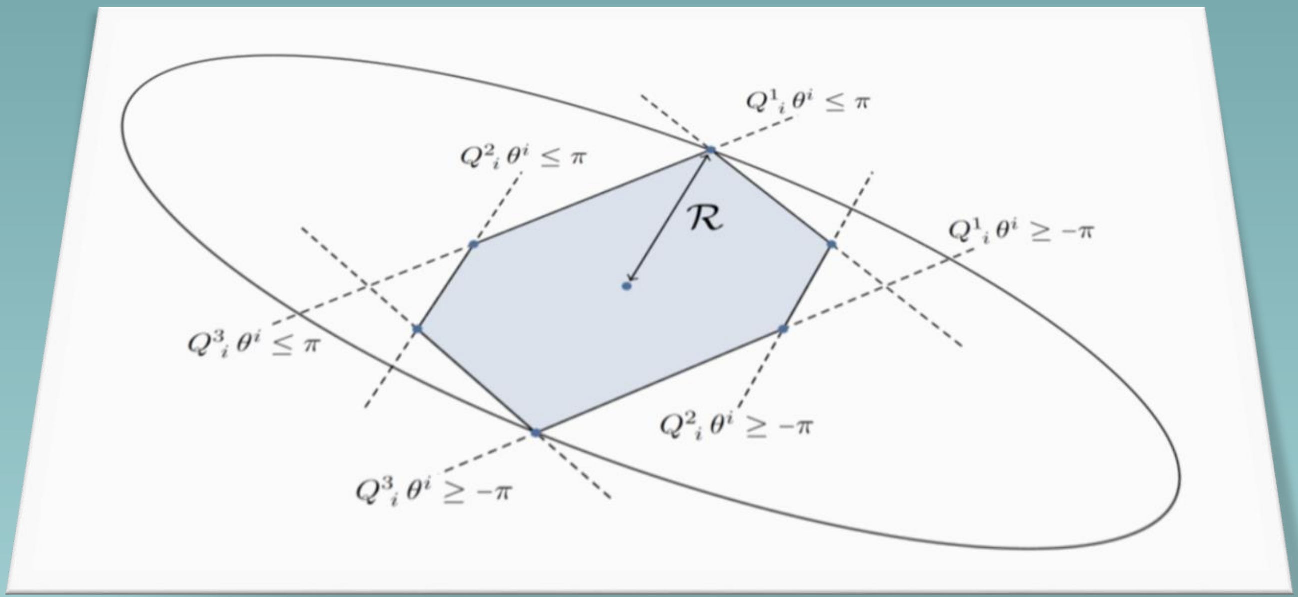
Axion Inflation in Calabi-Yau Hypersurfaces



Liam McAllister

Cornell

Weak Gravity Conjecture and Cosmology, Madrid

March 18, 2016

Based on:

Long, L.M., Stout,
"Systematics of Axion Inflation in Calabi-Yau Hypersurfaces," 1603.01259

Long, L.M., Stout, to appear.

Khrulkov, Long, L.M., Stillman, Sung, work in progress.

Bachlechner, Long, L.M.,
"Planckian Axions in String Theory," 1412.1093.

Bachlechner, Long, L.M.,
"Planckian Axions and the Weak Gravity Conjecture," 1503.07853.

Questions to Address

- Does quantum gravity allow super-Planckian inflationary displacements?

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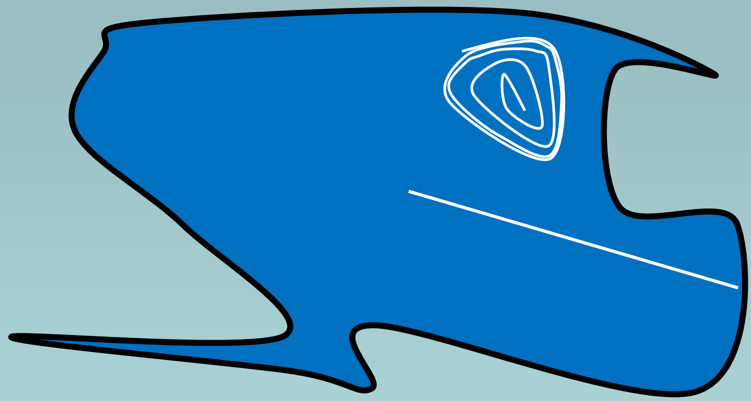
- Does string theory allow super-Planckian inflationary displacements?
  - simpler: specific, partially-computable quantum gravity theory.

Questions to Address

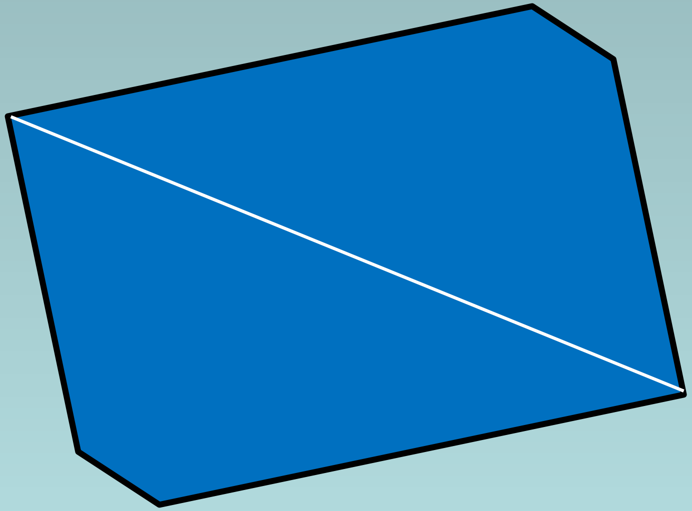
- Does quantum gravity allow super-Planckian inflationary displacements?
- Does string theory allow super-Planckian inflationary displacements?
- Does string theory allow super-Planckian diameters?
  - Inflationary displacements depend on dynamics, and on the structure of the potential. Long, tightly coiled paths possible. **Berg, Pajer, Sjors 09**



  - Study theories where the potential has enough structure so that a long inflationary path requires a large diameter. This is a purely geometric requirement.

Questions to Address

- Does quantum gravity allow super-Planckian inflationary displacements?
- Does string theory allow super-Planckian inflationary displacements?
- Does string theory allow super-Planckian diameters?
- Does string theory allow super-Planckian axion diameters?
  - Much simpler: all-orders shift symmetries structure the space and the potential.



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- Does string theory allow super-Planckian axion diameters?
- Does string theory allow super-Planckian axion fundamental domain diameters?
  - That is, exclude monodromy, the repeated traversal of axion fundamental domains.

    Silverstein, Westphal 08; L.M., Silverstein, Westphal 08; Kaloper, Sorbo 08; Flauger et al. 09; Kaloper, Lawrence, Sorbo 11; Palti, Weigand 14; Marchesano et al. 14; L.M. et al. 14
  - Backreaction by the source of monodromy is an issue. **Flauger et al. 09; Conlon 11; Palti 15**

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Kreuzer and Skarke

Questions to Address

- How large can the C_4 axion fundamental domain be in type IIB compactifications on O3/O7 orientifolds of CY threefold hypersurfaces in toric varieties, at (relatively) weak coupling and large volume?
- What do we learn about mechanisms for/against large diameters?
  - How much alignment occurs in such systems? **Kim, Nilles, Peloso 04**
  - How strong are the WGC constraints on these theories?

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  - How much alignment occurs in such systems? **Kim, Nilles, Peloso 04**
  - How strong are the WGC constraints on these theories?
- Answers, for $h^{1,1} \leq 4$, without seven-branes:
  - Maximum semi-diameter is ~ 0.3 M_{Pl}.
  - Maximum alignment enhancement is a factor ~2.5.
- We expect large diameters and large alignment to be most common at $h^{1,1} >> 1$, with seven-branes. Technically challenging; stay tuned.

Plan

I. Goal: geometry of fundamental domain
II. Axion geometry: K and Q
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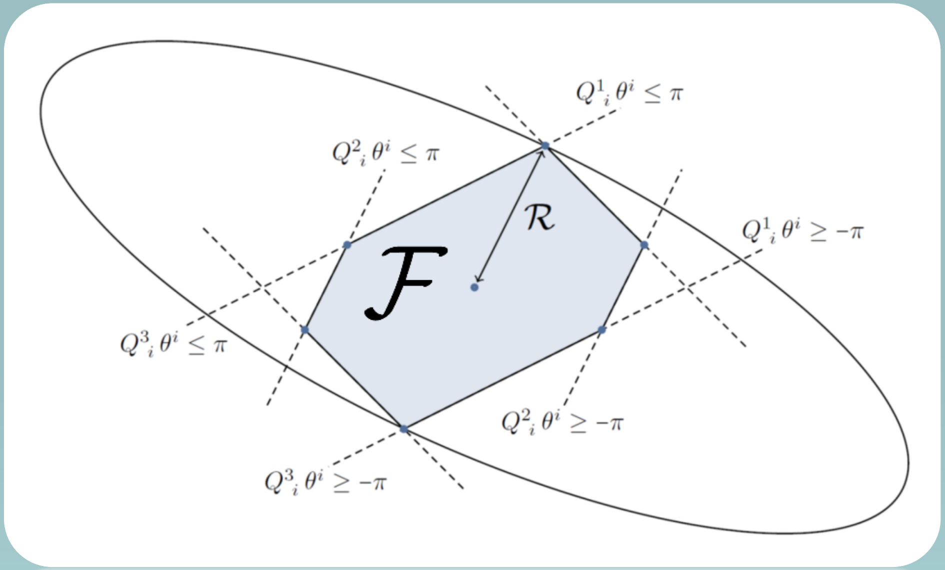
V. Hints at $h^{1,1}=100$
VI. Outlook

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\mathcal{R}_4 - \frac{M_{\rm pl}^2}{2}K_{ij}\partial^\mu\theta^i\partial_\mu\theta^j - \sum_{a=1}^{P}\Lambda_a^4\left(1-\cos(Q^a{}_i\theta^i)\right).$$

Hyperplanes:

$$-\pi \le Q^a{}_i\theta^i \le \pi$$



Fundamental domain, $\mathcal{F}$ of radius $\mathcal{R}$

$$\mathcal{R}^2 = \max_i\, \mathbf{d}_i^\intercal \cdot \mathbf{K} \cdot \mathbf{d}_i.$$

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}\mathcal{R}_4 - \frac{M_{\rm pl}^2}{2}K_{ij}\partial^\mu\theta^i\partial_\mu\theta^j - \sum_{a=1}^{P}\Lambda_a^4\left(1-\cos(Q^a{}_i\theta^i)\right).$$

When P=N, define: $\boldsymbol{\phi} = \mathbf{Q}\,\boldsymbol{\theta}$

$$\boldsymbol{\Xi} = (\mathbf{Q}^{-1})^\top \mathbf{K}\,\mathbf{Q}^{-1}$$

$$\mathcal{L} = \frac{-1}{2}\partial\boldsymbol{\phi}^\top\boldsymbol{\Xi}\,\partial\boldsymbol{\phi} - \sum_{i=1}^{N}\Lambda_i^4\left[1-\cos\left(\phi_i\right)\right]$$

Eigenvalues $\xi_1^2 \leq \ldots \leq \xi_N^2$

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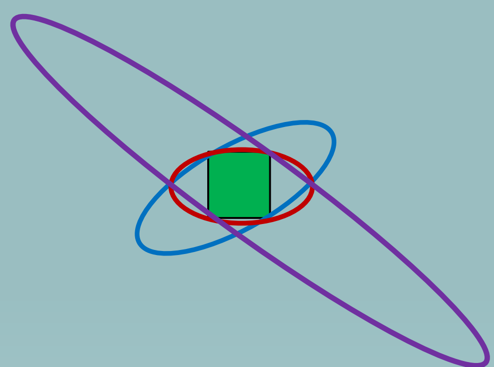
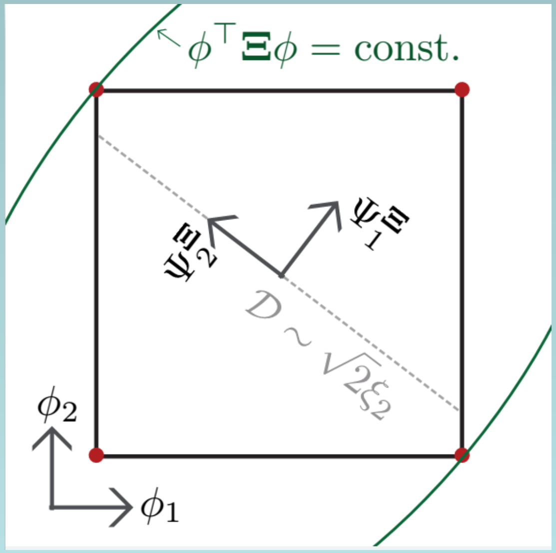


Diameter determined by eigenvectors of $\boldsymbol{\Xi}$

Eigenvector $\boldsymbol{\Psi}_N^{\boldsymbol{\Xi}}$,
largest eigenvalue ξ_N^2

$$\mathcal{D}_{\rm max} = 2\pi\xi_N\sqrt{N}$$



Lattice and kinetic alignment

$$\mathcal{D} \approx \xi_N \sqrt{N} = \frac{\xi_N}{f_N} \times \frac{f_N \sqrt{N}}{\sqrt{\sum f_i^2}} \times \sqrt{\sum f_i^2}$$

Enhancement from lattice alignment

Kim, Nilles, Peloso 04

Exponential.

Choi, Kim, Yun 14

Heavy-tailed.

Bachlechner, Long, L.M. 14

Enhancement from kinetic alignment

At most $\sqrt{N}$.

Bachlechner, Dias, Frazer, L.M. 14;

Naive estimate

Dimopoulos, Kachru, McGreevy, Wacker 05

$$\mathbf{K}$$

Eigenvalues $f_1^2 \leq \ldots \leq f_N^2$

$$\boldsymbol{\Xi} = (\mathbf{Q}^{-1})^{\top} \mathbf{K}\, \mathbf{Q}^{-1}$$

Eigenvalues $\xi_1^2 \leq \ldots \leq \xi_N^2$

See also: Czerny et al. 14; Tye, Wong 14; Long, L.M., McGuirk 14; Kappl et al. 14; Ben-Dayan et al. 14; Higaki, Takahashi 14; Junghans 15; Ruehle and Wieck 15.

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Lattice of charges Q

Orientation of eigenvectors

Varieties of Lattice Alignment

1. In an EFT of two axions, one can fine-tune the decay constants to achieve lattice alignment, and a super-Planckian effective period.

   Kim, Nilles, Peloso 04

2. In an EFT of N >> 1 axions, severely fine-tuned lattice alignment can give diameters $> e^N$. Choi et al. 14

3. In an EFT of N >> 1 axions, heavy-tailed lattice alignment occurs automatically. Bachlechner, Long, L.M. 14 $\mathrm{P}\Big(\mathcal{D} > 10^4 \langle\mathcal{D}\rangle\Big) \sim 10^{-2}$

In each case we define the degree of alignment as

$$\eta = \frac{\mathcal{R}_{\rm actual}}{\mathcal{R}_{Q=2\pi\mathbb{1}}}\,.$$

Are (1),(2),(3) possible in string theory? At what cost?
How are they restricted by WGC constraints?

Arkani-Hamed, Motl, Nicolis, Vafa 06; Cheung, Remmen 14; Rudelius 14,15; Brown, Cottrell, Shiu, Soler 15; Montero, Uranga, Valenzuela 15; Bachlechner, Long, L.M. 15; Heidenreich, Reece, Rudelius 15; et seq.

Achieving lattice alignment

What is the largest eigenvalue of $\boldsymbol{\Xi} = (\mathbf{Q}^{-1})^\top \mathbf{K}\,\mathbf{Q}^{-1}$?

Take $\mathbf{K}=f^2\mathbf{1}_{NxN}$, so $\boldsymbol{\Xi} = (\mathbf{Q}\mathbf{Q}^\top)^{-1}$ and $\xi_N^2 \equiv \lambda_N(\boldsymbol{\Xi}) = \lambda_1^{-1}(\mathbf{Q}\mathbf{Q}^\top)$

Q is the charge matrix. Can $\mathbf{QQ}^\mathrm{T}$ have a small eigenvalue?

Two approaches to lattice alignment in EFT:

1. Take **Q** to be a suitable random matrix and study the statistics of $\lambda_1(\mathbf{QQ}^\mathrm{T})$ Bachlechner, Long, L.M. 14
2. Argue for a structure in **Q** giving small $\lambda_1(\mathbf{QQ}^\mathrm{T})$. Choi, Kim, Yun 14; Kaplan and Ratazzi 15.

Both approaches show that exponential enhancements from lattice alignment are easy in EFT. Possible in quantum gravity?

Structure for lattice alignment

A structure in **Q** can cause **QQ**^{T} to have a small eigenvalue.

$$\mathcal{L} = -\frac{1}{2} f^2 \left(\partial\theta^i\right)^2 - \sum_{i=1}^{N} \Lambda_i^4 \cos\left(3\theta^{i+1} - \theta^i\right)$$

$$\mathbf{QQ}^T = \begin{pmatrix} 1 & -q & 0 & 0 & \dots \\ -q & 1+q^2 & -q & 0 & \dots \\ 0 & -q & 1+q^2 & -q & \dots \\ \vdots & \vdots & \vdots & \ddots & \dots \\ 0 & 0 & 0 & -q & q^2 \end{pmatrix}$$

$$\lambda_1(\mathbf{QQ}^\top) = q^{-2N}$$

But could such a Q arise? Why?

Kaplan and Ratazzi: q=3, with extradimensional locality as a heuristic justification. String embedding highly questionable.

Choi, Kim, Yun 14; Kaplan and Ratazzi 15.

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Kähler Metric

$$T^i = \frac{1}{2}\int_{D^i} J \wedge J + i \int_{D^i} C_4 \equiv \tau^i + i\theta^i, \qquad h^{1,1}_- = 0$$

$$\mathcal{K} = -2\log\mathcal{V} \qquad \mathcal{V} = \tfrac{1}{6}\kappa^{ijk} t_i t_j t_k \qquad \tau^i \equiv \partial\mathcal{V}/\partial t_i$$

Kähler metric K specified by intersection numbers.

Mori cone: set of holomorphic curves C, $\int_C J > 0$

We take the minimum curve volume to be $(2\pi)^2\alpha'$

corresponding to $\int_C J > 1$

Corrections $\Delta\mathcal{K} \sim \mathcal{V}^{-1}\, e^{-2\pi\sqrt{g_s}\, t}$ suppressed by $e^{-2\pi\sqrt{g_s}}$

Results homogeneous of degree -2 w.r.t. $t_i \rightarrow \lambda t_i$

Fourfold and Threefold Divisors

- Nonperturbative W from Euclidean D3-branes on suitable divisors.
- M-theory on fourfold:

$$\begin{array}{ccc} \hat{D} & \subset & Y_4 \\ & & \downarrow \\ D & \subset & B_3 \end{array}$$

Sufficient condition:

Witten 96

$$h^\bullet(\hat{D}, \mathcal{O}_{\hat{D}}) = (1,0,0,0).$$

Translate to D: $h^i(\hat{D}, \mathcal{O}_{\hat{D}}) = h^i(D, \mathcal{O}_D) + h^{i-1}(D, -\Delta|_D), \quad 0 \leq i \leq 3,$

If D does not intersect Δ, it suffices to have

Grassi 97

$$h^\bullet(D, \mathcal{O}_D) = (1,0,0)$$

Superpotential from Rigid Divisors

In the absence of worldvolume flux and bulk flux,
and neglecting intersections with seven-branes,
the leading terms in W arise from rigid divisors, i.e. D obeying

$$h^{\bullet}(D, \mathcal{O}_D) = (1, 0, 0)$$

Let D^i be a basis of $H_4(X, \mathbb{Z})$ and let $\text{Đ}^{\alpha} \equiv q^{\alpha}{}_{i} D^{i}$ be the rigid combinations.

Then:

$$W = W_0 + \sum_{\alpha=1}^{p} A_{\alpha}\, e^{-2\pi q^{\alpha}{}_{i} T^{i}}$$

Axion Data from Toric Geometry

To specify the geometry of the fundamental domain, compute:

- Kähler metric K, from intersection numbers
- Mori cone, or equivalently Kähler cone
- Leading rigid divisors $Đ^\alpha \equiv q^\alpha{}_i D^i$, yielding charges Q.

Process:

1. Select a reflexive polytope from Kreuzer-Skarke list.
2. Triangulate to reach a toric variety V with at most pointlike singularities. Anticanonical hypersurface in V is a CY3, X.
3. Compute Mori cone + intersection numbers of X.
4. Search cone of effective divisors for rigid divisors.
5. If enough are found so that Q is full rank, keep result.

Caveats

1. Select a reflexive polytope from Kreuzer-Skarke list.
2. Triangulate to reach a toric variety V with at most pointlike singularities. Anticanonical hypersurface in V is a CY3, X.
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- We study only favorable hypersurfaces, meaning $h^{1,1}(X,\mathbb{Z})$ is purely inherited from V.
- ‘Mori cone’ here means Mori(V). Can be >> Mori(X).
- Some rigid divisors we find are singular (‘normal crossing’).

Computational Issues

Triangulation.

Finding all (‘star, fine, regular’) triangulations of a polytope is costly at $h^{1,1}>10$. **`Sage`** fails.

For $10<h^{1,1}<30$ a better algorithm suffices. **Long, L.M., McGuirk 14**

For large $h^{1,1}$ one can find a single triangulation by a trick. **A. Braun**

We triangulate polytopes with $h^{1,1}=400$ in seconds.

Computing Mori cone + intersection numbers takes ~hours.

Divisor topology.

For $h^{1,1}<5$, **`Cohomcalg`** can compute $h^\bullet(D,\mathcal{O}_D)$ **Blumenhagen et al 10**

For larger $h^{1,1}$ we obtain topology of toric divisors from polytope data. We can find full-rank Q at $h^{1,1}=100$.

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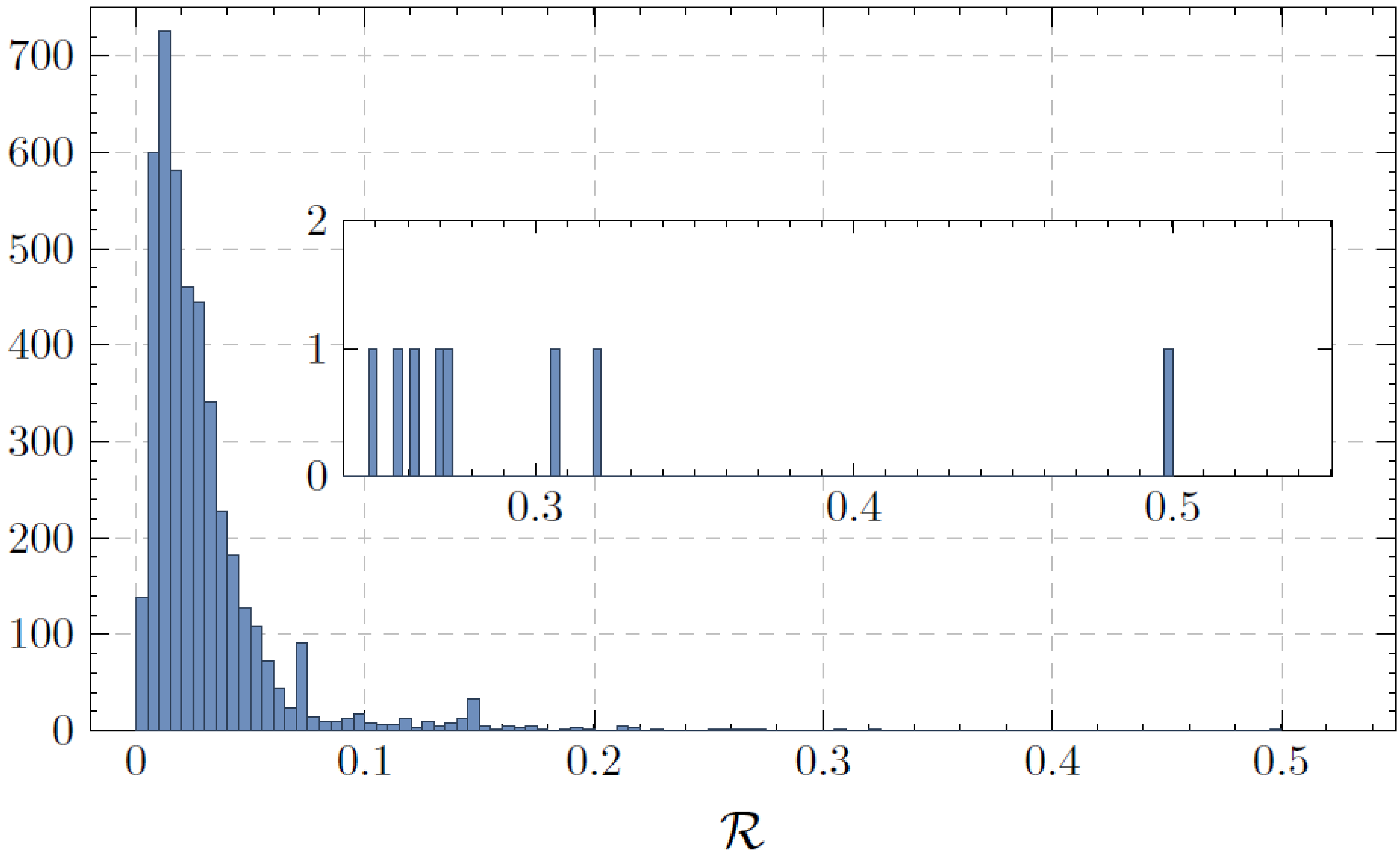
Results

$h^{1,1}(X)$	2	3	4
Number of polytopes	36	244	1197
Number of favorable polytopes	36	243	1185
Number of favorable triangulations	48	525	5330
Number of full-rank triangulations	24	262	4104
Full-rank with only smooth divisors	9	199	3214

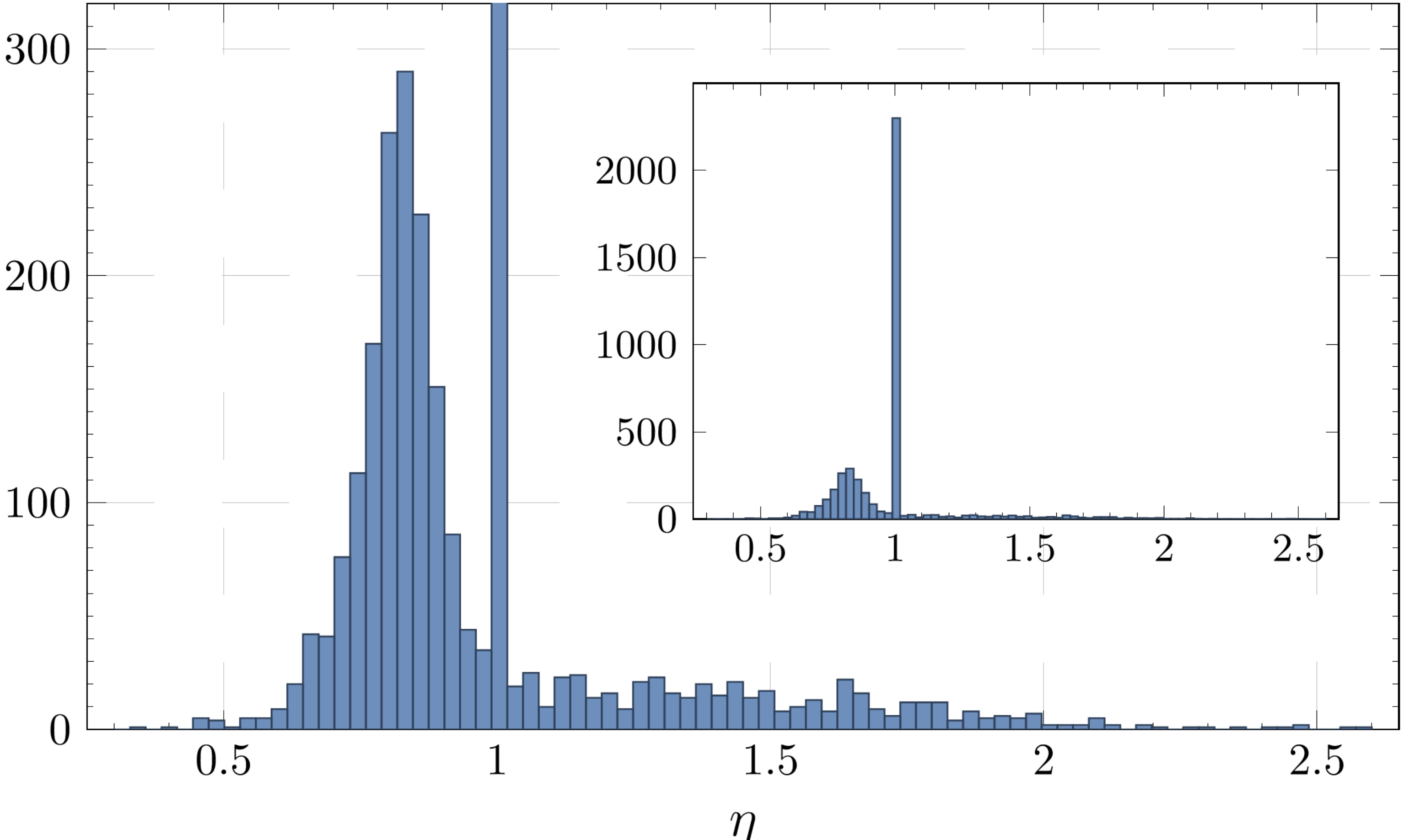
Results of the scan over reflexive polytopes with $h^{1,1}(X) \leq 4$.

4,390 theories with compact fundamental domains

Semi-diameters



Alignment



No alignment in 2,180 cases
Anti-alignment in 1,716 cases
Alignment in 494 cases

Max alignment = 2.55

Example with Alignment

$$h^{1,1} = 4 \qquad \mathbf{d}_i = \Big\{(-1,2,-1,-1),\ (-1,-1,2,1),\ (-1,-1,1,1),\ (1,0,-1,-1),$$
$$(-1,-1,1,2),\ (0,-1,1,1),\ (2,1,-2,-2)\Big\},$$

$$\mathcal{Q} = 2\pi \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 2 & 1 & -1 \end{pmatrix} .$$

Alignment by factor $\eta = 2.55$ extends range from

$$\mathcal{R} = .05 \text{ to } \mathcal{R} = 0.12.$$

Towards large $h^{1,1}$

- Computing divisor topology becomes challenging.
- But toric divisors' topology can be obtained from polytope data!
- Sometimes toric divisors suffice to give full-rank Q, giving an upper bound on the diameter and the degree of alignment.
- For each of $h^{1,1}$=50,60,…100 we found 10 threefolds in which toric divisors give a full-rank Q.
- Far too early to draw conclusions, but so far:
  - Q matrices tend to be very close to the identity.
  - Many curves, so extremely large $\mathcal{V}$. [Mori(X) vs. Mori(V)?]

$$\eta_{\max} = 7.86$$

Example with $h^{1,1}$=100

$\mathcal{V}$ = 4 x 10^{11} $\qquad$ $\eta_{\max} = 7.86$

$\mathcal{R}$= .00015

Q=97x97 identity, and:

$$\{\{-\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{3}{2}, -1, -\frac{1}{2}, \frac{1}{2}, 0, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{3}{2}, -1, -\frac{1}{2}, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{5}{2},$$
$$-2, -\frac{3}{2}, -1, -\frac{1}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{7}{2}, \frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{7}{2}, 1, \frac{1}{2},$$
$$0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1,$$
$$-\frac{3}{2}, 1, \frac{1}{2}, 0, -\frac{1}{2}, -1, 1, \frac{1}{2}, 0, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, 1, \frac{3}{2}, 0, -\frac{1}{2}, -4, -1, \frac{1}{2}, 0\},$$
$$\{-3, -\frac{7}{2}, -4, -\frac{9}{2}, -\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}, 0, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -2, -1, 0, -\frac{5}{2}, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5, -\frac{11}{2}, -6,$$
$$-5, -4, -3, -2, -1, -4, -\frac{9}{2}, -5, -\frac{11}{2}, -6, -\frac{13}{2}, -7, -\frac{15}{2}, -8, -\frac{17}{2}, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5, -\frac{11}{2}, -6, -\frac{13}{2},$$
$$-7, -\frac{15}{2}, -8, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5, -\frac{11}{2}, -6, -\frac{13}{2}, -7, -\frac{5}{2}, -3, -\frac{7}{2}, -4, -\frac{9}{2}, -5, -\frac{11}{2}, -6, -2, -\frac{5}{2}, -3, -\frac{7}{2},$$
$$-4, -\frac{9}{2}, -5, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{7}{2}, -4, -1, -\frac{3}{2}, -2, -\frac{5}{2}, -3, -\frac{1}{2}, -1, -\frac{3}{2}, -2, -\frac{1}{2}, -\frac{7}{2}, 0, -\frac{5}{2}, \frac{1}{2}, \frac{1}{2}, -9, -\frac{3}{2}, -2, -\frac{5}{2}\}$$
$$\{\frac{9}{2}, 5, \frac{11}{2}, 6, \frac{9}{2}, 3, \frac{3}{2}, -\frac{1}{2}, 0, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{5}{2}, 1, -\frac{1}{2}, \frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}, 8, \frac{13}{2}, 5, \frac{7}{2}, 2, \frac{1}{2}, 7,$$
$$\frac{15}{2}, 8, \frac{17}{2}, 9, \frac{19}{2}, 10, \frac{21}{2}, 11, \frac{23}{2}, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}, 8, \frac{17}{2}, 9, \frac{19}{2}, 10, \frac{21}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}, 8, \frac{17}{2},$$
$$9, 4, \frac{9}{2}, 5, \frac{11}{2}, 6, \frac{13}{2}, 7, \frac{15}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 5, \frac{11}{2}, 6, 2, \frac{5}{2}, 3, \frac{7}{2}, 4, \frac{9}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, 3, 0, \frac{1}{2}, 1, \frac{3}{2}, -\frac{1}{2},$$
$$\frac{13}{2}, -1, \frac{9}{2}, -1, -\frac{1}{2}, 12, 2, \frac{7}{2}, 4\}\}$$

Summary

We asked:

how large can C_4 axion fundamental domain diameters be,

and how much lattice alignment is possible,

in type IIB compactifications on CY hypersurfaces,

in a regime where all curves have volume > $(2\pi)^2 \alpha'$.

The instantons we included were Euclidean D3-branes without flux, wrapping rigid divisors that do not intersect seven-branes.

We studied 4,390 examples ($2 \leq h^{1,1} \leq 4$ in Kreuzer-Skarke).

Very modest alignment occurs, but is insufficient to allow a super-Planckian field range.

Outlook

Important limitations that should be relaxed:

- Include seven-branes rather than just Euclidean D3-branes. Gaugino condensate dual Coxeter numbers help.
- Impose Mori cone conditions of X, not V.
- Include non-favorable threefolds.
- Consider C_2 axions.
- Extend systematic scan to large $h^{1,1}$.
- Compute leading instantons in Kähler potential.

Implications:

- What can these theories teach us about Weak Gravity?

Can we sort this out before observations settle the question of primordial B-modes?

A symmetric orientifold

Orientifold of resolution of $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ Denef et al. 04 (DDFGK)
$h_{1,1} = 51$; 48 exceptional divisors, 12 SO(8) stacks on O7-planes
Explicit 3-form flux quanta.

$$W = W_0 + \sum_i A_i e^{-q^i{}_j T^j}$$

$$\sigma_{\mathcal{Q}} \approx 0.18$$

$$f_N \approx 0.013\, M_{\rm pl}$$

$$\mathbf{q} = \frac{\pi}{3} \begin{pmatrix}
1&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
1&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 \\
1&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 \\
1&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 \\
0&1&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&1&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&1&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&1&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 \\
0&0&1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 \\
0&0&1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & -1&-1&-1&-1 \\
0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
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0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
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0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
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0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
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0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6 & 0&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 6&0&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&6&0&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&6&0 \\
0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&0 & 0&0&0&6
\end{pmatrix}.$$

Possible corrections:

Renormalization of G_N? 1% from $\alpha'^3\mathcal{R}^4$; others?
Other α' corrections?
Further instantons, e.g. in K?

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