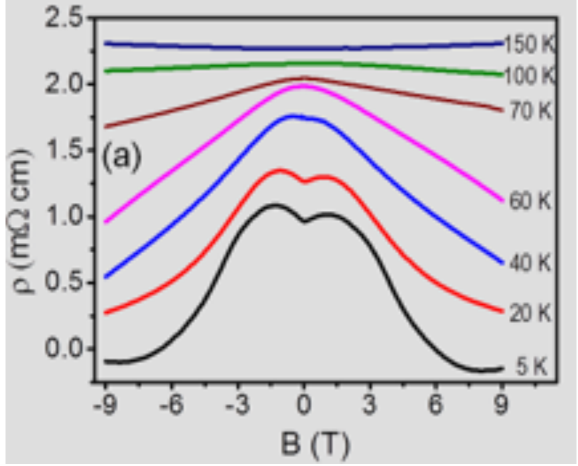
Motivation: negative magneto-resistivity in chiral fluid

- *Experimental observations on NMR*



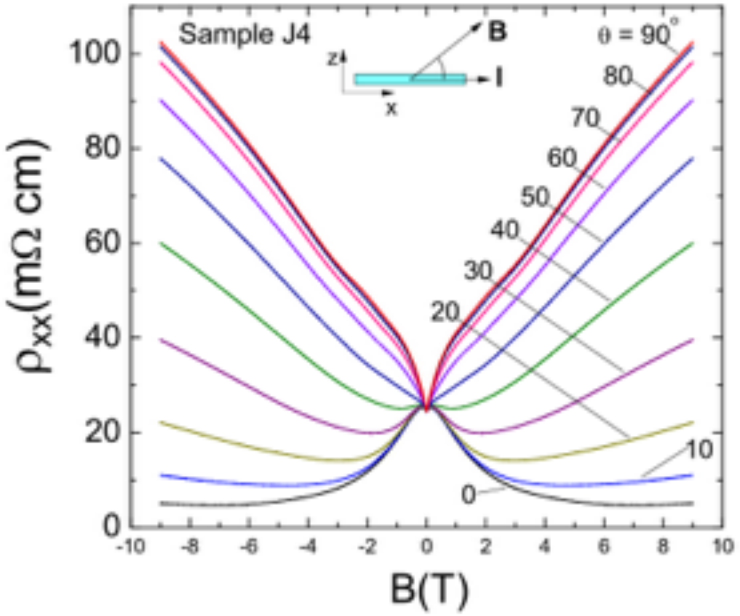
$Bi_{1-x}Sb_x$

Kim et al. 2013



$ZrTe_5$

Li, Kharzeev, et al. 1412.6543



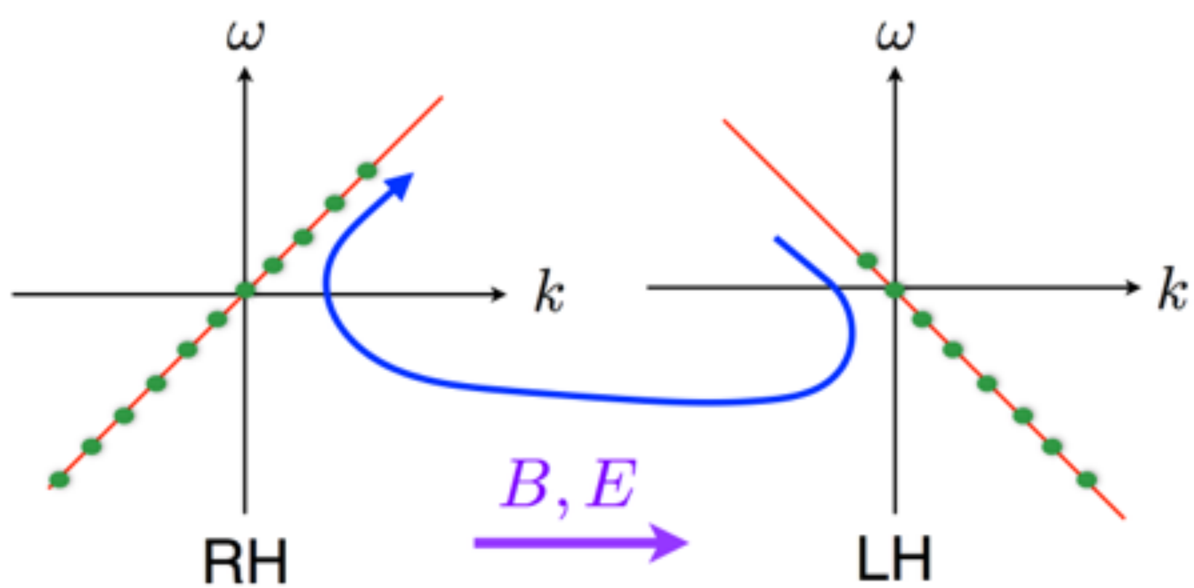
Na_3Bi

Xiong, et al. 2015

- NMR is also observed in TaAs, Cd3As2, TaP…

Motivation: NMR in the kinetic theory picture

- Mechanism: 3+1D Weyl fermions in the background magnetic field (Nielsen,Ninomiya, 1983)



- **infinite** DC magneto-conductivity due to chiral anomaly

- Infinite DC electric conductivity: momentum dissipation

Motivation: negative magneto-resistivity in chiral fluid

- ***Question I:*** How to obtain a physical and finite longitudinal magneto-conductivity?

    What kinds of dissipations do we need?

- ***Question II:*** What happens for strongly coupled anomalous system? Do we still have NMR?

    Holography

Outline:

- **Magneto-conductivity in chiral fluid:**
  universal hydro results with all possible dissipation effects

- **Holographic NMR without axial charge dissipations:** probe limit and with backreactions

- **Holographic axial charge dissipations: explicit breaking U(1)A :** probe limit and with backreactions

- **Conclusion and open questions**

I Magneto-conductivity in chiral fluid: dissipations

- 3+1D anomalous fluid with $U(1)_V \times U(1)_A$

- **Hydrodynamics** (D. Son, Surowka, 2009)

$$\partial_\mu T^{\mu\nu} = F^{\nu\alpha} J_\alpha \,, \quad \partial_\mu J^\mu = 0 \,, \quad \partial_\mu J_5^\mu = c E^\mu B_\mu \,,$$

constitutive equation

$$\begin{aligned} T^{\mu\nu} &= \epsilon u^\mu u^\nu + p P^{\mu\nu} + \tau^{\mu\nu} \,, \\ J^\mu &= \rho u^\nu + \nu^\mu \,, \\ J_5^\mu &= \rho_5 u^\nu + \nu_5^\mu \end{aligned}$$

where

$$\nu^\mu = -\sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) - \sigma_5 T P^{\mu\nu} \partial_\nu \left(\frac{\mu_5}{T}\right) + \sigma^{(E)} E^\mu + \sigma^{(V)} \omega^\mu + \sigma^{(B)} B^\mu \,,$$
$$\nu_5^\mu = -\sigma_5 T P^{\mu\nu} \partial_\nu \left(\frac{\mu}{T}\right) - \sigma T P^{\mu\nu} \partial_\nu \left(\frac{\mu_5}{T}\right) + \sigma_5^{(E)} E^\mu + \sigma_5^{(V)} \omega^\mu + \sigma_5^{(B)} B^\mu \,,$$

$$\sigma^{(B)} = c\mu_5 \left(1 - \frac{\mu\rho}{\epsilon + p}\right)$$

Erdmenger et.al,
Banerjee et. al, 2008;
D. Kharzeev et. al,
2008

I Magneto-conductivity in chiral fluid: dissipations

Linear response: determine transport coefficients by solving initial value problems in hydrodynamic equations (Kadanoff&Martin, 1963; Hartnoll et al. 2007).

Perturbations around thermal equilibrium

$$\begin{aligned} \mu(\vec{x},t) &= \mu + \delta\mu(\vec{x},t)\,, \\ \mu_5(\vec{x},t) &= \mu_5 + \delta\mu_5(\vec{x},t)\,, \\ T(\vec{x},t) &= T + \delta T(\vec{x},t)\,, \\ u^\mu(\vec{x},t) &= (1, \delta u_i(\vec{x},t))\,, \end{aligned}$$

to compute conductivity, $\delta E_i = \delta F^{0i} = -\delta F^{i0}$

with background magnetic field $F_{12} = -F_{21} = B, \quad E^\mu = 0$

I Magneto-conductivity in chiral fluid: dissipations

- Energy, momentum and axial charge dissipations

$$\partial_\mu \delta T^{\mu 0} = \delta F^{0\mu} J_\mu + \frac{1}{\tau_e} \delta T^{\mu 0} u_\mu \,,$$

$$\partial_\mu \delta T^{\mu i} = \rho \delta E^i + F^{i\lambda} \delta J_\lambda + \frac{1}{\tau_m} \delta T^{\mu i} u_\mu \,,$$

$$\partial_\mu \delta J^\mu = 0 \,,$$

$$\partial_\mu \delta J_5^\mu = c \delta E^\mu B_\mu + \frac{1}{\tau_5} \delta J_5^\mu u_\mu \,.$$

- Initial value problem:

$$(\delta\mu^0, \delta\mu_5^0, \delta T^0, \delta u_i^0, \delta E_i^0) \rightarrow (\delta T^{\mu\nu}, \delta J^\mu, \delta J_5^\mu)$$

- Transport coefficient:

$$\delta J^\mu(\vec{k}, \omega) = \frac{G_{J^\mu;J^\nu}(\vec{k}, \omega) - G_{J^\mu;J^\nu}(\vec{k}, 0)}{i\omega} \delta E_\nu^0(\vec{k}) + \ldots$$

electric conductivity

I Magneto-conductivity in chiral fluid: dissipations

Longitudinal magneto-conductivity

$$\Sigma = \sigma^{(E)} + \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c \sigma^{(B)}}{D} K_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p} \left[\rho - \frac{B^2 c}{D} K_1 \right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{D} K_2$$

D, K_0, K_1, K_2: functions of thermodynamic variables

$$\begin{aligned}
\delta\epsilon &\equiv e_5 \delta\mu_5 + e_1 \delta\mu + e_2 \delta T = \left(\frac{\partial \epsilon}{\partial \mu_5}\right)\Big|_{T,\mu} \delta\mu_5 + \left(\frac{\partial \epsilon}{\partial \mu}\right)\Big|_{T,\mu_5} \delta\mu + \left(\frac{\partial \epsilon}{\partial T}\right)\Big|_{\mu,\mu_5} \delta T\,, \\
\delta\rho &\equiv f_5 \delta\mu_5 + f_1 \delta\mu + f_2 \delta T = \left(\frac{\partial \rho}{\partial \mu_5}\right)\Big|_{T,\mu} \delta\mu_5 + \left(\frac{\partial \rho}{\partial \mu}\right)\Big|_{T,\mu_5} \delta\mu + \left(\frac{\partial \rho}{\partial T}\right)\Big|_{\mu,\mu_5} \delta T\,, \\
\delta\rho_5 &\equiv s_5 \delta\mu_5 + s_1 \delta\mu + s_2 \delta T = \left(\frac{\partial \rho_5}{\partial \mu_5}\right)\Big|_{T,\mu} \delta\mu_5 + \left(\frac{\partial \rho_5}{\partial \mu}\right)\Big|_{T,\mu_5} \delta\mu + \left(\frac{\partial \rho_5}{\partial T}\right)\Big|_{\mu,\mu_5} \delta T\,, \\
\delta p &= \rho_5 \delta\mu_5 + \rho \delta\mu + s \delta T\,,
\end{aligned}$$

I Magneto-conductivity in chiral fluid: dissipations

Longitudinal magneto-conductivity

$$\Sigma = \sigma^{(E)} + \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c \sigma^{(B)}}{D} K_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p}\left[\rho - \frac{B^2 c}{D} K_1\right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{D} K_2$$

- **Remarks (I):**

- When c=0 or B=0, it reduces to the ordinary electric conductivity

$$\sigma = \sigma_E + \frac{i}{\omega_m} \frac{\rho^2}{\epsilon + p}$$

- All dissipations are needed to get a finite DC longitudinal MC. In contrast, to get a finite transverse DC MC, we only need momentum dissipation.

I Magneto-conductivity in chiral fluid: dissipations

Longitudinal magneto-conductivity

$$\Sigma = \sigma^{(E)} + \frac{i}{\omega + \frac{i}{\tau_e}} \frac{B^2 c \sigma^{(B)}}{D} K_0 + \frac{i}{\omega + \frac{i}{\tau_m}} \frac{\rho}{\epsilon + p} \left[\rho - \frac{B^2 c}{D} K_1 \right] + \frac{i}{\omega + \frac{i}{\tau_c}} \frac{B^2 c^2}{D} K_2$$

- **Remarks (II):**

- *Energy dissipations* are only needed when there is a non-vanishing axial chemical potential;
- *momentum relaxation* is always related to finite density.
- We will focus on the zero density limit $\frac{\rho}{\epsilon + p} \to 0, \quad \frac{\rho_5}{\epsilon + p} \to 0,$ we have infinite DC conductivity without dissipation terms:

$$\sigma = \sigma_E + \frac{i}{\omega} \frac{B^2 c^2}{(\partial \rho_5 / \partial \mu_5)|_{T,\mu}}$$

This is consistent with the kinetic-physics picture.

For chiral fluid with axial charge dissipation: $\omega \to \omega + \frac{i}{\tau_5}$

II Holographic magneto-conductivity: probe limit

- *Holographic system in the probe limit:* 4+1D Schwartzschild black hole with thermal quantities $\epsilon = 3r_0^4\,,\quad s = 4\pi r_0^3\,,\quad T = \frac{r_0}{\pi}$

- *Chiral anomaly:* realised by a Chern-Simons term

$$S = \int d^5x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\Big(R+12\Big) - \frac{1}{4}\mathcal{F}^2 - \frac{1}{4}F^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\Big(F_{\nu\rho}F_{\sigma\tau} + 3\mathcal{F}_{\nu\rho}\mathcal{F}_{\sigma\tau}\Big)\Big]$$

- *Anomaly constant* in hydro equation $\quad c = 8\alpha$

- *Holographic calculations for the longitudinal conductivity in the probe limit via Kubo formula:* perturbations $\delta A_t, \delta V_z$

- Analytic result for longitudinal magneto-conductivity at small frequency

$$\sigma = \left[\frac{8\pi\alpha^2B^2}{r_0^3}\sec\left(\frac{\pi}{2}\sqrt{1-(8B\alpha/r_0^2)^2}\right) + \frac{i}{\omega}\frac{16B^2\alpha^2}{r_0^2}\right]\frac{\Gamma\left[\frac{3-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]\Gamma\left[\frac{3+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]}{\Gamma\left[\frac{5-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]\Gamma\left[\frac{5+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]}$$

II Holographic magneto-conductivity: probe limit

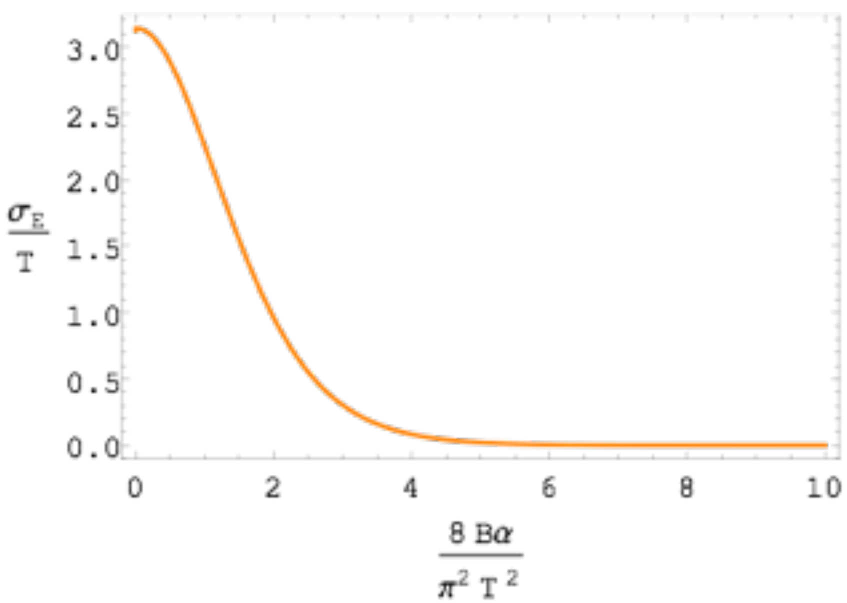
- The quantum critical conductivity affected by the background magnetic field

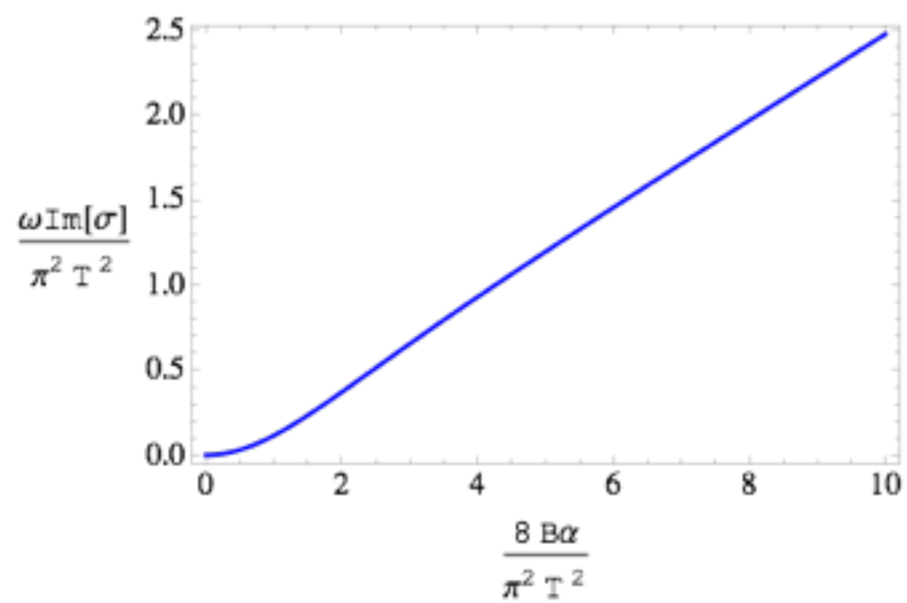
$$\sigma_E = \frac{8\pi\alpha^2 B^2}{r_0^3}\sec\left(\frac{\pi}{2}\sqrt{1-(8B\alpha/r_0^2)^2}\right)\frac{\Gamma\left[\frac{3-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]\Gamma\left[\frac{3+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]}{\Gamma\left[\frac{5-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]\Gamma\left[\frac{5+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\right]}$$

at small B, $\sigma_E = \pi T - \frac{c^2 B^2 \log 2}{2\pi^3 T^3} + \mathcal{O}(\alpha^4 B^4)$; at large B, $\sigma_E = e^{-\frac{cB}{2\pi T^2}}\left(\frac{cB}{T} + \mathcal{O}\left(\frac{1}{cB}\right)\right)$

The coefficient of the second term,

at small B, $\frac{c^2 B^2}{2\pi^2 T^2} + \mathcal{O}(\alpha^4 B^4)$; at large B, $cB + \mathcal{O}\left(\frac{1}{cB}\right)$



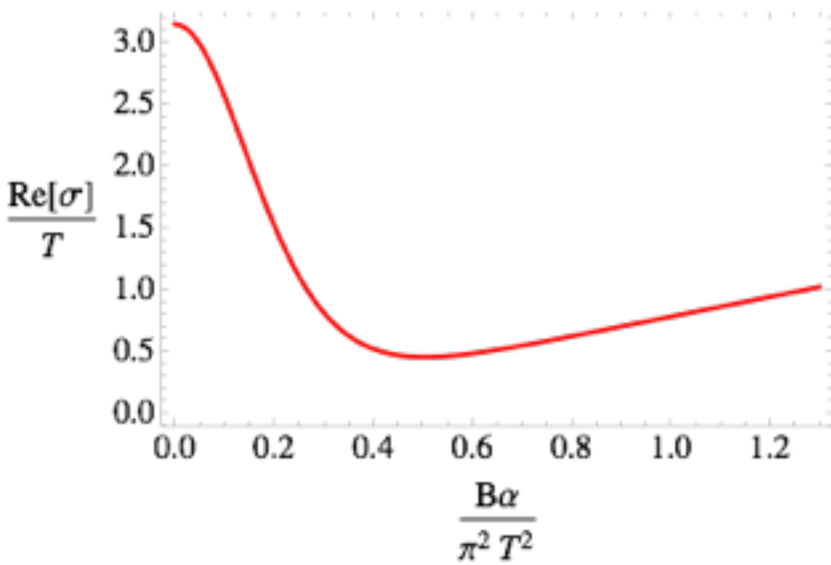


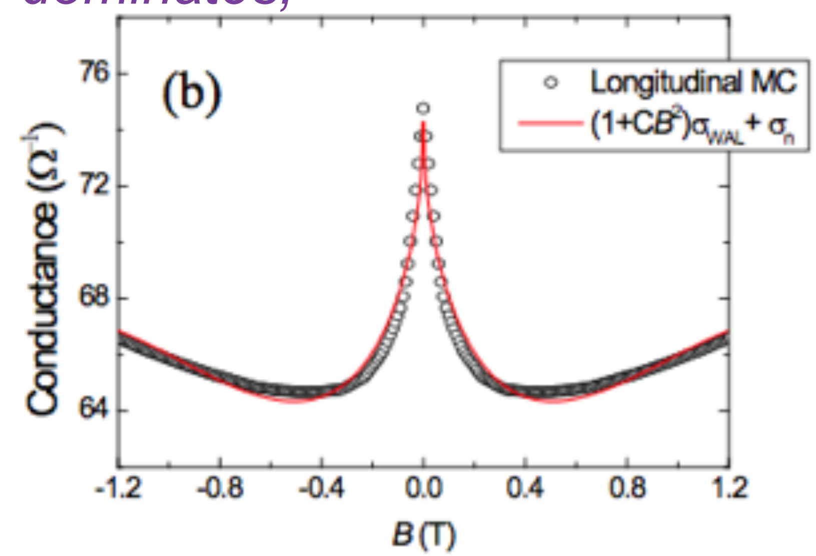
II Holographic magneto-conductivity: probe limit

- Introduce an axial relaxation time"by hand":

$$\sigma_{DC} = \sigma_E + \tau_5 \frac{B^2 c^2}{(\partial\rho_5/\partial\mu_5)|_{T,\mu}},$$

Small B: first term dominates;
Large B: second term dominates;





- Qualitatively the same as experimental result
- New hints from holography on small B: quantum critical conductivity due to chiral anomaly?

II Holographic magneto-conductivity: probe limit

- Compare with the hydrodynamic formula?
- Remember the hydrodynamic formula without axial charge dissipation $$\sigma = \sigma_E + \frac{i}{\omega}\frac{B^2c^2}{(\partial\rho_5/\partial\mu_5)|_{T,\mu}}$$
- The result involves thermodynamic quantities which depend on different systems.

- To solve for the thermodynamic quantities besides energy density and entropy density:

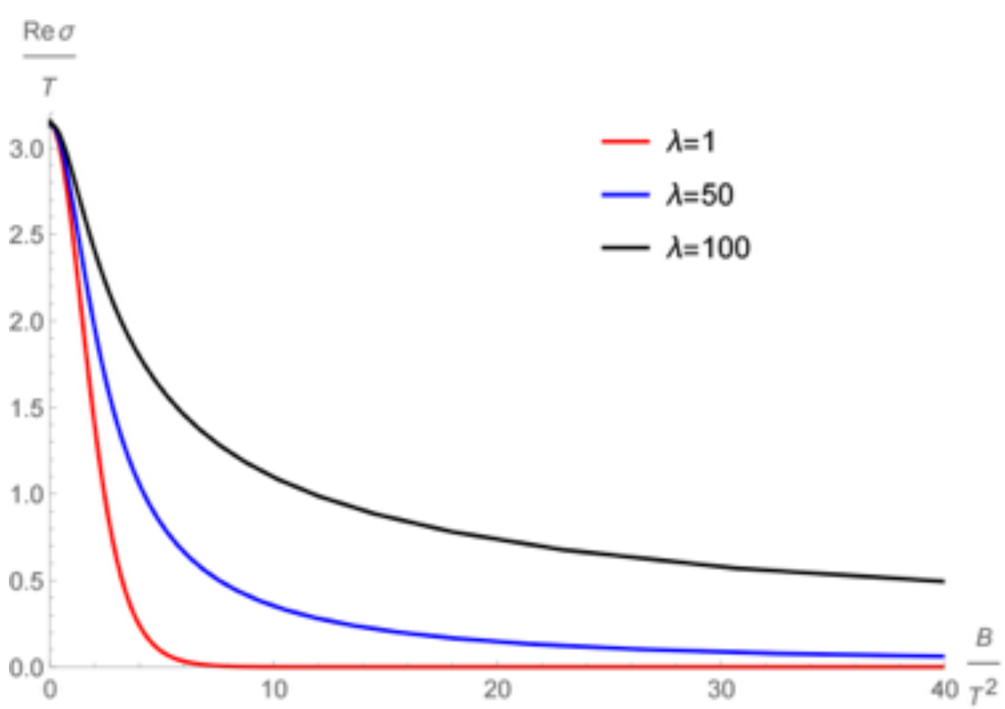
$$V_\mu = \big(V_t(r), By, 0, V_z(r), 0\big)\,, \quad A_\mu = \big(A_t(r), 0, 0, A_z(r), 0\big)$$

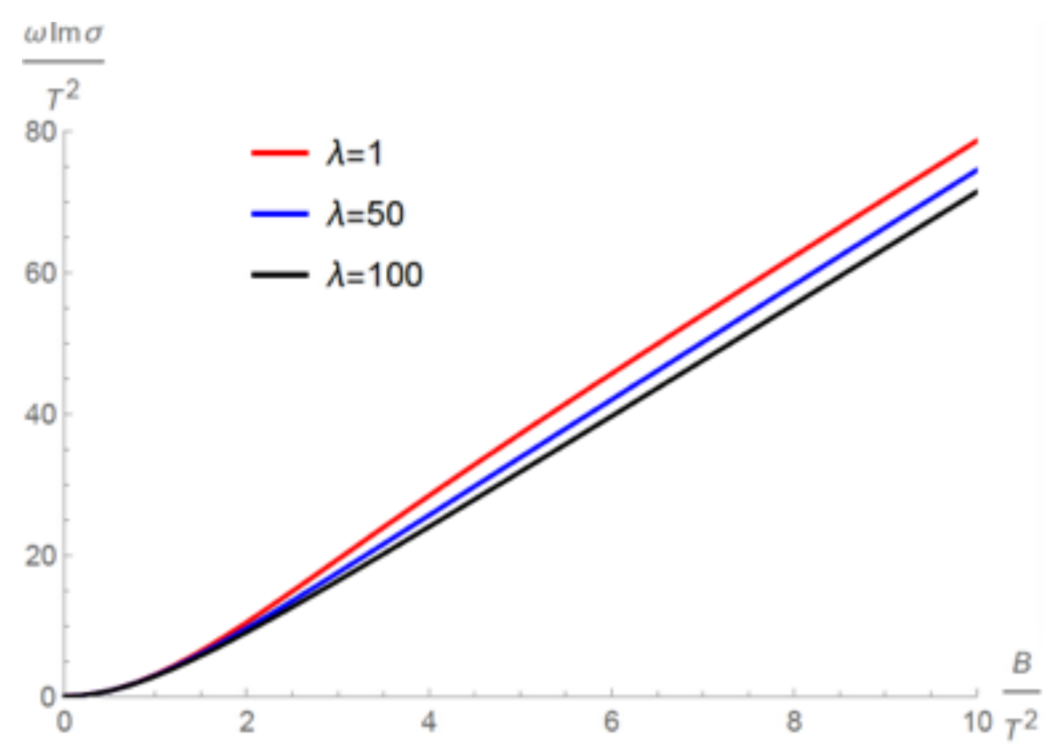
- MC from hydro method: still valid even at large B, i.e. outside the hydrodynamic regime!

$$\sigma = \sigma_E + \frac{i}{\omega}\frac{B^2c^2}{4r_0^2}\frac{\Gamma\big[\frac{3-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\big]\Gamma\big[\frac{3+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\big]}{\Gamma\big[\frac{5-\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\big]\Gamma\big[\frac{5+\sqrt{1-(8B\alpha/r_0^2)^2}}{4}\big]}\,.$$

II Holographic magneto-conductivity: zero densities with backreactions

- *Very large B behavior:* Backreactions
- Zero charge and axial charge densities; finite temperature
- $\lambda = \frac{2\kappa^2}{e^2}$ controls the strength of backreactions





II Holographic magneto-conductivity: zero densities with backreactions

- Numerical difficulties in the large B region;
- Zero temperature: infinite B/T^2 limit
- Near horizon: $\mathrm{AdS}_3\times\ \mathrm{R}^2$ +irrelevant deformations
- Surprisingly: the zero temperature magneto-conductivity goes back to the probe limit result for large B!

$$\sigma_{zz}(\omega\to 0) = 8\alpha B\frac{i}{w}$$ (for large enough α)

- Hydrodynamic formula still valid even at zero temperature

III Holographic axial charge dissipation: probe limit

- Realise an "intrinsic" axial charge relaxation time from holography:
  explicit breaking; massive gauge field;

Explicit breaking: breaking the axial symmetry $U(1)_A$ with a scalar source

$$S = \int d^5x\sqrt{-g}\Big[\frac{1}{2\kappa^2}\Big(R+12\Big) - \frac{1}{4}\mathcal{F}^2 - \frac{1}{4}F^2 + \frac{\alpha}{3}\epsilon^{\mu\nu\rho\sigma\tau}A_\mu\Big(F_{\nu\rho}F_{\sigma\tau} + 3\mathcal{F}_{\nu\rho}\mathcal{F}_{\sigma\tau}\Big)$$
$$-(D_\mu\Phi)^*(D^\mu\Phi) - m_s^2\Phi^*\Phi\Big]$$

scalar field charged under $U(1)_A$.

$m_s^2 = -3$: the conformal dimension of the non-renormalisation mode is 1.

$$\partial_\mu J_5^\mu = M\bar{\psi}\gamma_5\psi + c_{\rm em}\epsilon^{\mu\nu\rho\beta}F_{\mu\nu}F_{\rho\beta}\,.$$

$$\phi = \frac{M}{r} + \frac{\mathcal{O}}{r^3} + \dots,$$

III Holographic axial charge dissipation: probe limit

- Zero densities：
- Background: $V_\mu = (0, By, 0, 0, 0), \quad A_\mu = 0 \quad \phi = \frac{M}{T}\left(\frac{2}{\pi}\right)^{3/2} \frac{r_0 \Gamma[3/4]}{r\Gamma[1/4]} \mathrm{EllipticK}\left[\frac{1-\frac{r_0^2}{r^2}}{2}\right],$
- Applicable regime of probe limit: $T \gg \kappa\mu, \kappa\mu_5, \sqrt{\kappa B}, \kappa M$

  we could trust the regime T of order M.

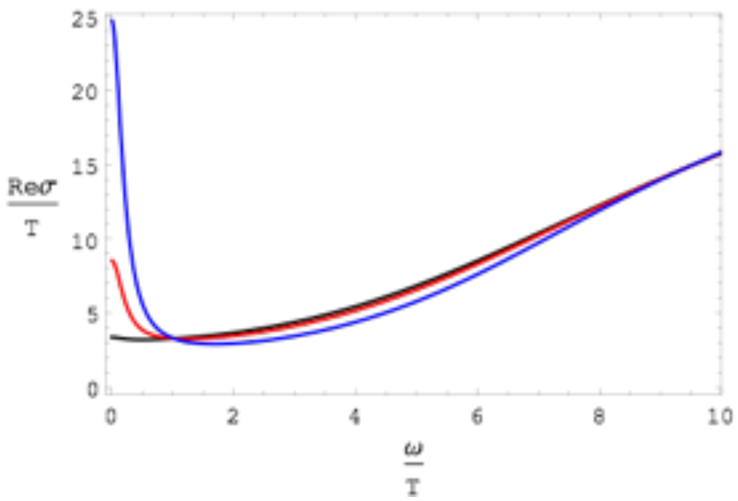
- Longitudinal fluctuations

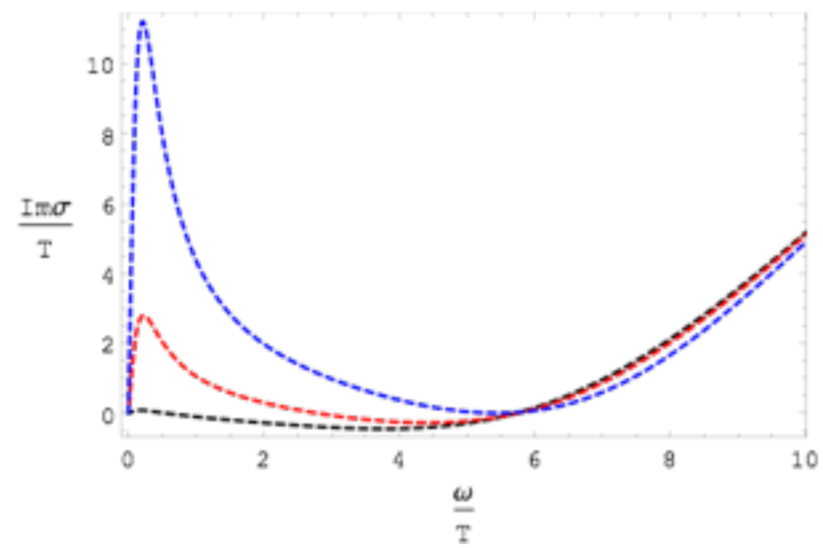
$$\delta V_z = v_z(r) e^{-i\omega t}, \quad \delta A_t = a_t(r) e^{-i\omega t}, \quad \delta\Phi = i\phi_2(r) e^{-i\omega t}.$$

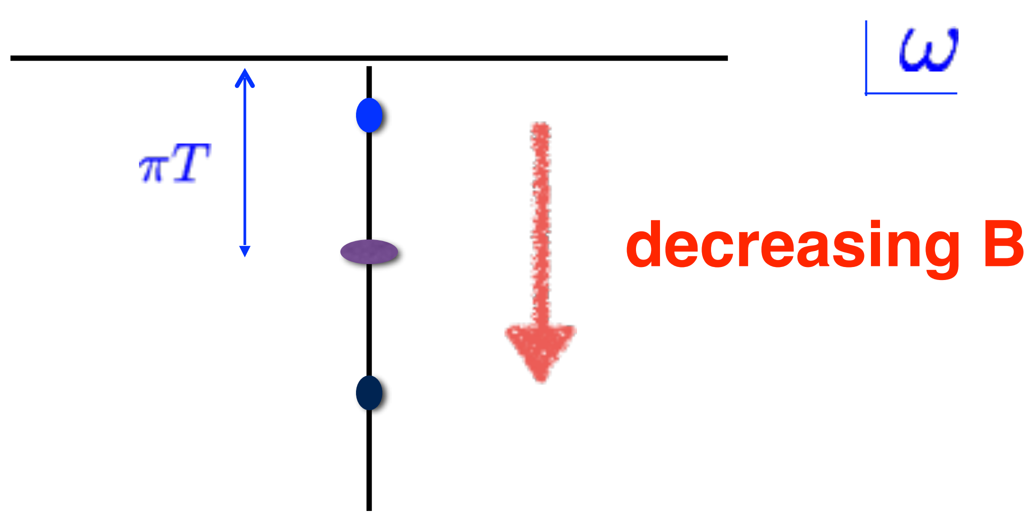
Longitudinal magneto-conductivity: $\sigma(\omega) = \frac{1}{i\omega} \frac{2v_z^{(2)}}{v_z^{(0)}}.$

III Holographic axial charge dissipation: probe limit

- Real and imaginary parts of longitudinal magneto-conductivity for fixing M while different B

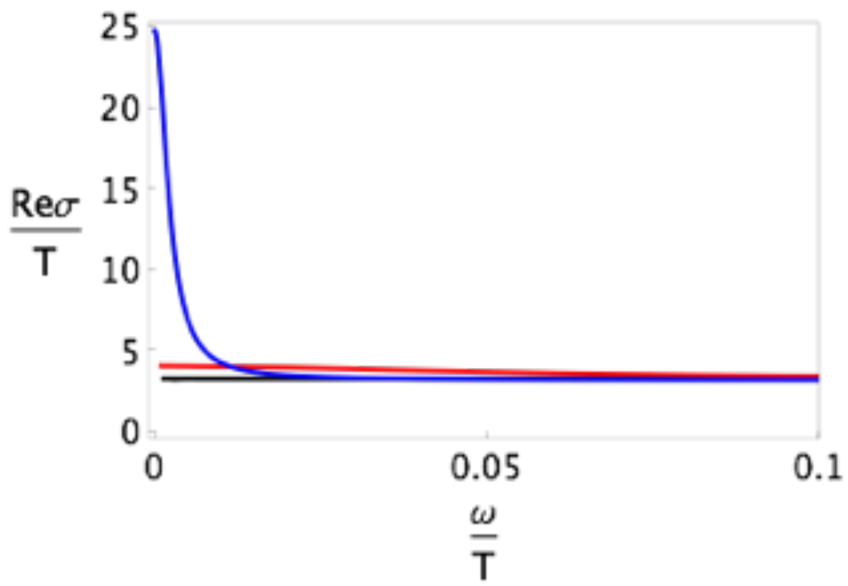


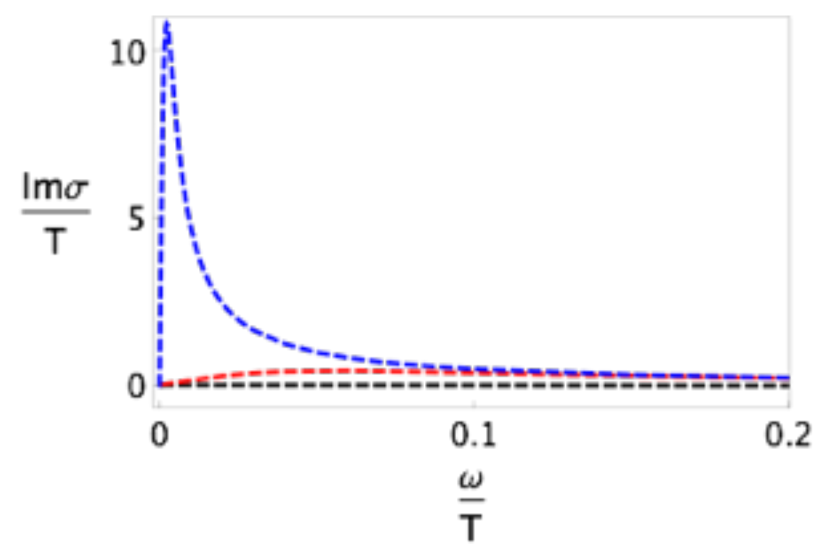


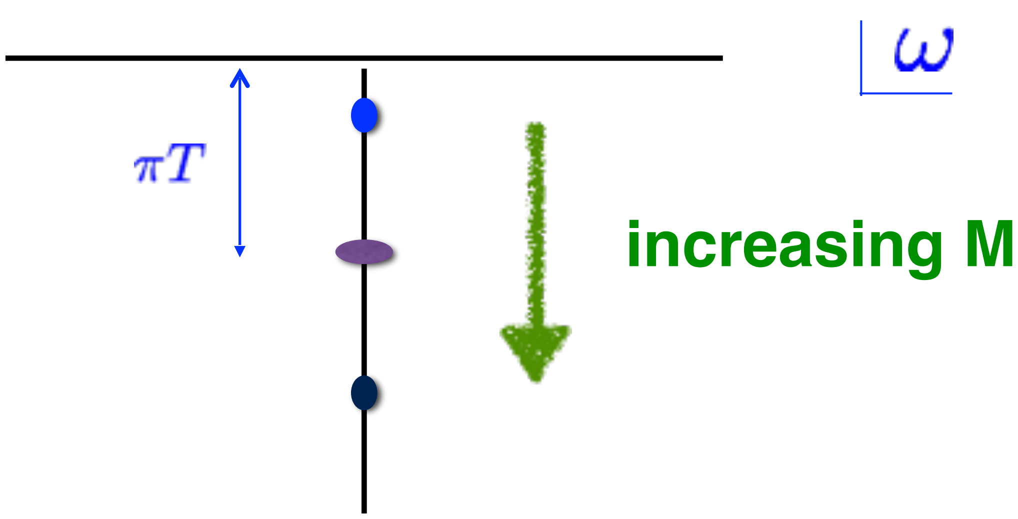


III Holographic axial charge dissipation: probe limit

- Real and imaginary parts of longitudinal magneto-conductivity for fixing B while different M



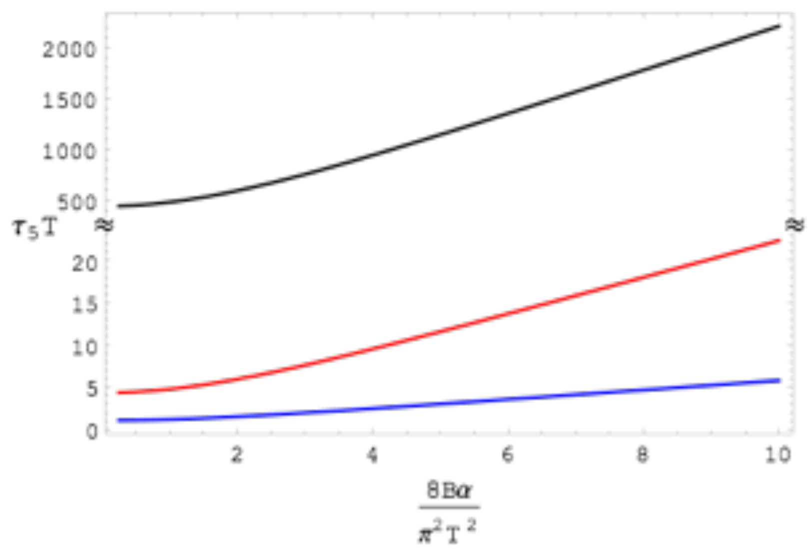


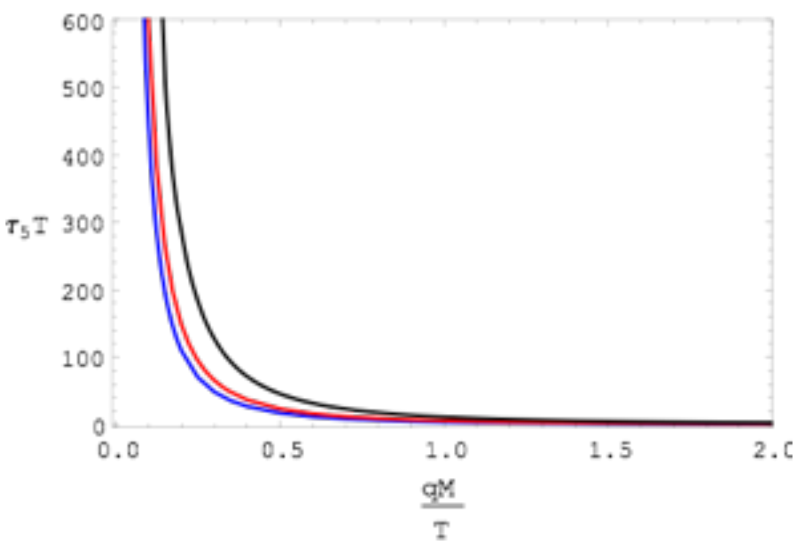


III Holographic axial charge dissipation: probe limit

- Relaxation time: directly calculated from the pole of the two-point correlator: $\langle J_5^0 J_5^0 \rangle_R$, which is obviously true for hydrodynamics $\quad \partial_\mu J^\mu = 0, \quad \partial_\mu J_5^\mu = -\frac{1}{\tau_5} J_5^0$

- Numerical results





- The relaxation time τ_5 decreases when M increases or B decreases: agrees with the behaviour in the AC conductivity

III Holographic axial charge dissipation: probe limit

DC conductivity: Negative Magnetoresistivity

- Radially conserved quantity (Donos, Gauntlett, 2013)

$$\delta V_\mu = (v_t(r), 0, 0, -Et + v_z(r), 0)\,, \quad \delta A_\mu = (a_t(r), 0, 0, a_z(r), a_r(r))\,, \quad \delta\phi = \phi_1(r) + i\phi_2(r)\,.$$

For zero densities:

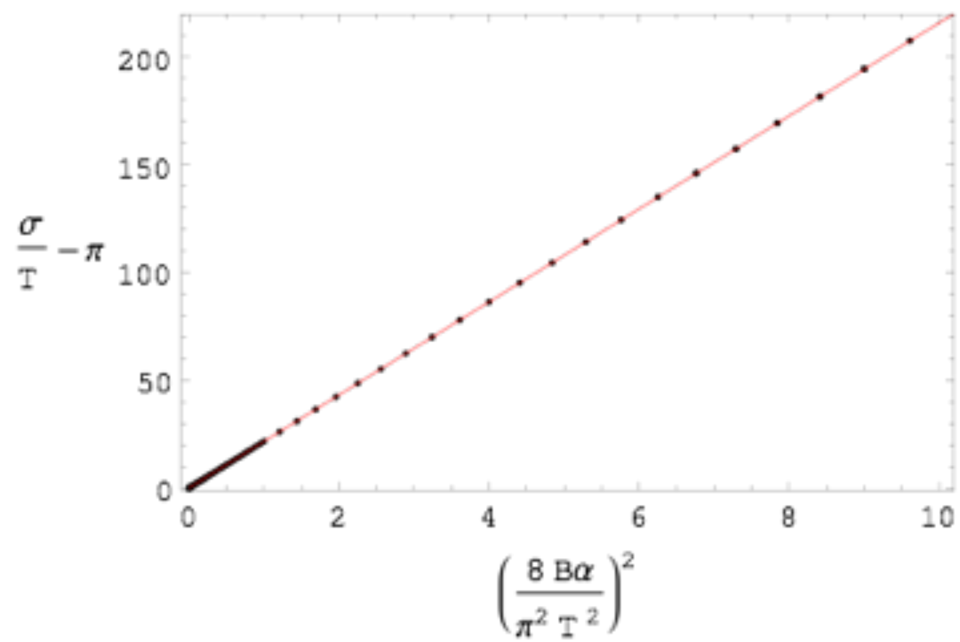
$$a_t'' + \frac{3}{r}a_t' - \frac{2q^2\phi^2}{r^2 f}a_t + \frac{8B\alpha}{r^3}v_z' = 0\,,$$
$$v_z'' + \left(\frac{3}{r} + \frac{f'}{f}\right)v_z' + \frac{8B\alpha}{r^3 f}a_t' = 0$$

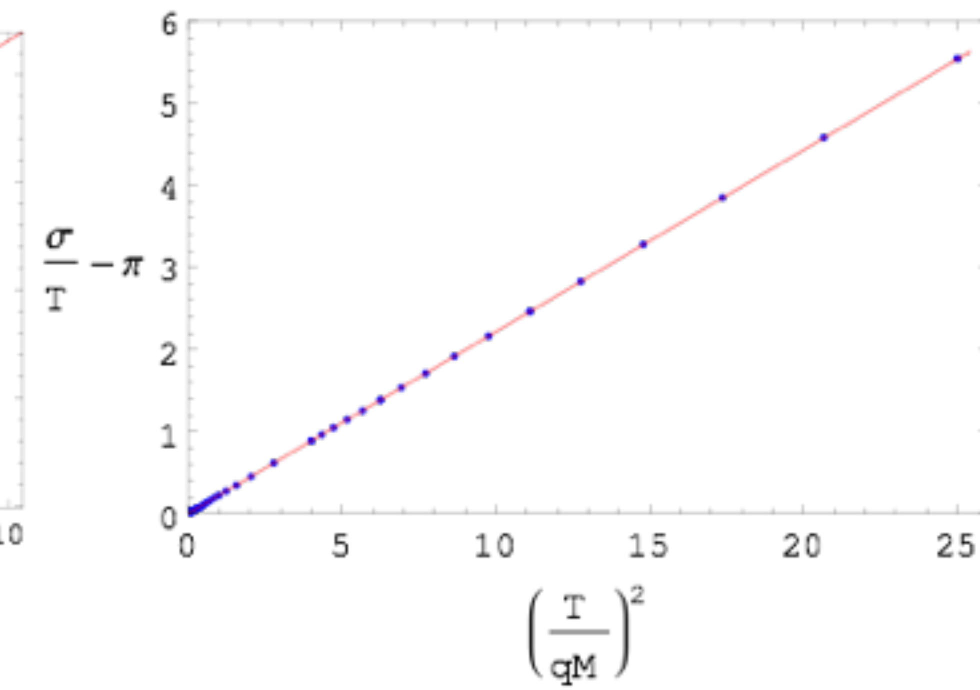
Conserved quantity: $J = -r^3 f v_z' - 8B\alpha a_t$

DC conductivity: $\sigma_{\rm DC} = \frac{J}{E} = \pi T + T\frac{\Gamma[1/4]^2}{4\pi^2\Gamma[3/4]^2}\frac{(8B\alpha/\pi^2T^2)^2}{(qM/T)^2}$

DC conductivity also can be computed from the zero frequency limit of AC conductivity via Kubo formula: exactly the same results

III Holographic axial charge dissipation: probe limit





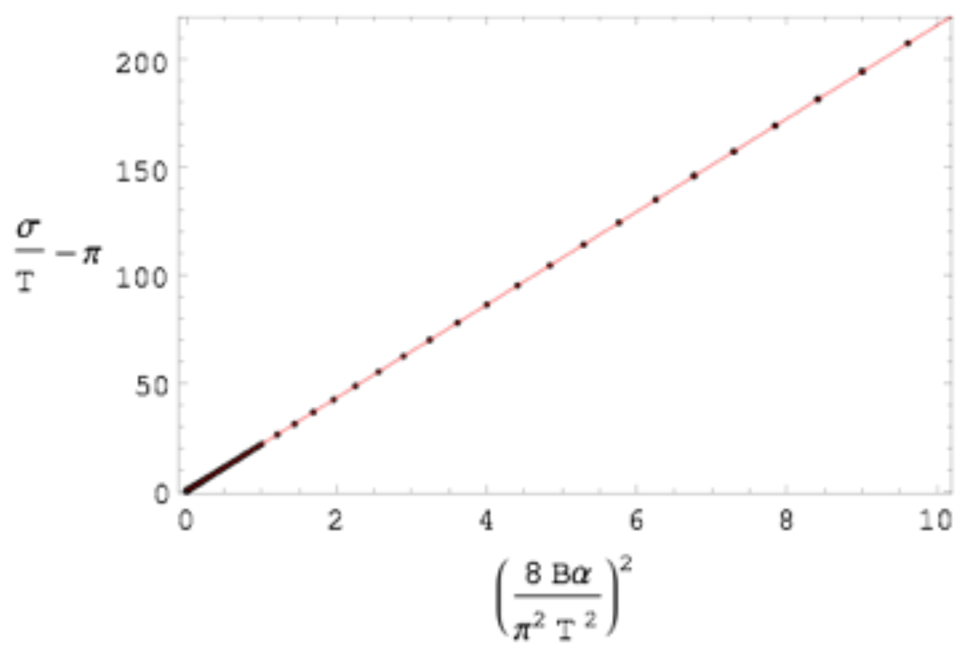
- **Comments**

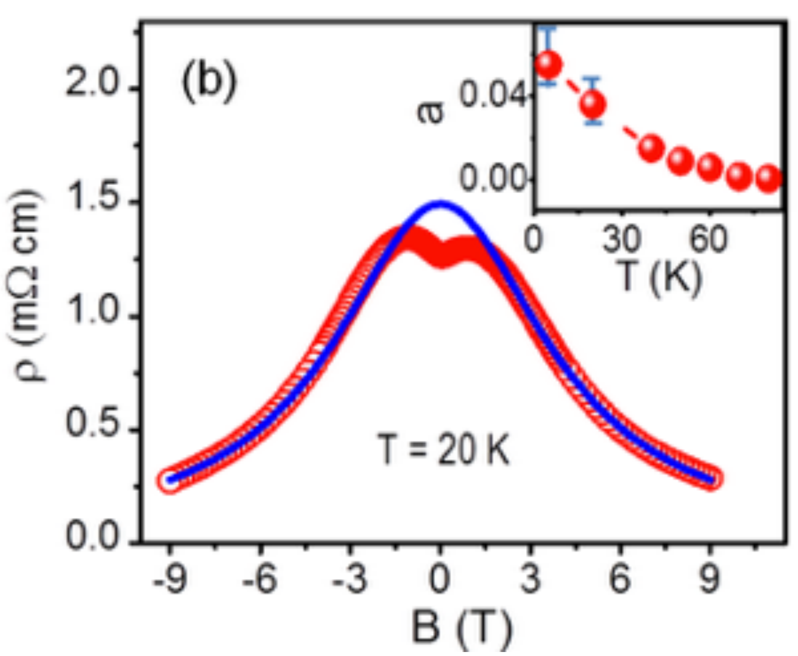
- Agrees with the hydrodynamic formula at large τ_5 , even when the magnetic field is large

III Holographic axial charge dissipation: probe limit

Comments

- Exact B-squared behaviour: the same as the experiment findings.





Li, Kharzeev, et al. 1412.6543

III Holographic axial charge dissipation: probe limit

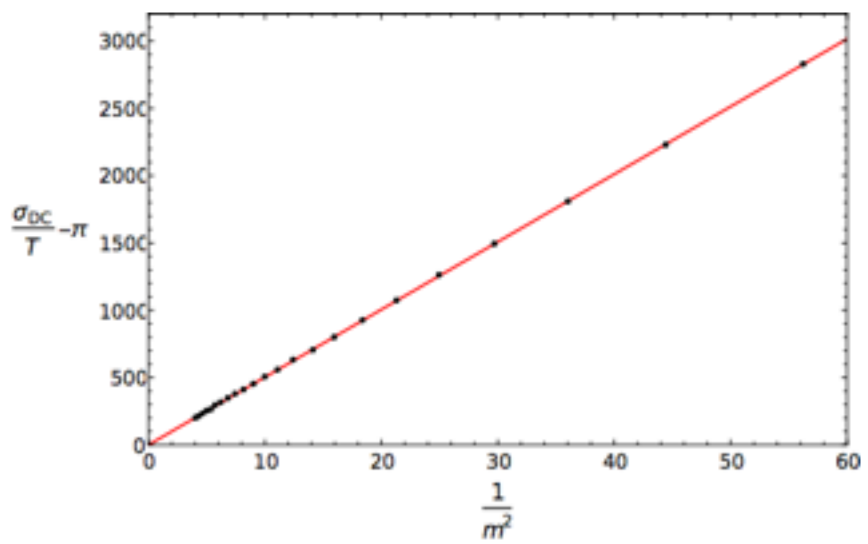
- Another way to realise the holographic axial charge dissipation model: adding a mass to the axial gauge field

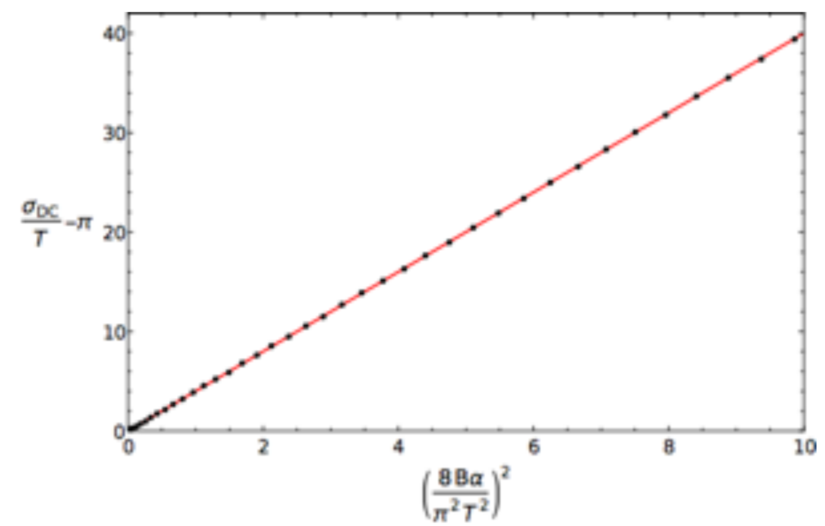
$$\mathcal{S} = \int d^5x\sqrt{-g}\left(\frac{1}{2\kappa^2}\left(R+\frac{12}{L^2}\right)-\frac{1}{4}F^2-\frac{1}{4}\mathcal{F}^2-\frac{m^2}{2}A_\mu A^\mu+\frac{\alpha}{3}\epsilon^{\mu\alpha\beta\gamma\delta}A_\mu\left(F_{\alpha\beta}F_{\gamma\delta}+3\mathcal{F}_{\alpha\beta}\mathcal{F}_{\gamma\delta}\right)\right),$$

- Due to the change of the conformal dimension of axial current, the dual hydrodynamics is not clear. However, we do obtain the finite DC magneto-conductivity.

III Holographic axial charge dissipation: probe limit

- Analytical result: $\sigma = \pi T + \frac{\pi T}{m^2}\left(\frac{8B\alpha}{\pi^2 T^2}\right)^2$
- Numerical result can be obtained as zero frequency limit of AC via Kubo formulae





III Holographic axial charge dissipation: probe limit

Comments

- Same equations for perturbations when choosing suitable gauges;

- The scalar operator in the explicit breaking model gives an effective mass to the gauge field. Similar physics happened for momentum dissipation case (Blake, et al. 2014). However, the other physical quantities behave differently, e.g. relaxation time, static susceptibility.

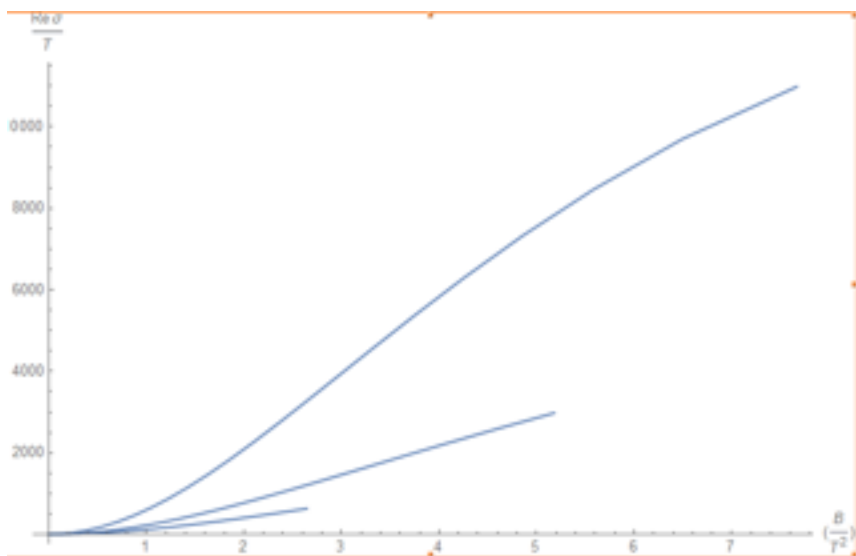
- When $\mu = \mu_5 = 0$, the massive gauge field model is in fact a special case for the explicit breaking case in which the bulk mass for the scalar is set to zero. The massless scalar is dual to a magical operator which is the same as A_r. However, there is an extra Higgs mode in the explicit breaking model.

III Holographic axial charge dissipation: backreactions

- Explicit breaking with an axially charged scalar source

$$\sigma_{zz} = J_z(\infty)/E = \frac{n_0}{\sqrt{h_0}} + \frac{32\alpha^2 B^2}{n_0\sqrt{h_0}e^2 q^2 \phi^2(r_0)}$$

depending on horizon values of various background fields: not always B-squared anymore



small B, still B-squared

Large B behavior: numerical difficulties; large B/M^2 limit of zero temperature

- Zero temperature: $\mathrm{AdS}_3 \times \mathrm{R}^2$ + a constant scalar+irrelevant deformations

—-not B-squared behaviour at large B (preliminary result)

Conclusions and open questions

- For chiral anomalous fluid with magnetic field: *momentum, axial charge and energy dissipations* are all needed to have a finite longitudinal DC magneto-conductivity;

- A universal formula for the longitudinal DC magneto conductivity in the hydrodynamic regime;

- Holographical check of the formula in the probe limit and with backreactions at zero densities;

- Two holographic models to realise axial charge dissipation. The dependence of the longitudinal DC magneto-conductivity on B is *universal B-squared in the probe limit*: the same as found in experiments, negative magnetoresistivity (NMR); different in the backreacted case

Conclusions and open questions

- Other ways to realise axial charge dissipation?
- Backreaction: including energy & momentum dissipations at finite densities;
- Relaxation times from Memory Matrix formalism;
- Adding all dissipations to the “No-drag” frame in the hydrodynamic limit (M.A. Stephanov, H.U. Yee, 2015);
- Holographic check of the hydrodynamic formula in the most general case;

Thank you!