Some theoretical motivations to introduce non standard initial conditions for LSS with inflationary vector field models

Workshop on Cosmic Microwave Background, Large Scale Structure and 21 cm Surveys, IFT, Madrid

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Some motivations

- General properties of vector fields in curved spacetime.
- Inflation driven by a vector field, Ford (1989).
- Cosmological “seed” magnetic fields, $e^{\alpha\Phi}F^2$, Ratra (1992).
- Primordial magnetic fields from psudo-Goldstone bosons, Garretson, Field & Carroll (1992).
- Vector curvaton mechanism, K. Dimopoulos (2006).
- Breaking of rotational invariance, Ackerman, Carroll & Wise (2007).
- Vector inflation (reloaded), Golovnev, Mukhanov & Vanchurin (2008).
- Inflation with “anisotropic hair”, Watanabe, Kanno & Soda (2009).
- Stability analysis of VF models, Contaldi, Peloso, Himmetoglu, Gumrukcuoglu (2007-2009).
- Vector curvaton without ghost instabilities, Dimopoulos, Karciauskas & Wagstaff (2009).
- CMB anomalies, WMAP 7-9, low multipole anomalies CMB, Planck 2013.
- ...

Some motivations

More recent/current motivations and expectations

- Primordial magnetic fields.
- Signatures of statistical anisotropies and parity violation in CMB correlators.
- Signatures of anisotropic and parity violating non-Gaussianity.
- Enhancement of gravitation waves.
- Effects on LSS. Non-Gaussian & anisotropic bias.
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Stable gauge invariant abelian models. Scalar + vector

- General scalar + vector models (allowing for derivative interactions):

$$S = \int d^4x\sqrt{-g}\,[R - \mathcal{L}(\phi, A_\mu)]$$

- Stable and causal gauge invariant models:

$$S = \int d^4x\sqrt{-g}\left[R - \mathcal{L}_\phi(\phi, \partial\phi) - \frac{1}{4}f_1(\phi)F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}f_2(\phi)F^{\mu\nu}\tilde{F}_{\mu\nu}\right]$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$$

$f_1(\phi)F^{\mu\nu}F_{\mu\nu} \Rightarrow$ Non-diluting anisotropic source (“anisotropic hair”).

$f_2(\phi)F^{\mu\nu}\tilde{F}_{\mu\nu} \Rightarrow$ Parity symmetry breaking.

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Consequences of vector fields during inflation

Anisotropic and parity violating correlations in models of the form (some type of) $f_1(\phi)F^2 + f_2(\phi)F\tilde{F}$, Dimopoulos, Karciauskas, Wagstaff, Bartolo, Dimastrogiovanni, Matarrese, Riotto, Liguori, Ricciardone, Peloso, Valenzuela-Toledo, Rodríguez, Lyth, Gumrukcuoglu, Himmetoglu, Shiraishi, Komatsu, Barnaby, Watanabe, Kanno, Soda, Sorbo, Emami, Firouzjahi...

$$P_\zeta(k) \quad \Rightarrow \quad P_\zeta(\vec{k}) = P_\zeta(k)\left[1 + g_\zeta(\hat{k}\cdot\hat{n})^2\right],$$

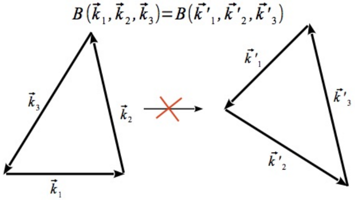
$$B_\zeta(k_1,k_2,k_3) \quad \Rightarrow \quad B_\zeta(\vec{k}_1,\vec{k}_2,\vec{k}_3) = B_\zeta\left[1 + g_\zeta b_1(\hat{k}_i,\hat{n}) + g_\zeta^2 b_2(\hat{k}_i,\hat{n})\right],$$

$$\ldots$$

$$g_\zeta \quad \sim \quad 0{,}3 \quad \text{(back in 2009)},\ \hat{n} \rightarrow \text{along the ecliptic}$$

$$-0{,}023 < \quad g_\zeta \quad < 0{,}036 \quad \underline{\text{Planck 2015.}}$$

Angle dependent correlations



Gauge symmetry breaking

Many forms to break gauge invariance. A simple way is by introducing a mass term. Other ways are, for instance, the introduction of derivative couplings of the fields $(\partial A)^2, A^2\partial \cdot A, \partial \cdot A(\partial\phi)^2, A \cdot \partial\phi, \ldots$.

$$S_{sv} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}f_1(\phi)F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}f_2(\phi)F^{\mu\nu}\tilde{F}_{\mu\nu} - \frac{1}{2}m(\phi)^2A^2 + \cdots\right]$$

- Statistical anisotropies, parity violating correlations and the mass term add scale dependent effects on the field perturbations.
- Free from ghost instability. Stability of the model for a massive vector curvaton carefully detailed by Dimopoulos, Karciauskas & Wagstaff (2009).

Equations of motion

Background metric

Background metric in conformal time

$$ds^2 = a(\tau)^2(-d\tau^2 + dx_i dx^i),$$

Assume nearly de Sitter geometry $a(\tau) \approx -1/H\tau$ with constant Hubble parameter H,

$$ds^2 = \frac{1}{H^2\tau^2}(-d\tau^2 + dx_i dx^i).$$

Equations of motion

$$S_{\phi A} = -\frac{1}{4}\int d^4x\sqrt{-g}\left[f_1(\phi)F^{\mu\nu}F_{\mu\nu} + f_2(\phi)F^{\mu\nu}\tilde{F}_{\mu\nu} + 2m^2(\phi)A^2\right]$$

$$\Downarrow$$

$$\nabla_\mu\left(f_1(\phi)F^{\mu\nu} + f_2(\phi)\tilde{F}^{\mu\nu}\right) - m^2(\phi)A^\nu = 0, \quad \& \quad \nabla_\mu\tilde{F}^{\mu\nu} = 0.$$

Equations of motion

Homogeneous scalar field $\partial_i \phi = 0 \Rightarrow f_1(\phi) = f_1(\phi(\tau))$, $f_2(\phi) = f_2(\phi(\tau))$ & $m(\phi) = m(\phi(\tau))$.
$\Rightarrow \partial_i f_1 = \partial_i f_2 = \partial_i m = 0$.

$$\begin{aligned}
\left[\nabla^2 - \partial_\tau^2 - \left(\frac{\partial_\tau m^2}{m^2} + \frac{\partial_\tau a^2}{a^2}\right)\partial_\tau - \left(\frac{\partial_\tau^2 m^2}{m^2} + \frac{\partial_\tau^2 a^2}{a^2} - \left(\frac{\partial_\tau m^2}{m^2}\right)^2 - \left(\frac{\partial_\tau a^2}{a^2}\right)^2 + \frac{m^2 a^2}{f_1}\right)\right] A_0 &= 0, \\
\left(\nabla^2 - \partial_\tau^2 - \frac{m^2 a^2}{f_1} - \frac{\partial_\tau f_1}{f_1}\partial_\tau + \frac{\partial_\tau f_2}{f_1}\nabla\times\right) A_i + \left[\frac{\partial_\tau f_1}{f_1} - \left(\frac{\partial_\tau m^2}{m^2} + \frac{\partial_\tau a^2}{a^2}\right)\right]\partial_i A_0 &= 0.
\end{aligned}$$

Further assume power law evolution of the coupling functions

$$m^2(a) \propto a^{2r}, \qquad f_1(a) \propto a^{2\alpha}, \qquad f_2(a) \propto a^{2\beta}.$$

Special case $\alpha = \beta$ (Sorbo & Caprini 2014) $\Rightarrow f_2 = \gamma f_1$ for constant γ. Going to Fourier space $F(\tau, \vec{x}) = \int \frac{d^3x}{(2\pi)^{3/2}} F(\tau, \vec{k}) e^{i\vec{k}\cdot\vec{x}}$ and choosing $\vec{k} = (k, 0, 0)$ the transverse components decouple:

$$\boxed{\left[\partial_\tau^2 + k^2 - \frac{\alpha(\alpha+1)}{\tau^2} + \frac{m^2 a^2}{f_1} \pm \frac{2\xi k}{\tau}\right] \tilde{A}_\pm = 0.} \tag{1}$$

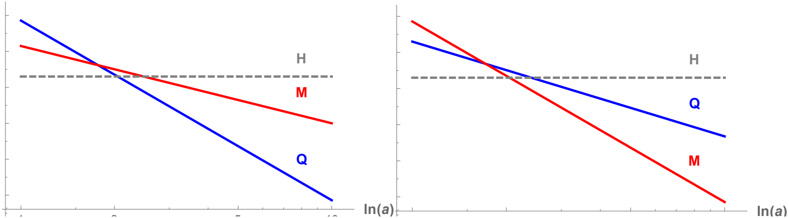
$$\tilde{A}_\pm = \sqrt{f_1}\frac{A_y \pm i A_z}{\sqrt{2}}, \qquad \boxed{\xi = -\alpha\gamma}$$

Solutions in fifferent regimes

Work in progress with Juan C. Bueno-Sánchez.

$$\left[\partial_\tau^2 + k^2 - \frac{\alpha(\alpha+1)}{\tau^2} + \frac{m^2a^2}{f_1} \pm \frac{2\xi k}{\tau}\right] A_\pm = 0.$$

$$M^2 = \frac{m^2a^2}{f_1}, \qquad Q^2 = \frac{|2\xi k|}{|\tau|}$$



Parity violation term dominates

Massless case (Dimopoulos & Karciauskas JHEP06(2012)040, Sorbo & Caprini JCAP10(2014)056, ...)

$$\tilde{A}_{\pm} = \frac{H}{\sqrt{2k}} W_{-i\xi,\alpha+1/2}(2ik\tau)$$

For $|k\tau| \ll \xi$ and $\xi \gg 1$, parity violation dominates the evolution and for superhorizon scales and $|k\tau| \ll 8/\xi \ll 1$

$$\tilde{A}_{+} \approx \sqrt{\frac{|\tau|}{2\pi}} e^{\pi\xi/2} \Gamma(|1+2\alpha|)(|2\xi k\tau|)^{-|\alpha+1/2|}$$

Exponential amplification of the $+1$ helicity mode.

Mass term dominates

Solution for massive case

$$\tilde{A}_{m\pm} = \sqrt{-\tau}\left[D_1 J_\nu\left(\frac{M}{(r-\alpha)H}\right) + D_2 J_{-\nu}\left(\frac{M}{(r-\alpha)H}\right)\right]$$

Introduces scale dependence in the perturbations spectrum in a non trivial way!

Super horizon evolution keep imprints of the scale dependence due to the mass term

$$\tilde{A}_{m+} \approx \sqrt{\frac{|\tau|}{2\pi}} G\left(\frac{M}{H}\right) e^{\pi\xi/2}(|2\xi k\tau|)^{-|\alpha+1/2|}$$

Power spectrum

Q dominates at the end of inflation

$$\begin{aligned}\mathcal{P}_- &= \frac{k^3}{2\pi^2}\lim_{V\to 0}|w_+|^2 \\ &= \left(\frac{H}{2\pi}\right)^2 \frac{(2|c|)^{2\nu_q-1}2|b||M_2^+|^2\Gamma^2(\nu_q)}{9\pi^2}\left(\frac{k}{aH}\right)^{3-\nu_q}\left(\frac{|\dot{f}_2|}{Hf_1}\right)^{-\nu_q}.\end{aligned}$$

(With $w = A/a$) Non trivial scale dependence in the perturbations spectrum. Flat spectrum for

$$\nu_q = 3 = \frac{1}{2|c|}|2\alpha+1| \implies \alpha_f = -\frac{1}{2}\pm 3|c|.$$

Exponential scale dependent amplification is imprinted in the PS

$$\begin{aligned}\mathcal{P}_+ &= \frac{k^3}{2\pi^2}\lim_{V\to 0}|w_-|^2 \\ &= \left(\frac{H}{2\pi}\right)^2 \frac{\Gamma^2(\nu_q)}{2}\frac{2|b|e^{2X/|c|}}{9\pi^2}\left|M_2^-\right|^2(2|c|)^{2\nu_q-1}\left(\frac{k}{aH}\right)^{3-\nu_q}\left(\frac{|\dot{f}_2|}{Hf_1}\right)^{-\nu_q}.\end{aligned}$$

Scale dependent amplification

$$\exp\left[\frac{2X}{|c|}\right] = \exp\left[\frac{2Q_{\bar{t}}}{|c|H}\right] = \exp\left[\frac{2}{|c|}\left(\frac{k}{aH}\right)^{b/2(b-c)}\left(\frac{|\dot{f}_2|}{Hf_1}\right)^{b/2(b-c)}\left(\frac{M}{H}\right)^{c/(c-b)}\right]$$

$Q^2 = \frac{k}{a}\frac{|\dot{f}_2|}{f_1} \propto a^{2c}$, $M^2 = \frac{m^2}{f_1} \propto a^{2b}$, with $b = \beta - \alpha$.

Some expectations

Multipolar expansion of the BS

$$B_\zeta = \sum_{L=0} c_L P_L(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) P(k_1) P(k_2) + perm.$$

c_L	=	observable parameters
P_L	=	Legendre polynomials

$c_L \rightarrow$ encodes information about statistical anisotropy and parity violation.

$$g = -\frac{48 N^2_{CMB} \rho_{Evev}}{\epsilon \rho_\phi},\ c_0 = -\frac{4N_{CMB}}{3\pi}\frac{e^{2\pi\xi}}{\xi^3} g,\ c_1 \neq 0 = -\frac{3c_0}{2},\ c_2 = \frac{c_0}{2}$$

Bartolo, Matarrese, Peloso, Shiraishi, JCAP07(2015)039

Planck 2015 $\rightarrow -0{,}0225 < g < 0{,}0363,\ -10{,}7 < c_0 < 16{,}7,$
$-89 < c_1 < 324,\ -57 < c_2 < 47\ .$

Optimal estimator for tracking the parity odd features.

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Gravitational waves production

Equation of motion for the tensor modes (using gauge $h_i{}^i = \partial_j h_{ij} = 0$):

$$\frac{d^2 h_{ij}}{d\tau^2} + 2\frac{a'}{a}\frac{dh_{ij}}{d\tau} - \Delta h_{ij} = \frac{2}{M_p^2}\mathrm{T}_{ij} = \frac{2}{M_p^2}\Pi_{ij}{}^{lm}T_{lm}.$$

Projector operator:

$$\Pi_{ij}{}^{lm} = \Pi_i{}^l\Pi_j{}^m - \frac{1}{2}\Pi_{ij}\Pi^{lm}, \quad \Pi_{ij} = \delta_{ij} - \frac{\partial_i\partial_j}{\Delta}.$$

- $\Pi_{ij}{}^{lm}$ projects the spatial energy-momentum tensor T_{ij} in the transverse direction.
- T_{ij} is traceless and divergenceless: $\mathrm{T}_{ii} = \partial_j\mathrm{T}_{ji} = 0$.
- Gravitational waves are only sourced by the transverse components $\tilde{A}_\pm$.

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Gravitational waves production

EM tensor for the vector field:

$$T_{\mu\nu} = f_1 \left(\frac{1}{4} g_{\mu\nu} F^2 - F_{\mu\alpha} F_\nu{}^\alpha \right) - m^2 (A_\mu A_\nu - \frac{1}{2} g_{\mu\nu} A^2)$$

Spacial components :

$$T_{ij} = -f_1 F_{i\alpha} F_j{}^\alpha - m^2 A_i A_j + \left(\frac{f_1}{4} F^2 - \frac{m^2}{2} A^2 \right) \delta_{ij}.$$

EM components that source tensor modes equation:

$$\mathrm{T}_{ij} = \Pi_{ij}{}^{lm} \left[-f_1 F_{l\alpha} F_m{}^\alpha - m^2 A_l A_m + (\cdots) \delta_{lm} \right]. \quad (2)$$

$(\cdots)$ terms are projected out by $\Pi_{ij}{}^{lm}$.

Canonical variables $\tilde{h}_{ij} = a h_{ij}$:

$$\frac{d^2 \tilde{h}_{ij}}{d\tau^2} - \frac{a''}{a} \tilde{h}_{ij} - \Delta \tilde{h}_{ij} = \frac{2a}{M_p^2} \mathrm{T}_{ij} = \frac{2a}{M_p^2} \Pi_{ij}{}^{lm} \left[-f_1 F_{l\alpha} F_m{}^\alpha - m^2 A_l A_m \right]. \quad (3)$$

$\Downarrow$Fourier transform

$$\left[\frac{d^2}{d\tau^2} + \left(k^2 - \frac{a''}{a} \right) \right] \tilde{h}_{ij}(\vec{k}) = \frac{2a}{M_p^2} \mathrm{T}_{ij}(\vec{k}).$$

Chiral GW

Enhancement of the $+$ polarisation (Sorbo JCAP06(2011)003, Barnaby, Moxon, Namba, Peloso, Shiu, Zhou PhysRevD.86.103508, Cook & Sorbo JCAP11(2013)047, ...)

$$\mathcal{P}^{+} = \frac{H^2}{\pi^2 M_P^2}(1 + 8{,}6\times 10^{-7}\frac{H^2}{M_P^2}\frac{e^{4\pi\xi}}{\xi^6})$$

$$\mathcal{P}^{-} = \frac{H^2}{\pi^2 M_P^2}(1 + 1{,}8\times 10^{-9}\frac{H^2}{M_P^2}\frac{e^{4\pi\xi}}{\xi^6})$$

Expectation

$$\mathcal{P}^{+} = \frac{H^2}{\pi^2 M_P^2}(1 + 8{,}6\times 10^{-7}\frac{H^2}{M_P^2}G_{+}(k,M)\frac{e^{4\pi\xi}}{\xi^6})$$

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GW statistics, anisotropic bias and anisotropic IC

Testing the statistics of GW, Shiraishi, Hikage, Namba, Namikawa, Hazumi arXiv:1606.06082. (Scale dependent and sizeable GW due to source fields like axions)

$\langle BBB \rangle$ at 3σ with LiteBIRD

Scale and c_L depending bias

$$P_g = b_1^2 P_m$$

$$P_g = b_1^2(g, c_0, c_1, c_2) P_m + \cdots$$

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Final remarks and hopes

- Interesting possibilities for the GW production mechanism with the mass term. Scale dependent GW.
- Statistical anisotropy, parity violating and scale dependent effect are interesting effects that naturally arise in vector field models.
- Going beyond shapes in NG, probing scale and angle dependence with LSS ($f_{NL} \sim 1$). Signatures on bias parameters?