The Weak Gravity Conjecture & Cosmology
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Gravitational Instantons, Moduli Stabilisation & Axion Inflation

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with
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Intro

- Large field inflation with axions faces many challenges.

[Arkani-Hamed, Motl, Nicolis, Vafa 2006]

Towards a general no-go principle: The Weak Gravity Conjecture

- General statement about Einstein-Maxwell / Einstein-axion theories.
- Only a conjecture.

[Brown, Cottrell, Shiu, Soler 2015]

- How does it manifest itself in explicit models? Loopholes?

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- How does it manifest itself in explicit models? Loopholes?

Gravitational Instantons — another general no-go principle?

[Giddings, Strominger 1988]

- If they exist, they are a universal feature of Einstein-axion theories.
- Could constrain axion inflation in a fairly model-independent way.

[Montero, Uranga, Valenzuela 2015]

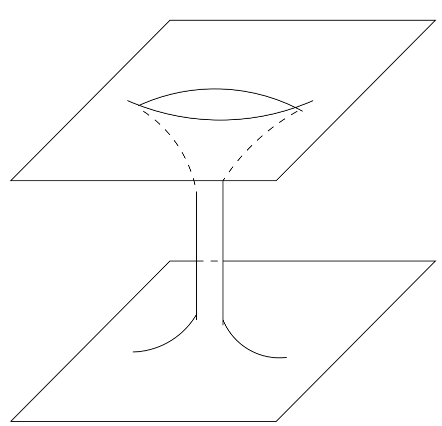
- Gravitational Instantons as a manifestation of the Weak Gravity Conjecture? [Bachlechner, Long, Mcallister 2015; Heidenreich, Reece, Rudelius 2015]

Intro

Gravitational Instantons [Giddings, Strominger 1988; Bergshoeff, Collinucci, Gran, Roest, Vandoren 2004]

- Euclidean solutions of 4-dimensional Einstein-axion(-dilaton) systems.
- Solution for metric: $ds^2 = \left(1 + \frac{C}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2$

$C < 0$

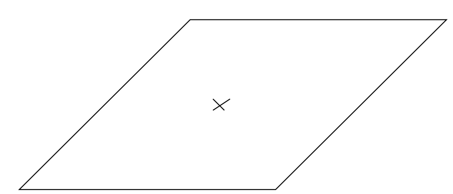


Wormhole

- narrowest radius:

$r_0 = |C|^{1/4}$

$C = 0$



Extremal Instanton

- requires a diatonic field
- space is flat

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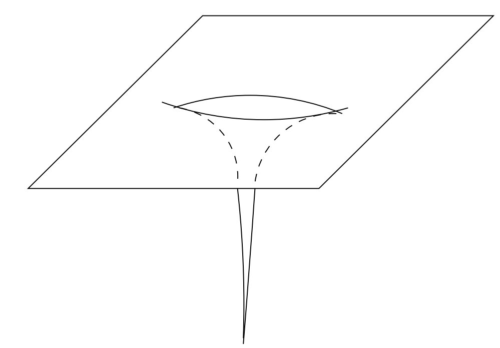

Cored Instanton

- requires a diatonic field
- singular for $r \to 0$

Intro

Gravitational Instantons

- Euclidean solutions of Einstein-axion(-dilaton) systems.
- Expected to give rise to a potential for the axion:

$$\delta V \sim \Sigma_n \, e^{-\frac{n}{f}} \cos(\tfrac{n\theta}{f})$$

- For natural inflation need $f > 1$.
- For transplanckian axion decay constants $f > 1$ gravitational instantons with $n \lesssim f$ should not be trusted and discarded.
- One should only trust such contributions with $n \gtrsim f$. Nevertheless, contributions with $n \sim f$ can easily disrupt large-field inflation by giving rise to unsuppressed higher harmonics. [Montero, Uranga, Valenzuela 2015]

Intro

Gravitational Instantons (Wormhole case)

Potential $\delta V \sim \Sigma_n \, e^{-\frac{n}{f}} \cos(\frac{n\theta}{f})$

Scalar Curvature at throat $R \sim \frac{f}{n}$

Unsuppressed potential = curvature approaches Planck scale

Trust solution only up to a cutoff $R < \Lambda^2$. $\Rightarrow$ $\delta V \sim e^{-1/\Lambda^2}$

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Relative correction to inflaton potential: $\frac{\delta V}{V_{inf}} \sim \frac{e^{-1/\Lambda^2}}{H^2}$

[Hebecker, Mangat, Rompineve, LW 2015]

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Outline

Do gravitational instantons constrain axion inflation?
(What is the cutoff?)

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1.) Review the theoretical basis for gravitational instantons.

Do gravitational instantons constrain axion inflation?
(What is the cutoff?)

2.) Include moduli: study Einstein-axion-moduli systems

3.) Calculate potential due to gravitational instantons.

1.
The Case For Gravitational Instantons

Review

Gravitational Instantons

- Euclidean solutions of Einstein-axion(-dilaton) systems.
- Start with theory of an axion θ_0 (ignore gravity and dilaton first):

$$S[\theta_0] = \int_M \frac{1}{2g_\theta^2} F_1 \wedge \star F_1 \ , \qquad F_1 = d\theta_0 \ ,$$

- Equivalently, have a dual description in terms of a field B_2 :

$$S[B_2] = \int_M \frac{1}{2g_B^2} H_3 \wedge \star H_3 \ , \qquad H_3 = dB_2,$$

where $H_3 = g_B^2 \star F_1$ and $g_B^2 = 1/g_\theta^2$.

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- This dualisation can also be done in the path integral.

Review

Now couple the θ_0 / B_2 theory to Einstein gravity:

- Using the θ_0 formulation the stress-energy tensor is

$$T^{(\theta)}_{\mu\nu} = \frac{1}{g_\theta^2}\left(-\frac{1}{2}g_{\mu\nu}(\partial\theta_0)^2 + \partial_\mu\theta_0\partial_\nu\theta_0\right) ,$$

- In the B_2 formulation we obtain:

$$T^{(B)}_{\mu\nu} = \frac{1}{g_B^2}\left(-\frac{1}{2\cdot 3!}g_{\mu\nu}H_3^2 + \frac{1}{2}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma}\right) .$$

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$$T_{\mu\nu}^{(B)} = \frac{1}{g_B^2}\left(-\frac{1}{2\cdot 3!}g_{\mu\nu}H_3^2 + \frac{1}{2}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma}\right) .$$

The Einstein equation $G_{\mu\nu} = T_{\mu\nu}^{(B)}$ together with the flux quantisation condition $\int_{S^3} H_3 = n$ have a non-trivial solution:

Giddings-Strominger wormhole

[Giddings, Strominger 1988]

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$$\begin{aligned} T^{(B)}_{\mu\nu} &= \frac{1}{g_B^2}\left(-\frac{1}{2\cdot 3!}g_{\mu\nu}H_3^2 + \frac{1}{2}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma}\right) = \\ &= -\frac{1}{g_\theta^2}\left(-\frac{1}{2}g_{\mu\nu}(\partial\theta_0)^2 + \partial_\mu\theta_0\partial_\nu\theta_0\right) = -T^{(\theta)}_{\mu\nu} , \end{aligned}$$

where we used $H_3 = g_B^2 \star F_1$ and $g_B^2 = 1/g_\theta^2$.

Obtain different Einstein equations for θ_0 and B_2 theories.

- Gravitational instanton solutions exist for the B_2 theory, but not for θ_0.

Review

Which theory is the "correct" one? Possible resolutions:

- The Euclidean θ_0 and B_2 theories coupled to gravity seem to differ.
- Gravitational instanton solutions exist for the B_2 theory, but not for θ_0.

For wormholes to exist…

- there must be a reason that the B_2 formulation is more fundamental.
- In the B_2 formulation strings are present in the path integral while fundamental instantons are not.

Review

A nod to the WGC:

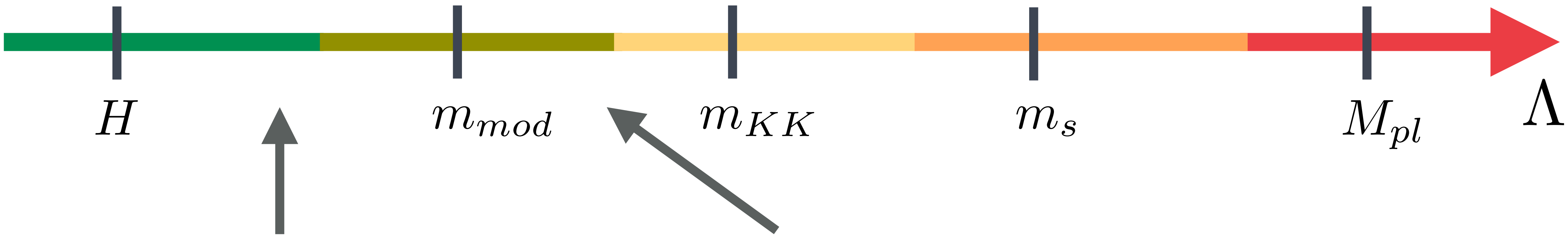
$$S[\theta_0] = \int_M \frac{1}{2g_\theta^2} F_1 \wedge \star F_1 + iQ_\theta \int_I \theta_0, \qquad F_1 = d\theta_0,$$

$$S[B_2] = \int_M \frac{1}{2g_B^2} H_3 \wedge \star H_3 + iQ_B \int_\sigma B_2, \qquad H_3 = dB_2,$$

- large axion decay constant: θ_0 theory is weakly coupled, while B_2 theory is strongly coupled: $f \sim 1/g_\theta \gg 1$.
- Taking WGC seriously, this is the regime of light instantons and heavy strings.
- More generally, expect the charged object on the perturbative side to be light.
- Tension with the requirement for wormholes…

2.
Gravitational Instantons & Moduli

Including Moduli



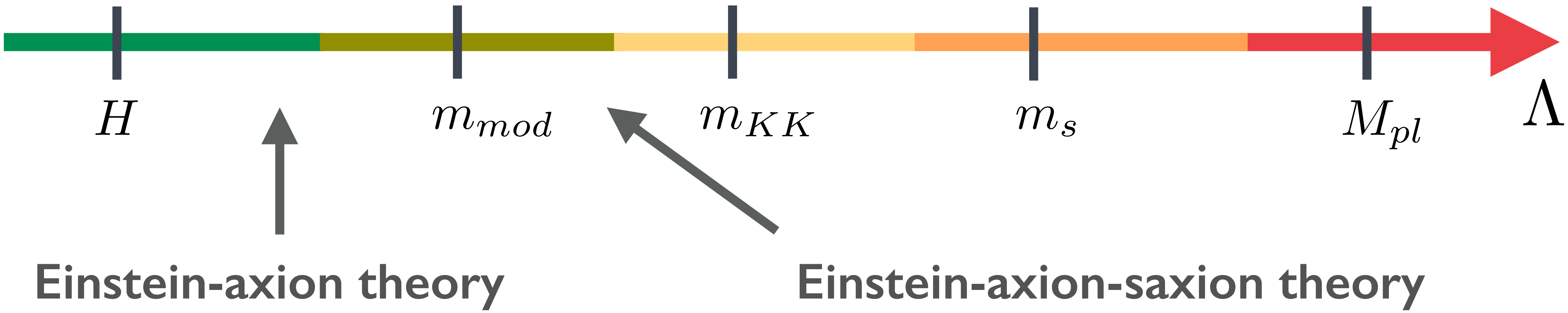
Einstein-axion theory

Einstein-axion-saxion theory

- For simplicity: include one additional scalar φ (lightest modulus).
- Einstein-axion-saxion theory with moduli potential:

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}K(\varphi)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right].$$

Including Moduli



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- Should rather study the dual Einstein-Maxwell-saxion theory:

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right],$$

where $\mathcal{F} = 1/(3!K)$.

Including Moduli

- Study the dual Einstein-Maxwell-saxion theory:

$$S = \int d^4x\sqrt{g}\left[-\frac{1}{2}R + \frac{1}{2}\mathcal{F}(\varphi)H^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + V(\varphi)\right],$$

Dangers:

- Far away from instanton throat / core the saxion sits at the minimum of the potential $\varphi = \langle\varphi\rangle \equiv 0$.
- Along the instanton throat $\varphi(r)$ will have a non-trivial profile.
- The saxion field value controls other parameters of the effective theory: couplings, masses, etc.

→ If φ departs significantly from $\langle\varphi\rangle$ within the instant throat, the validity of the effective Einstein-Maxwell-saxion theory is not guaranteed. Discard such gravitational instantons…

Including Moduli

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Simplifications:

- Take $\mathcal{F}$ to be of the form $\mathcal{F} = \frac{1}{3!f}e^{-\alpha\varphi}$.

  Can use known results & relevant for effective theories from strings.

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Example: axio-dilaton: $\mathcal{K} = -\ln(-i(S-\bar{S}))$, $S = C_0 + i/g_s$

$$\mathcal{L} = \tfrac{g_s^2}{4}(\partial\tfrac{1}{g_s})^2 + \tfrac{g_s^2}{4}(\partial C_0)^2 = \tfrac{1}{2}(\partial\varphi)^2 + \tfrac{f^2}{2}e^{2\sqrt{2}\varphi}(\partial\theta)^2$$

where $g_s = \langle g_s\rangle e^{\sqrt{2}\varphi}$, $C_0 = \theta$ and we defined $f^2 = \frac{\langle g_s\rangle^2}{2}$.

$$\longrightarrow \quad \mathcal{F} = \frac{1}{3!f}e^{-2\sqrt{2}\varphi}$$

Including Moduli

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- As we are only interested in controlled departures of φ from its vev we approximate the potential by its mass term only:

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 \ ,$$

with $m \ll 1$.

Including Moduli

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{2} R + \frac{1}{2} \mathcal{F}(\varphi) H^2 + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) \right],$$

- Grav. inst. solutions for the above system w/o a potential exist.
- Finding a general solution with $V = \frac{1}{2} m^2 \varphi^2$ is hard…

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Observation:

- For $r\lesssim 1/m$ the solution for $V=0$ becomes an increasingly good approximation to the solution of the full system.
- The boundary condition $\varphi(r\to\infty)=0$ is unchanged.
- One can show that the system is well-behaved for $r\gtrsim 1/m$:
  e.g. can show that $\varphi(r)$ either falls off monotonically or is bounded.

→ Take solution for $V=0$ as an approximate solution for $m\ll 1$.

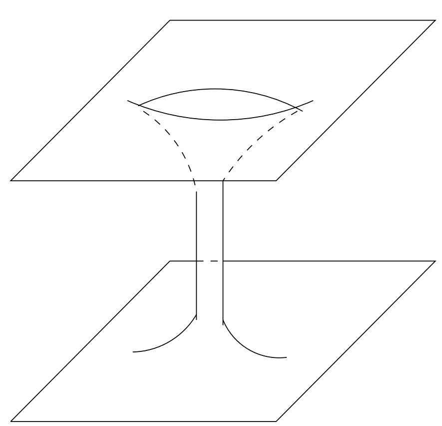
3.
How Large A Contribution?
Wormhole Case

Including Moduli

Gravitational Instantons

- Use known solutions for a massless dilaton.
- Solution for metric: $ds^2 = \left(1 + \frac{C}{r^4}\right)^{-1} dr^2 + r^2 d\Omega_3^2$

$C < 0$

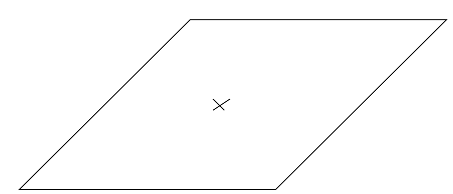


Wormhole

- narrowest radius:

$r_0 = |C|^{1/4}$

$C = 0$



Extremal Instanton

- requires a diatonic field
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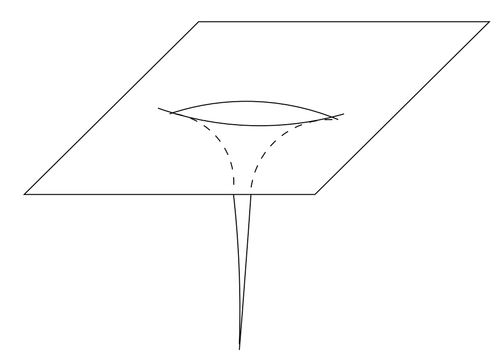

Cored Instanton

- requires a diatonic field
- singular for $r \to 0$

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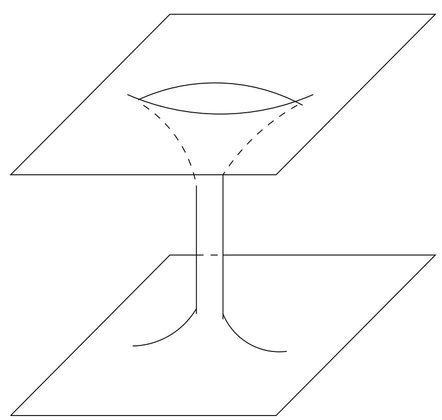

Wormhole

- narrowest radius: $r_0 = |C|^{1/4}$

Here: focus on wormholes.

Issues:

- Instantons or bounces?
- Do they give rise to a potential?

Wormholes

- Only exist for dilation couplings $\alpha < 2\sqrt{2/3}$.
- **Saxion profile:**

$$\varphi(r) = \frac{2}{\alpha} \ln \left\{ \frac{\cos\left(\sqrt{\frac{3}{2}}\frac{\alpha}{2} \operatorname{arccos}\left(\frac{\sqrt{|C|}}{r^4}\right)\right)}{\cos\left(\sqrt{\frac{3}{2}}\frac{\pi\alpha}{4}\right)} \right\}, \qquad C = -\frac{1}{3!4\pi^4}\frac{n^2}{f^2}\cos^2\left(\sqrt{\frac{3}{2}}\frac{\pi\alpha}{4}\right) .$$

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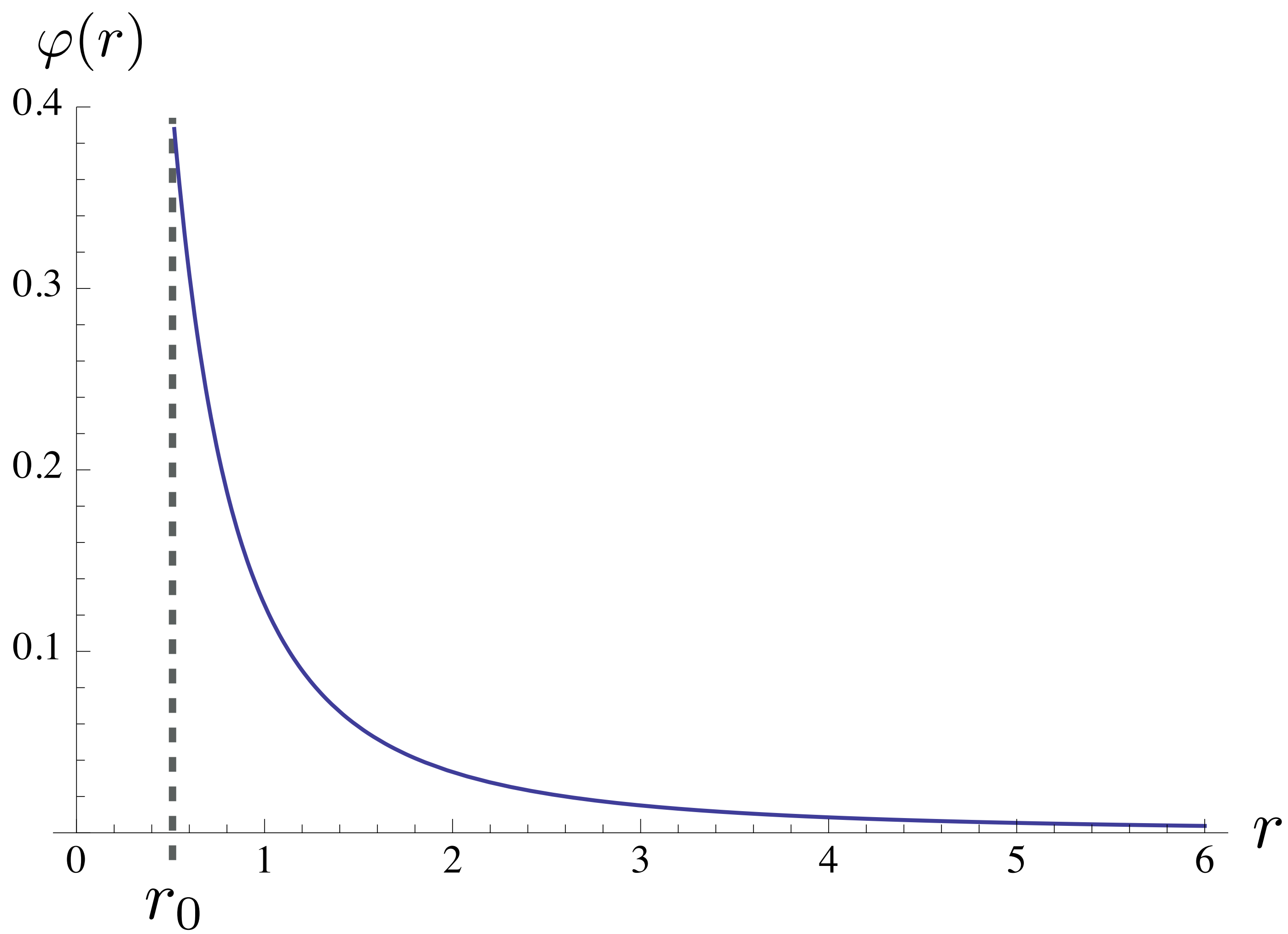

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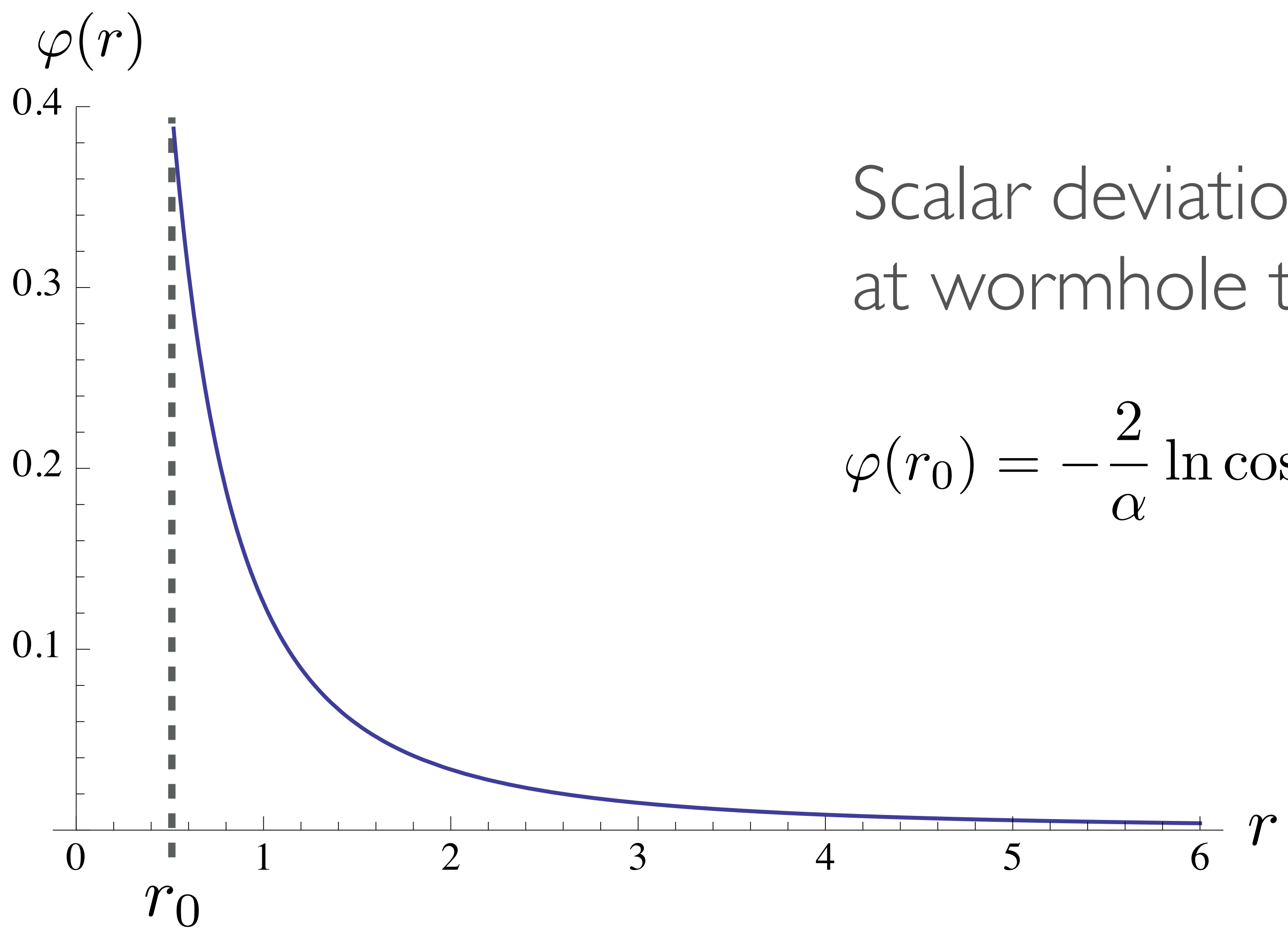
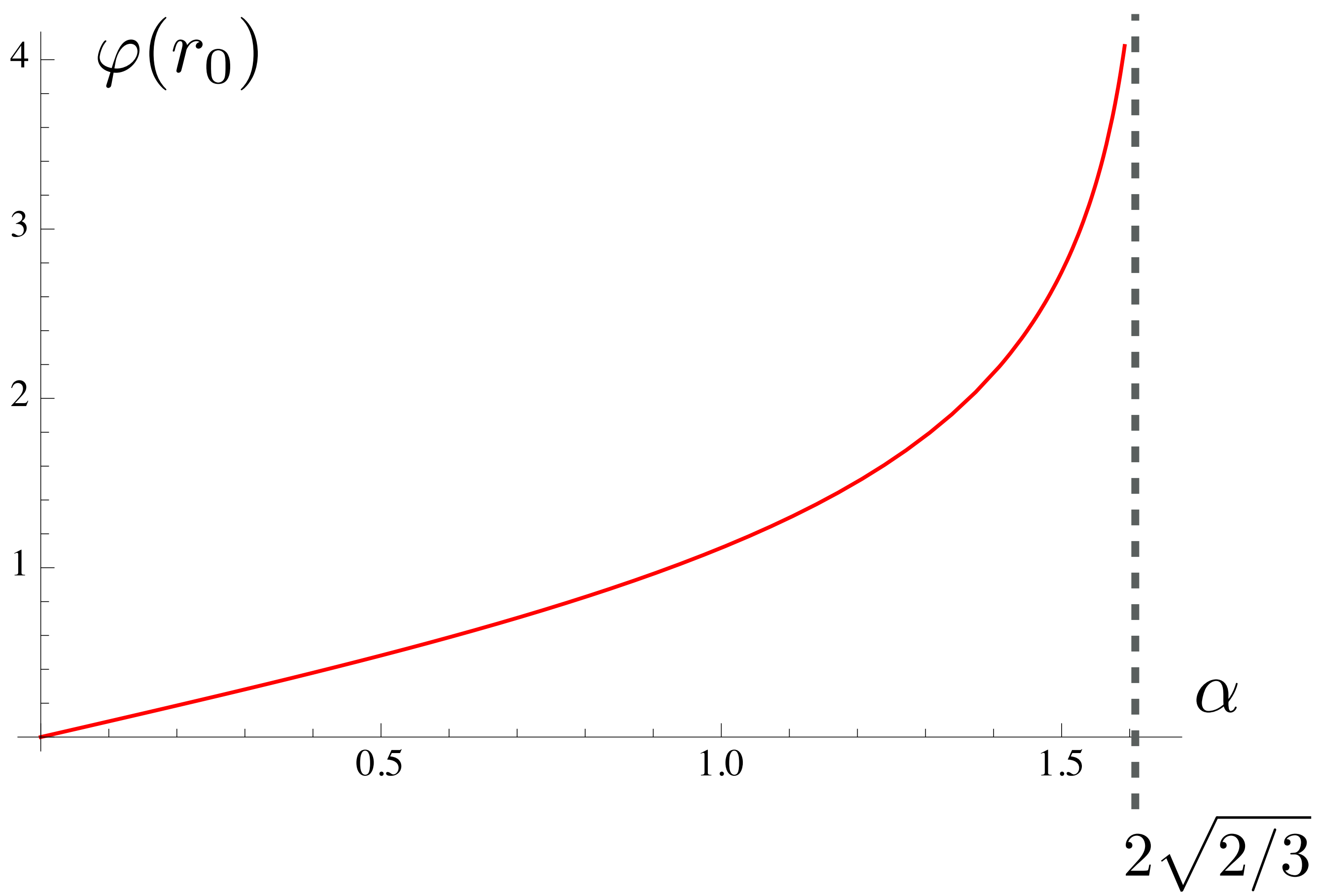


Scalar deviation largest at wormhole throat

$$\varphi(r_0) = -\frac{2}{\alpha} \ln\cos\left(\sqrt{\frac{3}{2}}\frac{\pi\alpha}{4}\right)$$

Wormholes

- Maximal scalar deviation at r_0: $\varphi(r_0) = -\frac{2}{\alpha} \ln \cos\left(\sqrt{\frac{3}{2}}\frac{\pi\alpha}{4}\right)$
- Do not want deviation to be larger than a certain amount $\Delta\varphi$.
- Note that $\varphi(r_0)$ grows monotonically with α :



Wormholes

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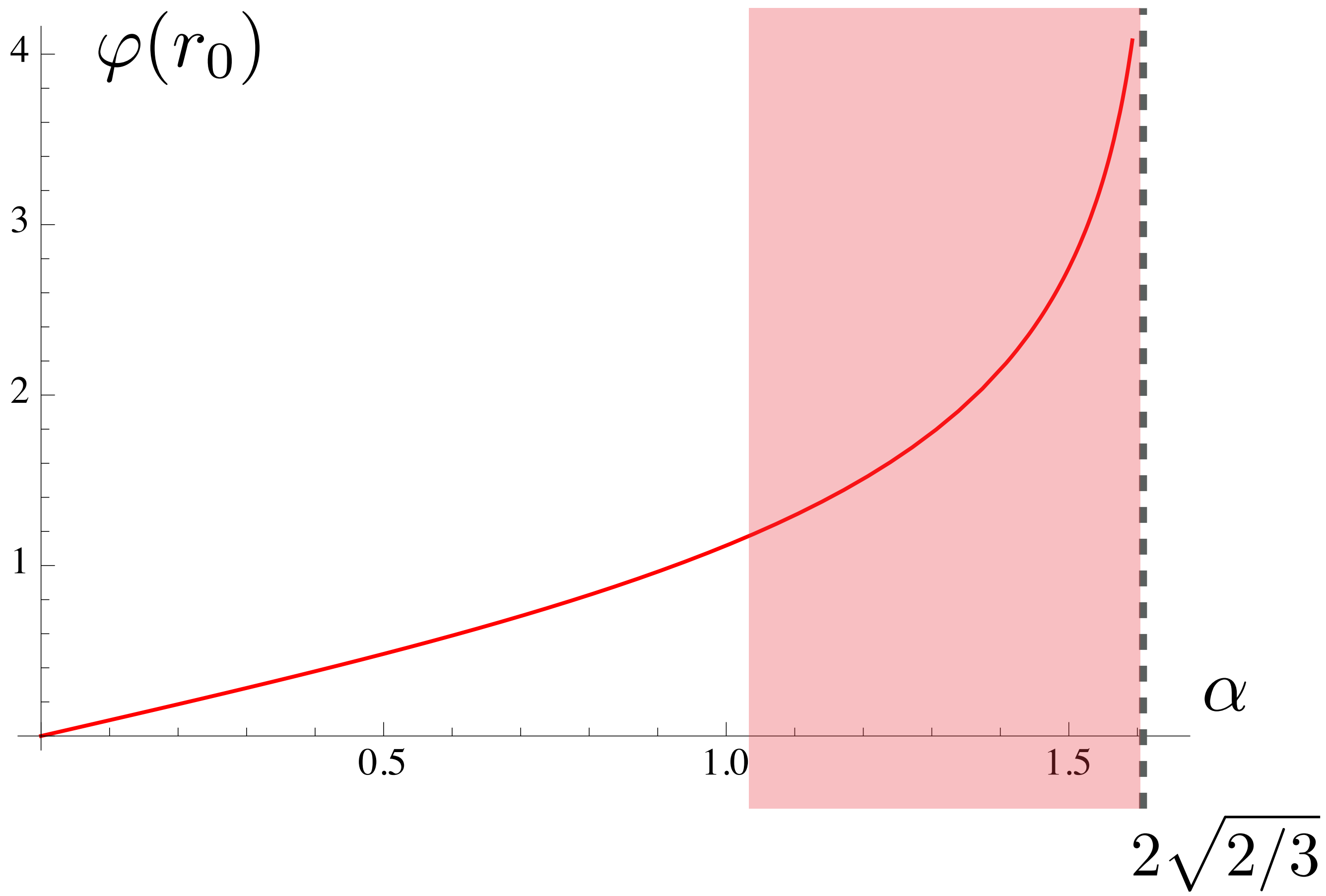

Enforcing $\varphi(r_0) < \Delta\varphi$ is equivalent to discarding wormholes in theories with a dilatons coupling $\alpha > \alpha_{max}$.

Wormholes

- Including a saxion seems in principle possible. Do not have to cut off analysis of gravitational instantons at the moduli scale.

- **Instanton action:** $S = (2\pi)^2\sqrt{6}\frac{1}{\alpha}\left(\tan\sqrt{\frac{3}{2}}\frac{\pi\alpha}{4}\right)r_0^2$

- Note that the action grows with α . The smallest action arises for $\alpha = 0$. In this case the saxion is stabilized hard at a high scale.

- Hence, once we allow for light moduli the gravitational instanton contributions become weaker.

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- Question remains: how small a value r_0 can one trust?

- The presence of moduli did not automatically lead to a cutoff.

Wormholes

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- In string theory the notion of the self-dual radius $r_{sd} = \sqrt{\alpha'} = \frac{l_s}{2\pi}$ in a circle / toroidal compactification provides a natural cutoff.
- Here the distance in question is the minimal radius r_0 of the S^3 at the throat of the wormhole solution. No notion of T-duality…

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- Consider two possibilities:

1.) Take r_0 as the self-dual radius of a circle compactification:

$$r_0 = l_s/2\pi$$

2.) Take the area of the S^3 to be unity in string lengths:

$$2\pi^2 r_0^3 = l_s^3 \quad \Rightarrow \quad r_0 = l_s/(2\pi^2)^{1/3}$$

Last, want to rewrite everything in Planck units $1 \equiv M_{pl}^2 = \frac{4\pi\mathcal{V}_s}{g_s^2 l_s^2}$:

Wormhole

e^{-S} for $\alpha = 0$	$\mathcal{V}_s = 1$	$\mathcal{V}_s = 100$
$r_0 = \frac{l_s}{2\pi}$	$4 \cdot 10^{-7}$	10^{-640}
$r_0 = \frac{l_s}{(2\pi^2)^{1/3}}$	$2 \cdot 10^{-35}$	10^{-3500}

Compare to typical potential for large-field inflation: $V \sim 10^{-8}$

Conclusions

- Examined gravitational instantons (here: only wormholes) in Einstein-axion/Maxwell-dilaton theories to study gravitational instantons in the presence of light moduli.
- Only considered moduli with dilatonic couplings. Would be interesting to generalize!
- In the presence of a light modulus gravitational instantons give weaker potential contributions compared to the case where all moduli are stabilized at a high scale.
- Without further input from string theory / quantum gravity it is difficult to decide whether gravitational instantons are strong enough to spoil axion inflation.
- The role of gravitational instantons for the Weak Gravity Conjecture is still to be clarified.

Many Thanks To The Organizers!

Especially the W Organizer.

Review

- This dualisation can also be done in the (Euclidean) path integral:

$$Z \sim \int d[B_2] \exp\left(-\int_M \frac{1}{2g_B^2} dB_2 \wedge \star dB_2\right).$$

- Write in terms of H_3 and enforce $dH_3 = 0$ via a Lagrange multiplier θ_0:

$$Z \sim \int d[H_3] d[\theta_0] \exp\left\{-\int_M \frac{1}{2g_B^2} \left(H_3 \wedge \star H_3 + 2ig_B^2 \theta_0 dH_3\right)\right\}.$$

Review

- This dualisation can also be done in the (Euclidean) path integral:

$$Z \sim \int d[B_2] \exp\left(-\int_M \frac{1}{2g_B^2} dB_2 \wedge \star dB_2\right).$$

- Write in terms of H_3 and enforce $dH_3 = 0$ via a Lagrange multiplier θ_0:

$$Z \sim \int d[H_3] d[\theta_0] \exp\left\{-\int_M \frac{1}{2g_B^2}\left(H_3 \wedge \star H_3 + 2ig_B^2 \theta_0 dH_3\right)\right\}.$$

- Integrate the last term by parts and complete the square:

$$Z \sim \int d[H_3] d[\theta_0] \;\exp\left\{-\int_M \frac{1}{2g_B^2}\left[\left(H_3 - ig_B^2 \star d\theta_0\right) \wedge \star \left(H_3 - ig_B^2 \star d\theta_0\right) + g_B^4 d\theta_0 \wedge \star d\theta_0\right]\right\}.$$

- Shift the integration variable and evaluate the Gaussian integral

$$Z \sim \int d[\theta_0] \exp\left(-\int_M \frac{1}{2g_\theta^2} d\theta_0 \wedge \star d\theta_0\right),$$

with $g_\theta^2 = 1/g_B^2$.