An Introduction to the Weak Gravity Conjecture and Cosmology

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Outline

Section 1

Axion Inflation

Inflation

Problem: Why is the universe so flat and homogenous?

Solution: Inflation.

(Period of quasi-exponential growth $a(t) \approx e^{Ht}$ in the early universe.)

Slow-Roll Inflation

- Inflation can be thought of as the theory of a ball rolling down a hill with friction.

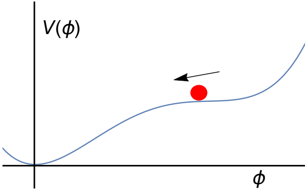


Figure 1 : The inflaton rolling down its potential.

- Slow roll parameters encode relevant features of potential:

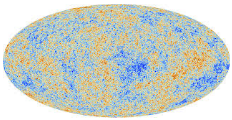
$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 , \quad \eta_V = M_p^2 \frac{V''(\phi)}{V(\phi)}. \qquad (1)$$

Slow-Roll Inflation

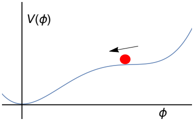
- Measurable quantities are determined by the slow-roll parameters,

$$r_* \approx 16\epsilon_V^* \quad (2)$$

$$n_s^* - 1 \approx 2\eta_V^* - 6\epsilon_V^*. \quad (3)$$



Experiment



Theory

Planck and BICEP2 Data

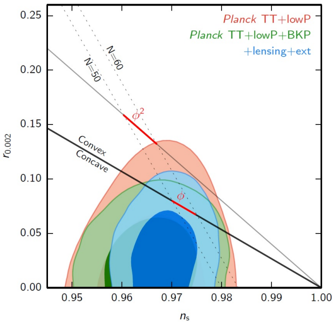


Figure 2 : *Planck* and BICEP2 measurements give a best fit value of $r_* = 0.05$. $r_* < 0.075$ at 95% CI when lensing + ΛCDM+noise+dust are taken into account. $r_* > 0$ at 92% CI [Ade et al. '15a, '15b].

Implications of a Large r_*

- A large r_* implies a large first derivative of the potential, and hence a fast-moving inflaton.
- Distance $=$ Rate $\times$ Time $\Rightarrow$ r_* is thus related to the distance traveled by the inflaton via the 'Lyth bound' [Lyth '96],

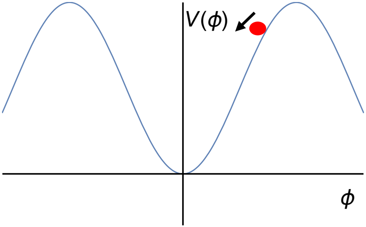
$$\Delta\phi \gtrsim \left(\frac{r_*}{0.01}\right)^{1/2} M_p. \tag{4}$$

- **A detectable tensor-to-scalar ratio implies a trans-Planckian traversal of the inflaton during the course of its slow-roll.**

Axions

- Axions (scalars with a perturbative shift symmetry) acquire a periodic "natural inflation" potential from instanton effects:

$$V(\phi) = \Lambda^4 (1 - \cos\frac{\phi}{f}) + \ldots \quad (5)$$



- $f > M_p$ is necessary for inflation.

Axions in String Theory

- Axions are ubiquitous in string compactifications.
- But...axion decay constants in string theory are constrained to be $\mathcal{O}(M_p)$ or smaller [Banks et al. '03], making them unsuitable for inflation.

Three Popular Solutions:

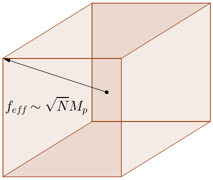


Figure 3 : *N*-flation [Dimopoulos et al. '05]

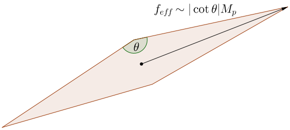


Figure 4 : Decay Constant Alignment [Kim, Nilles, Peloso '04]

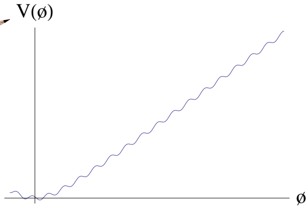


Figure 5 : Axion Monodromy [McAllister et al. '08, Silverstein, Westphal '08, Flauger et al. '09]

Section 2

The Weak Gravity Conjecture

The Weak Gravity Conjecture [Arkani-Hamed et al. '06]

The (Mild) Weak Gravity Conjecture

Any consistent gravitational theory with a $U(1)$ gauge field admitting a UV completion must contain a particle with charge to mass ratio greater than or equal to that of an extremal black hole.

Why Should the Weak Gravity Conjecture Be True?

- If not, extremal black holes will be unable to decay.

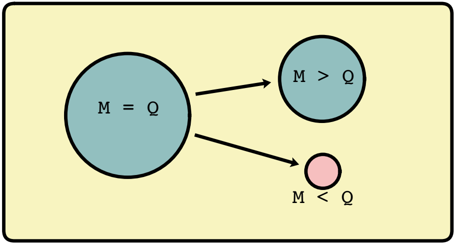


Figure 6 : Charged black hole decay.

- If not, near-extremal black holes move towards extremality, sequester information forever.

Why Should the Weak Gravity Conjecture Be True?

- Many examples in string theory and KK theory obey the WGC [Arkani-Hamed et al. ’06, Heidenreich et al. ’15, to appear].

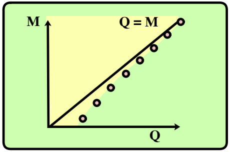


Figure 7 : Spectrum of charged particles.

- AdS/CFT factorization argument? [Harlow ’15]

The Generalized Weak Gravity Conjecture

- It is natural to generalize this to arbitrary p-forms and d spacetime dimensions.

The Generalized Weak Gravity Conjecture

Consider a p-form Abelian gauge field in any number of dimensions d. Then, there exists an electrically charged $p-1$ dimensional object and a magnetically charged $d-p-1$ dimensional object with tension,

$$T_{\rm el} \lesssim \left(\frac{g^2}{G_N}\right)^{1/2}, \quad T_{\rm mag} \lesssim \left(\frac{1}{g^2 G_N}\right)^{1/2}$$

Axions and the Weak Gravity Conjecture

- Consider the case of a 0-form ϕ (i.e. an axion) in 4d. The generalized WGC then says that there must exist a -1-dimensional object (instanton) with tension,

$$T \lesssim \frac{M_p}{f}. \tag{6}$$

- But, this T is just the instanton action S. If we impose $S > 1$, we find

$$1 < S \lesssim \frac{M_p}{f} \Rightarrow f < M_p. \tag{7}$$

- Thus,

The Generalized WGC $+$Instanton Action > 1	$\Rightarrow$	Decay constants larger than M_p are forbidden!

The N-Species Weak Gravity Conjecture

- The WGC was originally formulated only for theories with a single $U(1)$ gauge symmetry.
- In practice, we expect many $U(1)$s from a string compactification.
- Extending the WGC to such theories is non-trivial and has important implications for axion inflation.

The N-Species (Mild) Weak Gravity Conjecture

- Suppose we have not 1, but N 1-form gauge fields.
- The N-species WGC holds that the convex hull of the charge-to-mass vectors $\pm\vec{z}_i = \pm\frac{\vec{q}_i}{m_i}M_p$ must contain the N-dimensional unit ball. (Note: black holes have $|\vec{Z}| \leq 1$.)

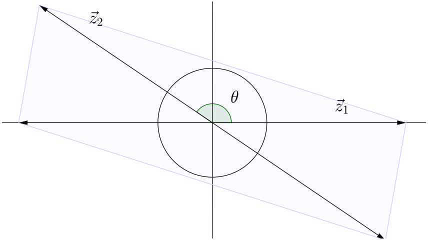


Figure 8 : The convex hull condition [Cheung, Remmen '14].

Section 3

WGC Constraints on Axion Inflation

The N-Species Axion WGC

- So far, we have seen two extensions of the WGC:
  - The "generalized" WGC for p-form gauge fields.
  - The "N-species" WGC for multiple gauge fields.
- It is natural to consider: what happens when we put these two together?

Axion Inflation Models and the WGC

The Main Point

"Vanilla" models of N-flation and decay constant alignment are both in conflict with the WGC. [Rudelius '15, Montero et al. '15, Brown et al. '15].

Assumptions:

- Instanton actions $S_i > 1$.
- Instantons satisyfing WGC give dominant contributions to inflationary potential

WGC Implications for Inflation:

Needed for Parametrically Large f_{eff}:

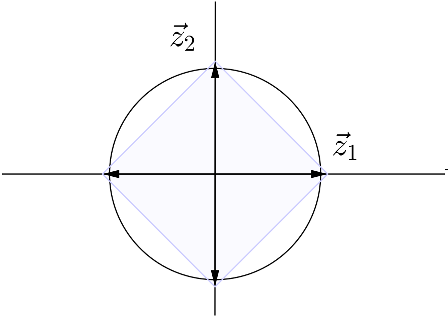


Figure 9 : *N*-flation

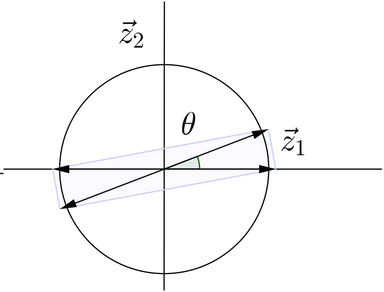


Figure 10 : Decay Constant Alignment

WGC Implications for Inflation:

Stipulated by the WGC:

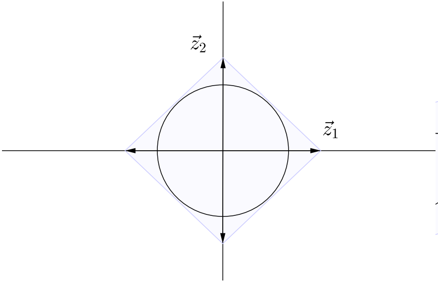


Figure 11 : *N*-flation

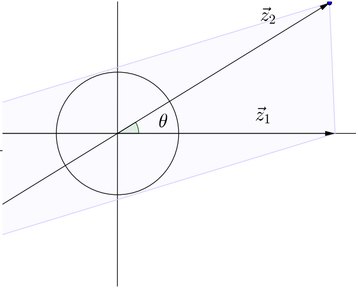


Figure 12 : Decay Constant Alignment

WGC Implications for Inflation:

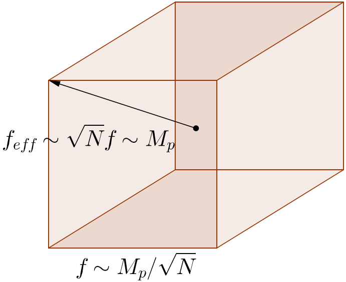


Figure 13 : *N*-flation

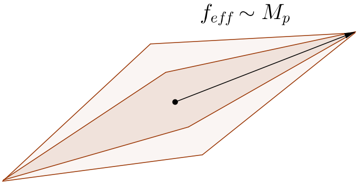


Figure 14 : Decay Constant Alignment

Section 4

Loopholes

Loopholes in the WGC

- Our derivation of the bound on axion moduli spaces relied crucially on two assumptions:
  1. Instanton actions larger than 1 $\Rightarrow$ the "small action loophole."
  2. No additional instantons satisfying the bound $\Rightarrow$ the "extra particle loophole."

The Small Action Loophole

- Recall: the bound dictated by the WGC is $fS < M_p$.
- In string theory, $S > 1$ is generally required for theoretical control.
- In extranatural inflation (5d theory with 1-form $U(1)$ compactified on a circle [Arkani-Hamed et al., '03]), S can be arbitrarily small, leaving f unbounded.

Closing the Small Action Loophole

- Applying the magnetic form of the WGC closes this loophole in the single-axion case [de la Fuente et al. '14]:

$$\text{WGC}: \frac{q_{\text{mag}}}{m_{\text{mag}}} \sim \frac{1/g}{\Lambda/g^2} > \frac{1}{M_p}$$

$$\text{Hierarchy of Scales:} \Lambda > \frac{1}{R}$$

$$\Rightarrow f \sim \frac{1}{gR} < M_p.$$

- N-flation also violates the magnetic WGC [Heidenreich et al. '15a].
- Decay constant alignment obeys the WGC at minima of the potential but violates it elsewhere in axion moduli space.

Closing the Small Action Loophole

The Extended Weak Gravity Conjecture (XWGC)

The weak gravity conjecture should be satisfied at any stationary point of the potential.

- If true, this conjecture would close the small action loophole.

The Extra Particle Loophole

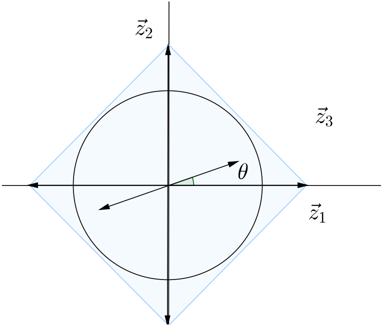


Figure 15 : A model with three charge vectors and two axions. Although the generalized weak gravity conjecture still constrains the size of moduli space, one could achieve a large inflaton traversal as long as the potential contributions from $\vec{z}_3$ dominate those from $\vec{z}_2$.

Closing the Extra Particle Loophole

- This loophole can also be closed by the magnetic WGC in some instances.
- In other cases, there will be additional modes coming in and out of the effective field theory over the course of the inflaton trajectory.

Closing the Extra Particle Loophole

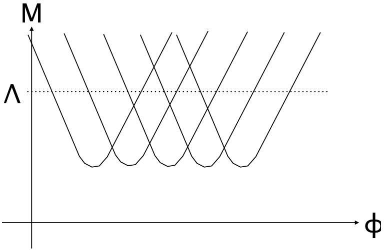


Figure 16 : Masses of KK modes in a model with extra particles as a function of the inflaton, ϕ. KK modes go in and out of the EFT as the inflaton rolls.

Section 5

Conclusions and Directions for Future Research

Conclusions

- The WGC strongly constrains models of N-flation and axion decay constant alignment.
- There are loopholes which would allow natural inflation consistent with the WGC, though these introduce other oddities and might not admit a UV completion.
- The N-species extension of the WGC is non-trivial, and there is evidence for both a mild version (the convex hull condition) and a strong version (the Lattice WGC–see Ben Heidenreich's talk).
- **Quantum gravity has more to say about inflation than has been previously appreciated.**

Outstanding Questions

- Is the mild WGC necessarily true in any consistent theory of quantum gravity?
  - If so, is the Lattice WGC true? The XWGC?
  - If not, what else could explain the sub-Planckian decay constants of string theory?
- Does the mild WGC place important constraints on realistic models in string theory, or are the aforementioned loopholes readily exploited?
- Can one place similar constraints on axion monodromy and/or relaxion models? (see talks from Westphal, Valenzuela)
- Is the WGC pointing us toward something even more fundamental about quantum gravity?

For Further Reading I

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For Further Reading VIII

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