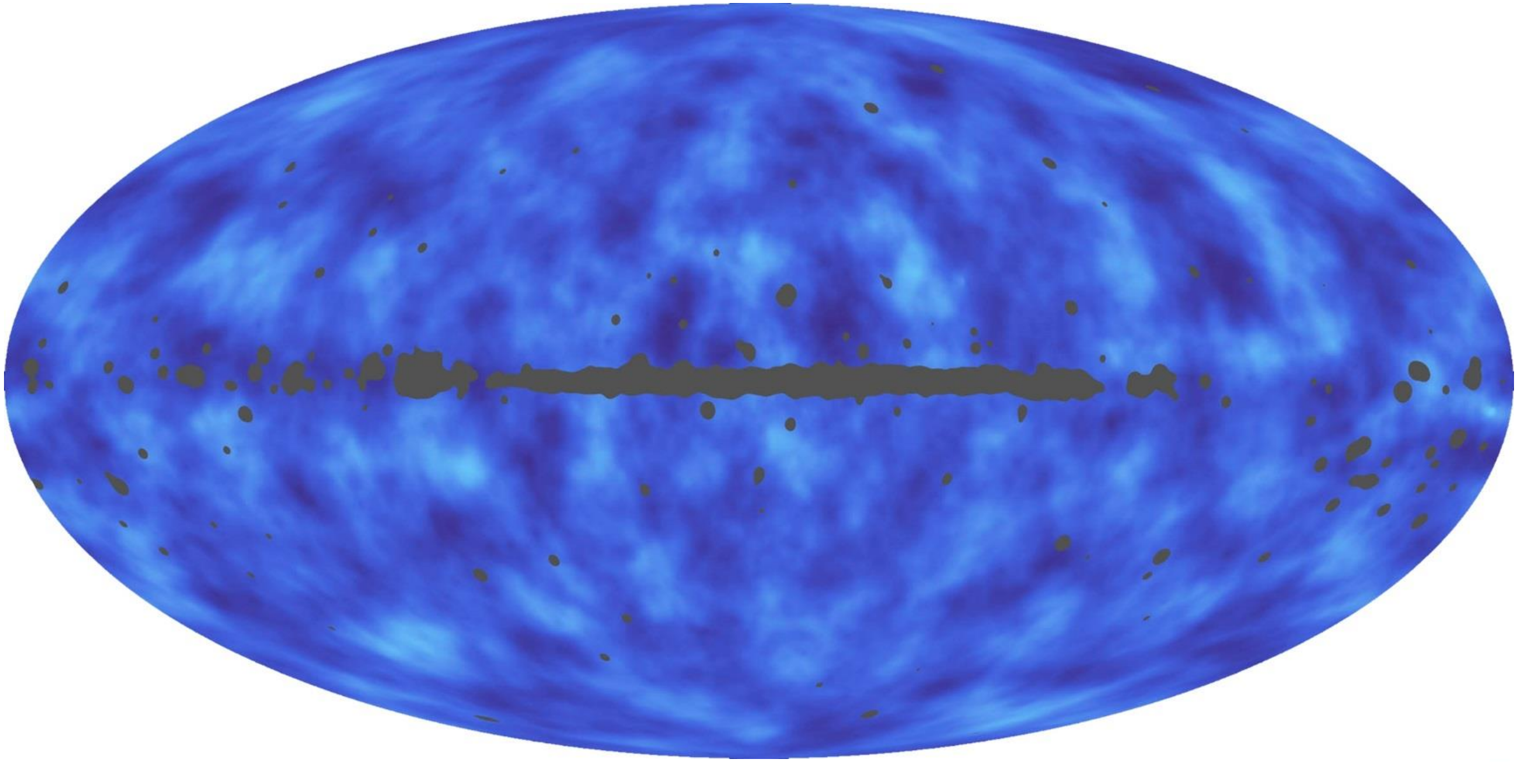
CMB Lensing

Antony Lewis

http://cosmologist.info/



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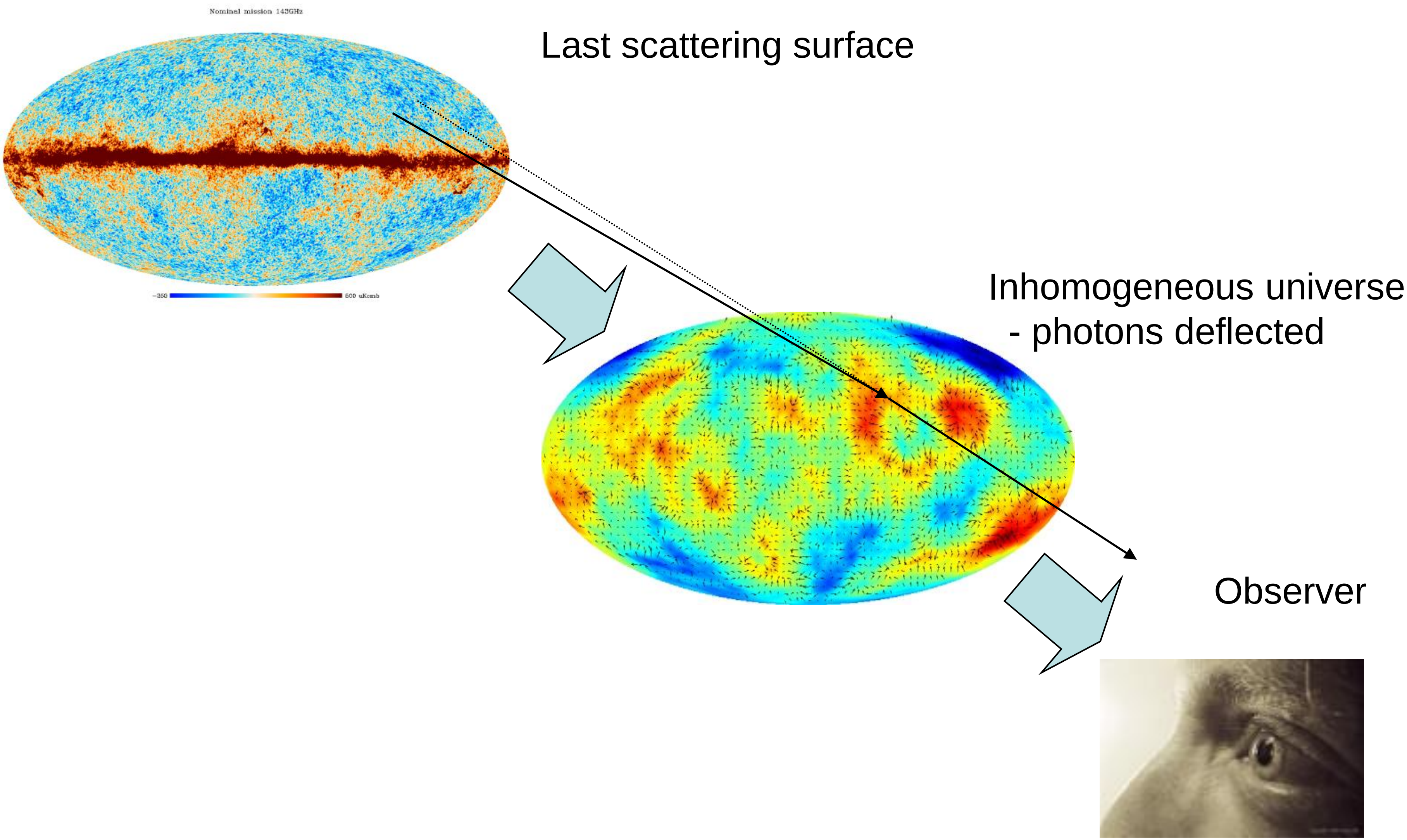
- Introduction

- Planck lensing
  (on behalf of Planck collaboration, 1502.01591; several slides credit D. Hanson)

- Post-Born lensing, Non-Gaussianity and lensing rotation B modes
  Pratten & Lewis 2016, 1605.05662

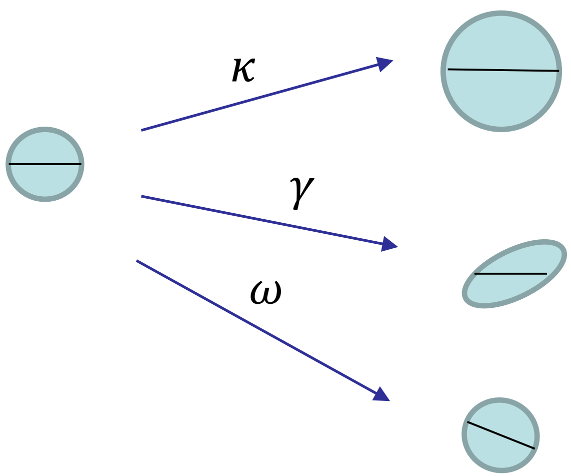
Weak lensing of the CMB



Deflection angle α, shear γ_i , convergence κ , and rotation ω

$$T(\hat{\mathbf{n}}) = T(\hat{\mathbf{n}}') = T(\hat{\mathbf{n}} + \boldsymbol{\alpha})$$

$$A_{ij} \equiv \delta_{ij} + \frac{\partial}{\partial \theta_i}\alpha_j = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2+\omega \\ -\gamma_2-\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$



Convergence $\kappa = -\frac{1}{2}\nabla \cdot \boldsymbol{\alpha}$

Shear

Rotation

Rotation $\omega = 0$ from scalar perturbations in linear perturbation theory
(because deflections from gradient of a potential)

Lensing warm up

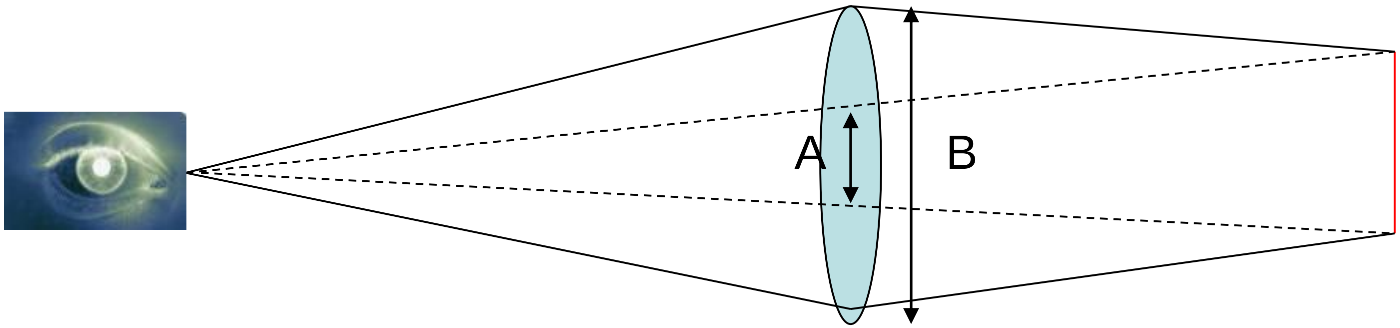
1) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

2) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

3) The lensing potential is nearly Gaussian because there are many lenses along the line of sight

4) Lensing rotates polarization, partly turning E modes into B modes

5) The CMB lensing power spectrum peaks at $L \sim 60$, so temperature lensing reconsutruction is sensitive to large-scale galactic foregrounds

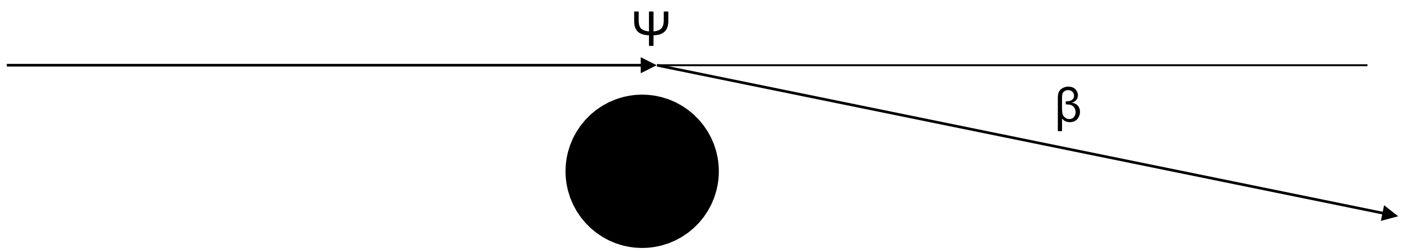


$$\frac{\text{Number of photons before lensing}}{\text{Number of photons after lensing}} = \frac{A^2}{B^2} = \frac{\text{Solid angle before lensing}}{\text{Solid angle after lensing}}$$

<u>Conservation of surface brightness</u>: number of photons per solid angle unchanged

→ uniform CMB lenses to uniform CMB – so no observable effect

CMB lensing order of magnitudes



Newtonian argument: $\beta = 2\,\Psi$
General Relativity: $\beta = 4\,\Psi$

($\beta \ll 1$)

Potentials linear and approx Gaussian: $\Psi \sim 2 \times 10^{-5}$

$\beta \sim 10^{-4}$

(in matter domination $\Psi \sim$ const and decays in DE era until non-linear)

Characteristic size from peak of matter power spectrum ~ 300Mpc

Comoving distance to last scattering surface ~ 14000 Mpc

⇨ pass through ~50 lumps ⇨ total deflection $\sim 50^{1/2} \times 10^{-4}$

assume uncorrelated

~ 2 arcminutes

(neglects angular factors, correlation, etc.)

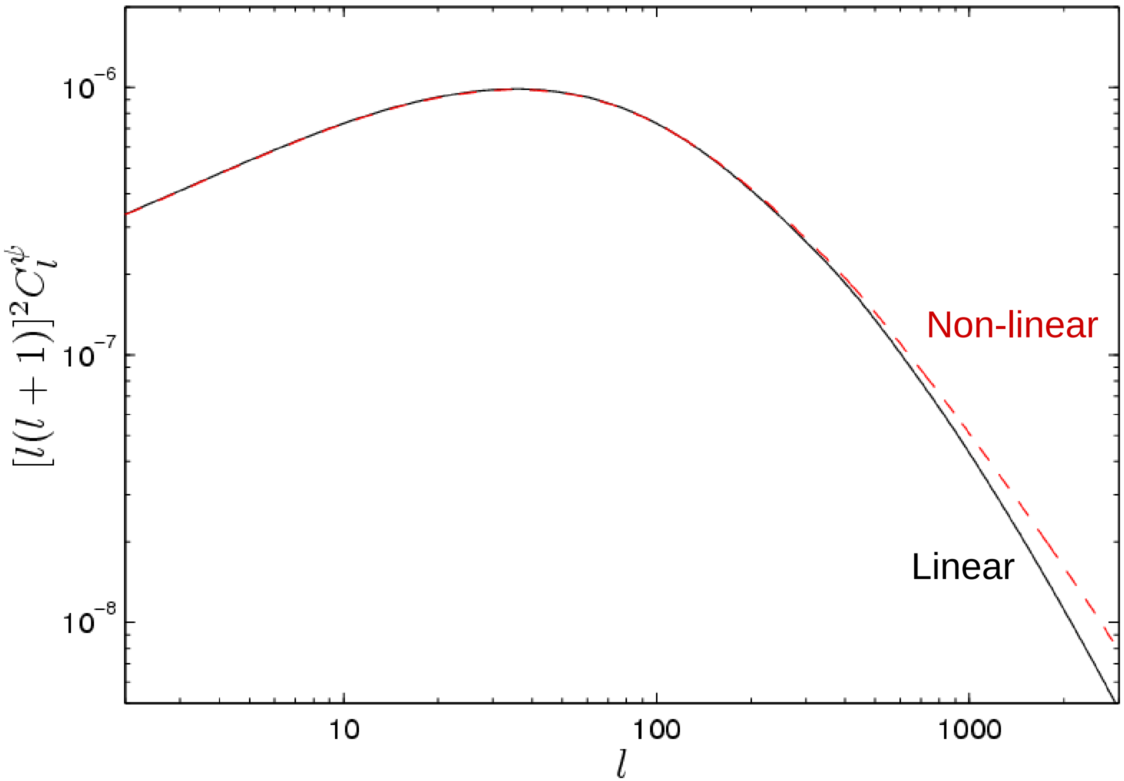
Deflection angle power spectrum

On small scales
(Limber approx, $k\chi \sim l$)

$$C_l^\psi \approx \frac{8\pi^2}{l^3}\int_0^{\chi_*} \chi \mathrm{d}\chi\, \mathcal{P}_\Psi(l/\chi;\eta_0-\chi)\left(\frac{\chi_*-\chi}{\chi_*\chi}\right)^2$$

(better: $l \to l + 1/2$)

Deflection angle power ~ $l(l+1)C_l^\psi$



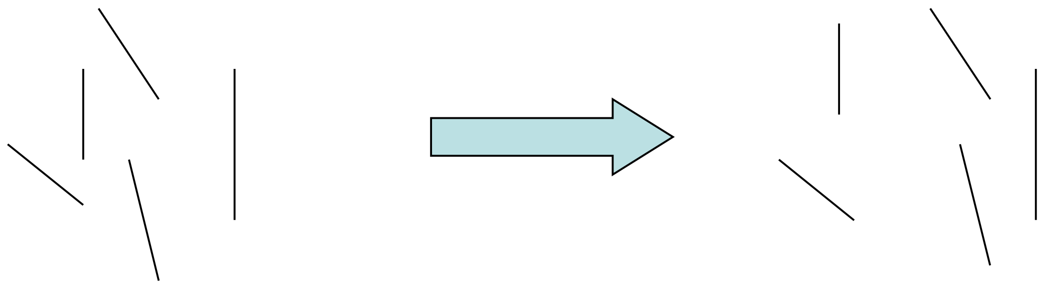
Deflections O(10^{-3}), but coherent on degree scales → important!

Lensing of polarization

- Polarization not rotated w.r.t. parallel transport (vacuum is not birefringent)
- Q and U Stokes parameters simply re-mapped by the lensing deflection field

No rotation with scalar perturbations

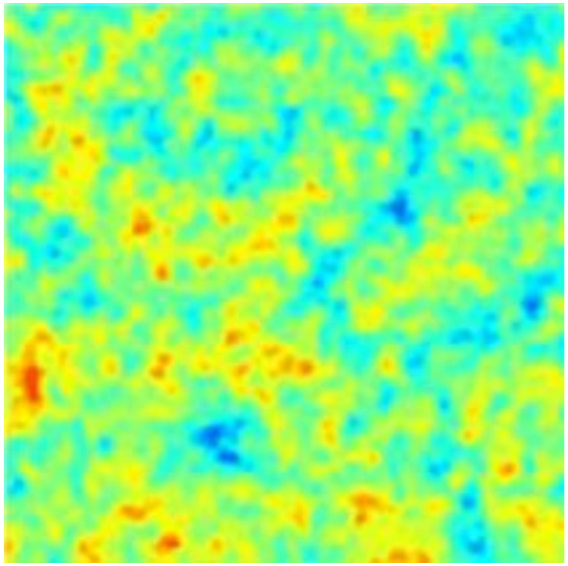
e.g.



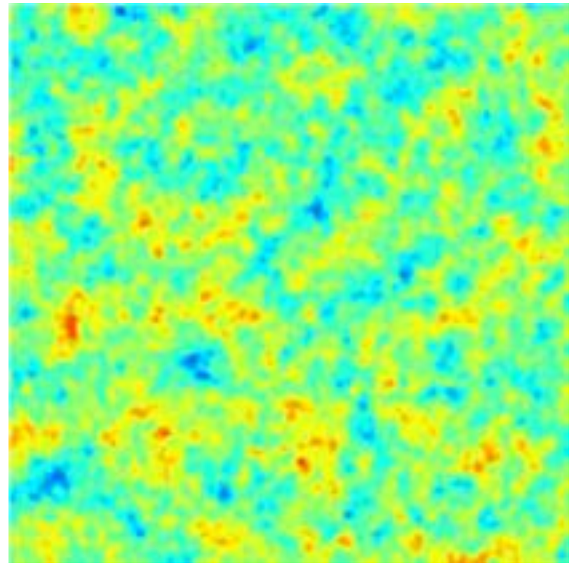
Last scattering Observed

How to we measure the lensing field?

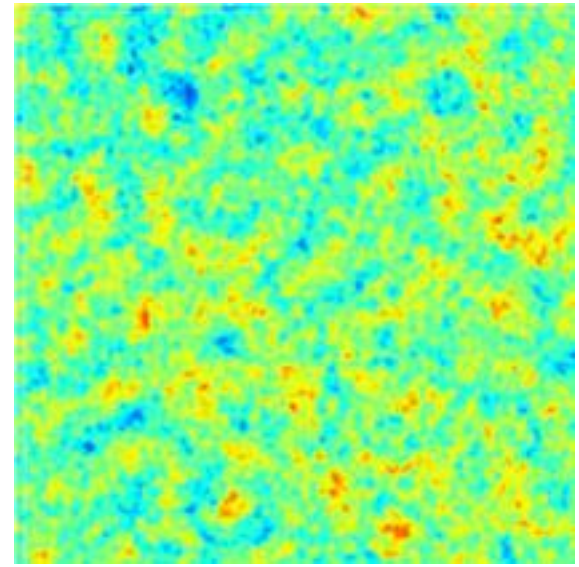
Magnified

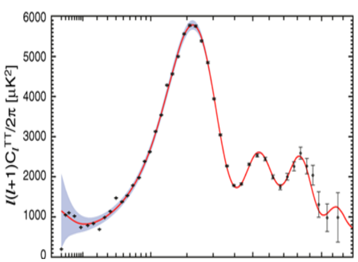


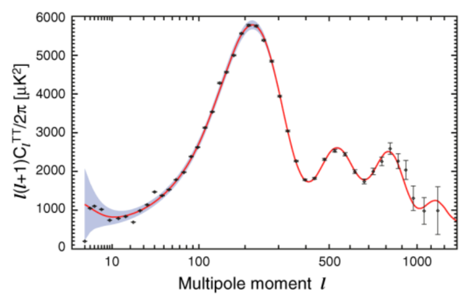
Unlensed

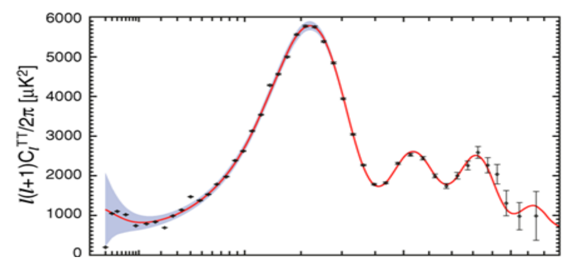


Demagnified





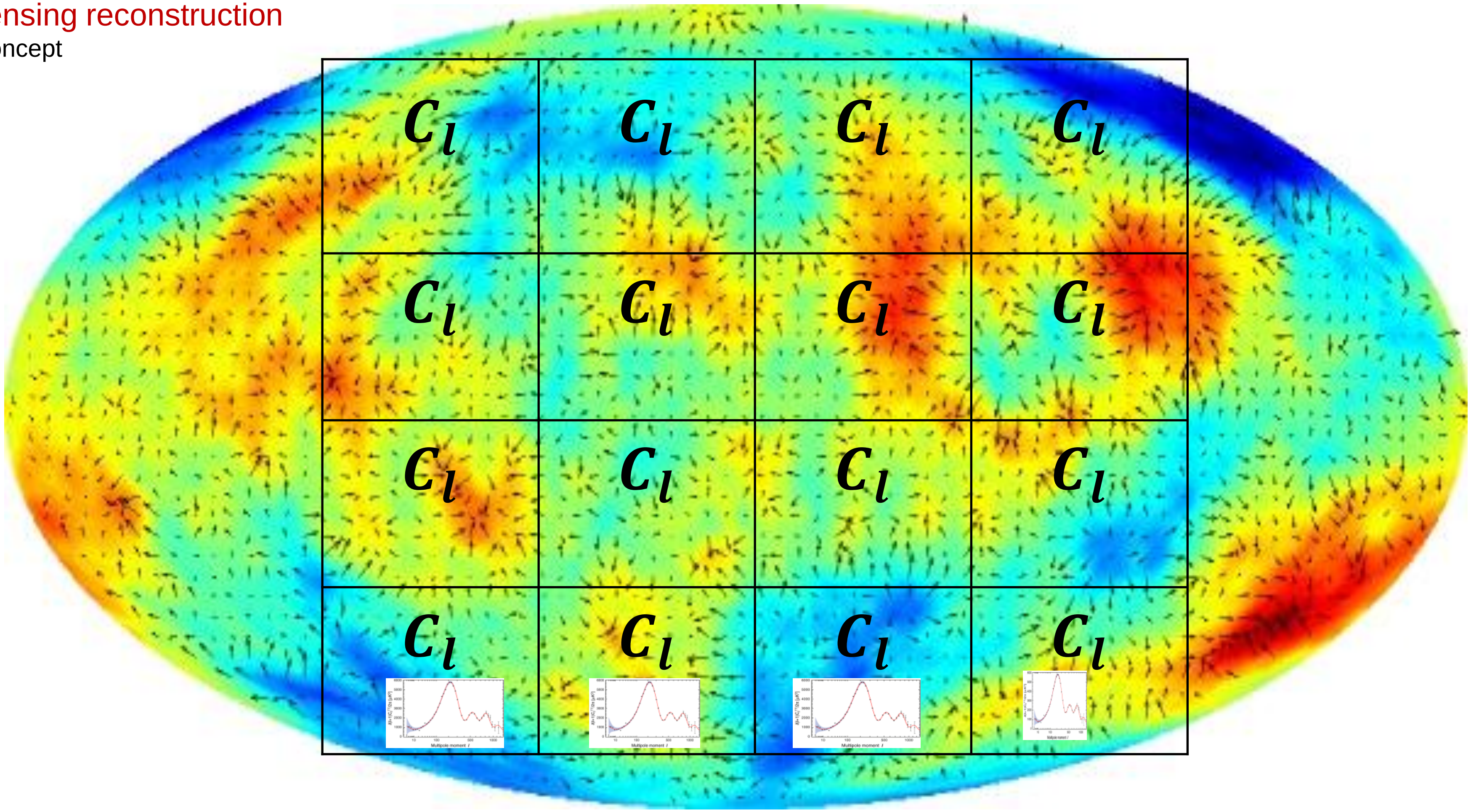




Fractional magnification ~ convergence $\kappa = -\boldsymbol{\nabla} \cdot \frac{\boldsymbol{\alpha}}{2}$ + shear modulation:

Lensing reconstruction

-concept



Variance in each C_l measurement $\propto 1/N_{\rm modes}$

$N_{\rm modes} \propto l_{\rm max}^2$ - dominated by smallest scales

⇒ measurement of angular scale (⇒ κ) in each box nearly independent
⇒ Uncorrelated variance on estimate of magnificantion κ in each box
⇒ Nearly white 'reconstruction noise' $N_l^{(0)}$ on κ , with $N_l^{(0)} \propto 1/l_{\rm max}^2$

Lensing reconstruction

- Maths and algorithm sketch

For a *given* (fixed) lensing field, $T \sim P(T|X)$:

X here is lensing potential, deflection angle, or κ

Flat sky approximation: modes correlated for $\mathbf{k}_2 \neq \mathbf{k}_3$

First-order series expansion in the lensing field:

$$\langle \tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\rangle_{P(\tilde{T}|X)} \approx \int \mathrm{d}\mathbf{K} X(\mathbf{K})^* \underbrace{\left\langle \frac{\delta}{\delta X(\mathbf{K})^*} \left(\tilde{T}(\mathbf{k}_2)\tilde{T}(\mathbf{k}_3)\right) \right\rangle}_{\mathcal{A}(K,k_2,k_3)\delta(K+k_2+k_3)} \approx \mathcal{A}(K,k_2,k_3)\, X(\mathbf{K})^*|_{\mathbf{K}=-\mathbf{k}_2-\mathbf{k}_3}$$

function easy to calculate for $X(\mathbf{K}) = 0$

$$A(L,l_1,l_2) \sim \left(\mathbf{l}_1 \cdot \mathbf{L}\tilde{C}_{l_1} + \mathbf{l}_2 \cdot \mathbf{L}\tilde{C}_{l_2}\right)$$

Can reconstruct the modulation field X

Full sky analysis similar, summing modes with optimal weights gives

$$\hat{\psi}^*_{l_1 m_1} = N^{(0)}_{l_1} \sum_{l_2 l_3}^{l_1 \le l_2 \le l_3} \Delta^{-1}_{l_1 l_2 l_3} \mathcal{A}^{TT}_{l_1 l_2 l_3} \sum_{m_2 m_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \frac{\tilde{T}_{l_2 m_2}\tilde{T}_{l_3 m_3}}{\tilde{C}^{TT}_{\mathrm{tot}\ l_2}\tilde{C}^{TT}_{\mathrm{tot}\ l_3}}$$

Zaldarriaga, Hu, Hanson, and others.

Warm up summary

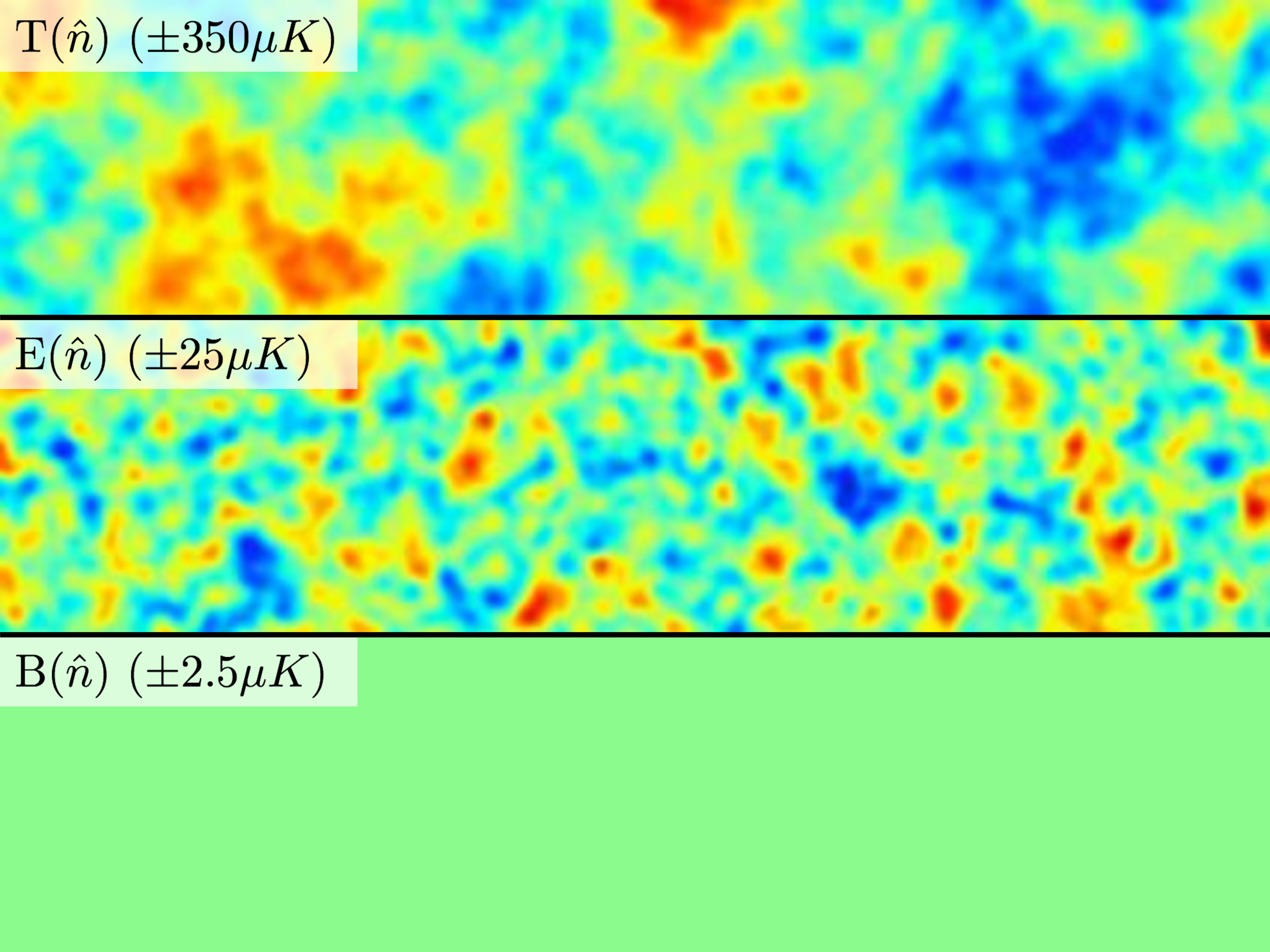
✗ 1) Overdensities focus light rays, so the CMB looks hotter where there are overdensities along the line of sight

✗ 2) Even in linear theory lensing is mostly at low redshift because density perturbations grow with time

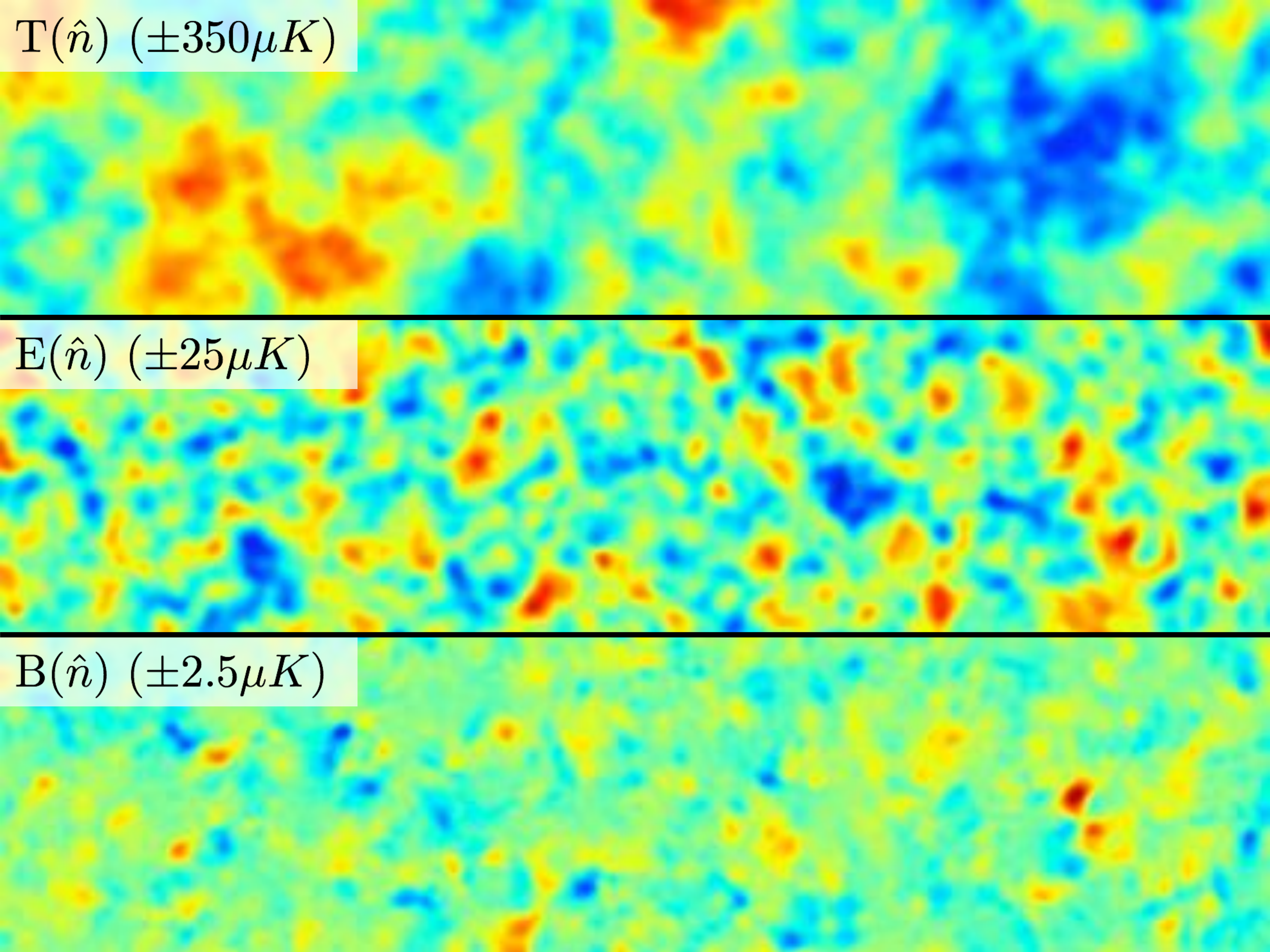
✓ 3) The lensing potential is nearly Gaussian because there are many lenses along the line of sight

✗ 4) Lensing rotates polarization, partly turning E modes into B modes

✗ 5) The CMB lensing power spectrum peaks at $L \sim 60$, so is sensitive to large-scale galactic temperature foregrounds



T($\hat{n}$) ($\pm 350\mu K$)
E($\hat{n}$) ($\pm 25\mu K$)
B($\hat{n}$) ($\pm 2.5\mu K$)

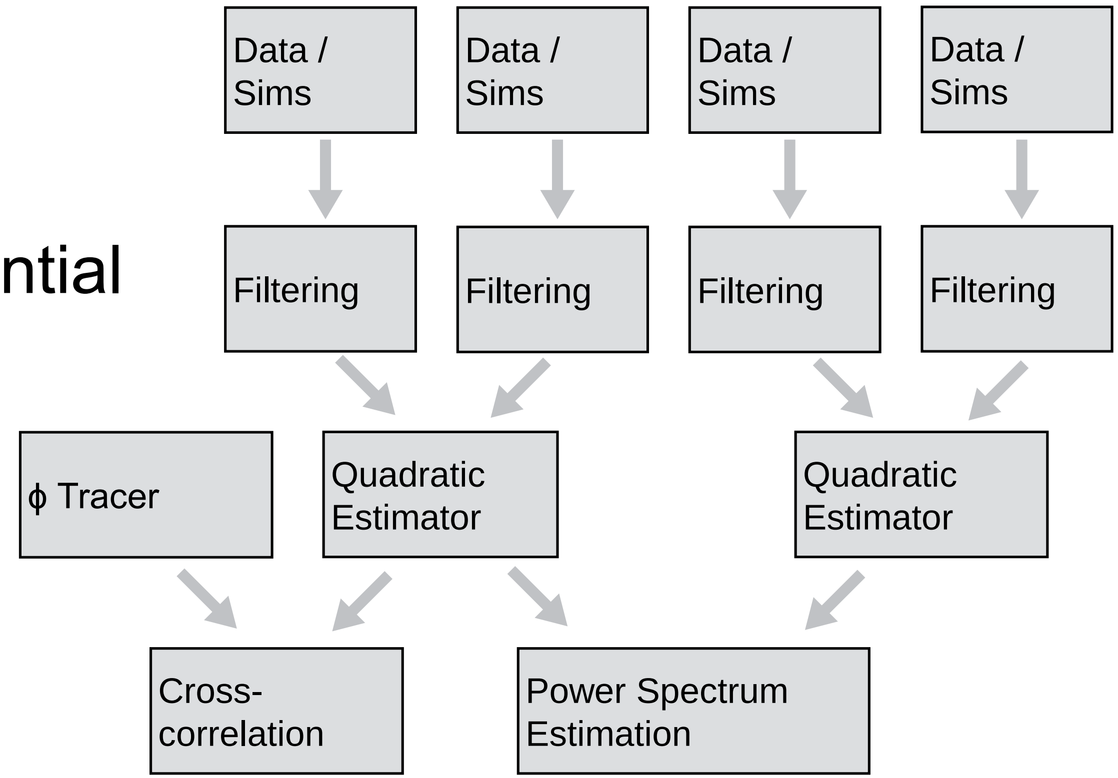

T($\hat{n}$) ($\pm 350\mu K$)
E($\hat{n}$) ($\pm 25\mu K$)
B($\hat{n}$) ($\pm 2.5\mu K$)

Lens Reconstruction Pipeline

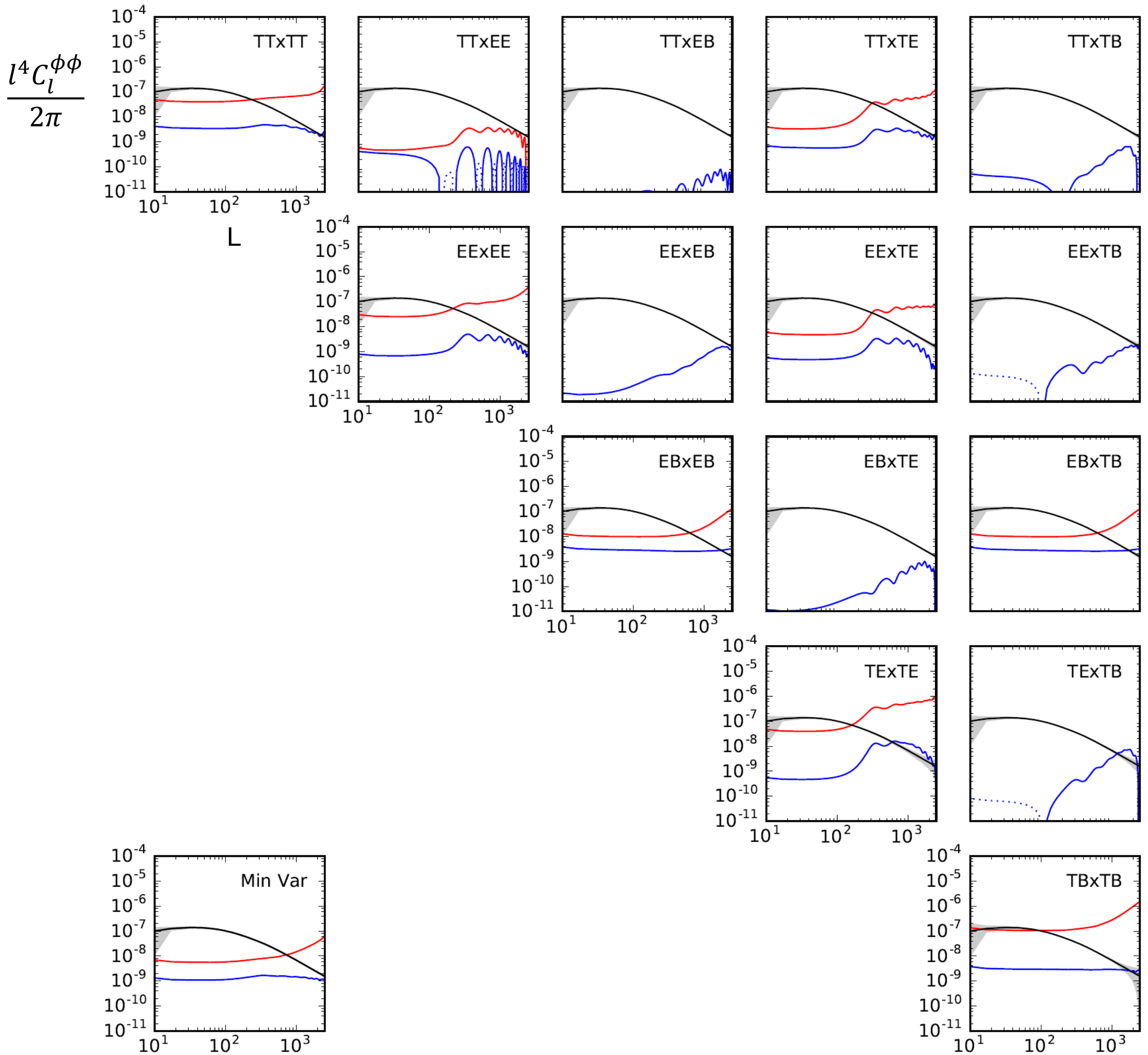
→ process input maps

→ estimate lensing potential from anisotropic 2-point

→ estimate lensing power spectrum.



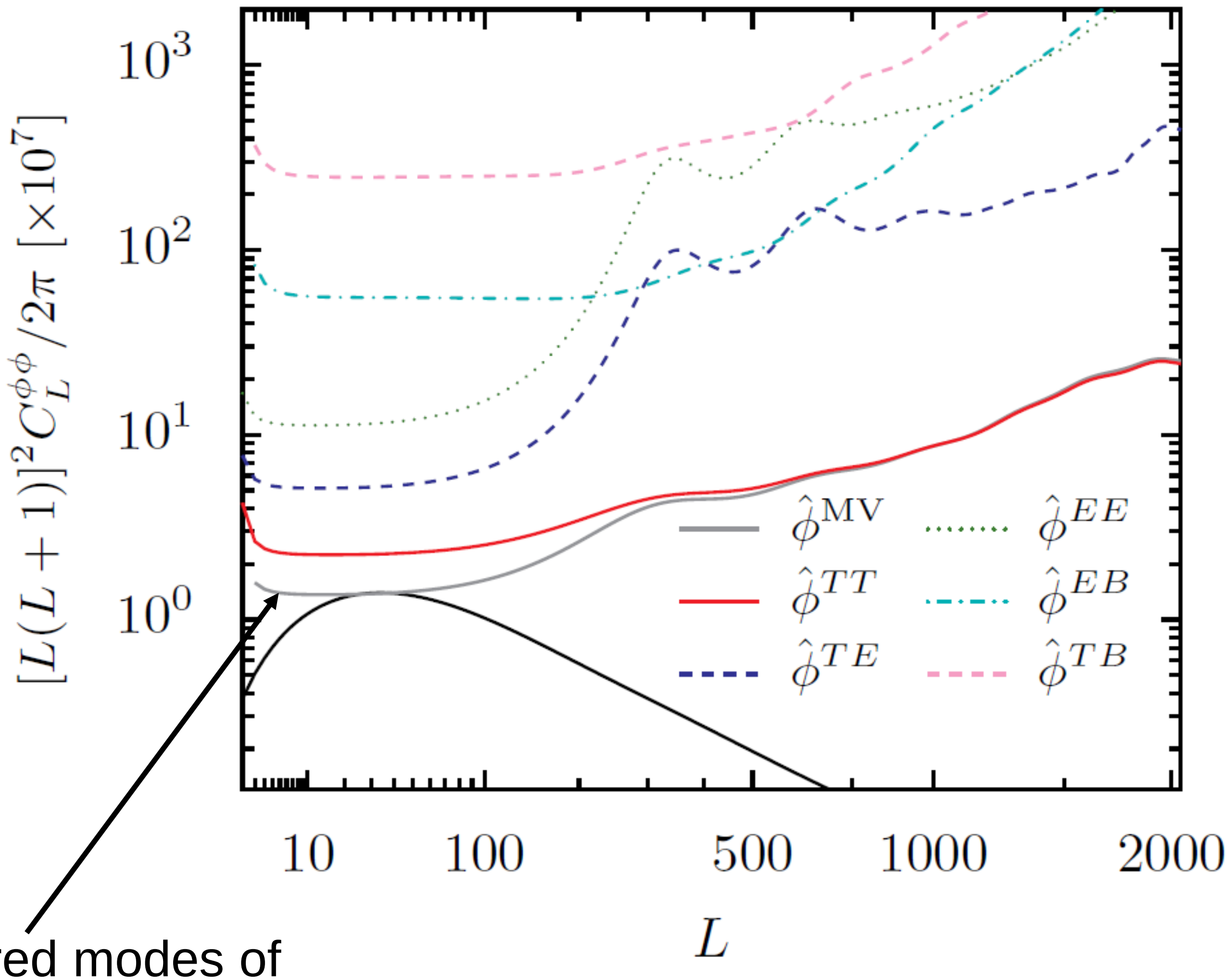
Large set of possible estimators, e.g. for S4 several nearly-independent probes



Noise

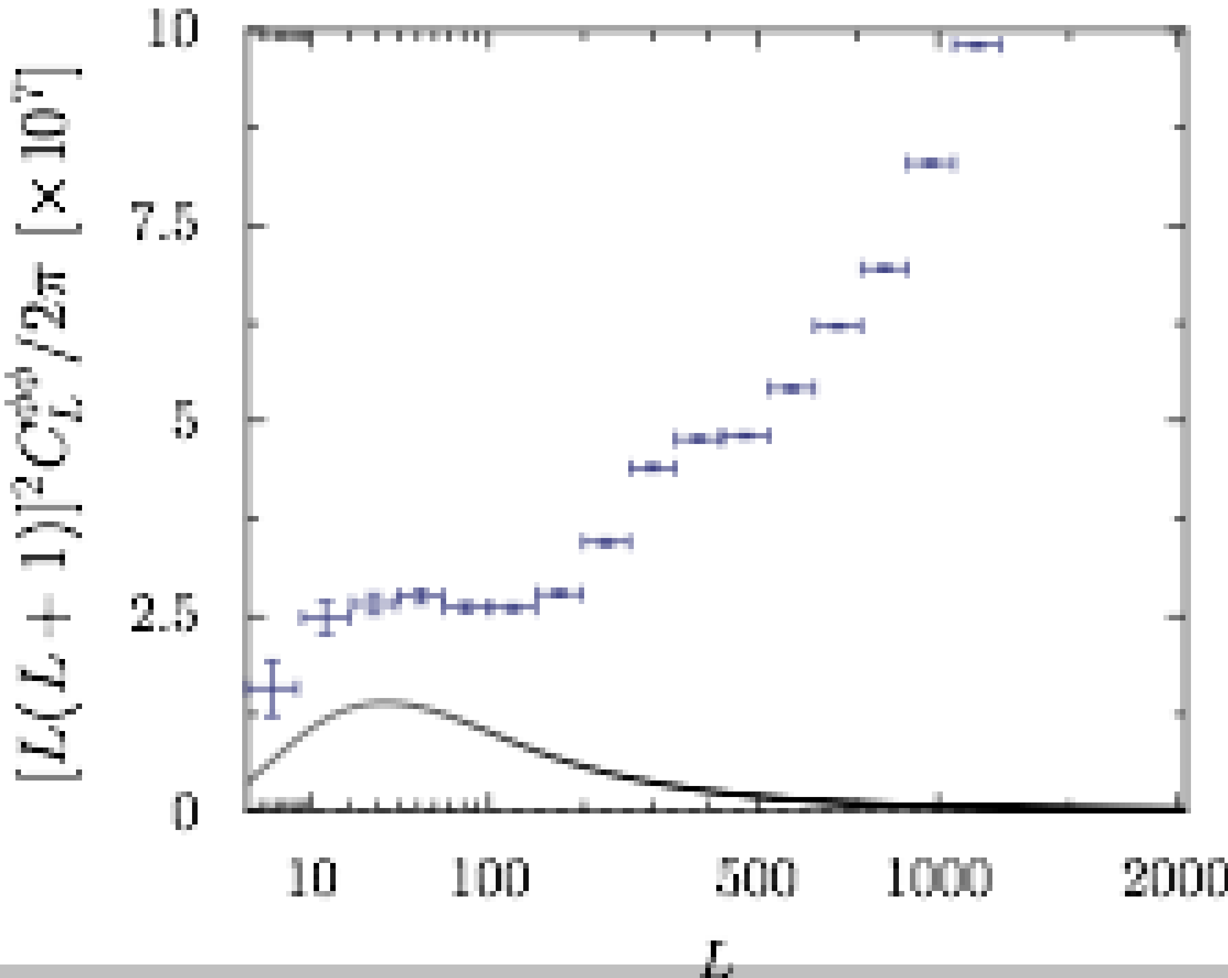
N1

Planck noise power spectra for lensing estimators.



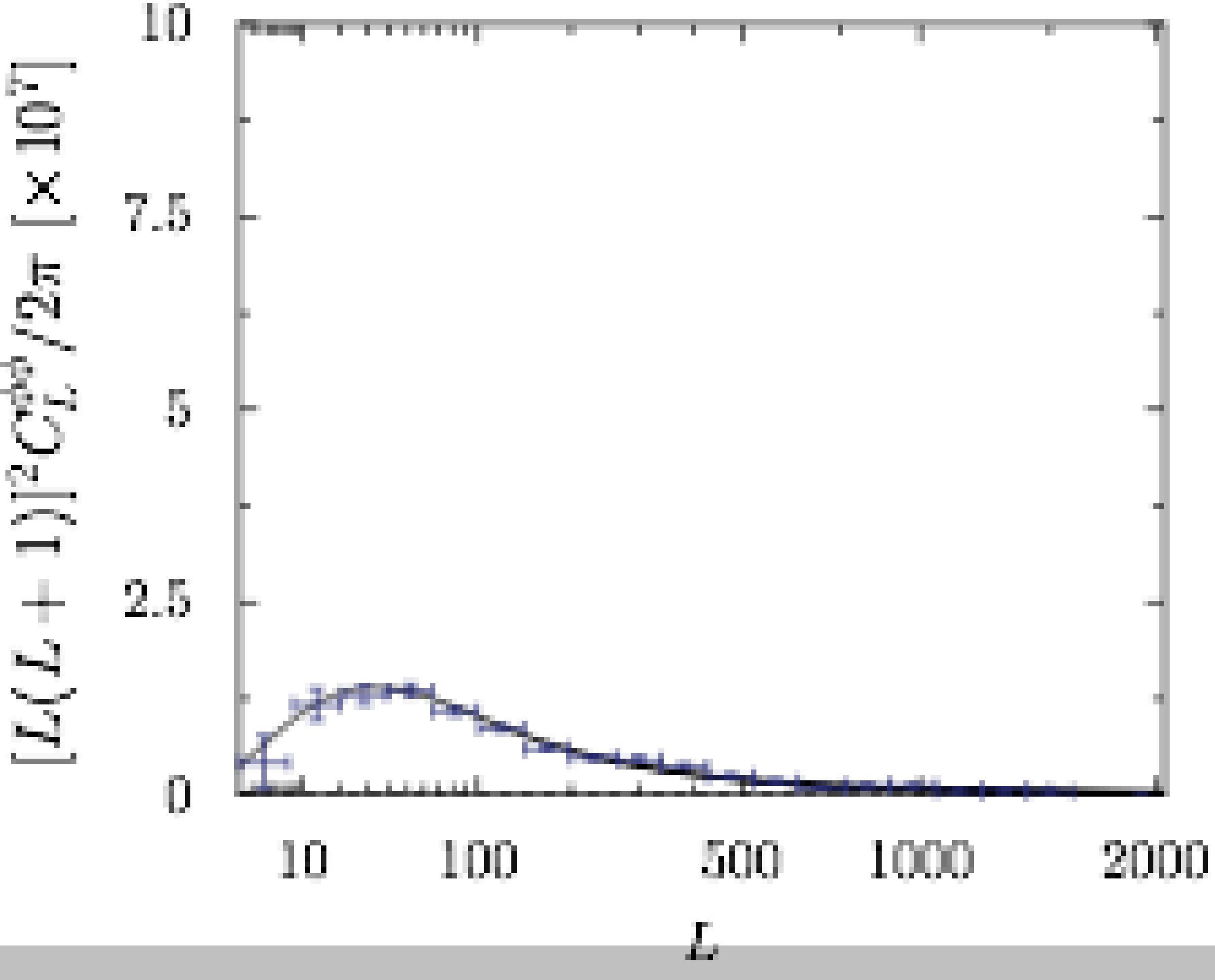
Best measured modes of
MV estimator have S/N=1.

Power Spectrum Estimation



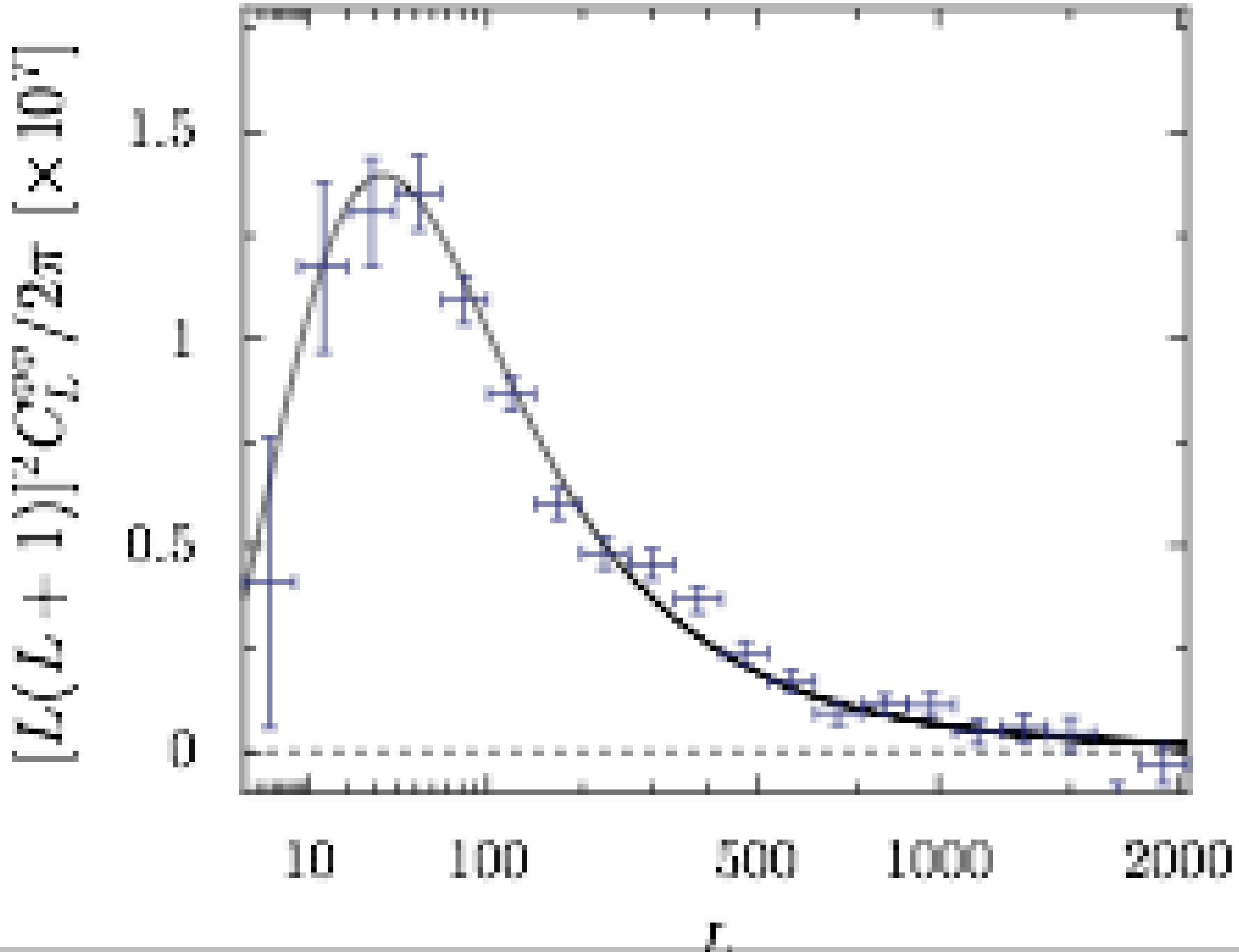
1) Raw power spectrum of quadratic estimates.

Power Spectrum Estimation



2) Correct for noise bias estimated from sims.

Power Spectrum Estimation



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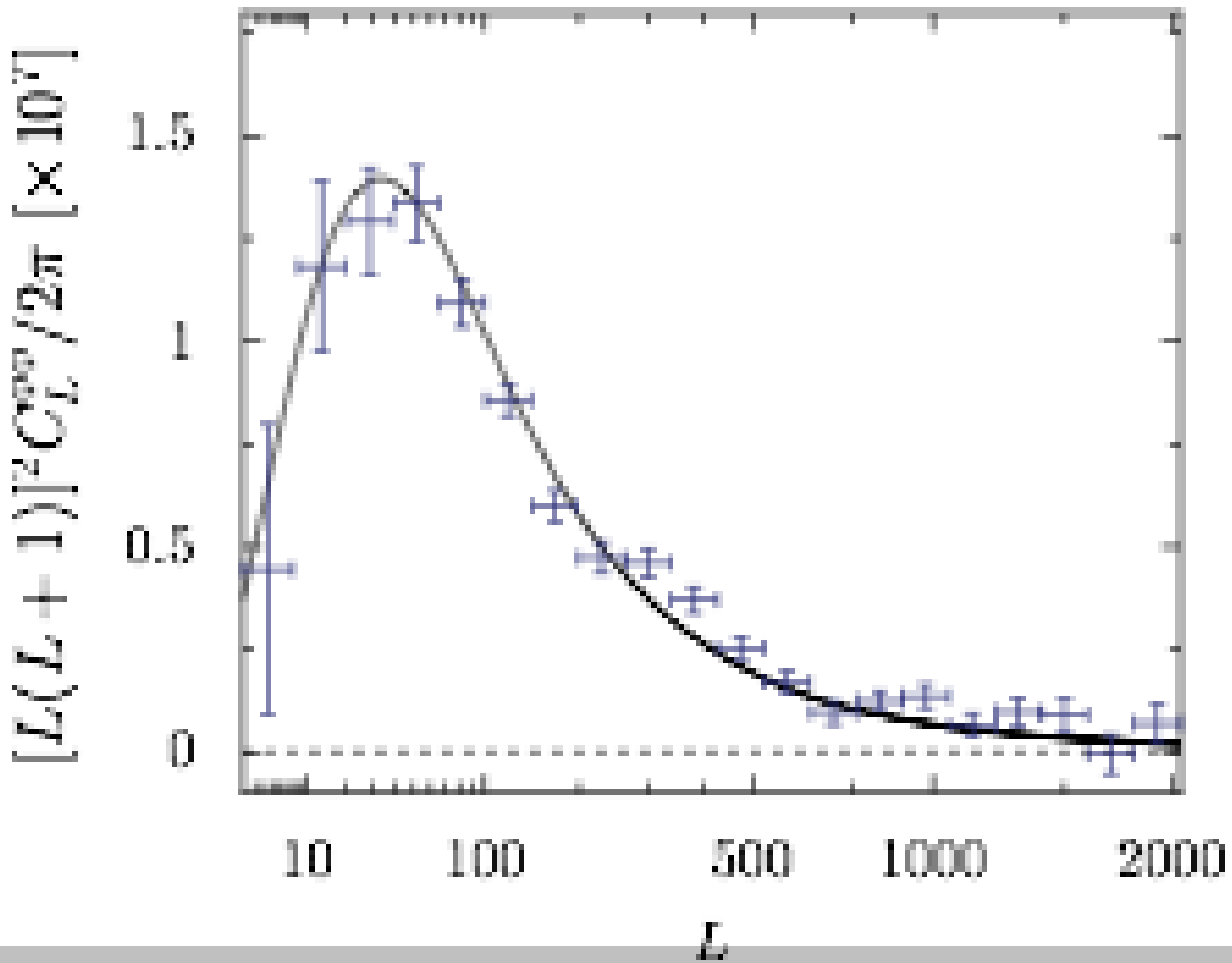
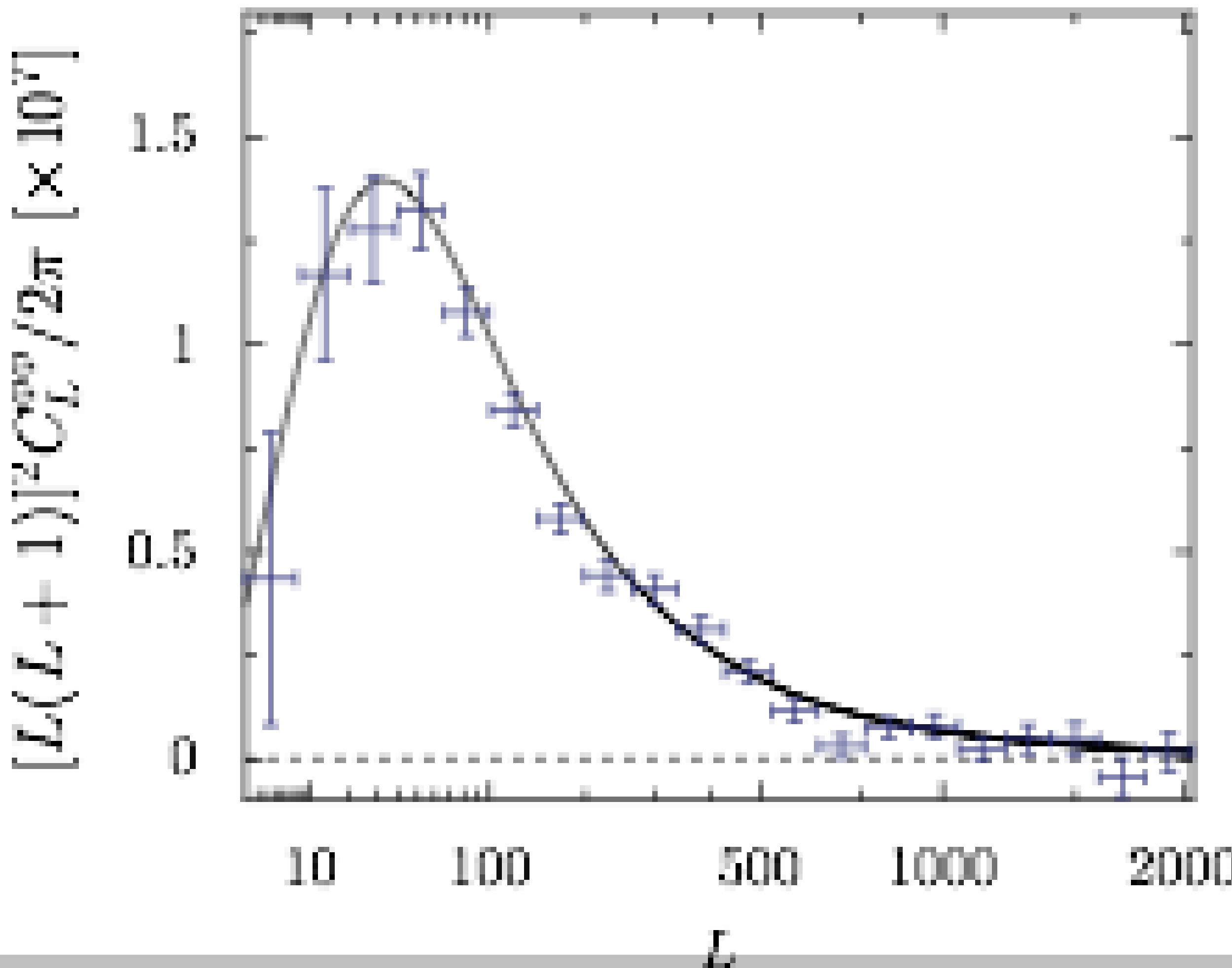
Power Spectrum Estimation



3) Apply further data-based estimate of noise bias to reduce sensitivity to inaccuracy of sims.

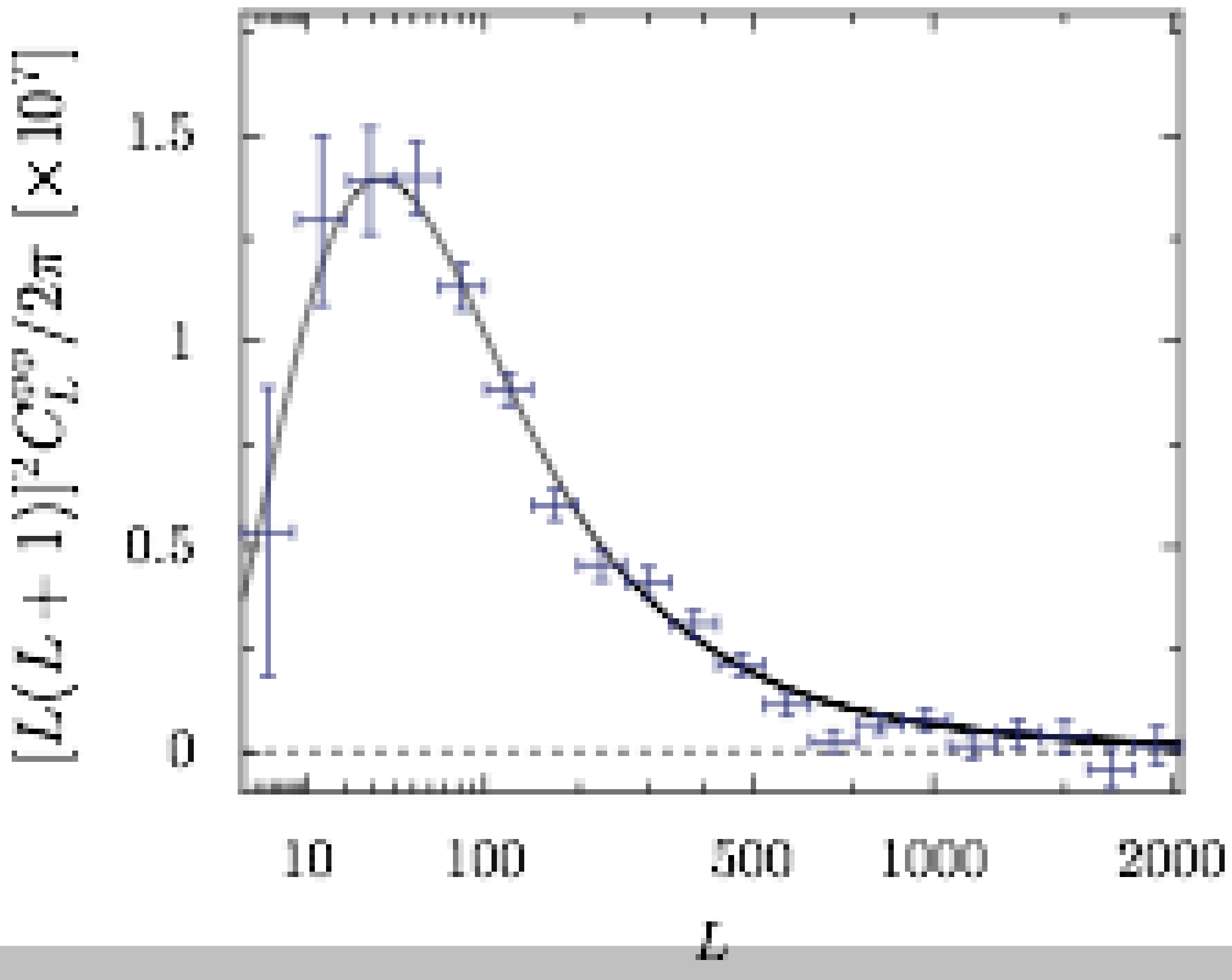
Power Spectrum Estimation



4) Correct for "N1" bias.

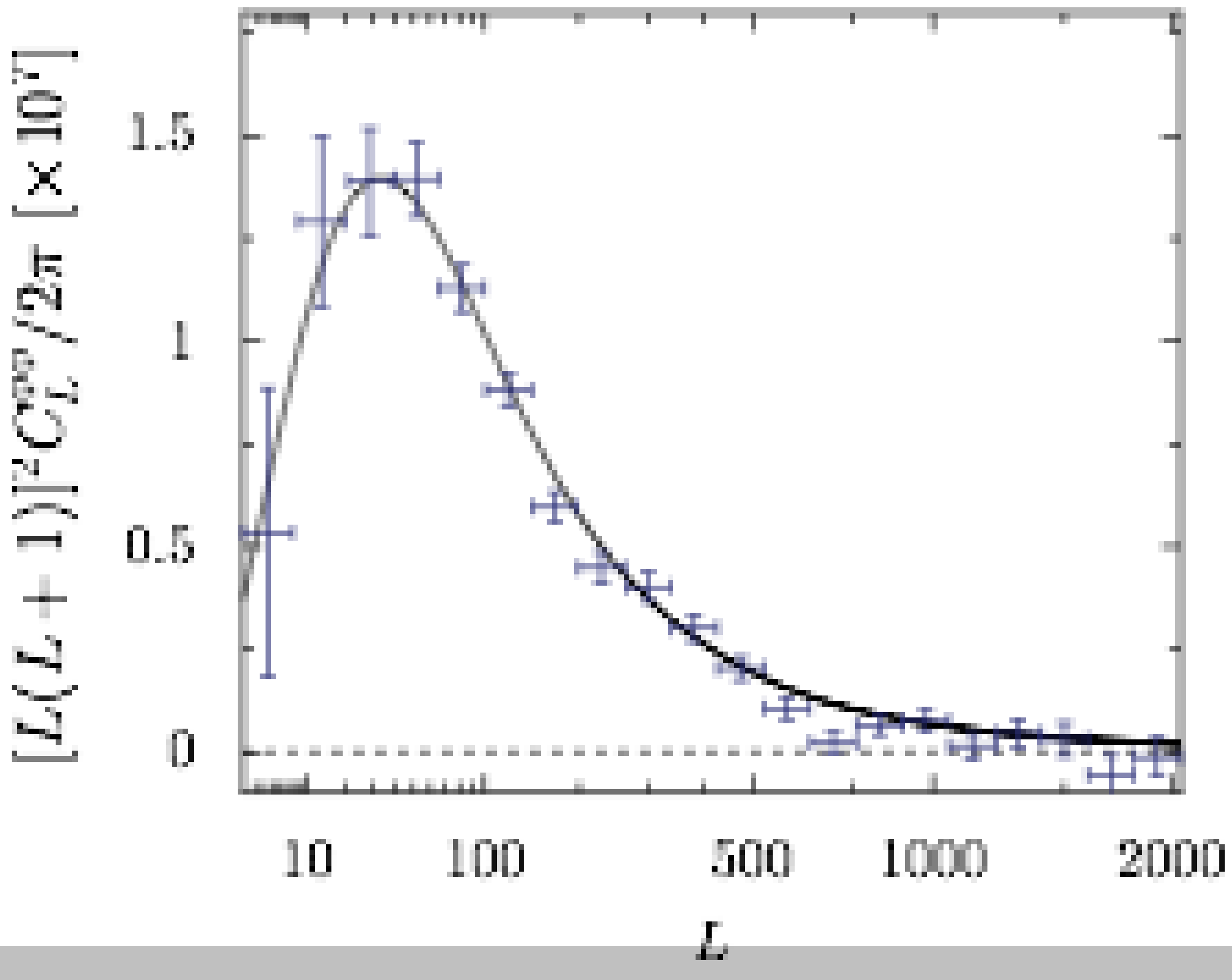
(cosmetic: likelihood uses full result and calculates N1)

Power Spectrum Estimation



5) MC correction for mode mixing / inaccuracies in normalization.

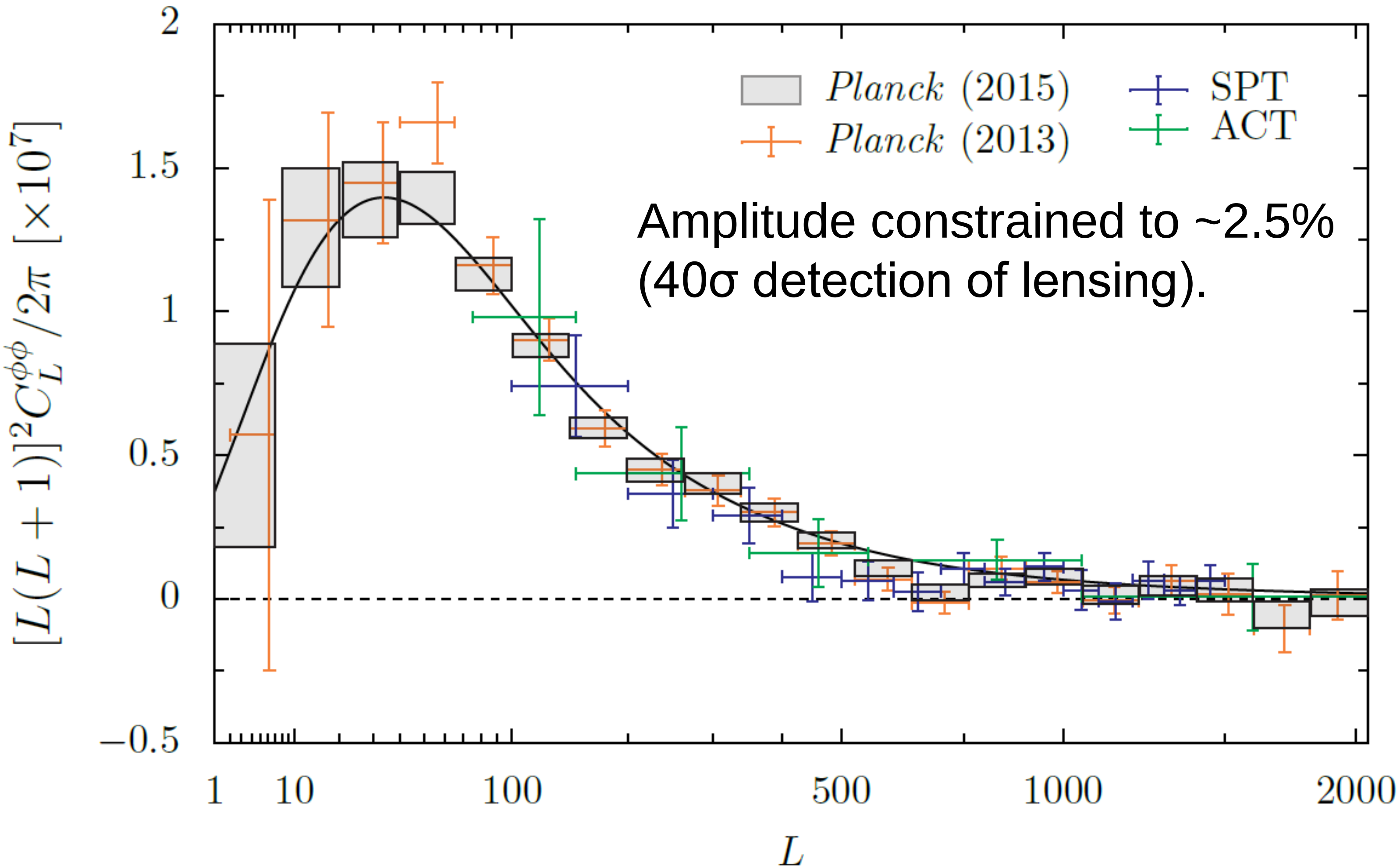
Power Spectrum Estimation



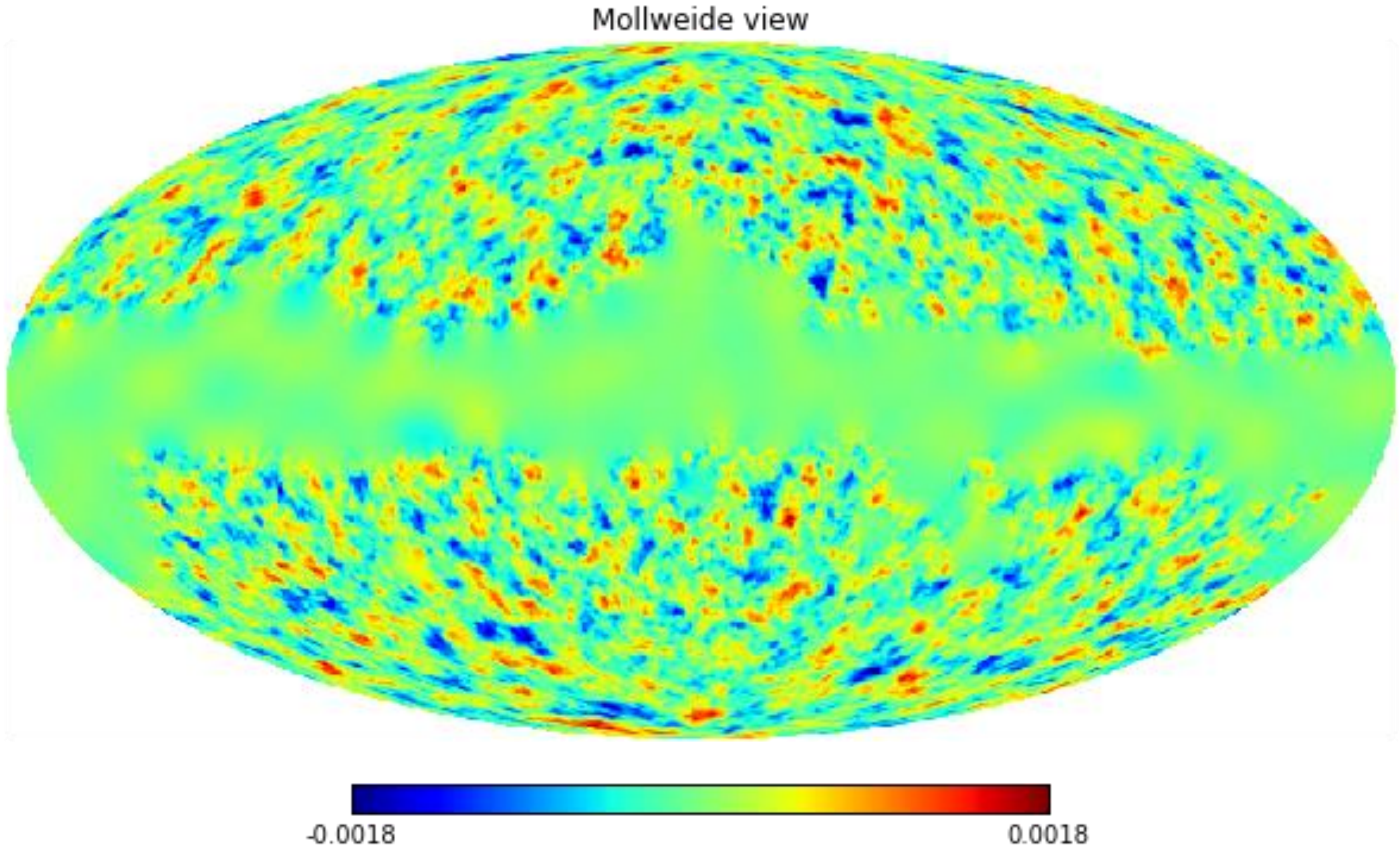
6) Correct for "PS" bias.

Done!

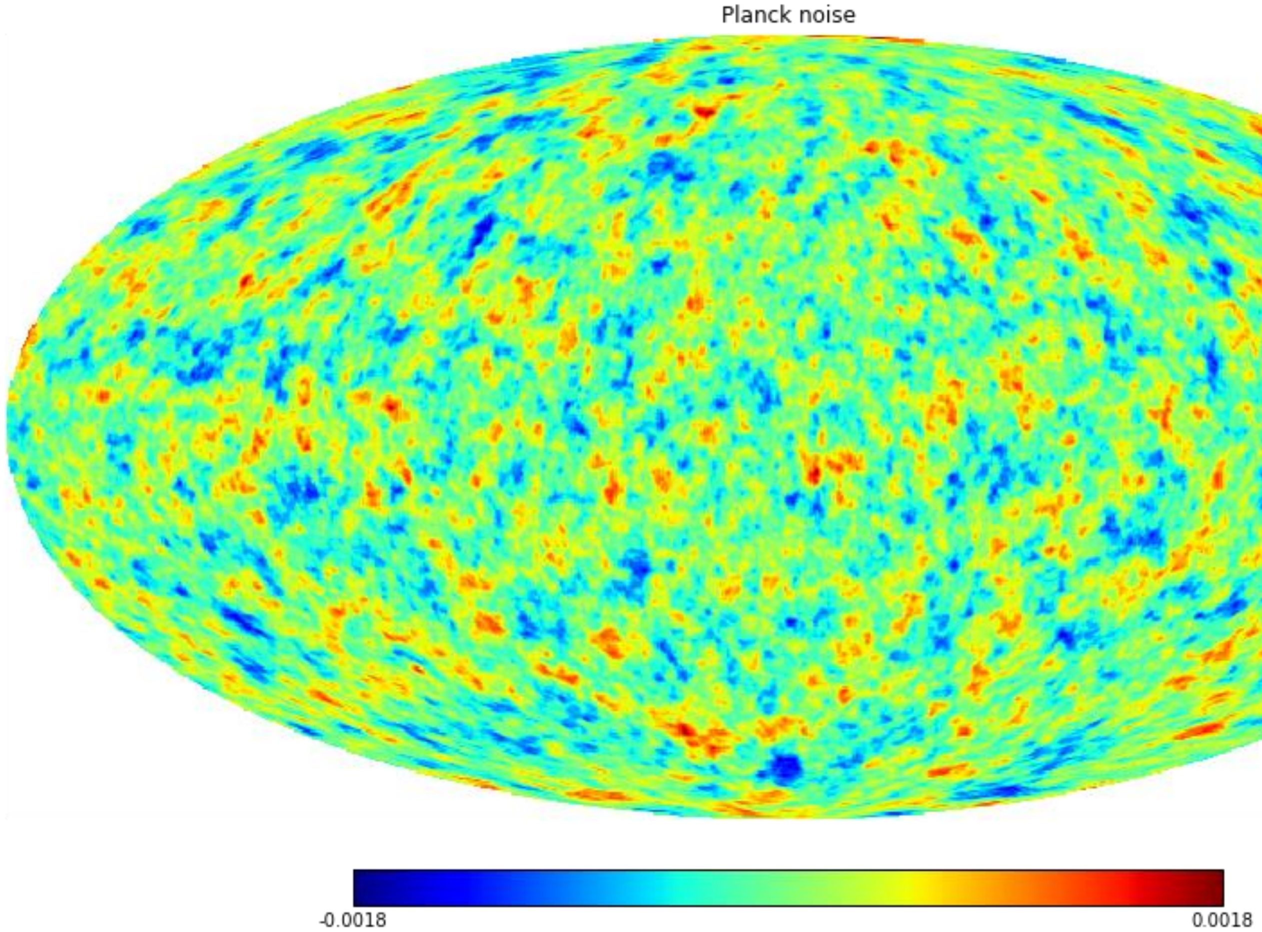
Lensing Power Spectrum



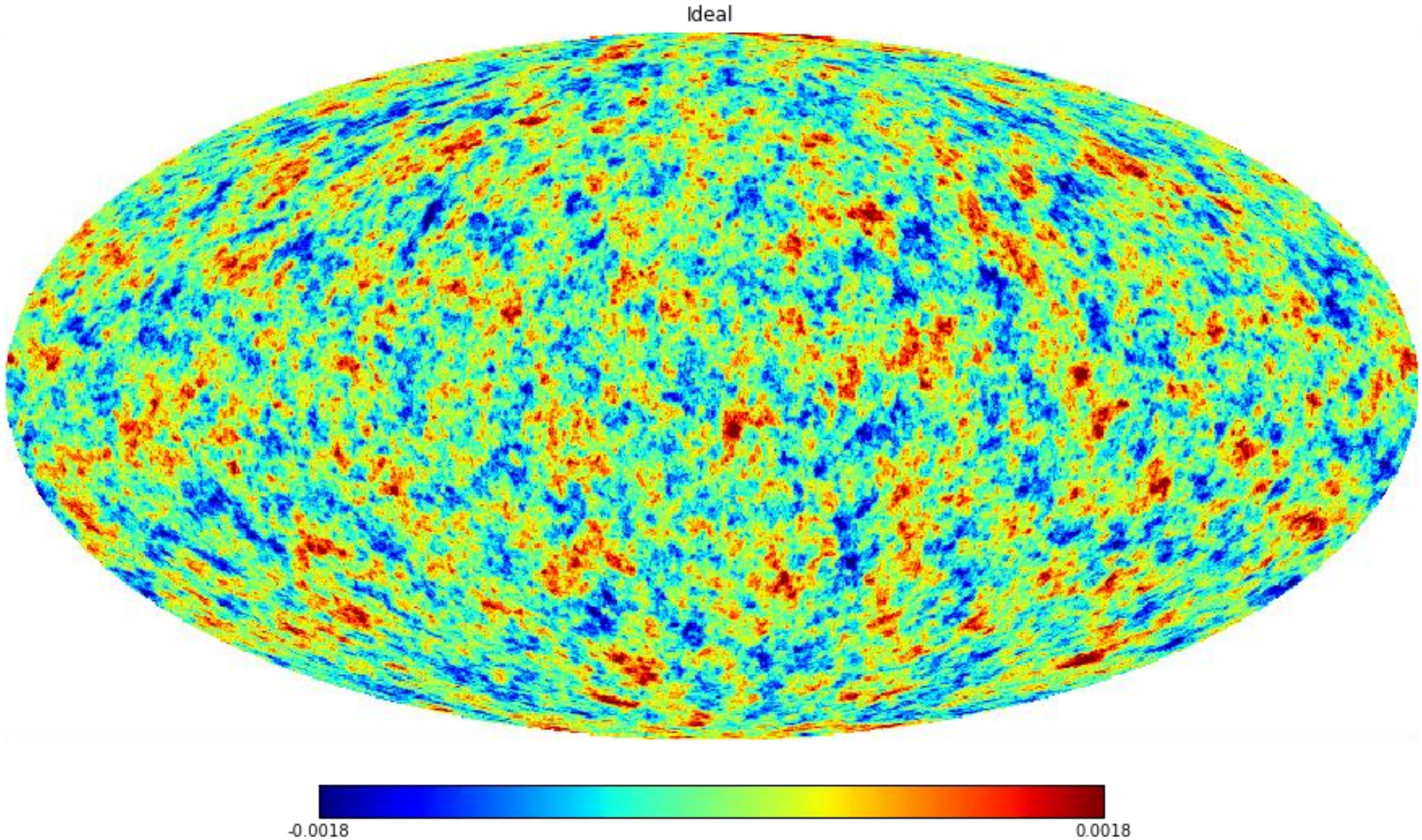
Planck 2015 lensing ($E_{\nabla\Phi}$)



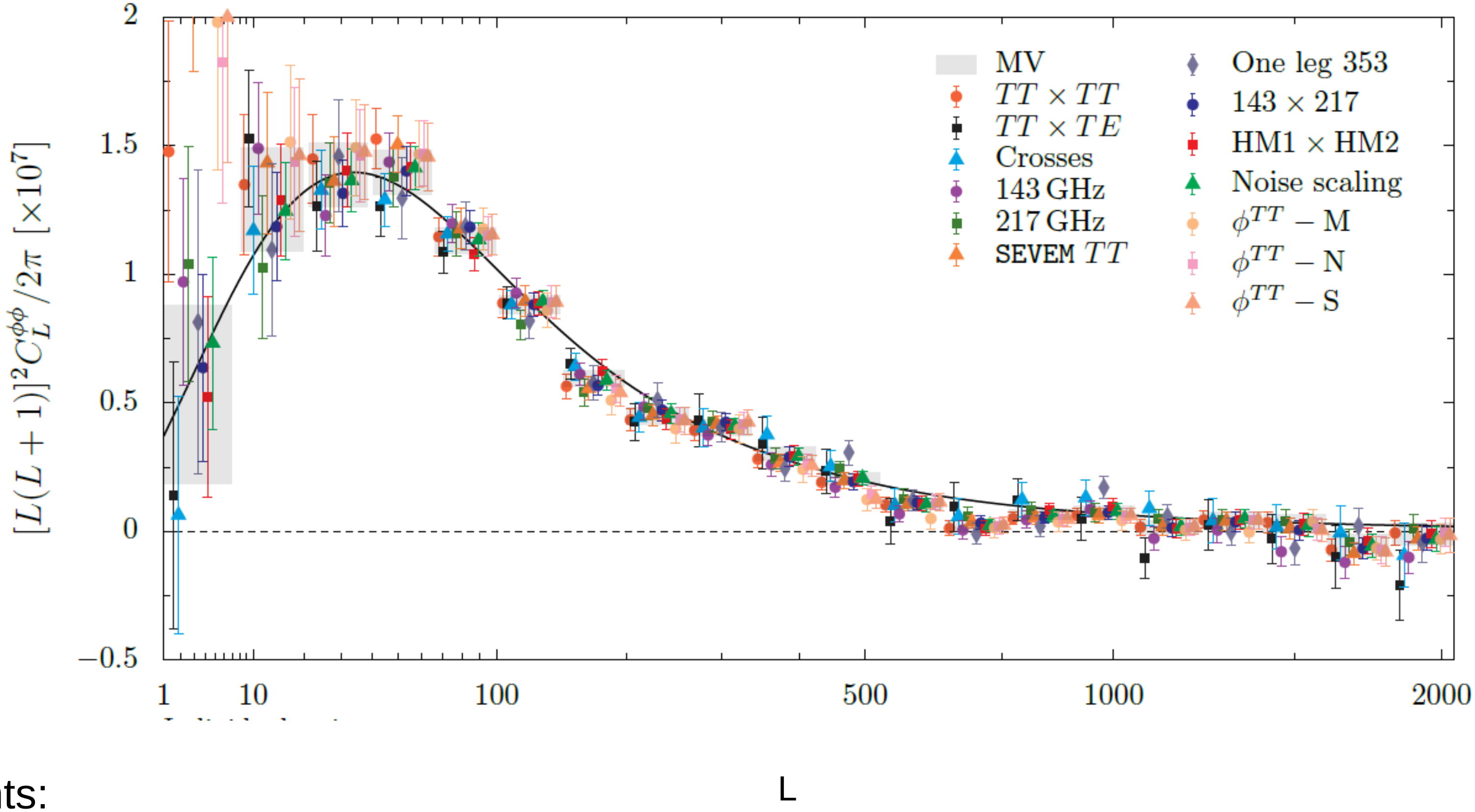
Planck lensing sim $E_{\nabla\Phi}$



True lensing $E_{\nabla\Phi}$



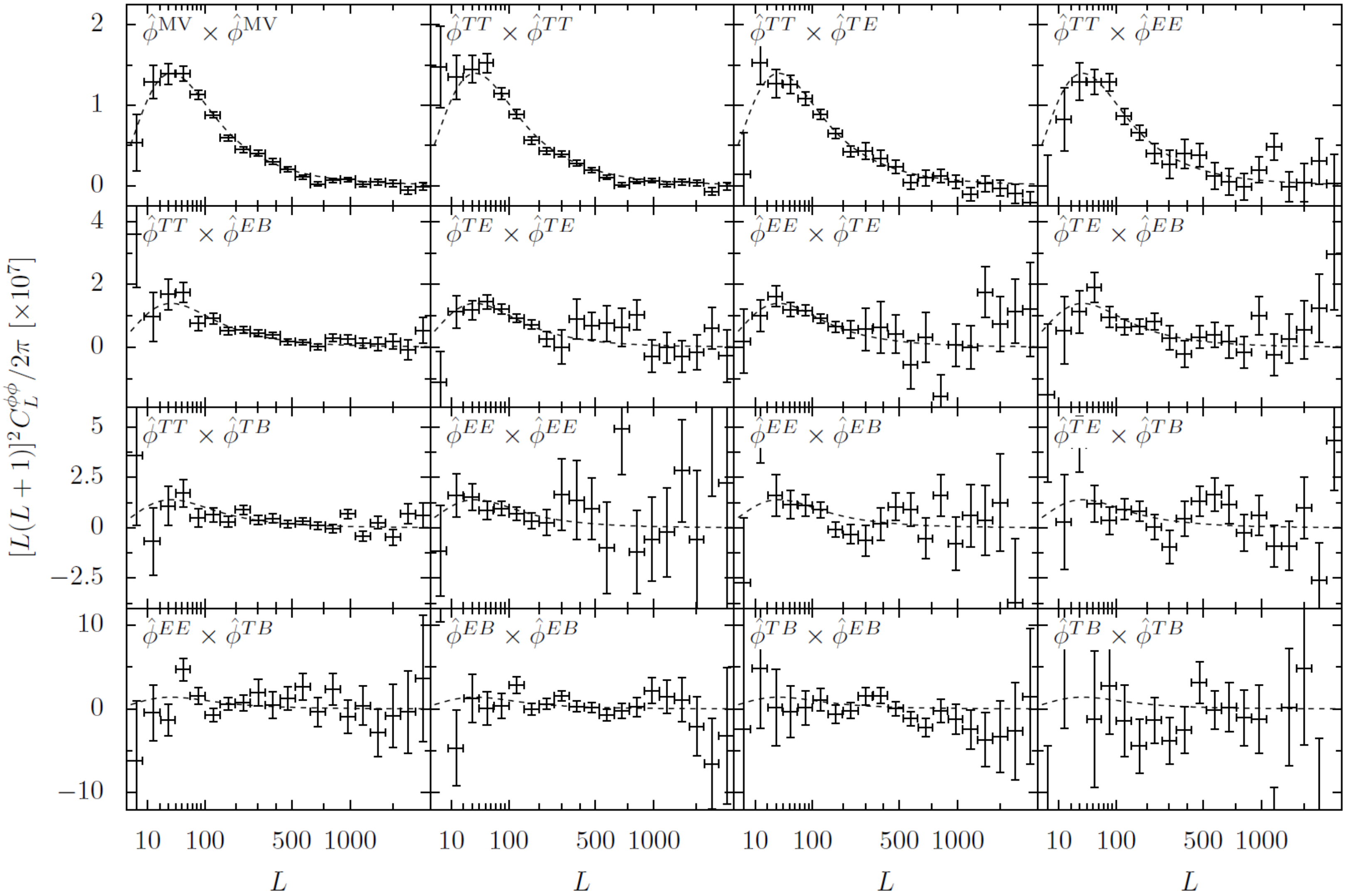
Reconstruction passes many internal consistency tests.



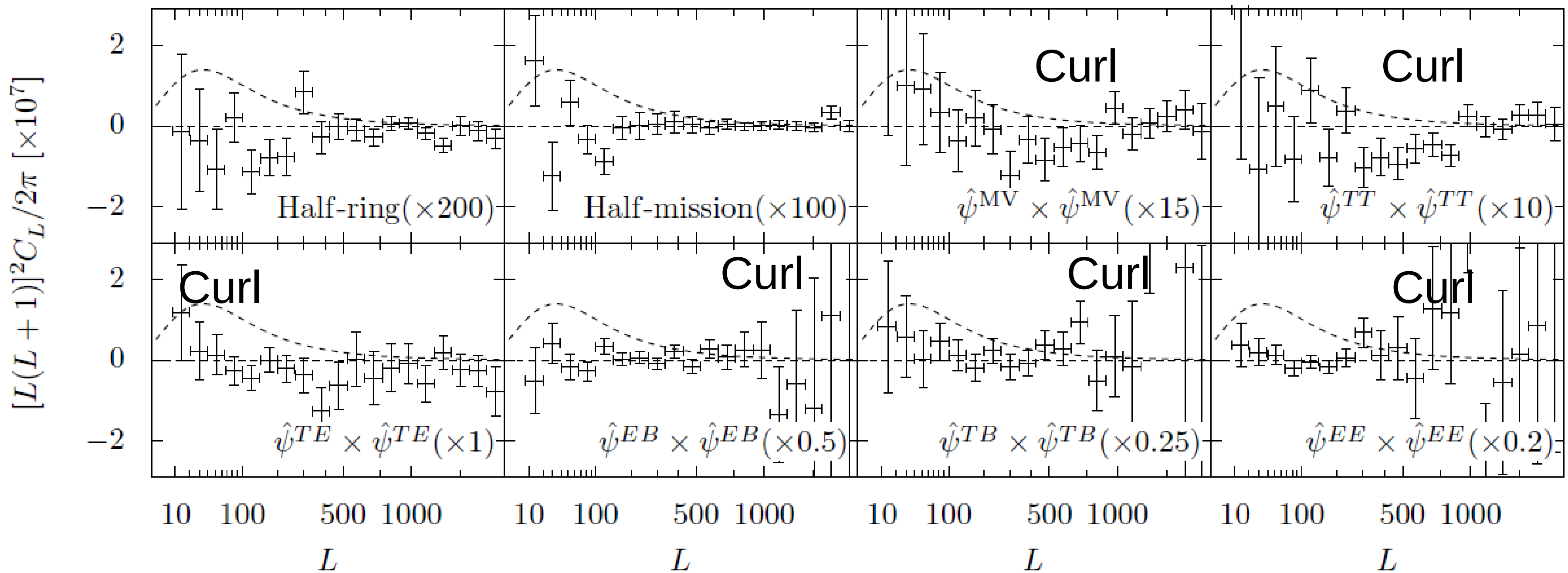
Highlights:

- Half-mission cross.
- Individual estimators.
- Replace one of four points in trispectrum with 353GHz.

Individual Cross-spectra

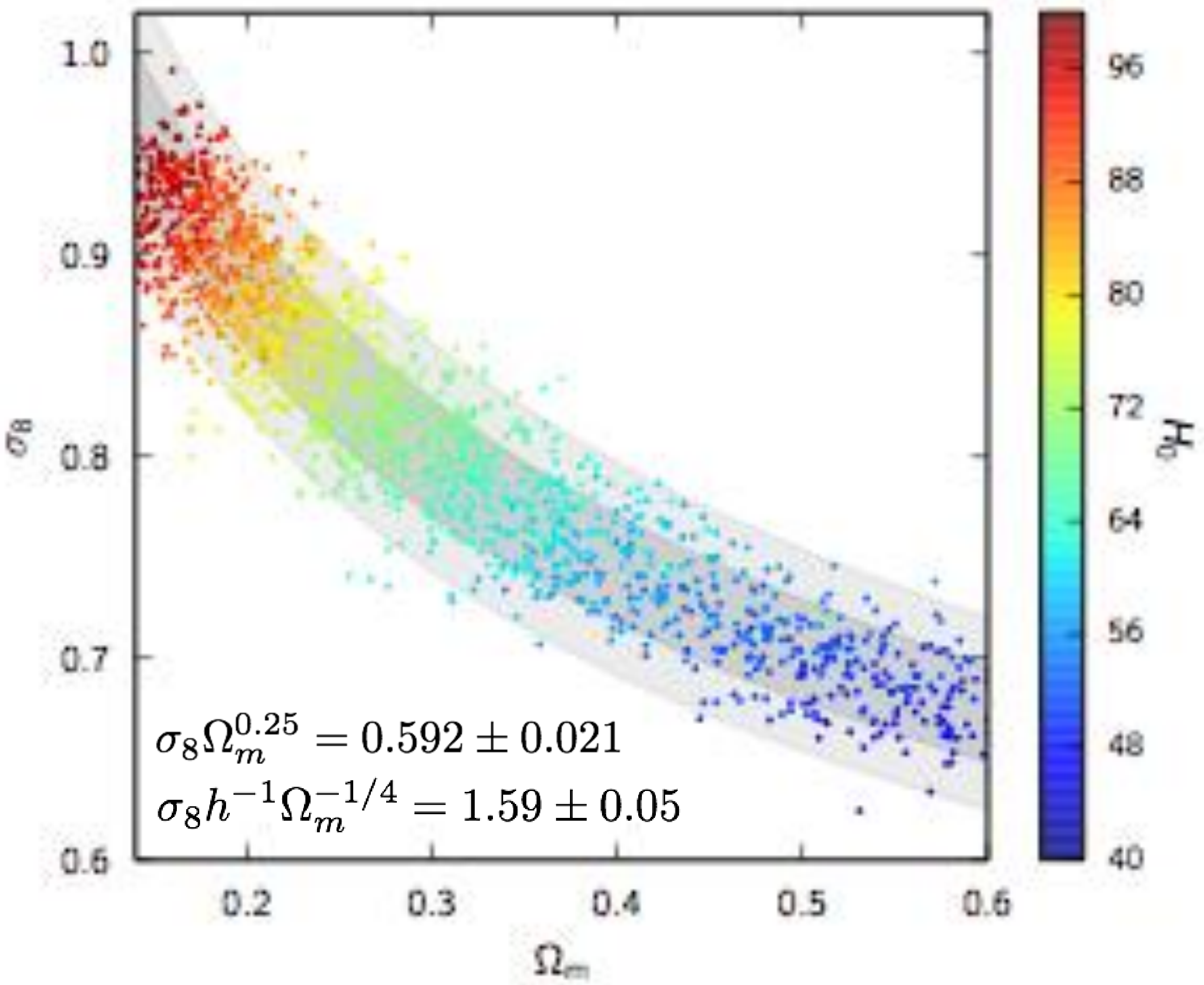


Null Tests

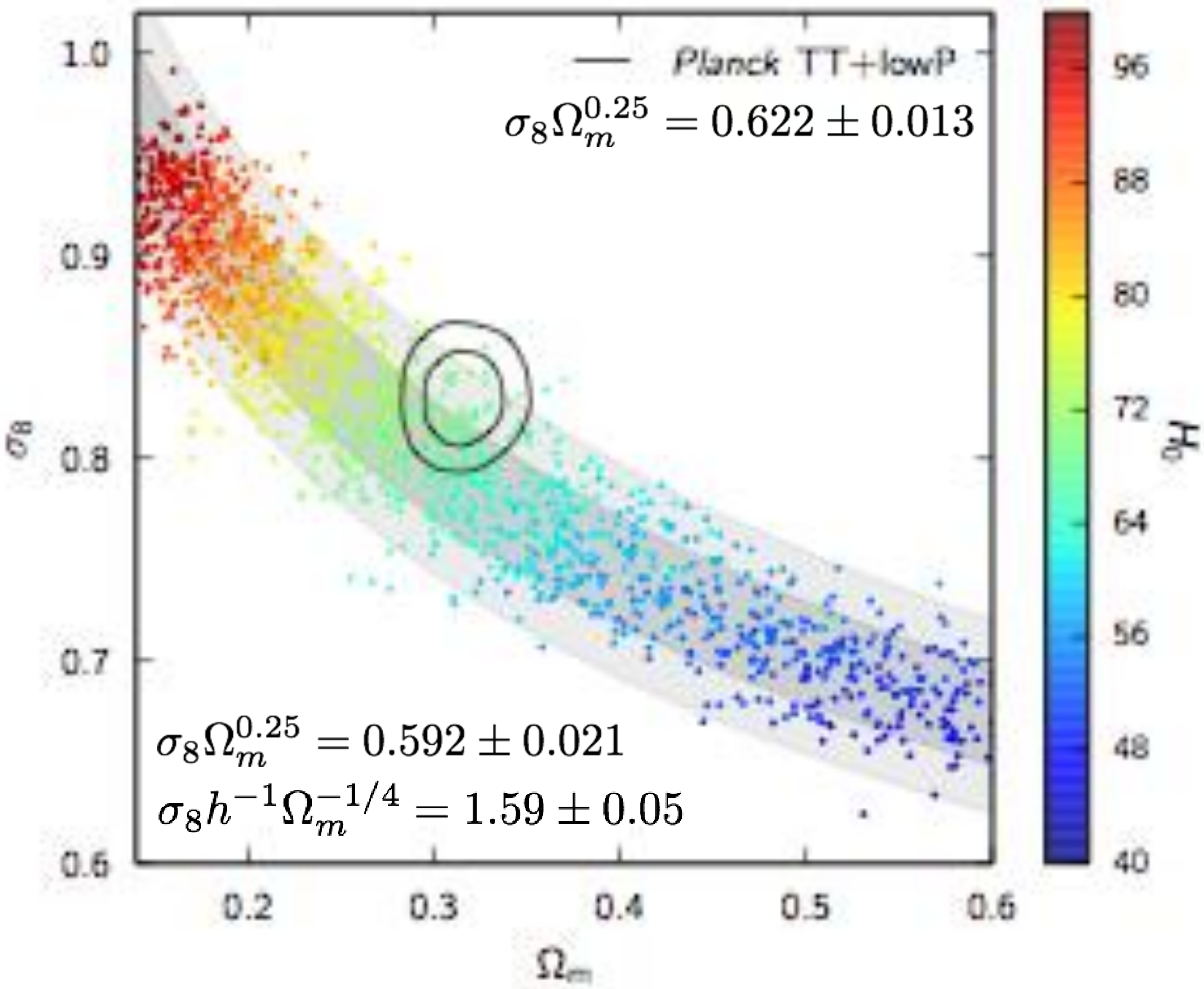


Conservative likelihood uses 40 ≤ L ≤ 400

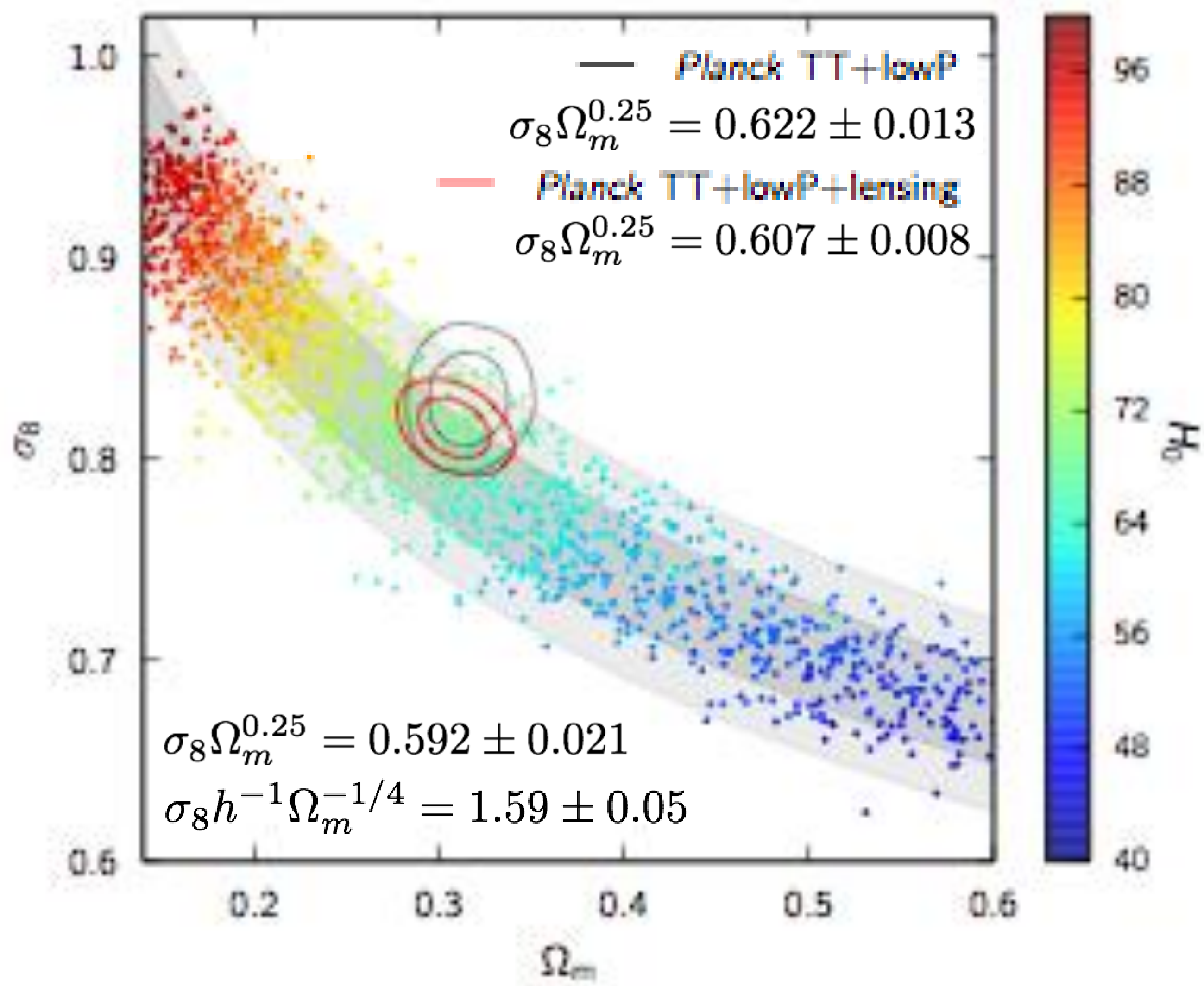
LCDM Parameter Constraints from CMB Lensing Only



LCDM Parameter Constraints from CMB Lensing Only

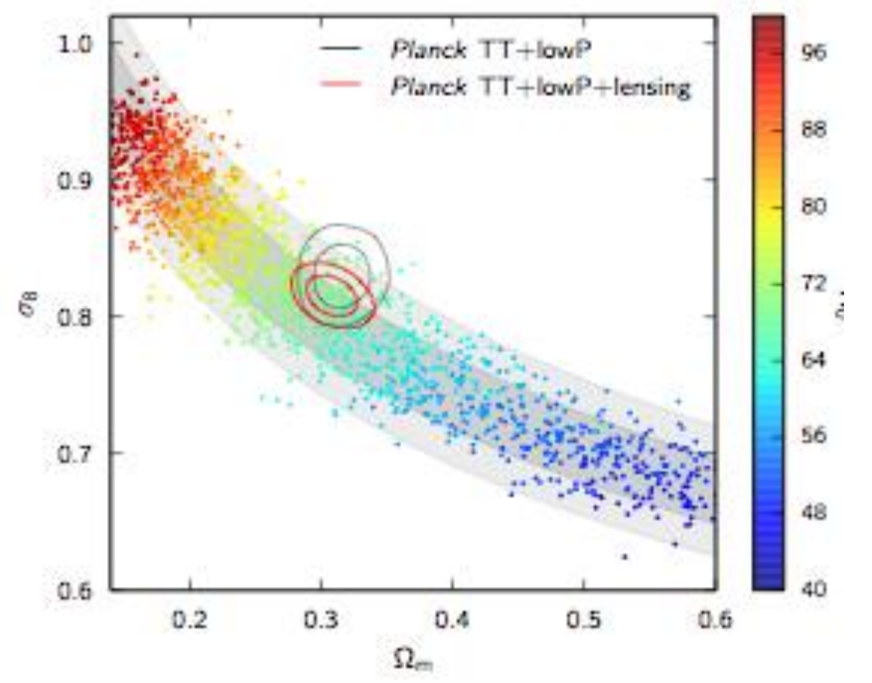


LCDM Parameter Constraints from CMB Lensing Only



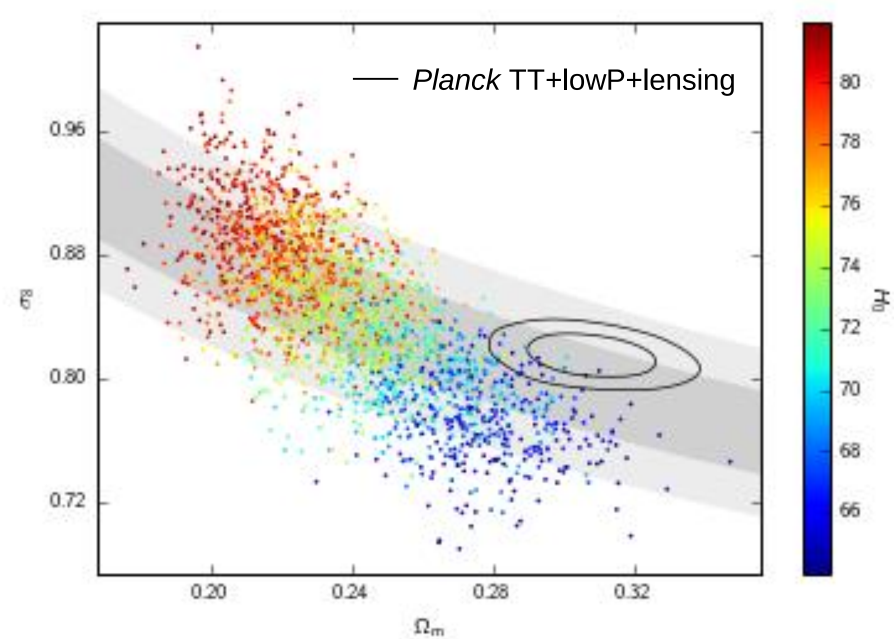
c.f. galaxy lensing (cosmic shear)

Planck Lensing



Kids-450 2016 (1606.05338)

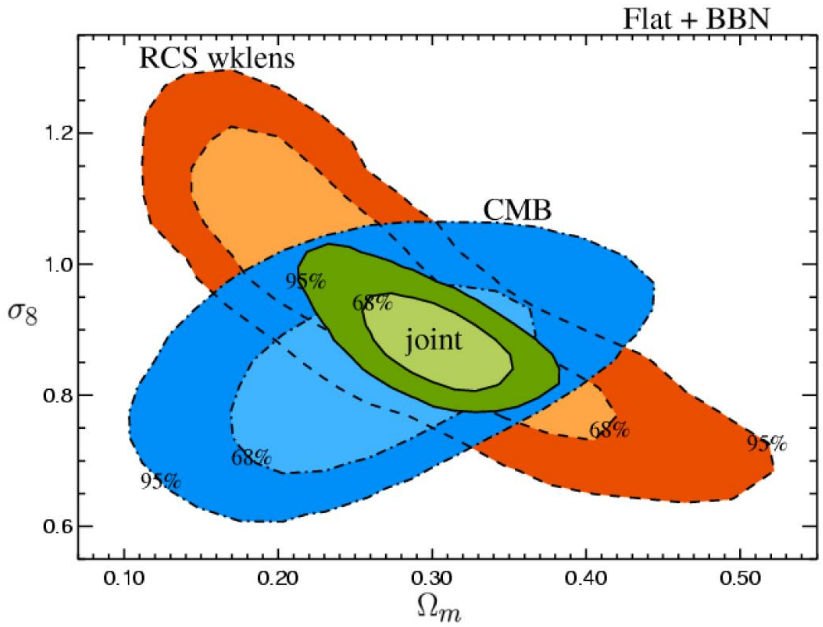
+ weak Planck A_s prior



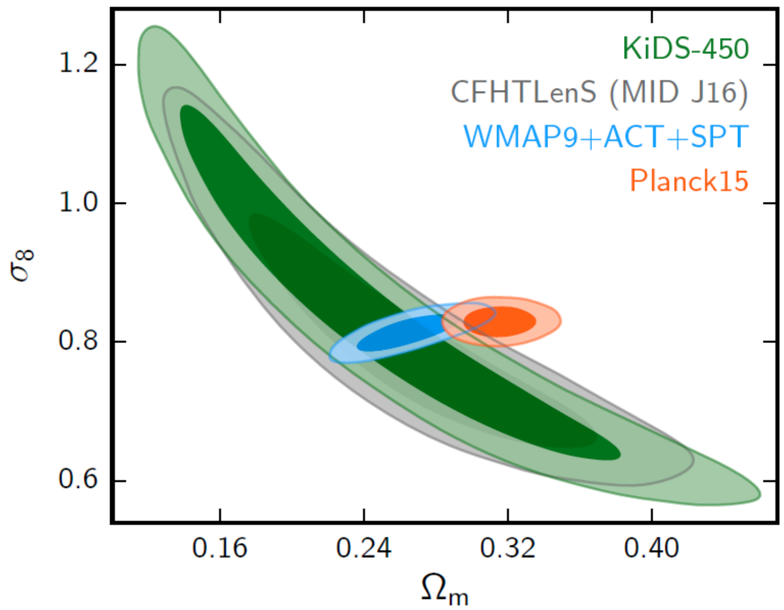
Are we heading for a clear breakdown of $LCDM$?
(or statistical fluctuation, better understanding of galaxy shear systematics…?)

CMB lensing progress looks relatively good.. so far fewer systematics

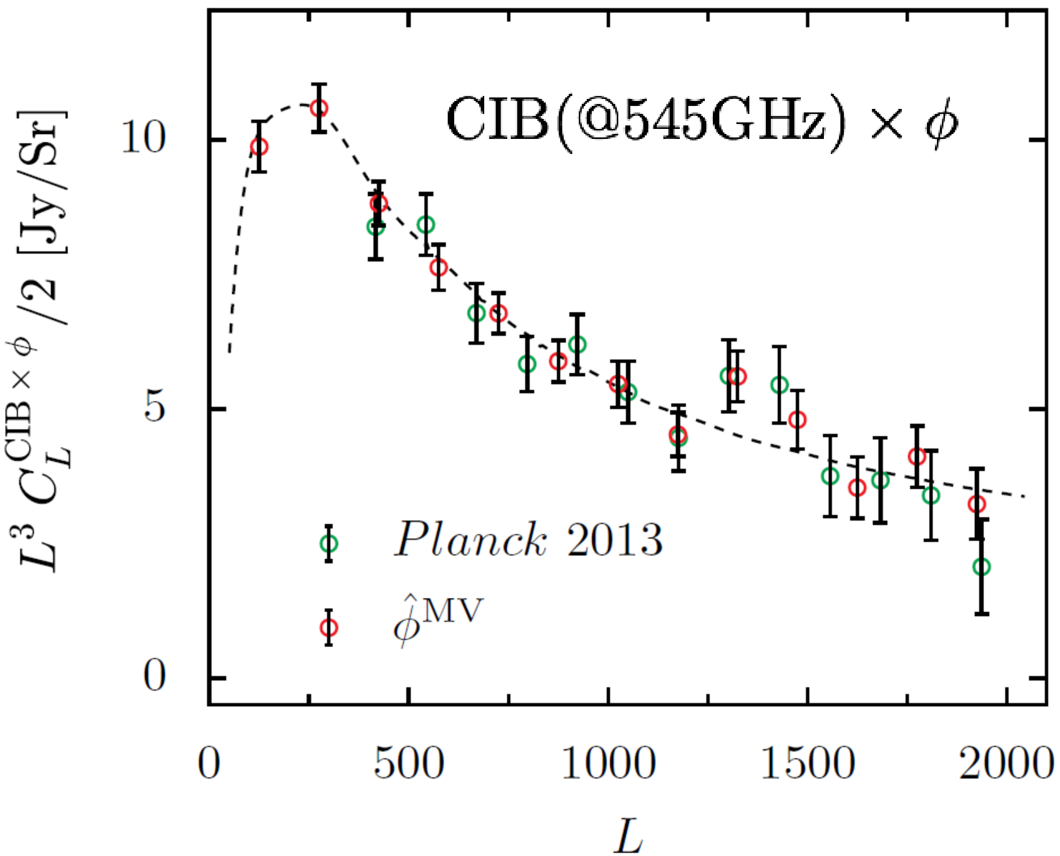
RCS 2003 (astro-ph/0302435)



Kids-450 2016 (1606.05338)



Cross-correlation with the Infrared Background



Now detected at ~50σ.

CIB provides an independent, high S/N probe of ϕ, useful for lensing B-mode estimates.

Other things you can do

Cross correlation with LSS (DES etc.)

Cross correlation with SZ (Hill et al)

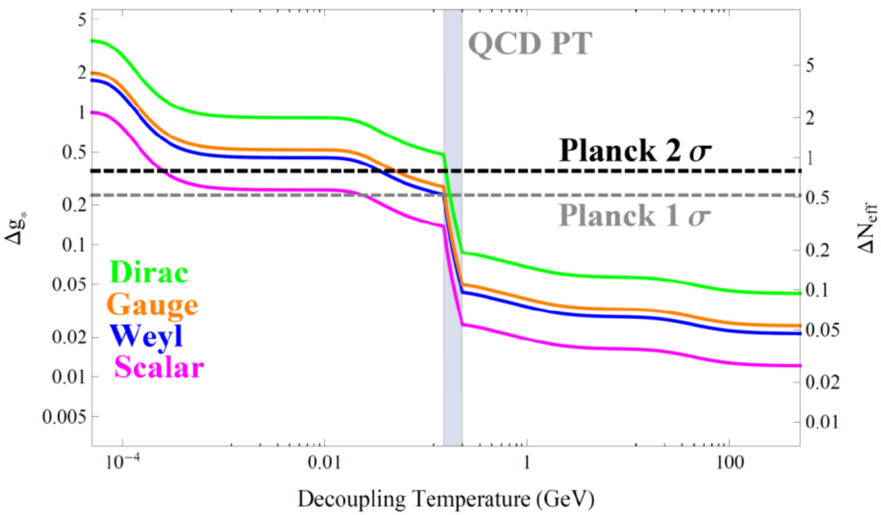
Cross-correlation with x, y, z…

Calibration of multiplicative biases in galaxy lensing estimates

Delensing (B modes, but also T, E?)

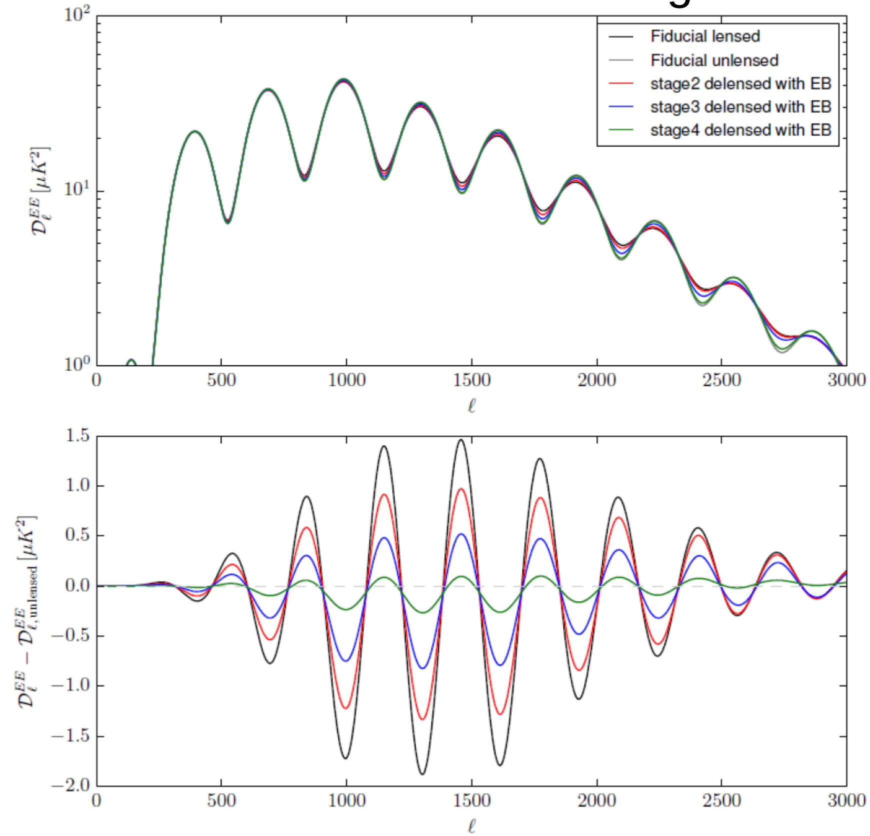
…

For high$-l$ E and Φ, clear physics targets may be (just) within reach of S4…



1303.5379

Can use E-mode delensing!



Experiment	Timeline	$\sigma(N_{eff})$	$\sigma(\Sigma m_\nu)$ (eV)
Planck	Present	0.18	0.23
AdvACT/SPT3G	2016-2019	0.06	0.06
CMB-S4	2020-?	0.02	0.016 (with DESI)

Joel Meyers

Impact of post-Born lensing on the CMB

Geraint Pratten[1] and Antony Lewis[1]

[1]*Department of Physics & Astronomy, University of Sussex, Brighton BN1 9QH, UK*

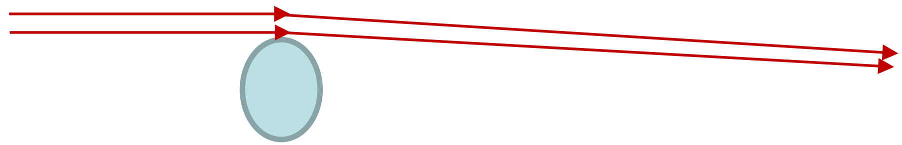
Lensing of the CMB is affected by post-Born lensing, producing corrections to the convergence power spectrum and introducing field rotation. We show numerically that the lensing convergence power spectrum is affected at the $\lesssim 0.2\%$ level on accessible scales, and that this correction and the field rotation are negligible for observations with arcminute beam and noise levels $\gtrsim 1\,\mu\mathrm{K\,arcmin}$. The field rotation generates $\sim 2.5\%$ of the total lensing B-mode polarization amplitude (0.2% in power on small scales), but has a blue spectrum on large scales, making it highly subdominant to the convergence B modes on scales where they are a source of confusion for the signal from primordial gravitational waves. Since the post-Born signal is non-linear, it also generates a bispectrum with the convergence. We show that the post-Born contributions to the bispectrum substantially change the shape predicted from large-scale structure non-linearities alone, and hence must be included to estimate the expected total signal and impact of bispectrum biases on CMB lensing reconstruction quadratic estimators and other observables. The field-rotation power spectrum only becomes potentially detectable for noise levels $\ll 1\,\mu\mathrm{K\,arcmin}$, but its bispectrum with the convergence may be observable at $\sim 3\sigma$ with Stage IV observations. Rotation-induced and convergence-induced B modes are slightly correlated by the bispectrum, and the bispectrum also produces additional contributions to the lensed BB power spectrum.

arXiv:1605.05662

Ray-deflection: first lens changes location of second lensing event

$$\Psi(\mathbf{x}_0 + \delta\mathbf{x}) \approx \Psi(\mathbf{x}_0) + \Psi_{,a}(\mathbf{x}_0)\delta x_a + \frac{1}{2}\Psi_{,ab}(\mathbf{x}_0)\delta x_a \delta x_b + \mathcal{O}(\Psi^4)$$

Linear approximation



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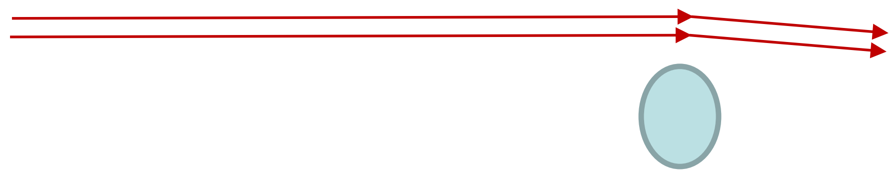

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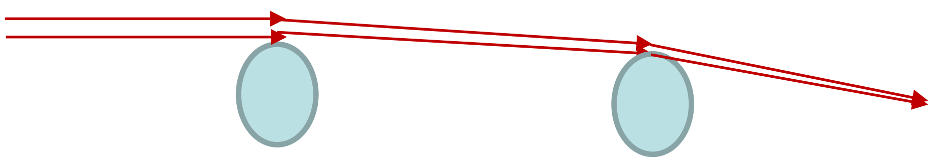
Post-Born lensing

e.g. more net lensing



Lens-Lens coupling: Beam size (and shape) affected by first lensing event

$$\psi_{ab}(\boldsymbol{\theta},\chi) = 2\int_0^{\chi} d\chi'\,\chi'^2\,W(\chi',\chi)\,\Psi_{,ac}(\mathbf{x}')\,[\delta_b^c - \psi_b^c(\boldsymbol{\theta},\chi')]$$

Linear approximation

+

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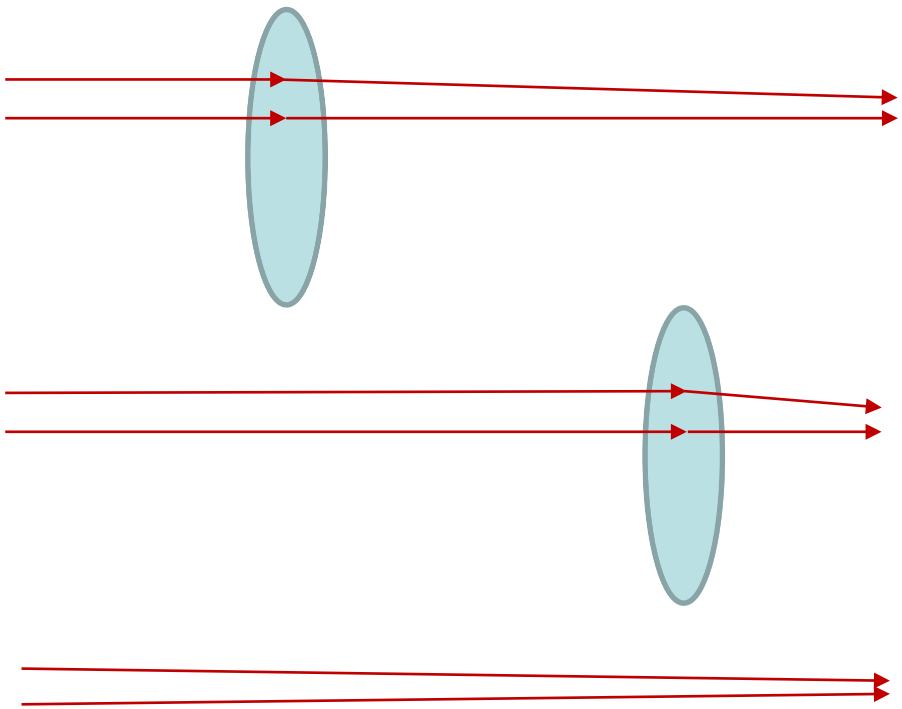
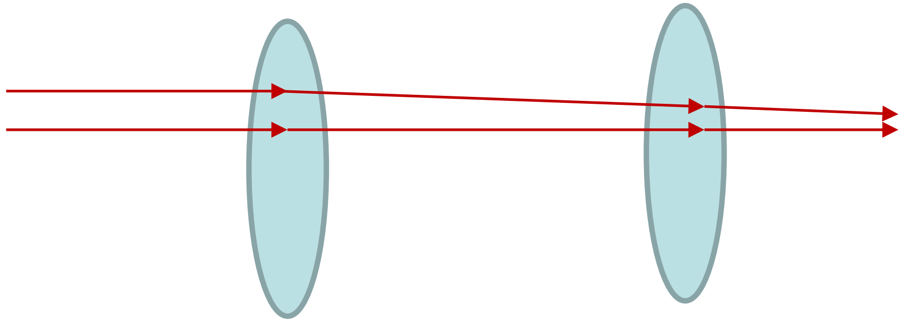
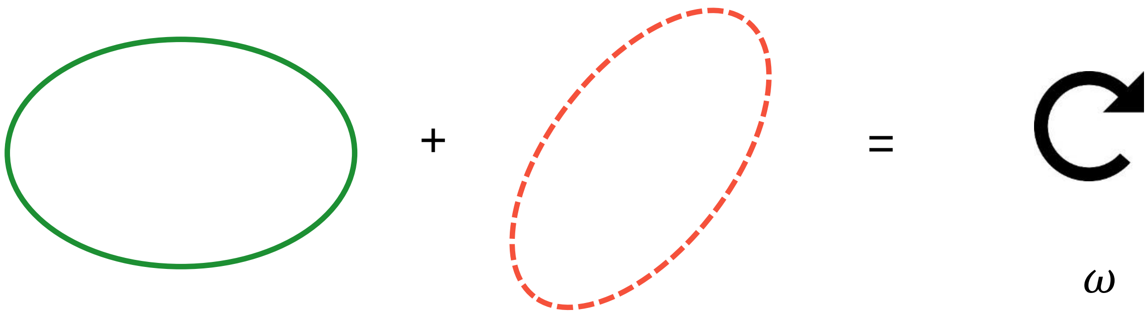


Post-Born lensing

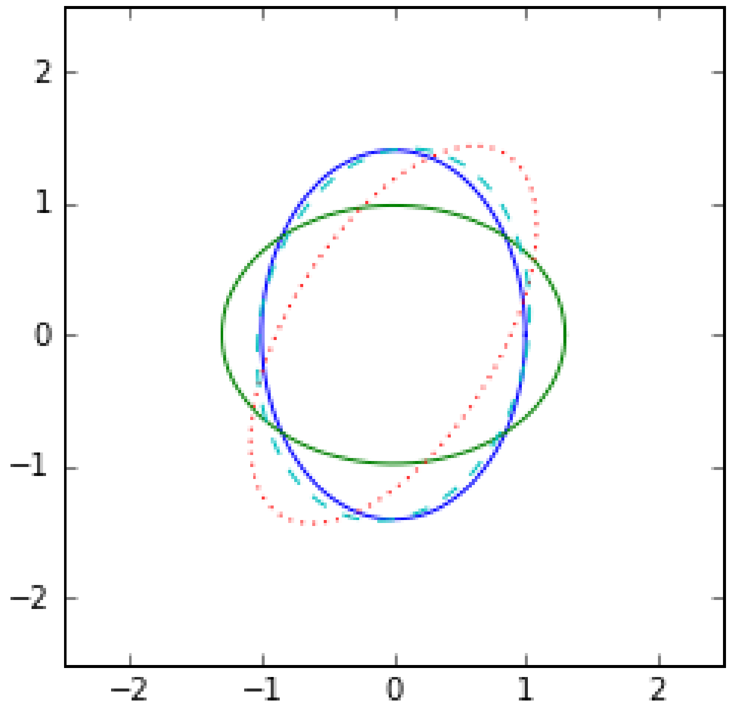


e.g. less net lensing

Lens-Lens with two non-aligned shears ⇒ rotation



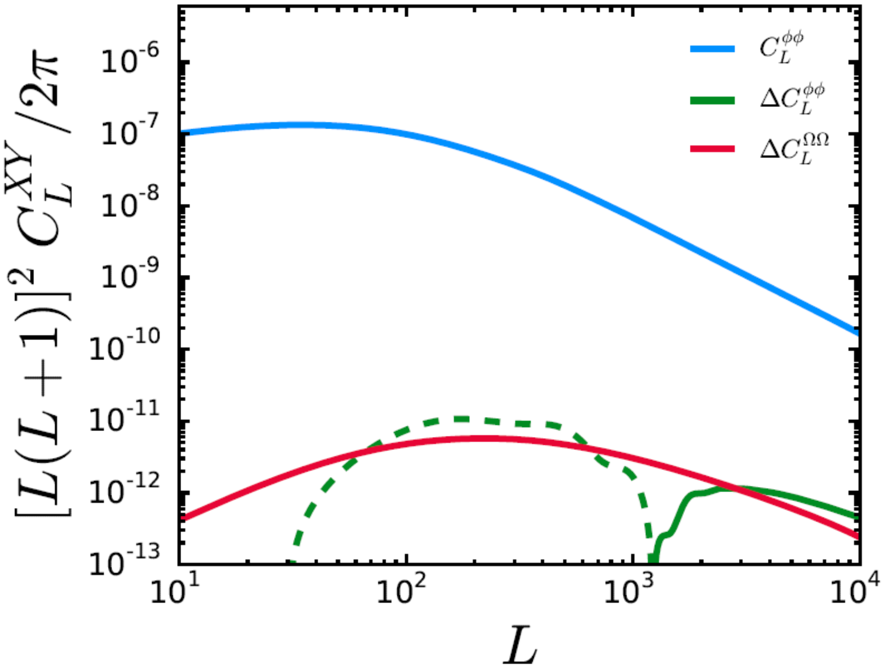
ω $+ \kappa + \gamma$

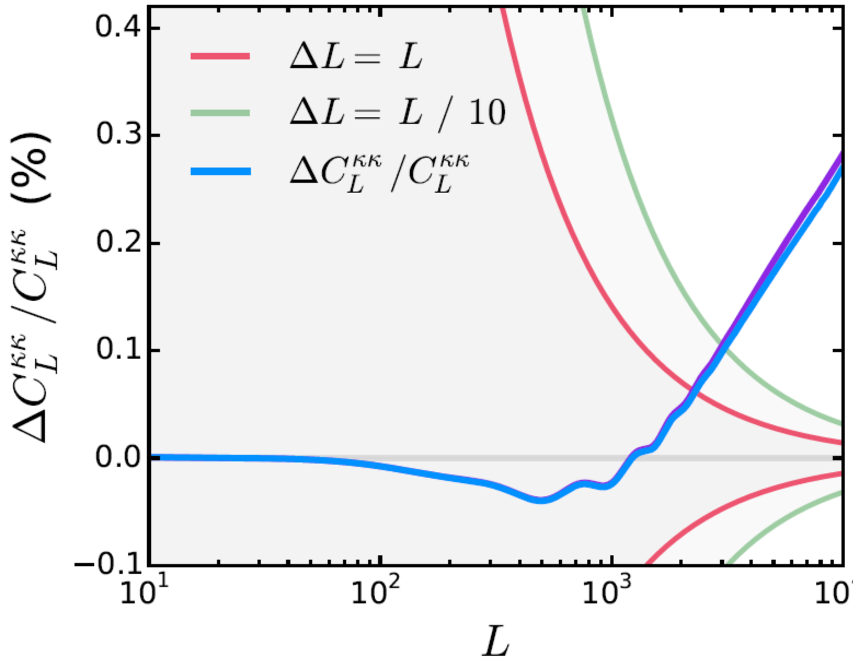


$$\alpha_a = \nabla_a \phi + \epsilon_{ab} \nabla^b \Omega$$

$$\kappa = -\frac{1}{2}\nabla^a \alpha_a = -\frac{1}{2}\boldsymbol{\nabla}^2\phi, \quad \text{and} \quad \omega = -\frac{1}{2}\epsilon^{ab}\nabla_a\alpha_b = -\frac{1}{2}\boldsymbol{\nabla}^2\Omega$$

$$\mathcal{A}^{ab} = \frac{\partial \theta^a_{\mathcal{S}}}{\partial \theta^b} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$





- Negligible change to convergence spectrum

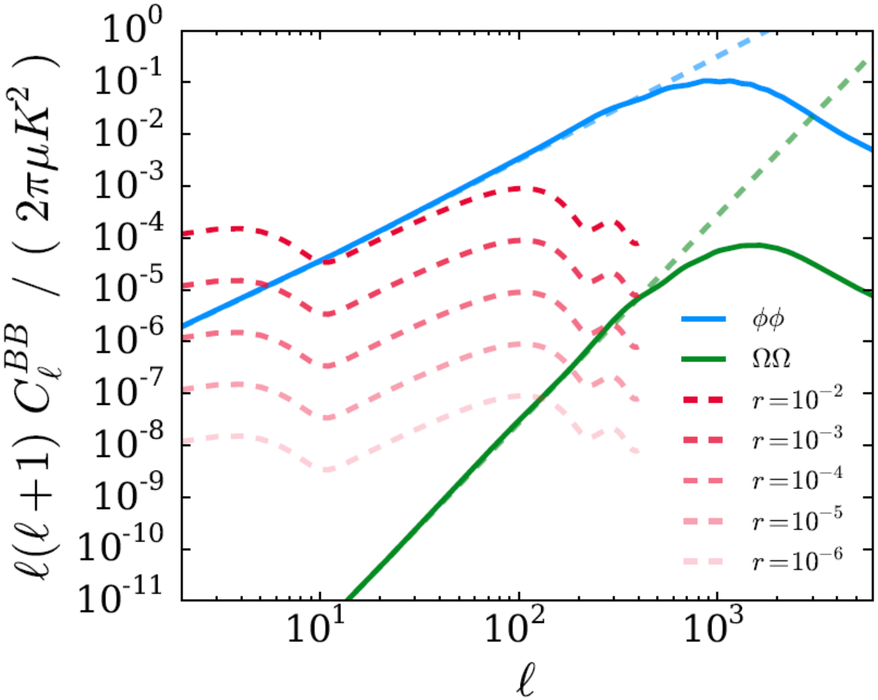
- Non-zero rotation spectrum

Impact on CMB polarization

$$\tilde{P}_{ab}(\boldsymbol{\theta}) = P_{ab}(\boldsymbol{\theta}+\boldsymbol{\alpha}) \approx P_{ab}(\boldsymbol{\theta}) + \underbrace{\boldsymbol{\alpha}_a \nabla^a P_{ab}(\boldsymbol{\theta})}$$

$$\alpha_a = \nabla_a \phi + \epsilon_{ab} \nabla^b \Omega$$

For rotation: $= \epsilon_{cd}\nabla^d(\Omega\nabla^c P_{ab})$



Large scales:

$$\tilde{C}_\ell^{BB}(\text{convergence}) \approx \frac{1}{\pi}\int d\ln\ell' C_{\ell'}^{\kappa\kappa}\ell'^2 C_{\ell'}^{EE} \qquad \tilde{C}_\ell^{BB}(\text{rotation}) \approx \frac{\ell^2}{2\pi}\int d\ln\ell' C_{\ell'}^{\omega\omega} C_{\ell'}^{EE}$$

$$\tilde{C}_\ell^{BB}(\text{convergence}) \approx 2.0\times10^{-6}\mu\text{K}^2, \qquad \tilde{C}_\ell^{BB}(\text{rotation}) \approx 1.7\times10^{-11}\left(\frac{\ell}{100}\right)^2\mu\text{K}^2$$

arXiv:1605.05662

How Gaussian is the lensing potential field?

Non-Gaussianity potentially important:

- Useful extra signal? (Namikawa 2016)
- Biases on lensing quadratic estimators (Boehm et al 2016)
- Corrections to the lensed CMB power spectra (Marozzi et al 2016)

Expected to be quite small:

Large distance to CMB $\Rightarrow$ many independent lenses

$\Rightarrow$ Gaussianization by central limit theorem

But how small, and what shape?...

Beyond Gaussianity – general possibilities

Flat sky approximation: $\Theta(x) = \frac{1}{2\pi}\int d^2l\,\Theta(l)e^{ix\cdot l}$ $(\Theta = T)$

Gaussian + statistical isotropy

$$\langle\Theta(l_1)\Theta(l_2)\rangle = \delta(l_1 + l_2)C_l$$

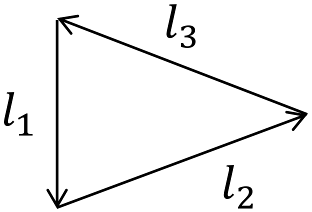
- power spectrum encodes all the information
- modes with different wavenumber are independent

Higher-point correlations

Gaussian: can be written in terms of C_l

Non-Gaussian: non-zero connected n-point functions

Bispectrum



$$\boldsymbol{l_1} + \boldsymbol{l_2} + \boldsymbol{l_3} = \boldsymbol{0}$$

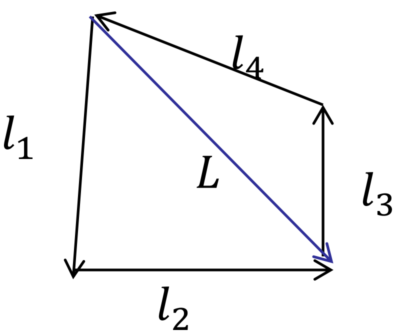
Flat sky approximation: $$\langle \Theta(l_1)\Theta(l_2)\Theta(l_3) \rangle = \frac{1}{2\pi}\delta(l_1 + l_2 + l_3) b_{l_1 l_2 l_3}$$

If you know $\Theta(l_1), \Theta(l_2)$, sign of $b_{l_1 l_2 l_3}$ tells you which sign of $\Theta(l_3)$ is more likely

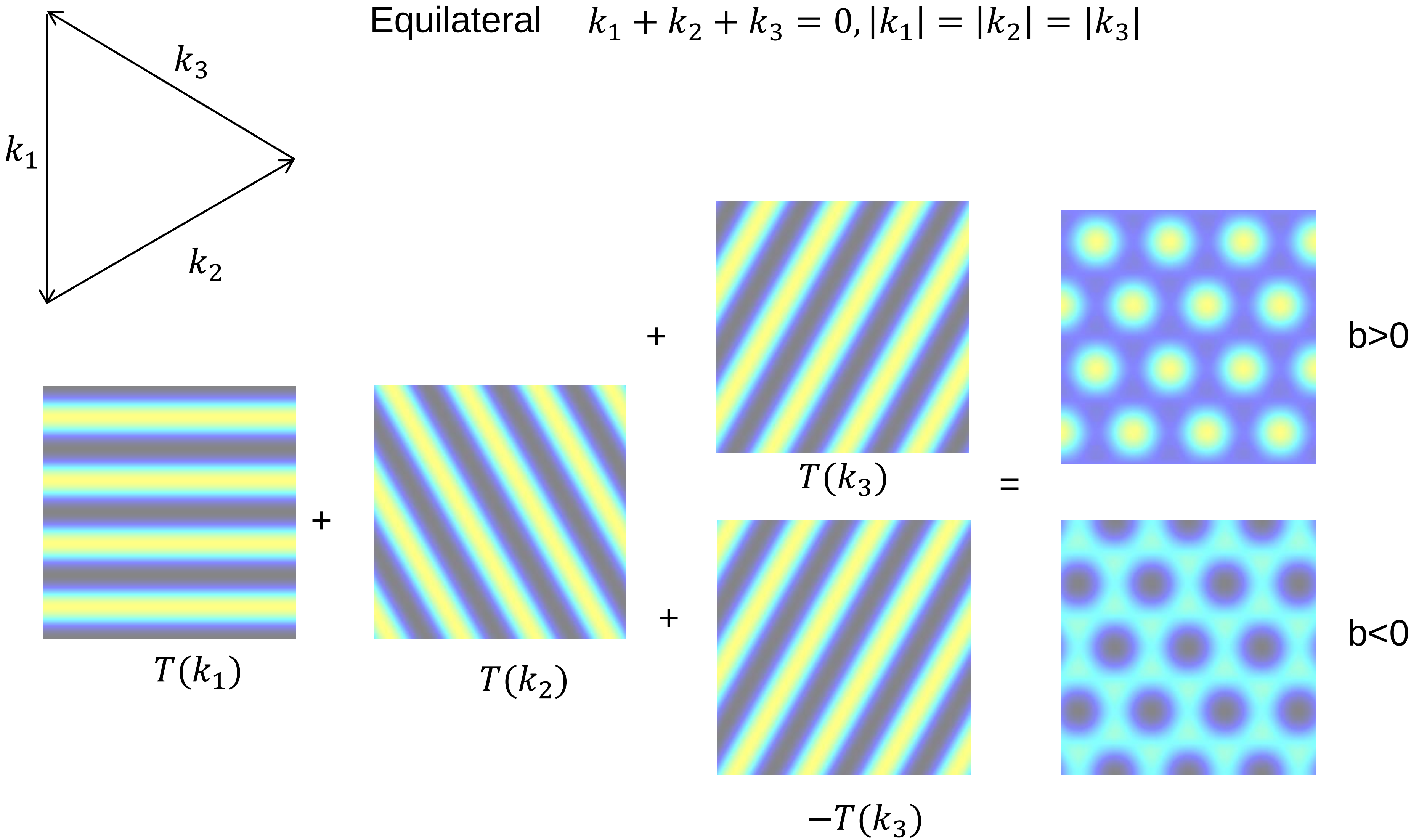
Trispectrum

$$\langle \Theta(\mathrm{l}_1)\Theta(\mathrm{l}_2)\Theta(\mathrm{l}_3)\Theta(\mathrm{l}_4)\rangle_C = (2\pi)^{-2}\delta(\mathrm{l}_1 + \mathrm{l}_2 + \mathrm{l}_3 + \mathrm{l}_4) T(\mathrm{l}_1, \mathrm{l}_2, \mathrm{l}_3, \mathrm{l}_4)$$

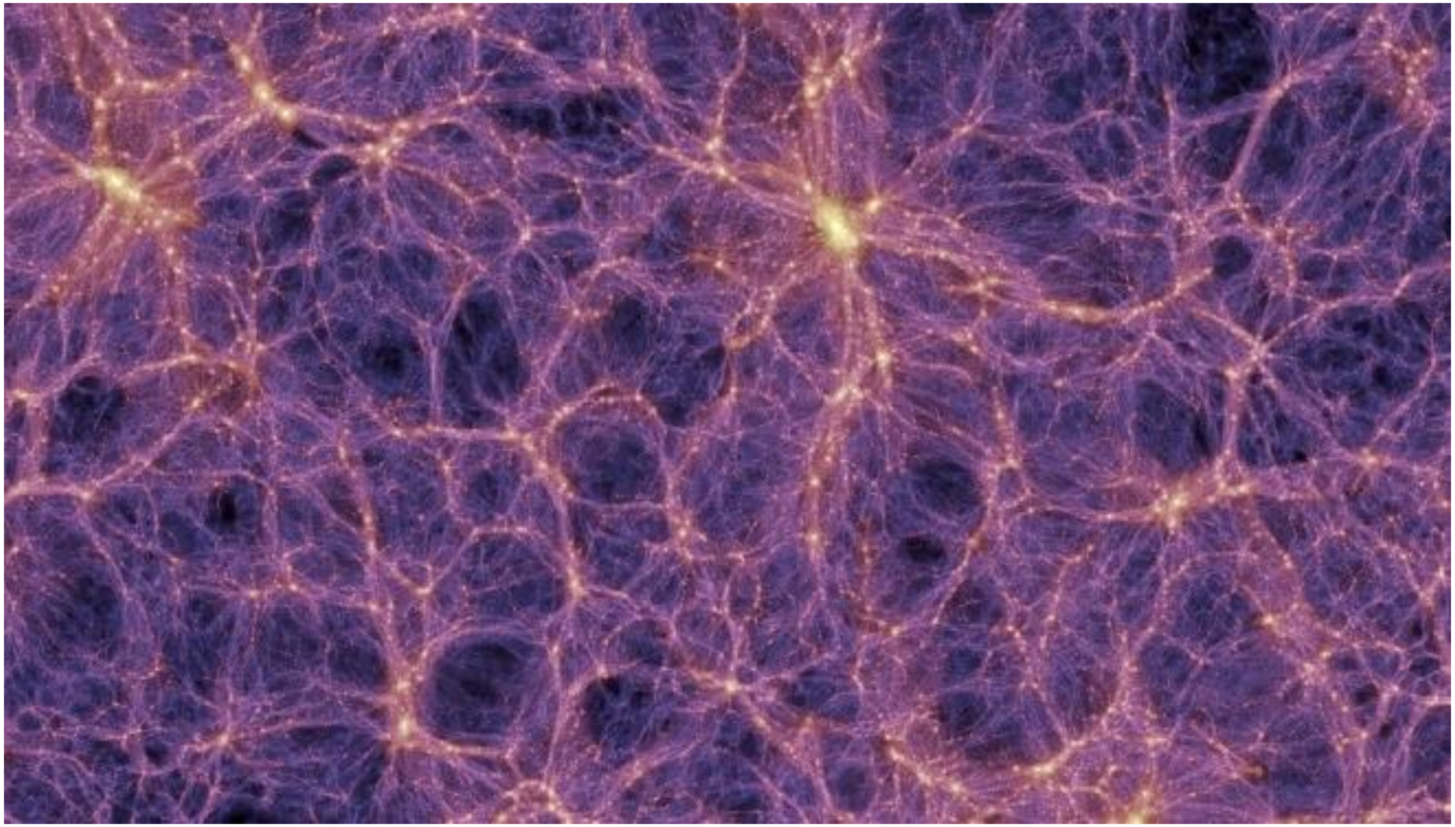
$$\langle \Theta(\mathrm{l}_1)\Theta(\mathrm{l}_2)\Theta(\mathrm{l}_3)\Theta(\mathrm{l}_4)\rangle_C = \frac{1}{2}\int \frac{d^2\mathbf{L}}{(2\pi)^2}\delta(\mathrm{l}_1 + \mathrm{l}_2 + \mathbf{L})\delta(\mathrm{l}_3 + \mathrm{l}_4 - \mathbf{L})\mathbb{T}^{(\ell_1 \ell_2)}_{(\ell_3 \ell_4)}(L) + \mathrm{perms.}$$



N-spectra…



AL: The Real Shape of Non-Gaussianities, arXiv:1107.5431

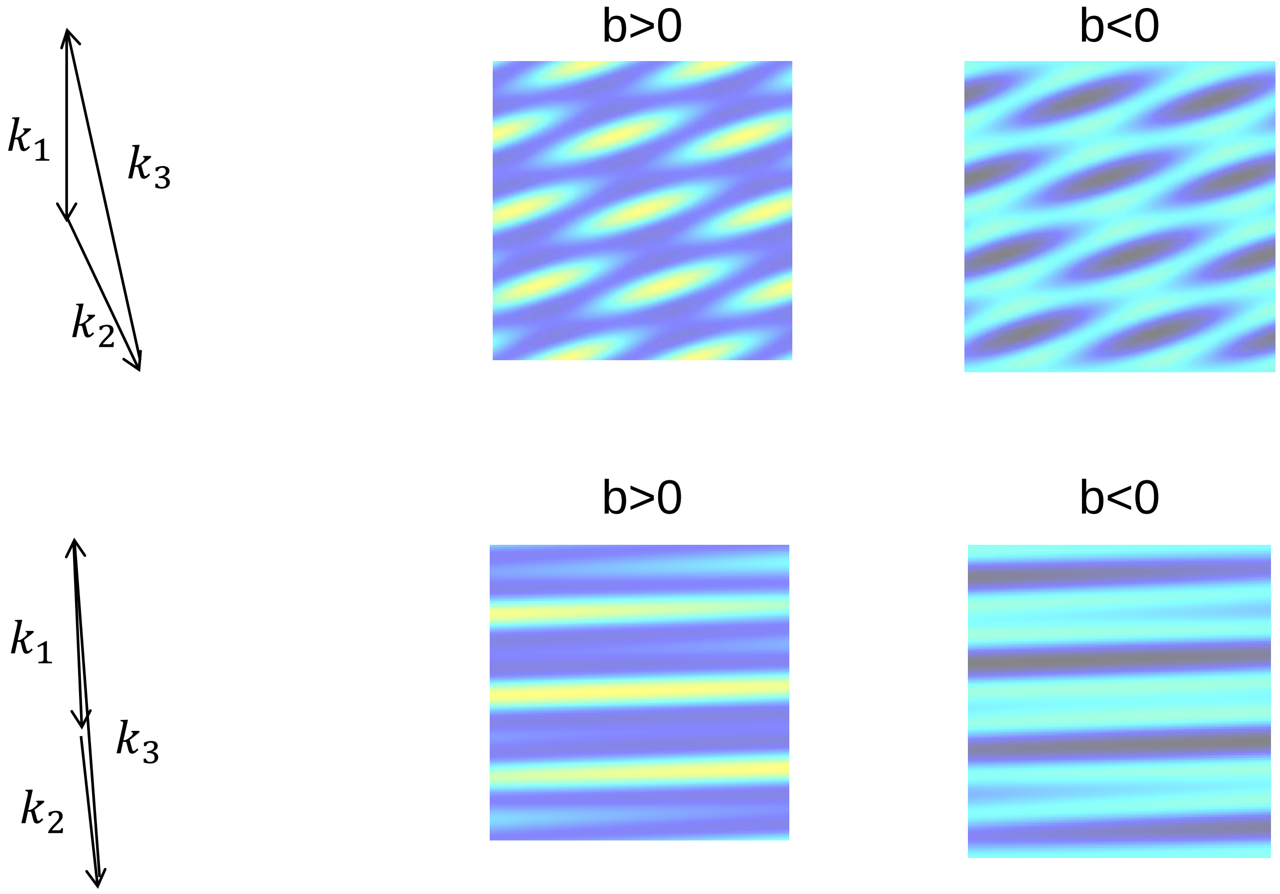


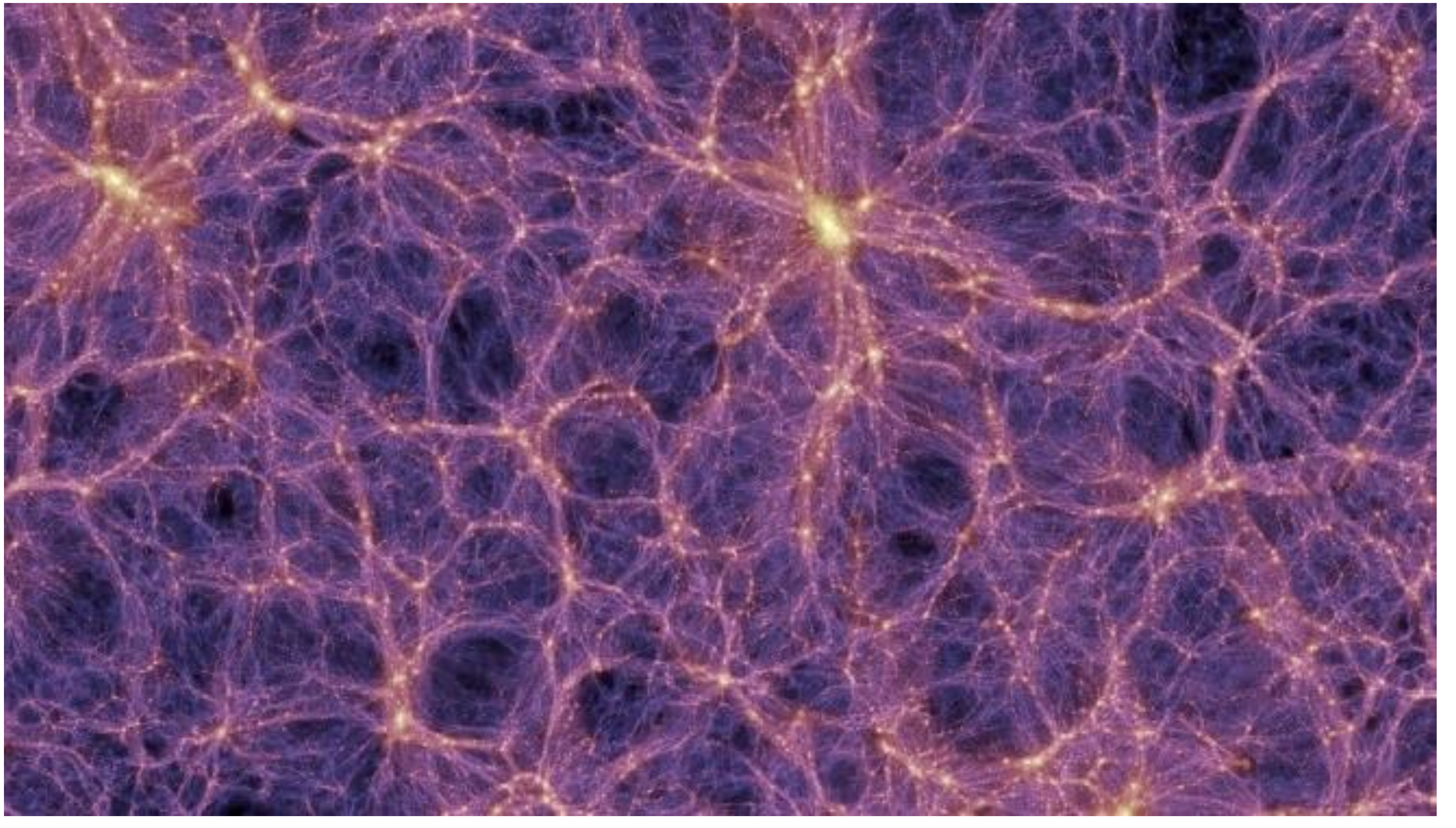
In 2D projection (e.g. lensing)

Clusters → Positive equilateral bispectrum

(+sight-aligned filaments)

Near-equilateral to flattened/folded:



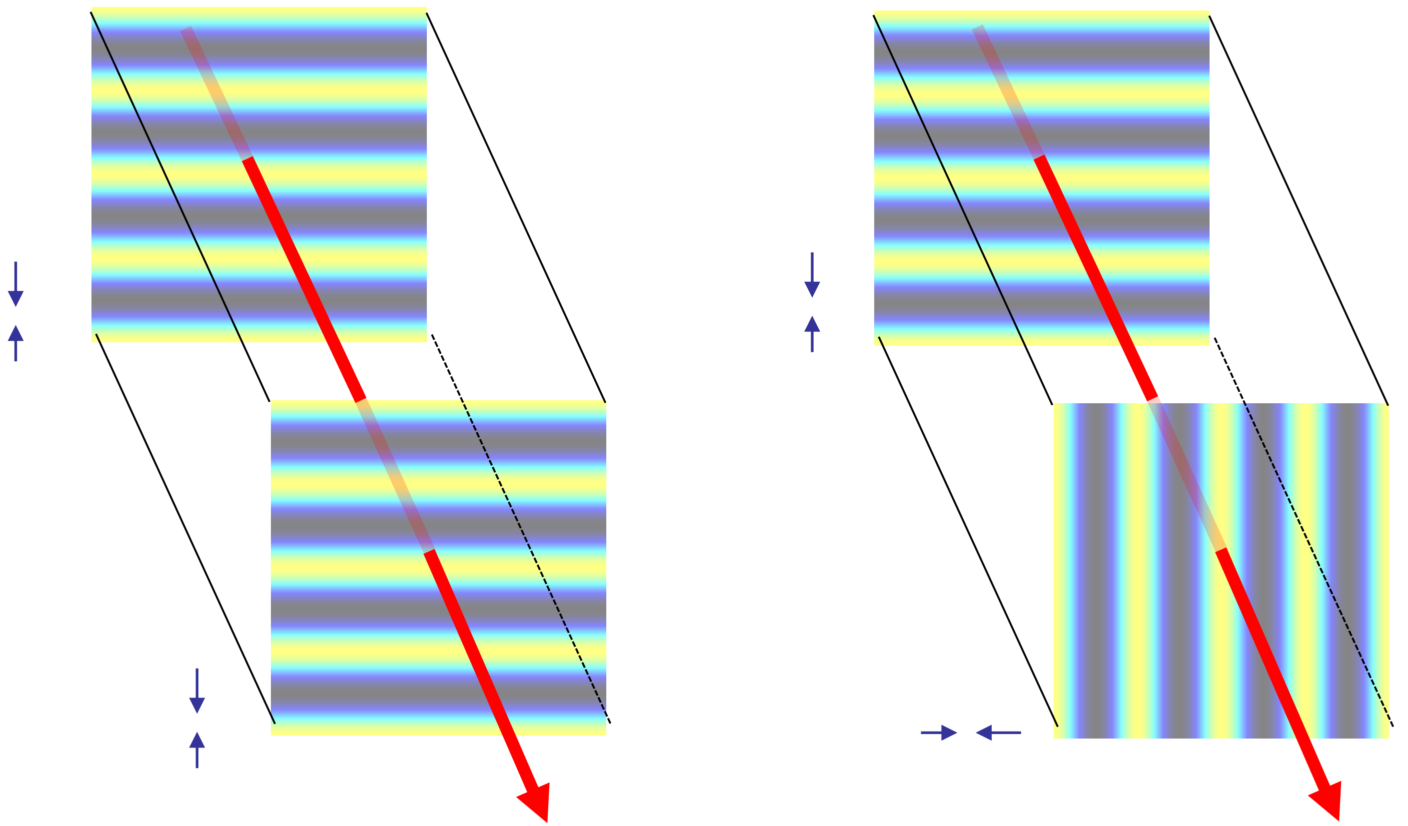


In 2D projection (e.g. lensing)

Filaments → Positive flattened bispectrum

(+sight-aligned sheets)

LSS has positive bispectrum, hence κ bispectrum from LSS growth also positive.
What about post-Born?

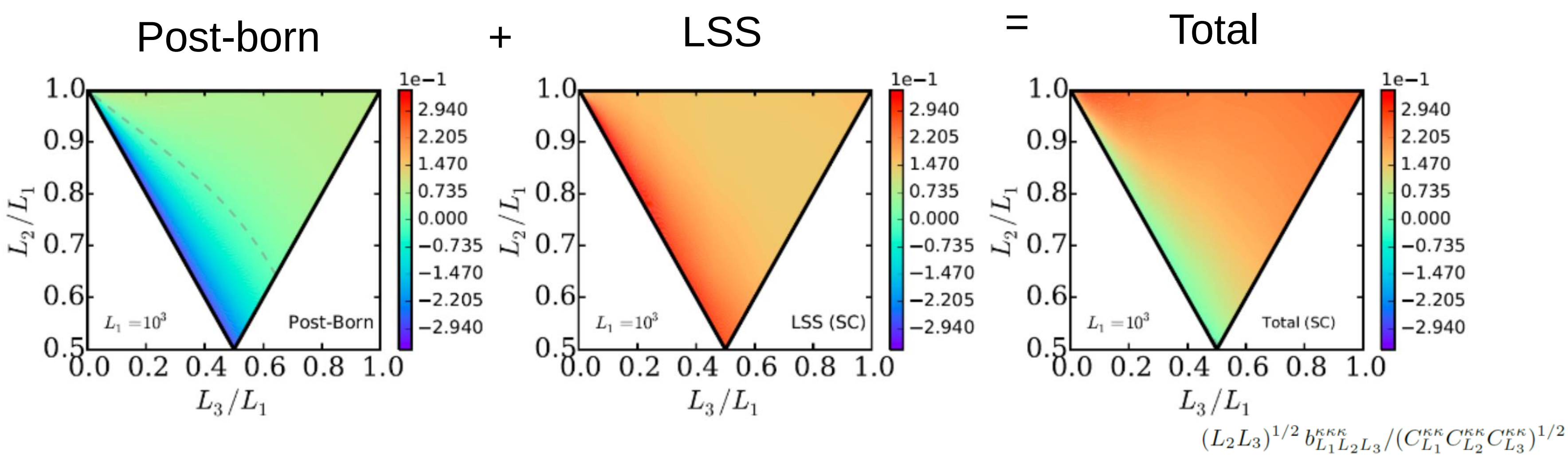


Big negative lens-lens effect

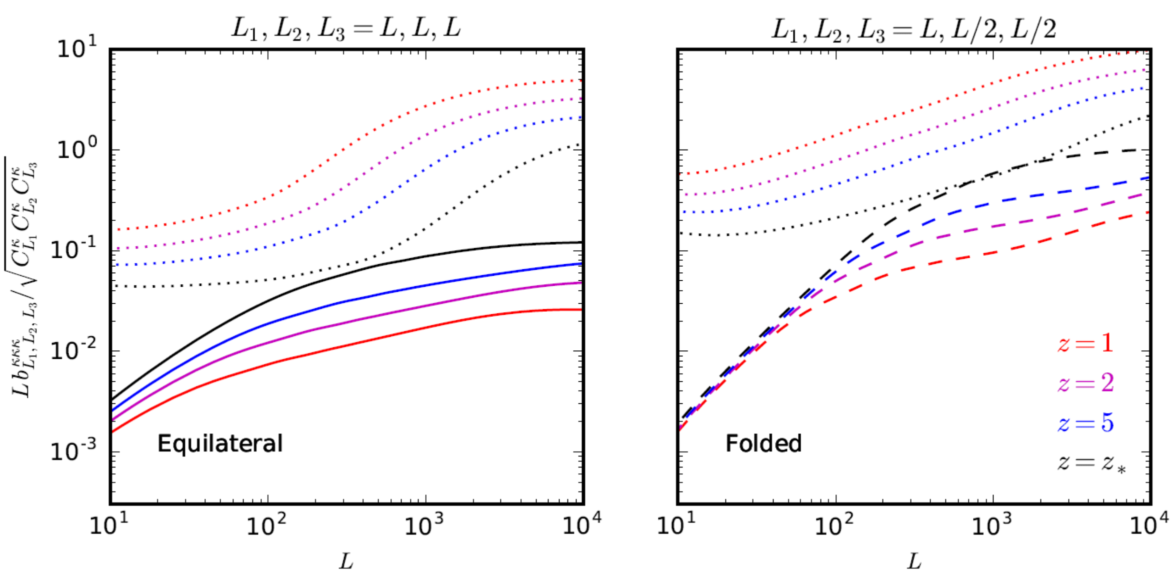
Zero lens-lens effect

Negative flattened bispectrum

Convergence Bispectrum



This cancellation is a fluke, LSS dominates at lower redshifts



Unexpectedly small folded Gaussianity of the CMB lensing convergence!

Naïve S/N for post-Born and total bispectrum

	noise [μK arcmin]	beam [arcmin]	$\ell_{\rm max}$	$f_{\rm sky}$	$\Delta\kappa\kappa$ S/N	$\omega\omega$ S/N	$\kappa\kappa\kappa$ S/N	$\kappa\kappa\omega$ S/N
Planck	33	5	2000	0.7	0.0	0.0	0.8	0.1
Simons Array	12	3.5	4000	0.65	0.0	0.0	3.4	0.4
SPT 3G	4.5	1.1	4000	0.06	0.0	0.0	2.3	0.4
S4	1	3	4000	0.4	0.2	0.7	25	3.1
S5	0.25	1	4000	0.5	0.8	2.7	99	8.8

Conclusions

- T
- E
- B

Plenty of modes still to go!

non-Gaussian ⇒ quadratic estimators for lensing field

- κ
- ω

Only just started! Lots to do. Nearly Gaussian.

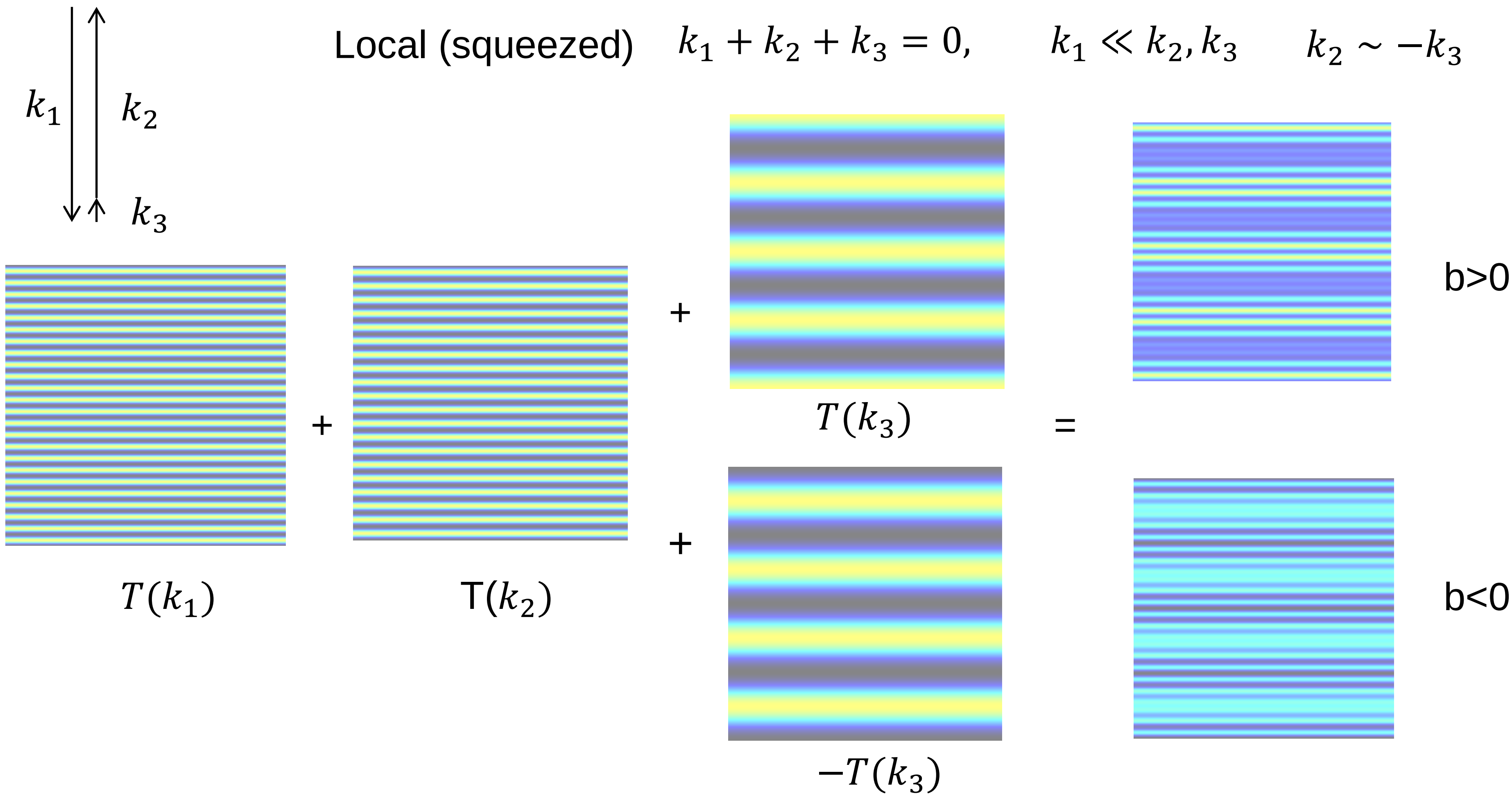
Negligible for near future

But lensing rotation is highly non-Gaussian as entirely quadratic.

⇒ Can measure rotation by correlation with quadratic combinations of densities, e.g. $\omega\kappa\kappa$ bispectrum

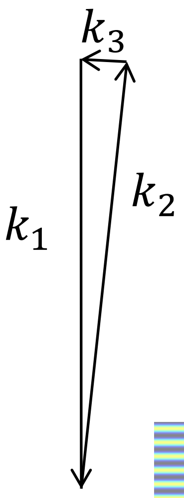
The scientific results that we present today are a product of the Planck Collaboration, including individuals from more than 100 scientific institutes in Europe, the USA and Canada.

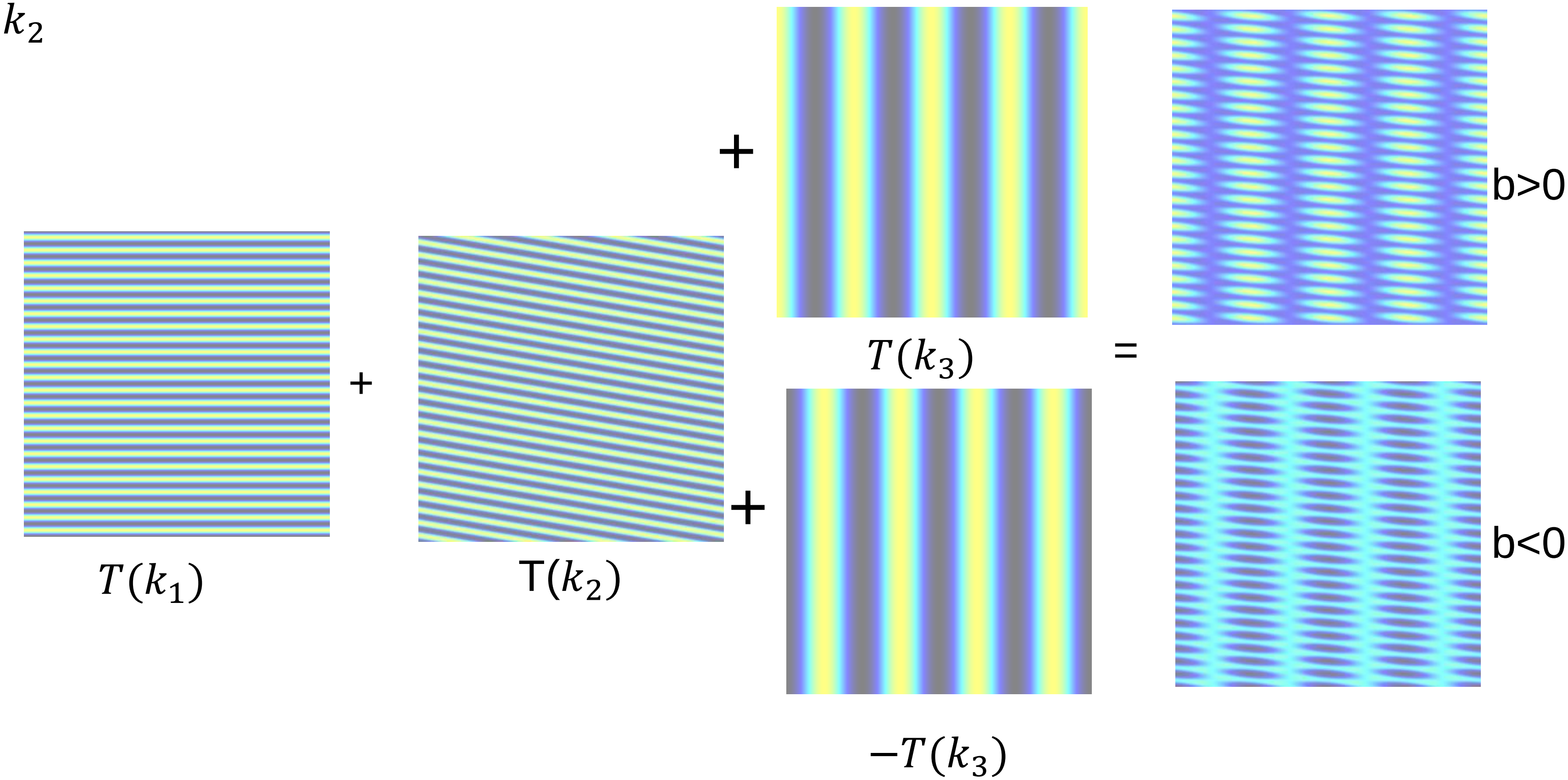
Planck is a project of the European Space Agency, with instruments provided by two scientific Consortia funded by ESA member states (in particular the lead countries: France and Italy) with contributions from NASA (USA), and telescope reflectors provided in a collaboration between ESA and a scientific Consortium led and funded by Denmark.



Squeezed bispectrum is a *correlation* of small-scale power with large-scale modes

Local (squeezed) $k_1 + k_2 + k_3 = 0,\quad k_1 \ll k_2, k_3 \quad k_2 \sim -k_3$

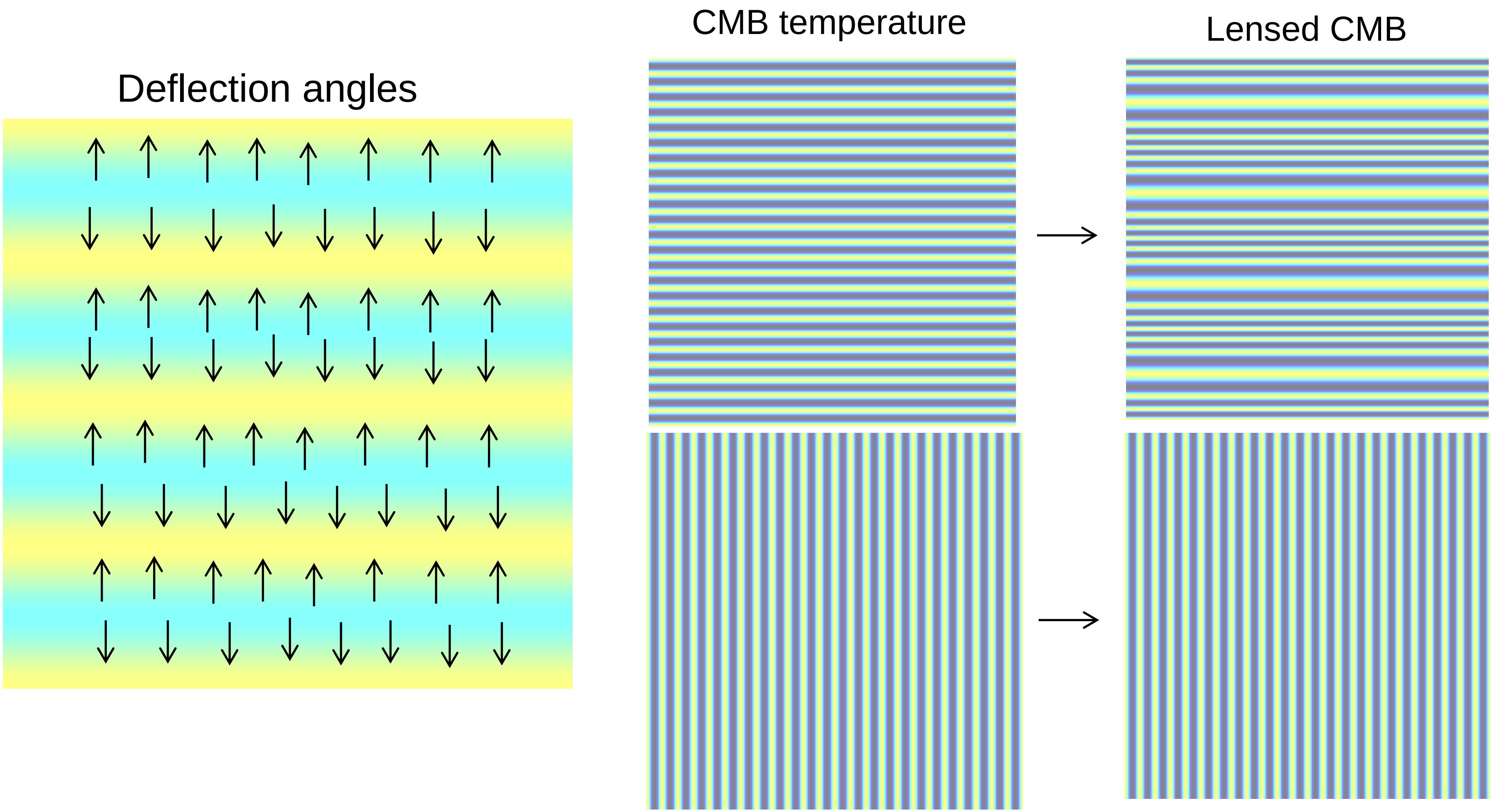




Possible direction-dependent modulation.

Local modulations (e.g. f_{NL}) are isotropic, but e.g. CMB lensing is not

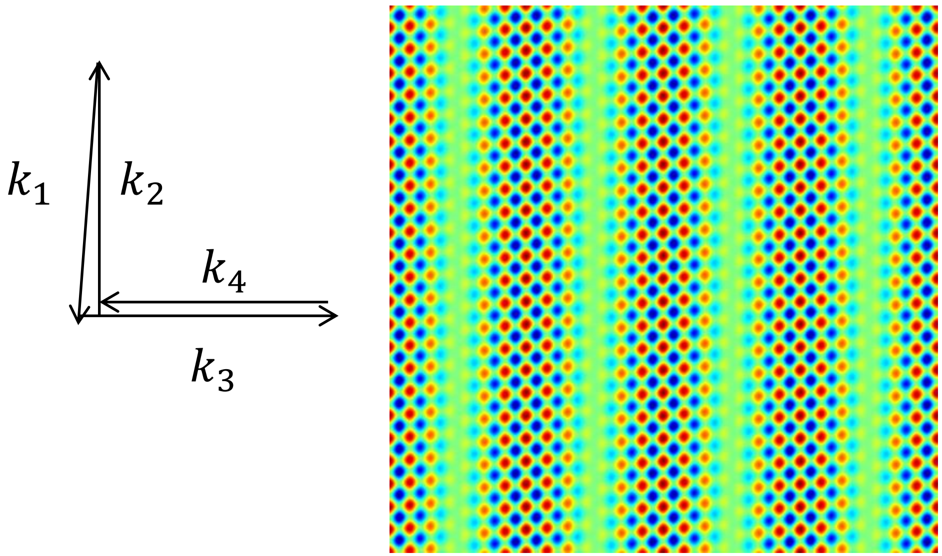
Why is lensing anisotropic?



Modulation depends on relative orientation

⇒ anisotropic ψTT bispectrum

Diagonal squeezed trispectra $|k_1| \sim |k_2|, |k_3| \sim |k_4|, |k_1 + k_2| = |k_3 + k_4| \ll |k_2|, |k_3|$



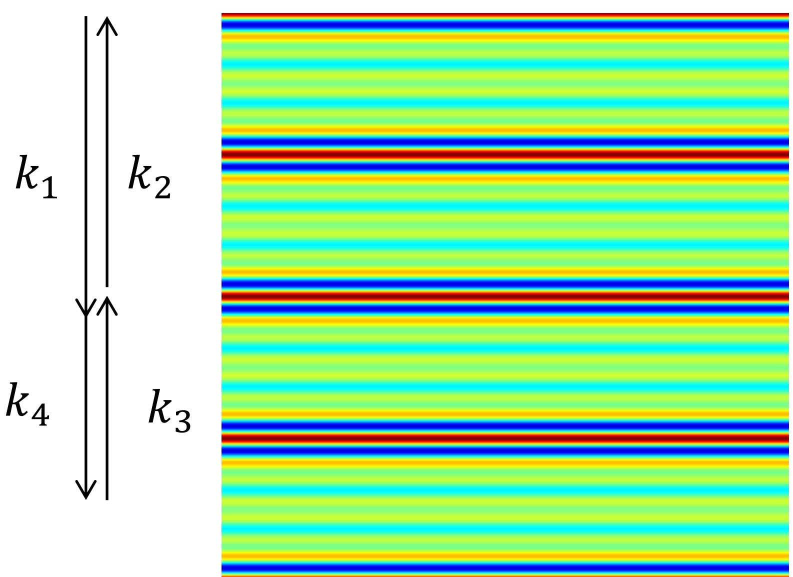
Trispectrum = power spectrum of modulation

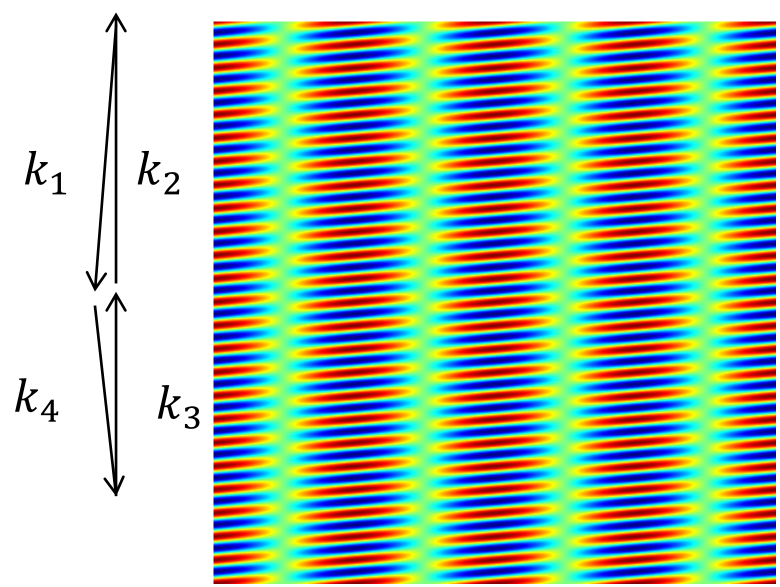
e.g. $\chi = \chi_0\left(1 + f_{NL}\chi_0\ \right)$

$\tau_{NL} \sim f_{NL}^2$

or $\chi = \chi_0(1 + \phi)$

(any correlation, $\tau_{NL} > f_{NL}^2$)



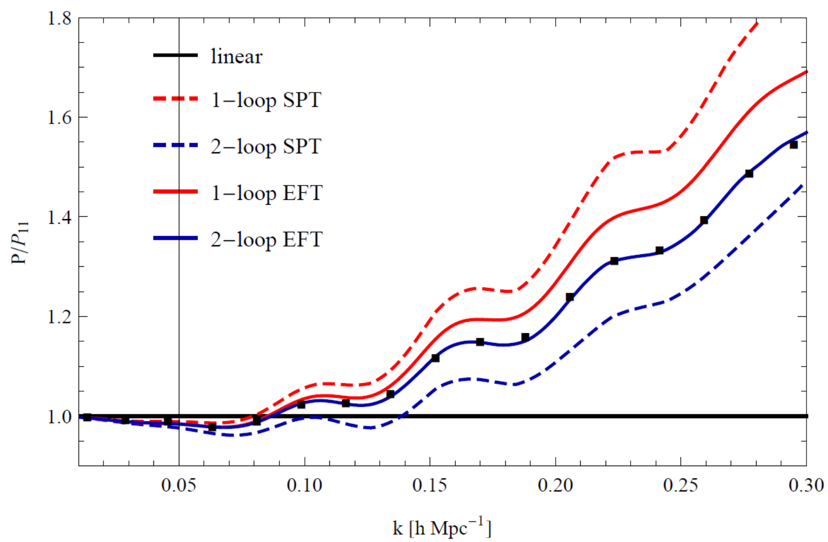


Can we predict the CMB lensing power spectrum accurately enough?

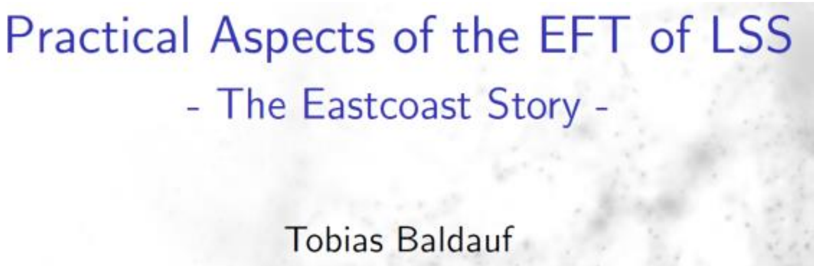
- Relatively high-redshift kernel, quite large lenses $\Rightarrow$ mostly linear
- Potential probes total matter P(k): no bias modelling issues

Effective Field Theory (EFT) good enough?

$$P_{\rm mm}(k,t) = P_{\rm lin}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)D^2(t)k^2P_{\rm lin}(k)$$



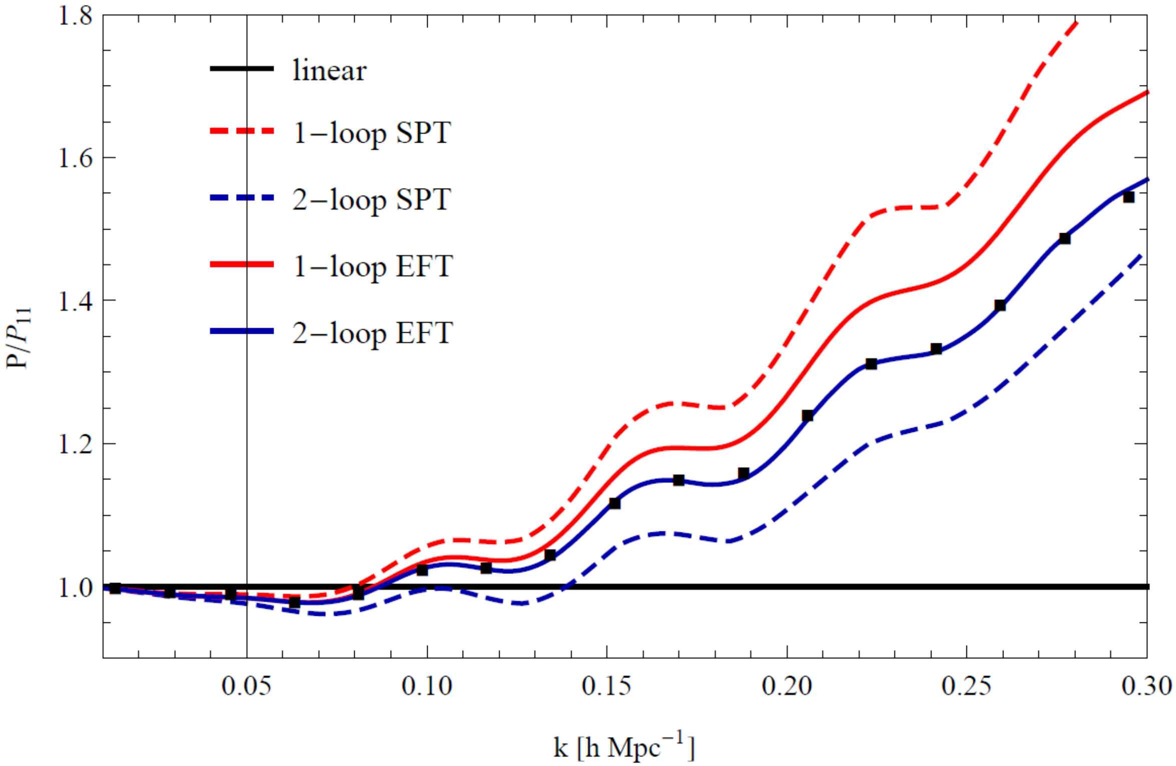
Practical Aspects of the EFT of LSS
- The Eastcoast Story -
Tobias Baldauf



EFT=Effective Field Theory

Systematic model of large-scale perturbations, nuisance parameters encoding effect of small scales

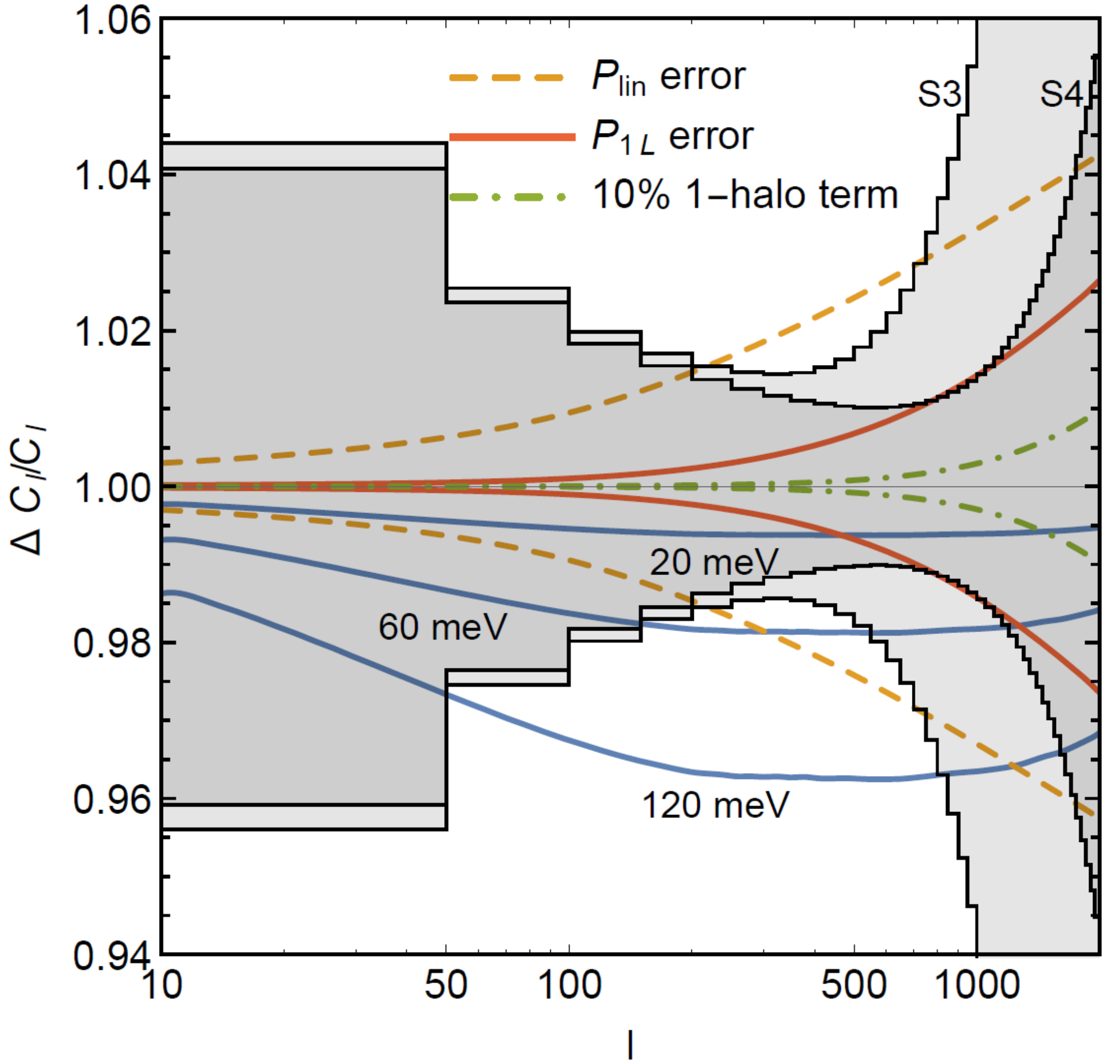
$$P_{\rm mm}(k,t) = P_{\rm lin}(k) + P_{22}(k,\Lambda) + 2P_{13}(k,\Lambda) - 2c_s^2(\Lambda)D^2(t)k^2P_{\rm lin}(k)$$



This is just matter.

Lots more parameters for general bias…

Theory Errors



$\sigma(M_\nu) \sim 30 \text{ meV}$

with

$\sigma_A = 0.5\%$

Marko Simonović

EFT just about good enough for CMB lensing