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Emergent Supersymmetry

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IS SUSY ALIVE AND WELL?

Disclaimer:

I have not done any original research in this area

Emergent symmetry in condensed matter

- The microscopic degrees of freedom of condensed matter systems break symmetries that we take for granted in particle theory (boost invariance, rotation invariance) and have lots of short-range complexity

- However close to critical points, these systems develop long-range correlations, corresponding to a light degree of freedom, e.g. mass $\sim (T - T_c)/T_c$

- The effective theory of the light degree/s of freedom is much simpler, described e.g. by a handful of critical exponents

- Indeed condensed matter systems that are wildly different at the microscopic level can belong to the same universality class of effective theories near a critical point

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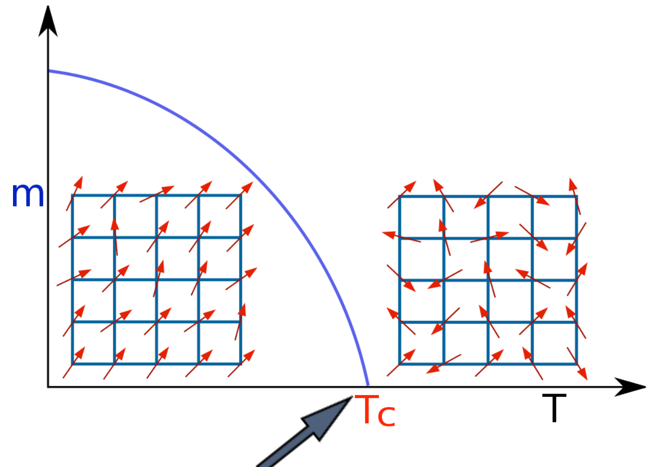
Emergent symmetry in condensed matter

- Not surprisingly, as you approach a critical point broken symmetries can be restored, since e.g. an order parameter vanishes
- In addition to restoring rotational symmetry, it is possible to see emergent Poincare invariance…



Full rotational symmetry at the critical point

The Ising model

- This is easy to see heuristically in the 2D Ising model:

$$\mathbf{H} = -\mathbf{J} \sum_{<\mathbf{i},\mathbf{j}>} \sigma_{\mathbf{i}}\sigma_{\mathbf{j}} - \mathbf{h}\sum_{\mathbf{i}} \sigma_{\mathbf{i}}$$

- Here $\sigma_{\mathbf{i}} = \pm\mathbf{1}$ is the Ising spin operator, $\mathbf{h}$ is an external field, and the first sum is over nearest neighbors

- Well below some critical temperature T_c the spins align, and at high temperatures the system is disordered

The Ising model

- The order parameter is the mean magnetization, the expectation value of the spin operator

$$\mathbf{m} = < \sigma_{\mathbf{i}} >$$

- The phase transition at the critical point is second order

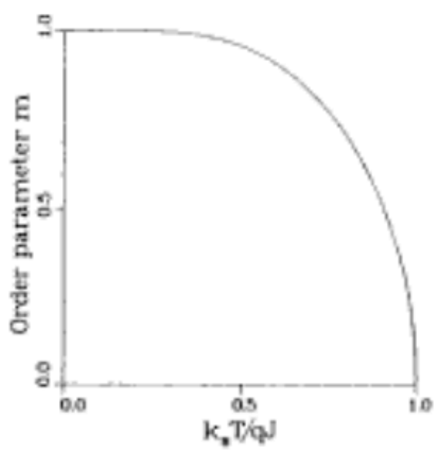


Figure 3.2: Temperature dependence of order parameter below T_c.

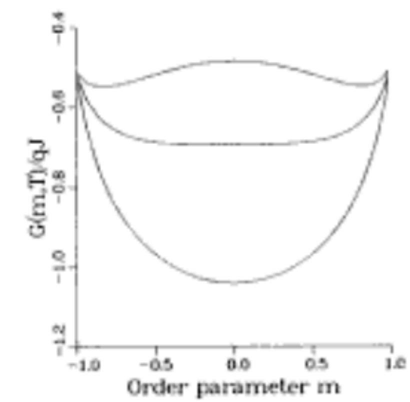
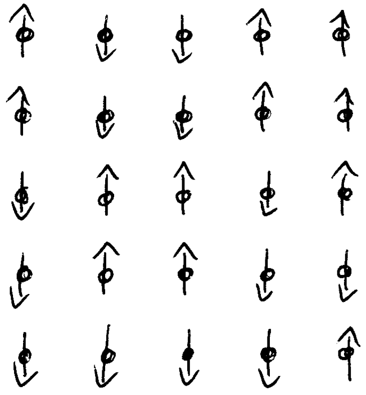


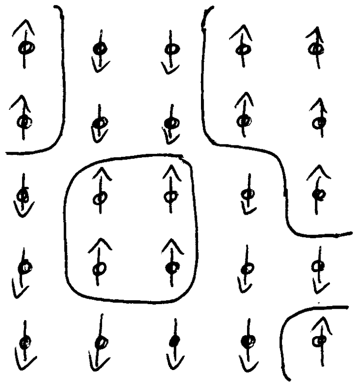
Figure 3.1: Free energy in the Bragg–Williams approximation for the Ising model. The three curves correspond to, respectively, $T = 1.5T_c$, $T = T_c$, and $T = 0.77T_c$.

The Ising model

any spin configuration



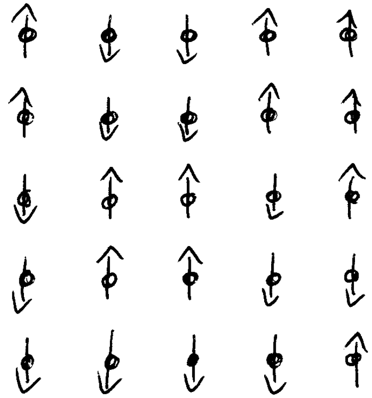
is completely specified by



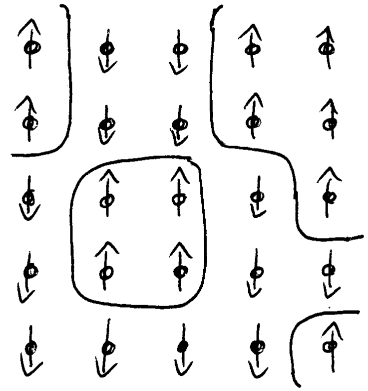
Erich Poppitz

The Ising model

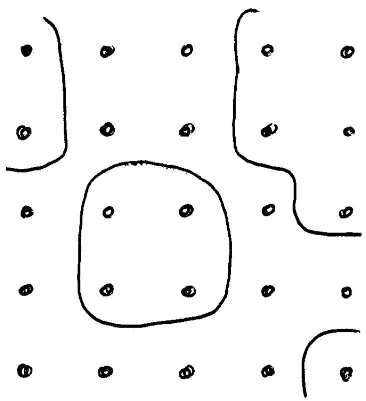
any spin configuration



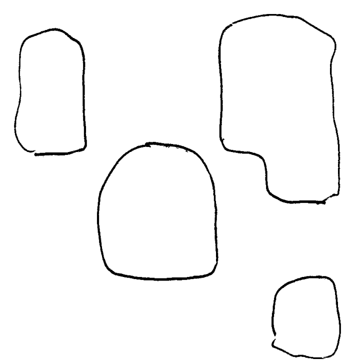
is completely specified by



the loops around up-spin islands,



so the Ising model Z is really a Boltzmann-weighted sum over all possible closed-loop configurations



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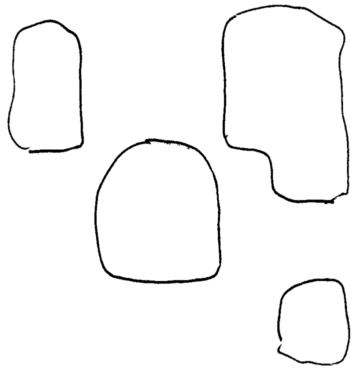
closed loops = world lines of virtual particle-antiparticle pairs -



- the stuff the quantum vacuum is made of!

thus:

2-dim Ising = (1+1)-dim Euclidean QFT

a closer look reveals that:

2-dim Ising = 2 (1+1)-dim massive free Majorana **fermions** of spins 1/2 & 3/2

now, we know that: low-T - magnetization, mostly large loops
high-T - random orientations, mostly small loops
in between - T_C - loops of all sizes

when loops of all sizes contribute to Z -> no scale in the problem at criticality

spin-1/2 fermion becomes massless -> scale and conformal invariance "emerge" at critical temperature

Onsager, 1944

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Emergent space-time symmetry

- Thus the critical 2D Ising model can be described by a conformally invariant 1+1 dimensional relativistic field theory

- This is even **more** space-time symmetry than 2D rotational or 2D translational or 2D Lorentz invariance

- This CFT is of course the c=1/2 minimal model of Friedan, Qiu, and Shenker; besides the identity operator it has only two scalar operators, with conformal weights (1/2,1/2) and (1/16,1/16)

Germán Sierra

$$c = 1 - \frac{6}{m(m+1)}, \quad m = 3,4,\cdots$$

$$h_{r,s} = \frac{\left[(m+1)r - ms\right]^2 - 1}{4m(m+1)}, \quad 1 \le r < m, \quad 1 \le s \le r$$

$$I \Leftrightarrow \phi_{1,1} \; or \; \phi_{2,3}, \quad h_0 = 0$$

$$\psi \Leftrightarrow \phi_{2,1} \; or \; \phi_{1,3}, \quad h_\psi = 1/2$$

$$\sigma \Leftrightarrow \phi_{2,2} \; or \; \phi_{1,2}, \quad h_\sigma = 1/16$$

Emergent space-time symmetry

- In Ising language these correspond to the energy operator and the spin operator, respectively
- In the continuum Majorana fermion language they correspond to a mass term and a twist operator (the latter relates the vacua of the fermion defined with periodic vs antiperiodic boundary conditions}

Germán Sierra

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Emergent supersymmetry?

- This example certainly suggests the possibility that some 2D condensed matter systems might demonstrate **emergent supersymmetry** close to a critical point
- Indeed the next-simplest of the FQS minimal models, with c=7/10, is in fact a 1+1 dimensional superconformal theory!
- Furthermore the Tri-critical Ising model is described by this SCFT near its tri-critical point

Tri-critical Ising Model (TIM)

- Tri-critical behavior can arise in models whose critical behavior is determined by more than one parameter and more than one order parameter

- A simple example is to modify the Hamiltonian of the 2D Ising model as follows:

$$\mathbf{H} = -\mathbf{J} \sum_{<\mathbf{i},\mathbf{j}>} \sigma_{\mathbf{i}}\sigma_{\mathbf{j}} - \mathbf{h} \sum_{\mathbf{i}} \sigma_{\mathbf{i}} + \boldsymbol{\Delta}\sigma_{\mathbf{i}}^{\mathbf{2}}$$

- Where now we also allow vacancies on the 2D lattice, i.e.

$$\sigma_{\mathbf{i}} = \pm\mathbf{1}, \mathbf{0}$$

- And the critical behavior involves both $< \sigma_{\mathbf{i}} >$ and $< \sigma_{\mathbf{i}}^{\mathbf{2}} >$

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Tri-critical Ising Model (TIM)

$$\mathbf{H} = -\mathbf{J} \sum_{<\mathbf{i},\mathbf{j}>} \sigma_\mathbf{i}\sigma_\mathbf{j} - \mathbf{h}\sum_\mathbf{i} \sigma_\mathbf{i} + \boldsymbol{\Delta}\sigma_\mathbf{i}^\mathbf{2}$$

- Depending on the values of the parameters in the Hamiltonian, the model can have either a second order or a first order phase transition

- The tri-critical point is the dividing point between these two kinds of behaviors, it occurs for

$$\mathbf{J} = \frac{\mathbf{3}}{\mathbf{4}}\mathbf{T_c} \qquad \mathbf{h} = \mathbf{0} \qquad \boldsymbol{\Delta} = -\mathbf{2}\log\mathbf{2}\ \mathbf{T_c}$$

Tri-critical Ising Model (TIM)

- There are many condensed matter systems that exhibit tri-critical behavior, including some in the same universality class as the TIM
- For example, consider a mixture of liquid ^{4}He and ^{3}He:
- For small concentrations of ^{3}He, as the temperature is lowered there is a continuous transition of the ^{4}He to a superfluid state, with the mixture remaining homogeneous
- For large concentrations, the transition is discontinuous, accompanied by phase separation of superfluid and normal fluid
- The tri-critical point occurs when the relative concentration of ^{3}He = 0.670

Tri-critical Ising Model (TIM)

- In 1984 FQS showed that the the critical behavior of the TIM is equivalent to a 1+1 D superconformal field theory

- The "disorder" operator μ is a chain of spin operators coming in from infinity

- The fermion ψ is a product of σ and μ

Table 2. Operators in the TIM with periodic boundary conditions.

conf. dim.	field	alias	phys. role	LG field
$(0,0)$	$\mathbb{I}$		identity	
$(\frac{3}{80},\frac{3}{80})$	σ	φ_1	magnetization	Φ
$(\frac{1}{10},\frac{1}{10})$	ε	φ_2	energy	$:\Phi^2:$
$(\frac{7}{16},\frac{7}{16})$	σ'	φ_3	submagnetization	$:\Phi^3:$
$(\frac{3}{5},\frac{3}{5})$	t	φ_4	chemical potential	$:\Phi^4:$
$(\frac{3}{2},\frac{3}{2})$	ε''		(irrelevant)	$:\Phi^6:$

Table 4. Operators in the TIM with $\mathbb{Z}_2$-twisted boundary conditions.

conf. dim.	field	phys. role
$(\frac{3}{80},\frac{3}{80})$	μ	disorder field
$(\frac{7}{16},\frac{7}{16})$	μ'	subleading disorder field
$(\frac{3}{5},\frac{1}{10})$	ψ	fermion
$(\frac{1}{10},\frac{3}{5})$	$\bar{\psi}$	anti-fermion
$(\frac{3}{2},0)$	G	SuSy generator
$(0,\frac{3}{2})$	$\bar{G}$	SuSy generator

The energy operator, fermions, and t (the chemical potential or vacancy operator) form a single superfield

Emergent supersymmetry?

SUPERSYMMETRY, TWO-DIMENSIONAL CRITICAL PHENOMENA AND THE TRICRITICAL ISING MODEL*†

Zongan QIU[1,2]

Enrico Fermi and James Franck Institutes and Department of Physics, University of Chicago, Chicago, IL 60637, USA

Received 22 October 1986

We discuss the general properties of supersymmetrical two-dimensional critical phenomena, i.e. superconformal field theory. We first review the consequences of ordinary conformal invariance. We then discuss general features of superconformal invariance and the superdifferential equation that correlation functions in certain theories obey. We give the explicit form of this differential equation in the even sector of the tricritical Ising model at its tricritical point. We find the solution of this equation. The physical realizations of this model, e.g. helium adsorbed on krypton-plated graphite, are the first observable supersymmetric field theories in nature.

Seems to imply that emergent supersymmetry was first observed in a real physical system more than 30 years ago…

Possible Ising Transition in a ^{4}He Monolayer Adsorbed on Kr-Plated Graphite

M. J. Tejwani, O. Ferreira, and O. E. Vilches
Phys. Rev. Lett. **44**, 152 – Published 21 January 1980

ABSTRACT

The specific heat of ^{4}He adsorbed on Kr-plated graphite indicates the existence of an order-disorder transition from a (1×1) [½] triangular structure on a honeycomb lattice of adsorption sites. The specific heat singularity is logarithmic, as predicted from universality considerations, different from the power law observed in the $(\sqrt{3}\times\sqrt{3})R30°$ order-disorder transition of ^{4}He on the bare graphite.

Received 24 October 1979

“Despite extensive searches the Tri-critical Ising quantum criticality and the associated supersymmetry has yet to be observed experimentally in spin models” – X. Zhu and M. Franz arXiv:1602.08172

The New York Times

January 5, 1993

315 Physicists Report Failure In Search for Supersymmetry

By MALCOLM W. BROWNE

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Condensed Matter

January 5, 1993

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Majorana fermions as qubits

- In the past few years there has a been a lot of activity, both theory and experiment, related to realizing **Majorana fermions in condensed matter systems**
- The motivation for this is **quantum computing**, i.e. the ability to construct robust qubits
- This progress may also lead to the first explicit demonstrations of **supersymmetry** in condensed matter

Transverse Ising Model

- This is a 1D quantum version of the Ising model:

$$\mathbf{H} = -\mathbf{J}\sum_{\mathbf{i}} \sigma^{\mathbf{x}}_{\mathbf{i}}\sigma^{\mathbf{x}}_{\mathbf{i+1}} - \mathbf{h_z}\sum_{\mathbf{i}} \sigma^{\mathbf{z}}_{\mathbf{i}}$$

- Where now we have introduced Pauli matrices to describe spins +-1/2 quantized along the x-axis, and flipped by $\sigma^{\mathbf{z}}_{\mathbf{i}}$
- The quantum phase structure is equivalent to that of the 2D classical Ising model at finite temperature
- The ground state is doubly degenerate modulo exponentially suppressed solitons that propagate the length of the system

$$|\psi_{\rightarrow}\rangle = |\rightarrow\rightarrow\rightarrow\cdots\rightarrow\rangle,\ |\psi_{\leftarrow}\rangle = |\leftarrow\leftarrow\leftarrow\cdots\leftarrow\rangle$$

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Transverse Ising Model -> superconducting fermions

$$\mathbf{H} = -\mathbf{J}\sum_{\mathbf{i}} \sigma_{\mathbf{i}}^{\mathbf{x}} \sigma_{\mathbf{i+1}}^{\mathbf{x}} - \mathbf{h_z} \sum_{\mathbf{i}} \sigma_{\mathbf{i}}^{\mathbf{z}}$$

- A Jordan-Wigner transformation turns this model into a model of fermions. Similar to the 2D Ising model, “string-like” products of spin operators behave like fermions
- Define creation and annihilation operators:

$$\mathbf{c_i} = \left(\prod_{\mathbf{j=1}}^{\mathbf{i-1}} \sigma_{\mathbf{j}}^{\mathbf{z}}\right) \sigma_{\mathbf{i}}^{+} \qquad \mathbf{c_i^\dagger} = \left(\prod_{\mathbf{j=1}}^{\mathbf{i-1}} \sigma_{\mathbf{j}}^{\mathbf{z}}\right) \sigma_{\mathbf{i}}^{-}$$

- Then just from the properties of Pauli matrices we get the anti-commutation relations:

$$\left\{\mathbf{c_i}, \mathbf{c_j^\dagger}\right\} = \delta_{\mathbf{ij}}$$

Transverse Ising Model -> superconducting fermions

Thus we get a Hamiltonian describing a 1D chain of spinless fermions:

$$\mathbf{H} = \sum_{\mathbf{i}} \left[-\mathbf{t}\,(\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i+1}} + \mathrm{h.c.}) - \mu\,(\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i}} - \frac{\mathbf{1}}{\mathbf{2}}) + (\boldsymbol{\Delta}\,\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i+1}}^{\dagger} + \mathrm{h.c.}) \right]$$

- Here $\mathbf{t}$ is the strength of a hopping term, μ is a chemical potential, and $\boldsymbol{\Delta}$ is the strength of the analog of Cooper pairing in a BCS superconductor.
- Note that for spinless fermions (or spin-polarized fermions) the pairing involve fermions of **different** (adjacent) sites.
- This is thus a model for (what is called) SPSC or p-wave superconducting chains

Majorana zero modes from superconducting electrons

$$\mathbf{H} = \sum_{\mathbf{i}} \left[-\mathbf{t}\,(\mathbf{c_i^\dagger c_{i+1}} + \mathrm{h.c.}) - \mu\,(\mathbf{c_i^\dagger c_i} - \frac{\mathbf{1}}{\mathbf{2}}) + (\boldsymbol{\Delta}\,\mathbf{c_i^\dagger c_{i+1}^\dagger} + \mathrm{h.c.}) \right]$$

- How to realize a Majorana fermion mode from electrons?
- First define two sets of Majorana fermions $\gamma_{\mathbf{j},\mathbf{1}}, \gamma_{\mathbf{j},\mathbf{2}}$ at each site:

$$\gamma_{\mathbf{j},\mathbf{1}} = \mathbf{c_j^\dagger} + \mathbf{c_j} \qquad \gamma_{\mathbf{j},\mathbf{2}} = \mathbf{i}(\mathbf{c_j^\dagger} - \mathbf{c_j})$$

$$\{\gamma_{\mathbf{i},\alpha}, \gamma_{\mathbf{j},\beta}\} = \mathbf{2}\delta_{\mathbf{ij}}\delta_{\alpha\beta} \qquad \gamma_{\mathbf{i},\alpha}{}^\dagger = \gamma_{\mathbf{i},\alpha}$$

A. Kitaev

Majorana zero modes from superconducting electrons

$$\mathbf{H} = \sum_{\mathbf{i}} \left[-\mathbf{t}\,(\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i+1}} + \mathrm{h.c.}) - \mu\,(\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i}} - \frac{\mathbf{1}}{\mathbf{2}}) + (\boldsymbol{\Delta}\,\mathbf{c}_{\mathbf{i}}^{\dagger}\mathbf{c}_{\mathbf{i+1}}^{\dagger} + \mathrm{h.c.}) \right]$$

- Now rewrite the Hamiltonian in the special case $\mathbf{t} = \boldsymbol{\Delta}, \mu = \mathbf{0}$

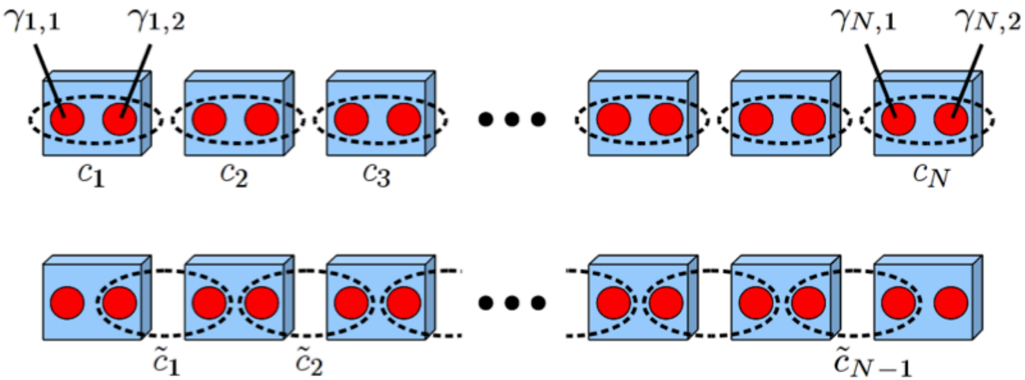
$$\mathbf{H} = -\mathbf{it} \sum_{\mathbf{j=1}}^{\mathbf{N-1}} \gamma_{\mathbf{j,2}} \gamma_{\mathbf{j+1,1}}$$

- In terms of the Majorana modes the pairing is again at adjacent sites

- However two of the Majorana modes, $\gamma_{\mathbf{1,1}}$ and $\gamma_{\mathbf{N,2}}$ do not get paired!

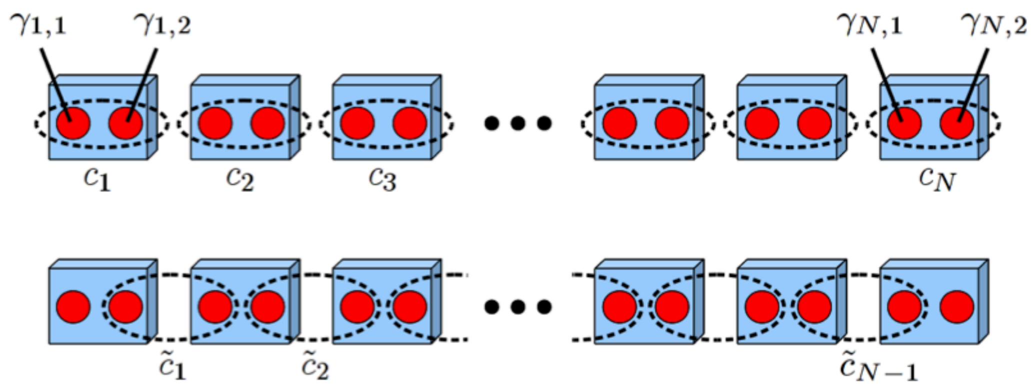
A. Kitaev

Majorana zero modes in a topological superconductor



- The unpaired Majorana zero modes **persist** when we relax the tuning on the parameters, provided $|\mathbf{2t}| > |\mu|$
- This is called the topological phase of the superconductor
- In the Transverse Ising construction, the degenerate ground state can be split by turning on a longitudinal magnetic field
- But here we see that no local disturbance can disturb the doubly degenerate ground state

Majorana zero modes as qubits



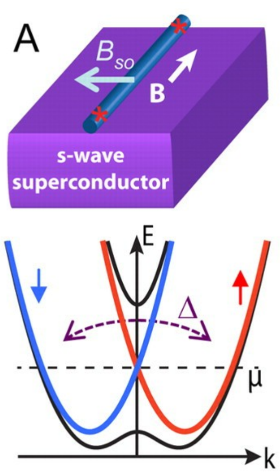
- If one could create and manipulate a large number of these Majorana zero modes, they could be the basis of error-resistant quantum computers

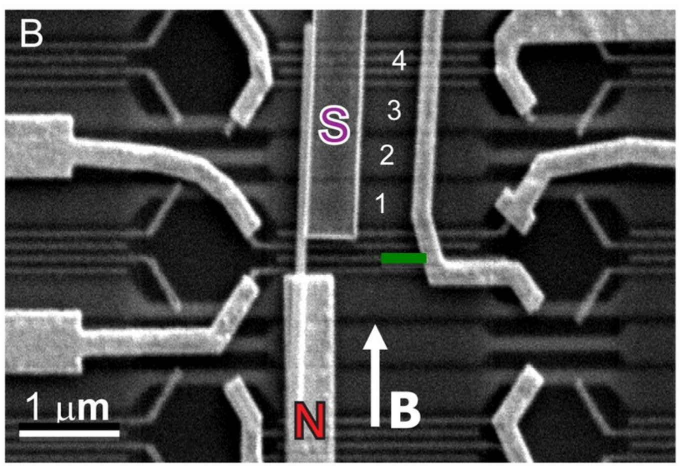
- Here the qubits would be intrinsically non-local entities

The first step is to show that you can make these at all…

Majorana zero modes in real systems

- The most progress so far is by using semiconductor nanowires, proximity coupled to ordinary (s-wave) superconductors
- Since 2012 at least six groups have presented evidence for unpaired Majorana quasiparticles at the ends of nanowires





Mourik et al., Science 2012 (Kouwenhoven's group, Delft) following proposals by Lutchyn, Sau & Das Sarma, 2010;
Oreg, Refael & von Oppen, 2010.

Emergent supersymmetry redux?

- Recently several groups have tried to revive the idea of demonstrating emergent SUSY in a condensed matter system, now focusing on the various setups thought to give topological superconductors with Majorana quasiparticles

T. Grover et al, arXiv:1301.7449
I. Affleck et al, arXiv:1504.05192
X. Zhu, arXiv:1602.08172

- The simplest example is to add a strong attractive 4-fermion interaction to our previous hamiltonian

$$\mathbf{H} = -it \sum_{j=1} \gamma_{j,2}\gamma_{j+1,1} + g \sum_{j=1} \gamma_{j,2}\gamma_{j+1,1}\gamma_{j+2,2}\gamma_{j+3,1}$$

I. Affleck et al, arXiv:1504.05192

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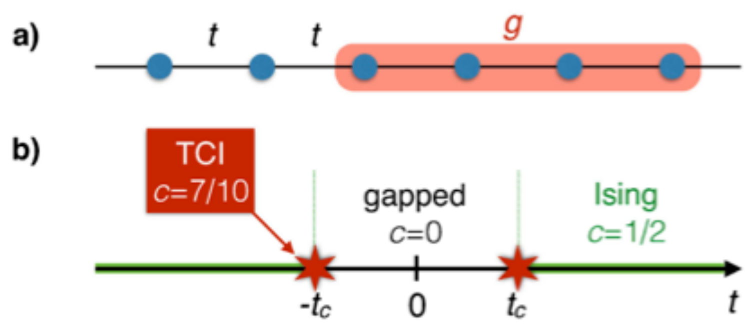
Emergent supersymmetry redux?

- The authors show numerically that this model has a critical point for t/g = 0.00405 that matches the scaling behavior of the Tri-critical Ising model
- As they say, if you can realize Majorana modes in superconductors, why not also an attractive interaction between them?



I. Affleck et al, arXiv:1504.05192

Emergent spontaneously broken supersymmetry?

- In the TIM the supersymmetry is only realized in the critical limit
- As I noted before, in this limit the energy operator, fermions, and t (the chemical potential or vacancy operator) form a single superfield:

$$\mathbf{\Phi}(z) = \varepsilon(z) + \theta\psi(z) + \bar{\theta}\psi(z) + \theta\bar{\theta}\, t(z)$$

- What happens when you perturb away from the critical limit by turning on one or more of the relevant operators of the superconformal CFT?
- Most perturbations explicitly break SUSY; in the massless Majorana fermion realization of the SCFT this corresponds to introducing a mass

Emergent spontaneously broken supersymmetry?

$$\mathbf{\Phi}(z) = \varepsilon(z) + \theta\psi(z) + \bar{\theta}\psi(z) + \theta\bar{\theta}\, t(z)$$

- The exception is to perturb by the relevant operator t, since this preserves the supersymmetry
- The theory perturbed this way is a massless Majorana fermion with self-interactions
- This is the Goldstino of the Volkov-Akulov nonlinear realization of spontaneously broken SUSY!

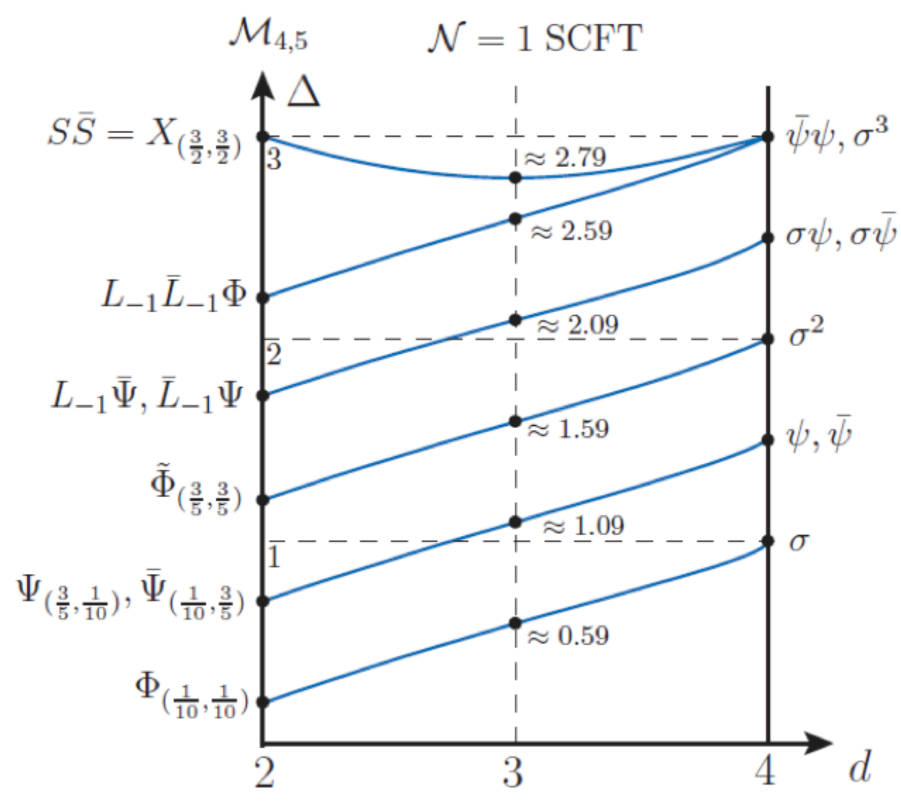
Kastor, Martinec, Shenker

In the Affleck et al proposal, it is not implausible that one could eventually realize this also in a superconductor

Emergent supersymmetry in 2+1 dimensions?

$$\mathcal{L}_{\mathrm{GNY}} = \frac{1}{2}(\partial_\mu \sigma)^2 + \bar{\psi}_j \not{\partial} \psi^j + g_1 \sigma \bar{\psi}_j \psi^j + \frac{1}{24} g_2 \sigma^4$$

- Consider a Majorana fermion having Yukawa interaction with a scalar in $4-\varepsilon$ dimensions
- Pade extrapolation of the epsilon expansion to d=2 connects this to the Tri-critical Ising Model SCFT
- In between there also seems to be a 2+1D SCFT
- Could also be realized in a condensed matter system?



S. Giombi I. Klebanov arXiv 1409.1937

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Conclusion

- We hope that Supersymmetry and Supercolliders turn out to be deeply connected
- It may be that Supersymmetry and Superconductors are also deeply connected