The Higgs mass and naturalness in SUSY models

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The Higgs mass in the MSSM

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1. Heavy stops
2. Large (maximal) stop mixing

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Both directions rise new questions:

1. Naturalness?
2. Vacuum stability?

Checks for the vacuum stability

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$$A_t^2 < 3(m_{H_u}^2 + m_{\tilde{t}_l}^2 + m_{\tilde{t}_R}^2)$$

are used to identify dangerous directions.

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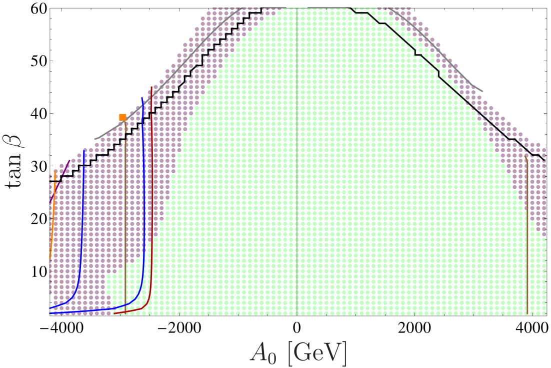
Comparison with numerical calculations

It has been found that these conditions are usually **too weak**!

Vacuum stability in the CMSSM



[Camargo-Molina, O'Leary,Porod,FS,1309.7212]

$m_0 = 1$TeV

$M_{1/2} = 1$ TeV

$\mu > 0$

- Dots: Vevacious results
- lines: (semi-) analytical conditions

- (1): $[A_t]$
- (2): $[A_\tau]$
- (3): $[c \cdot A_t]$
- (4): $[A_u]$
- (5): $[A_0]$
- (6): $[\mu \tan\beta]$
- (7): $[Y_\tau]$
- $\tilde{\chi}^0$ LSP
- Fittino (1204.4199)

Vacuum stability in natural MSSM

[Camargo-Molina,Garbrecht,O'Leary,Porod,FS,1405.7376]

When Thermal corrections are included, four possibilities exist

- A point is stable
- A point is long lived at zero and finite temperature
- A point is long lived at zero, but short-lived at finite temperature
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Parameter scan:

$\tan\beta$	$m_{\tilde{t}_L}, m_{\tilde{t}_R}$ [GeV]	μ [GeV]	T_t [GeV]
5-60	500–1500	100 – 500	-3000 – 3000

$M_A = 1000$ GeV, $M_{1,2,3} = 100, 300, 1000$ GeV, $m_{\tilde{f}} = 1500$ GeV

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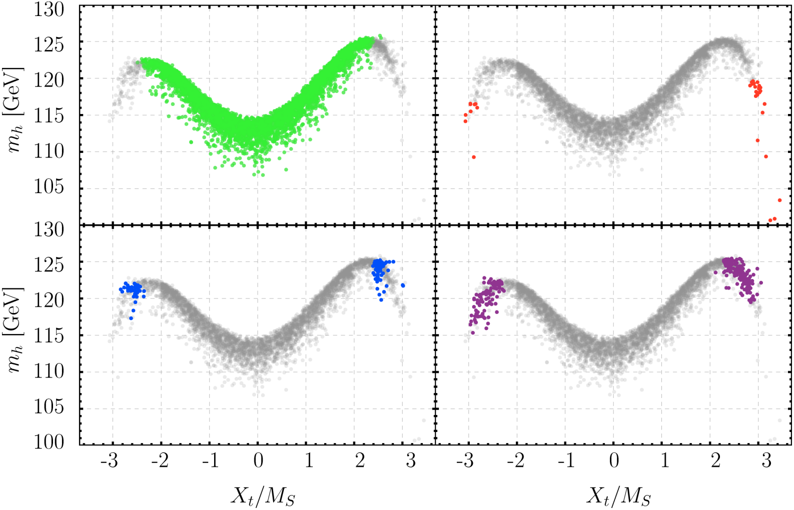

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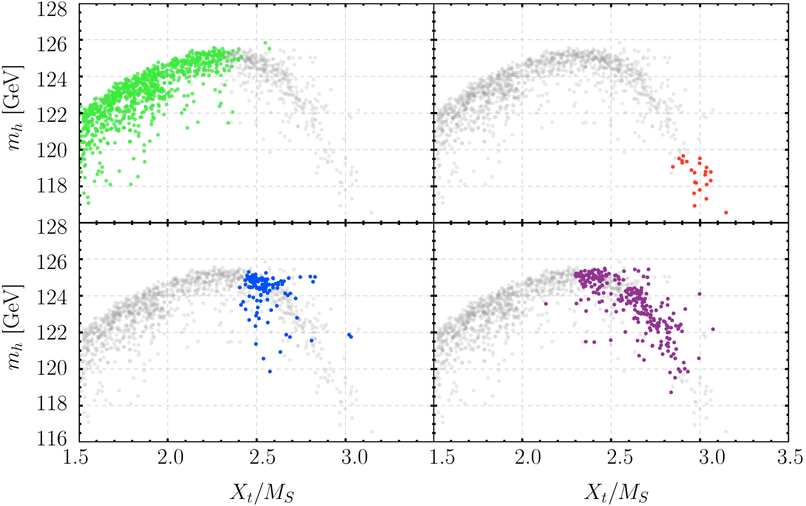

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Vacuum stability constraints

The constraints from the vacuum stability puts strong constraints on natural SUSY with large stop mixing!

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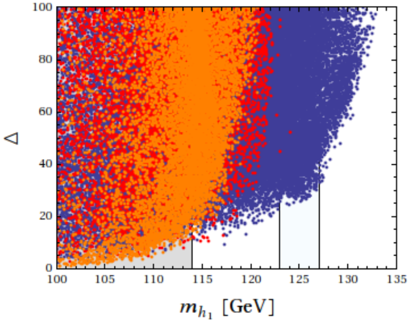
One of many motivations to look beyond the MSSM!

(Collider limits, neutrino masses, GUT/String motivated models, . . .)

The Higgs mass and fine-tuning

The Higgs mass prediction is crucial to determine the fine-tuning



(blue: GNMSSM, orange: MSSM
[Ross,Schmidt-Hoberg,FS,1205.1509])

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$\rightarrow$ **strong need to catch up (at least) with MSSM precision in non-minimal SUSY models!**

Fully automatised two-loop calculations

The combination SARAH/SPheno provides a fully automatised two-loop calculation of the Higgs mass in SUSY models.

Approach

[Goodsell,Nickel,FS,1411.0675,1503.03098]

- Generic one- and two-loop calculations which are matched on concrete models.
- Auto-generated Fortran code for numerical evaluation

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Approximations @2-loop: gaugeless limit ($g_1 = g_2 = 0$), $p^2 = 0$:

- similar precision as most public tools provide for MSSM
- All available ($\overline{\text{DR}}$) two-loop results (MSSM, NMSSM) are exactly reproduced!

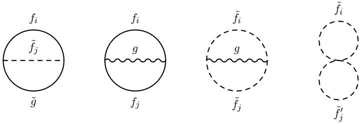
New results for the Higgs mass

The setup was used to calculate many new two-loop results:

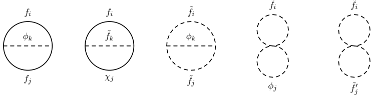
- Contributions from trilinear RpV [Dreiner,Nickel,FS,1411.3731]
- Missing corrections in the NMSSM [Goodsell,Nickel,FS,1411.4665]
- CP violating NMSSM beyond $O(\alpha_s \alpha_t)$ [Goodsell,FS, 1604.05335]
- Contributions from non-holomorphic soft-terms [Ün, Tanyildizi,Kerman Solmaz,1412.1440]
- MRSSM [Diessner,Kalinoswki,Kotlarski,Stöckinger,1504.05386]
- Contributions from vectorlike (s)tops [Nickel,FS,1505.06077]
- Other vector-like states [Basirnia, Macaluso, Shih, 1605.08442]
- The MSSM beyond MFV [Goodsell,Nickel,FS,1511.01904]
- ...

Two-loop corrections to m_h in the NMSSM

$\alpha_S(\alpha_b + \alpha_t)$ (known before)

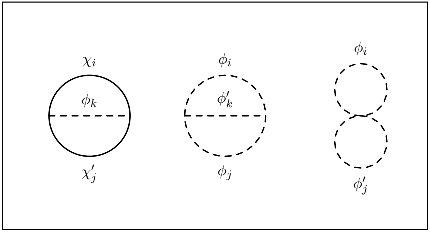


MSSM-like



NMSSM-specific

(available for first time!)

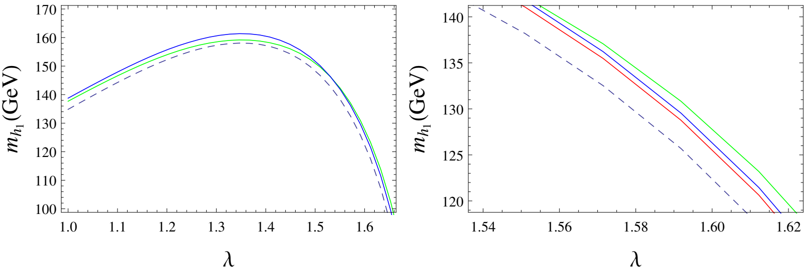


NMSSM results I: heavy singlet & large λ

[Goodsell,Nickel,FS,1411.4665]

$\kappa = 1.6 \quad \tan\beta = 3 \quad T_\lambda = 600$ GeV $\quad T_\kappa = -2650$ GeV $\quad \mu_{eff} = 614$ GeV

$m^2_{\tilde{f}} = 2 \cdot 10^6$ GeV^2 $\quad T_i = 0 \quad M_1 = 200$ GeV $\quad M_2 = 400$ GeV $\quad M_3 = 2000$ GeV



1–loop / $\alpha_S(\alpha_b + \alpha_t)$ / full / MSSM approx.

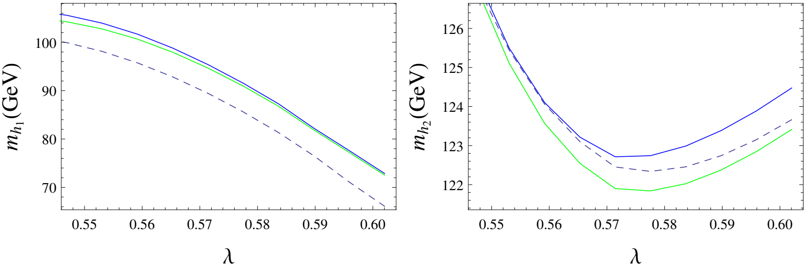
- Additional corrections crucial for (very) large λ
- Using MSSM results not a good approximation anymore

NMSSM results II: light singlet

[Goodsell,Nickel,FS,1411.4665]

$\kappa = 0.596$ $T_\lambda = -27$ GeV $T_\kappa = -240$ GeV $\mu_{eff} = 130$ GeV

$T_t = -3050$ GeV $T_b = T_\tau = -1000$ GeV $m^2_{\tilde{t}_L} = 9.0 \cdot 10^5$ GeV^2 $m^2_{\tilde{t}_R} = 1.05 \cdot 10^6$ GeV^2



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- Corrections can be larger than the ones $\sim \alpha_S$
- Again, MSSM approximations fail

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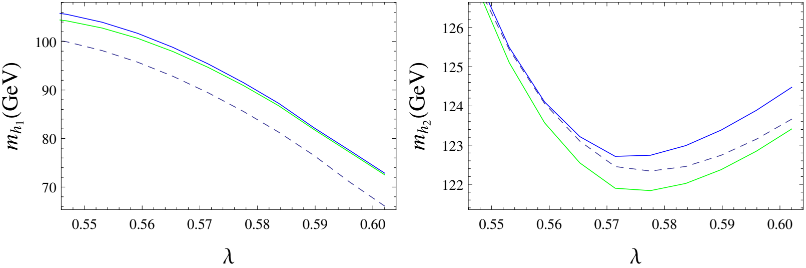

1–loop / $\alpha_S(\alpha_b + \alpha_t)$ / full

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- Again, MSSM approximations fail

$\rightarrow$ New corrections can have sizeable impact on FT prediction!

Vectorlike top partners

[Nickel,FS,1505.06077]

MSSM with vectorlike top partners

$$W = W_{MSSM} + Y^i_{t'} \hat{Q}_i \hat{T}' \hat{H}_u + M_{T'} \hat{T}' \hat{\bar{T}}' + m^i_{t'} \hat{U}_i \hat{\bar{T}}'.$$

→ Only 1-loop eff. pot results available before

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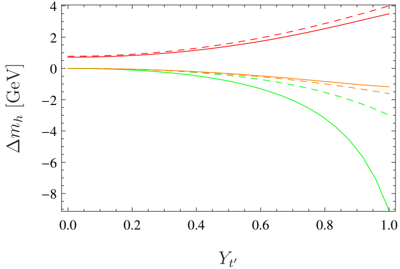
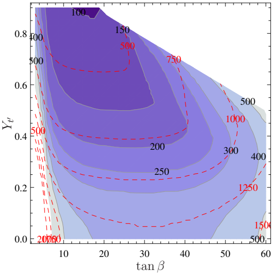
Impact of additional corrections:



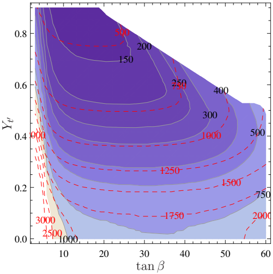
$B_{T'} = 0$ (dashed), $B_{T'} = (1.5\ \text{TeV})^2$ (full), $\tan\beta = 3$, $M_{T'} = 1.0$ TeV

shifts by momentum dependence, one-loop thresholds to Y_{top}, two-loop corrections

Impact on the fine-tuning in minimal GMSB



$m_h = 122$ GeV



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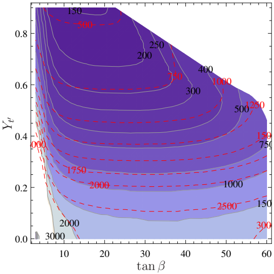

$m_h = 128$ GeV

$M = 10^7$GeV, $M_V(M) = 0.5$TeV, Λ varied to obtain Higgs mass
Red: gluino mass

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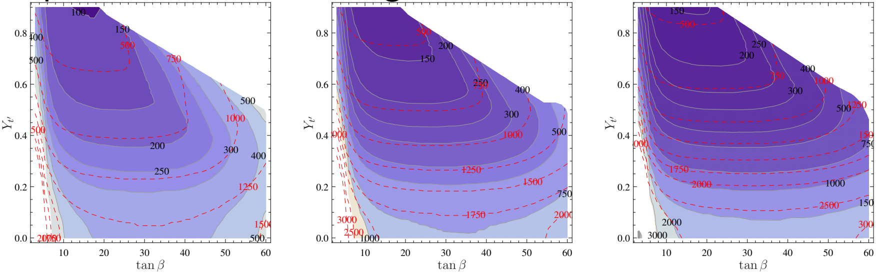

$m_h = 122$ GeV $\qquad$ $m_h = 125$ GeV $\qquad$ $m_h = 128$ GeV

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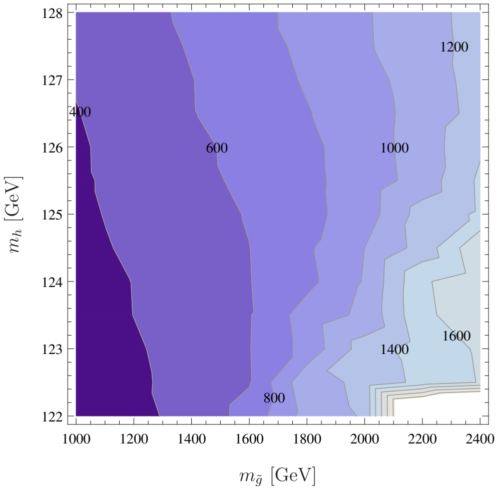
Fine-Tuning

- Even with this precision a sizeable uncertainty in FT visible!
- Higgs mass can be obtained easily, but gluino mass drives FT.

Impact on the fine-tuning in minimal GMSB



Models with Dirac gauginos

Models with Dirac gauginos have nice features:

- Suppressed flavour constraints
- Suppressed production of coloured SUSY states at the LHC
- Running of $m_{H_u}^2$ independent of gluino mass

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The effects on the Higgs mass are very different compared to the MSSM:

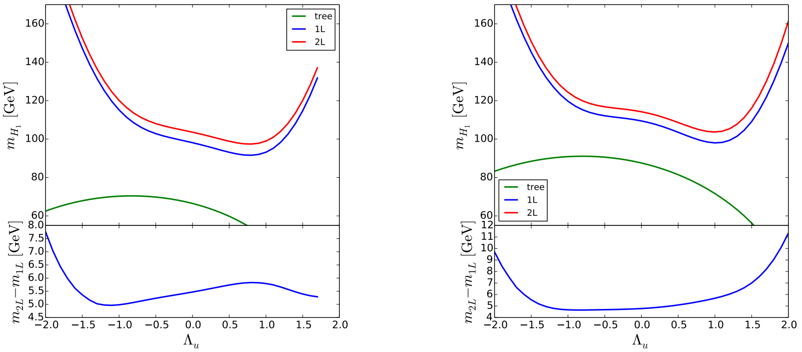
- Tree-level Higgs mass is usually suppressed
- No A-terms: stop corrections are suppressed
- New sgluon corrections at two-loop
- Other, potentially large couplings (depending on the model)

MRSSM: Higgs mass and fine-tuning

$$W = W_Y + \mu_D\,\hat{R}_d\hat{H}_d + \mu_U\,\hat{R}_u\hat{H}_u + \hat{S}(\lambda_d\,\hat{R}_d\hat{H}_d + \lambda_u\,\hat{R}_u\hat{H}_u) + \Lambda_d\,\hat{R}_d\hat{T}\,\hat{H}_d + \Lambda_u\,\hat{R}_u\hat{T}\,\hat{H}_u\ .$$

- New superpotential terms to increase Higgs mass

[Diessner,Kalinoswki,Kotlarski,Stöckinger,1504.05386]



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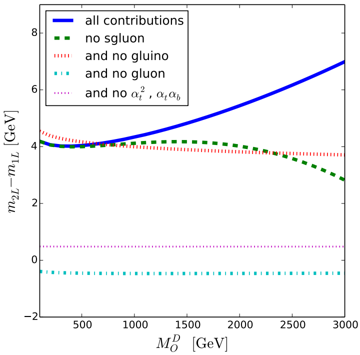

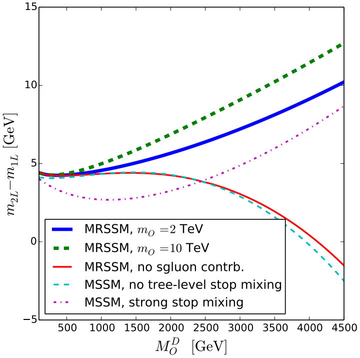

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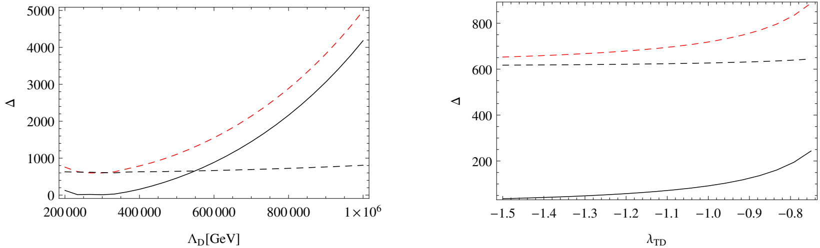
- New superpotential terms to increase Higgs mass
- New coloured corrections
- Single scale SUSY breaking

[Ding,Li,FS,Zhu,1510.01328]



$\Delta(\mu_U)$ (dashed red), $\Delta(\Lambda)$(dashed black), Δ^C(full black line)

Tiny FT if one assumes correlation between Λ and μ_U

$\rightarrow$ not easy to built concrete model to realise this ☹

Summary

- The MSSM, the measured Higgs mass and naturalness don't fit perfectly together
- There are many attractive and well motivated SUSY scenarios beyond the MSSM
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- There are many attractive and well motivated SUSY scenarios beyond the MSSM
- The Higgs mass prediction is very important in all of these models
- With the combination `SARAH/SPheno` one gets a Higgs mass precision similar to the MSSM
- Model-specific 1- and 2-loop corrections are needed to get trustworthy results (not only) for the fine-tuning

Backup

Summary of conditions proposed in literature I

In the case that large Yukawa couplings are involved the following two conditions are proposed

[Nilles,Srednicki,Wyler],[Alvarez-Gaume,Polchinski],[Derendinger,Savoy],[Claudson,Hall,Hinchliff]

- Stau VEVs:

$$A_\tau^2 < 3(m_{H_d}^2 + |\mu|^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2) \tag{1}$$

- Stop VEVs:

$$A_t^2 < 3(m_{H_u}^2 + |\mu|^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\,. \tag{2}$$

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A algorithm to constrain stop vacua in the limit $\tan\beta \to \infty$ was proposed in `hep-ph/0103341`, it results in

[C. Le Mouel]

$$A_t^2 < (0.65 - 0.85)^2(3(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + 2m_t^2)) \tag{3}$$

Summary of conditions proposed in literature II

For first and second generation of squarks the conditions are changed to [Casas, Lleyda, Munoz,hep-ph/9507294]

$$A_u^2 < 3(m_{H_u}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2)\,. \tag{4}$$

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$$A_u^2 < 3(m_{H_u}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2)\,. \quad (4)$$

This can be translated into a condition for the GUT parameters in the C(N)MSSM [Ellwanger, Hugonie,hep-ph/9902401]

$$(A_0 - 0.5 M_{1/2})^2 < 9 M_0^2 + 2.67 M_{1/2}^2 \quad (5)$$

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Validity

These two rules are derived under the assumption that no large Yukawa coupling is involved.

→ They were not supposed to be used for stops

→ They should be over-constraining if applied for stops

Summary of conditions proposed in literature III

For light staus additional conditions are proposed:

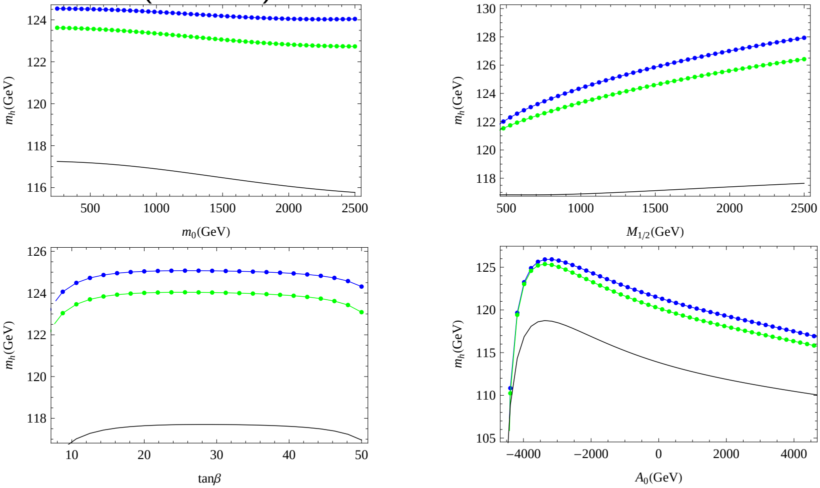
[Hisano,Sugiyama,1011.0260],[Kitahara,Yoshinaga,1303.0461]

$$\begin{aligned} \mu \tan\beta &< 213.5\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} - 1.30\times 10^4 \text{ GeV} \\ & -17.0(m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}) + 4.52\times 10^{-2} \text{ GeV}^{-1}(m_{\tilde{\tau}_L} - m_{\tilde{\tau}_R})^2 \quad (6) \\ |(Y_\tau v_u \mu)/(\sqrt{2} m_\tau)| &< 56.9\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} + 57.1(m_{\tilde{\tau}_L} + 1.03 m_{\tilde{\tau}_R}) - 1.28\times 10^4 \text{ GeV} \\ & + \frac{1.67\times 10^6 \text{ GeV}^2}{m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}} - 6.41\times 10^6 \text{ GeV}^3 \left(\frac{1}{m^2_{\tilde{\tau}_L}} + \frac{0.983}{m^2_{\tilde{\tau}_R}}\right) \quad (7) \end{aligned}$$

Assumptions

These conditions are derived for the large $\tan\beta$ limit in the pMSSM and neglect A-terms.

Validation I (MSSM)

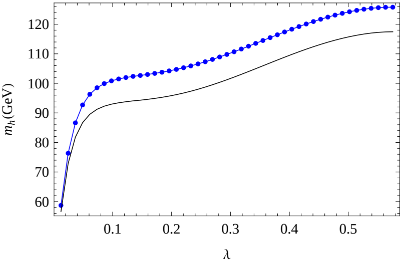


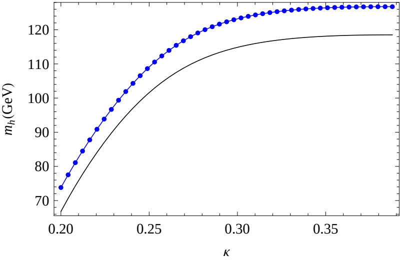
full lines: `SARAH`, dots: Brignole,Dedes,Degrassi, Slavich, Zwirner ([hep-ph/0112177,0206101,0212132,0305127])

1–loop / $\alpha_S(\alpha_b + \alpha_t)$ / full 2-loop

Validation II

real NMSSM:



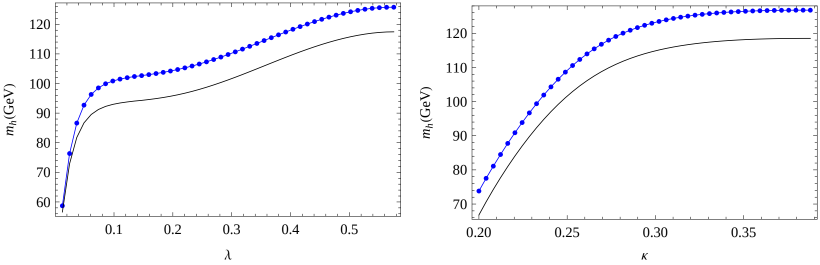


full lines: `SARAH`, dots: Degrassi, Slavich ([0907.4682])

1–loop / $\alpha_S(\alpha_b + \alpha_t)$

Validation II

real NMSSM:



full lines: `SARAH`, dots: Degrassi, Slavich ([0907.4682])

1–loop / $\alpha_S(\alpha_b + \alpha_t)$

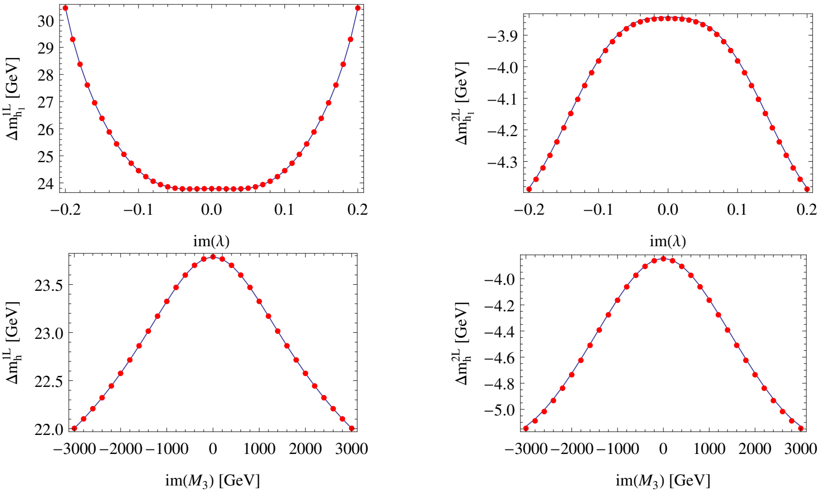
Dirac Gauginos:

full agreement with non-public code for $\alpha_S(\alpha_b + \alpha_t)$ corrections

[Goodsell,Slavich]

Validation III: complex NMSSM

[Goodsell,FS,1604.05335]



SPheno/SARAH, (modified) NMSSMCALC

Variants of models with Dirac gauginos

1. R-symmetry *not* conserved in Higgs sector (MDGSSM)

[Benakli,Goodsell,FS,1211.0552] [Braathen,Goodsell,Slavich,1606.09213]

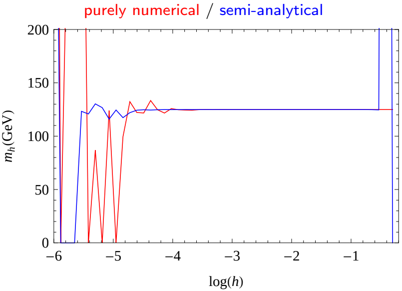
$$W = W_Y + \lambda_S \hat{S}\hat{H}_u \cdot \hat{H}_d + 2\lambda_T \hat{H}_d \cdot \hat{T}\hat{H}_u + \mu \hat{H}_u \cdot \hat{H}_d$$

2. R-symmetry conserved in Higgs sector (MRSSM)

[Diessner,Kalinoswki,Kotlarski,Stöckinger,1504.05386]

$$\begin{aligned} W =& W_Y + \mu_D \hat{R}_d \hat{H}_d + \mu_U \hat{R}_u \hat{H}_u + \hat{S}(\lambda_d \hat{R}_d \hat{H}_d + \lambda_u \hat{R}_u \hat{H}_u) \\ &+ \lambda_d^T \hat{R}_d \hat{T} \hat{H}_d + \lambda_u^T \hat{R}_u \hat{T} \hat{H}_u \quad . \end{aligned}$$

Numerical stability



- Numerical derivation dependence on initial step-size
  - There is a large plateau which can be used
  - we implemented a 'safe mode' which varies the step-size and checks the stability

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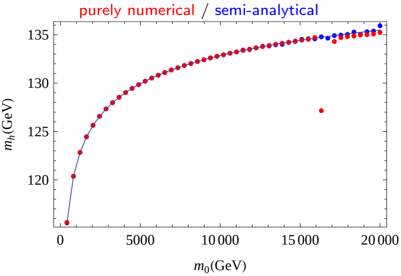

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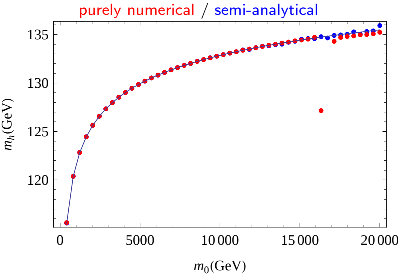

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  - There is a large plateau which can be used
  - we implemented a 'safe mode' which varies the step-size and checks the stability
- Numerics worse for $M_{SUSY} \gg v$ (No SUSY calculation should be used anyway!)
- Problems can appear for models with small VEVs (e.g. RpV)

Intrinsic problem of eff. pot. in Landau gauge

Goldstone boson catastrophe

The second derivative of the one-loop effective potential

$$V^{(1)} \sim (m^2)^2 \left[\log(m^2/Q^2) + c\right]$$

diverges for massless particles

$$\Pi^{(1)} \equiv \frac{\partial^2 V^{(1)}}{\partial m^2 \partial m^2} \to \infty \quad \text{for} \quad m^2 \to 0$$

At two-loop already the first derivative diverges for $m^2 \to 0$

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  $\to$ ew corrections are not considered in the MSSM at 2-loop

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Possible solution: include p^2 dependence

Higgs mass calculation for singlet extensions

$$W_{NMSSM} = W_Y + \lambda \, \hat{H}_d \hat{H}_u \hat{S} + \frac{1}{3} \kappa \, \hat{S}^3$$

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Two-loop corrections available for the NMSSM in public codes

- $\alpha_S \alpha_t$: `NMSSMCALC`
- $\alpha_S(\alpha_t + \alpha_b)$ & MSSM approx. for $\alpha_t(\alpha_t + \alpha_b)$, $(\alpha_b + \alpha_\tau)^2$: `NMSSMTools`, `FlexibleSUSY`, `SoftSUSY`
- $\alpha_s(\alpha_b + \alpha_t)$, $(\alpha_t + \alpha_b + \alpha_\lambda)^2$, $\alpha_\tau(\alpha_\tau + \alpha_b)$, $\alpha_\kappa(\alpha_\kappa + \alpha_\lambda)$: `SPheno`

Higgs mass predictions in the NMSSM of public codes

[FS,Ahtron,Ellwanger,Gröber,Mühlleitner,Slavich,Voigt,1507.05093]

Differences fully understood and due to

- threshold corrections
- renormalisation scheme
- Two-loop calculations

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- Two-loop calculations

	Q	$\tan\beta$	λ	κ	A_λ	A_κ	μ_{eff}	M_1	M_2	M_3	A_t	A_b	$m_{\tilde{t}_L}$	$m_{\tilde{t}_R}$
TP1	1500.	10.	0.1	0.1	-10.	-10.	900.	500.	1000.	3000.	3000.	0.	1500.	1500.
TP2	1500.	10.	0.05	0.1	-200.	-200.	1500.	1000.	2000.	2500.	-2900.	0.	2500.	500.
TP3	1000.	3.	0.67	0.1	650.	-10.	200.	200.	400.	2000.	1000.	1000.	1000.	1000.
TP4	750.	2.	0.67	0.2	405.	0.	200.	120.	200.	1500.	1000.	1000.	750.	750.
TP5	1500.	3.	0.67	0.2	570.	-25.	200.	135.	200.	1400.	0.	0.	1500.	1500.
TP6	1500.	3.	1.6	1.61	375.	-1605.	614.	200.	400.	2000.	0.	0.	1500.	1500.

SM-like Higgs mass:

	TP1	TP2	TP3	TP4	TP5	TP6
FlexibleSUSY	123.55	122.83	126.58	127.62	125.08	126.46
NMSSMCalc	120.34	118.57	124.86	126.37	123.14	123.45
NMSSMTOOLS	123.52	121.83	127.28	127.30	126.95	126.63
SOFTSUSY	123.84	123.08	126.59	127.52	125.12	126.67
SPHENO	124.84 ($\sim$0.0)	124.74 ($\sim$0.0)	126.77 (-0.5)	126.62 (-1.2)	125.61 (-0.3)	131.29 (+3.3)

Shift from additional two-loop corrections in SPheno/SARAH

Point	original	Y	g	v	modified
TP1	124.84	123.65	123.61	123.84	123.84
TP2	124.74	123.18	123.13	123.05	123.05
TP3	126.77	126.06	126.00	126.10	126.10
TP4	126.62	126.21	126.16	126.33	126.33
TP5	125.61	124.89	124.84	124.85	124.85
TP6	131.29	130.06	130.01	129.91	129.91

Table : The Higgs prediction for the SM-like Higgs mass by SPheno after applying successively the different adjustments for the Yukawas (Y), gauge couplings (g), and the electroweak VEV (v). Here, "original" refers to the results when using the code without any modification, while for "modified" all adjustments are turned on.

Point	original	Y	δ_1	δ_2	modified
TP1	120.34	124.41	124.85	124.85	124.85
TP2	118.57	123.31	123.82	123.82	123.82
TP3	124.86	127.55	127.50	127.56	127.56
TP4	126.37	128.32	128.18	128.23	128.23
TP5	123.14	126.21	126.03	126.12	126.12
TP6	123.45	127.26	127.55	127.73	127.73

Table : The prediction for the SM-like Higgs mass by NMSSMCalc after adjusting the DR Yukawa couplings (Y); performing a DR renormalization of the EW sector by changing the values of g_1, g_2 and v and removing the finite parts of the one-loop counter-terms δM_W, δM_Z and δe (δ_1); removing also the finite counterterm for v at the two-loop level (δ_2). The same conventions as for Tab. 1 are used.

Point	original	$Q' = Q$	Y	g	v	modified
TP1	123.52	123.52	123.96	123.99	123.84	123.84
TP2	121.83	121.44	123.46	123.52	123.08	123.08
TP3	127.28	127.28	127.43	127.43	126.60	126.60
TP4	127.30	127.30	127.13	127.07	127.52	127.52
TP5	126.95	126.95	127.34	127.45	125.12	125.12
TP6	126.63	126.63	127.56	127.66	126.67	126.67

Table : Changes in the prediction by `NMSSM-Tools` for the SM-like Higgs mass after forcing the SUSY scale and the scale for the mass calculation to be identical ($Q' = Q$), changing the Yukawa couplings (Y), the gauge couplings (g), and the electroweak VEV (v). The same conventions as for Tab. 1 are used.

Summary of conditions proposed in literature I

In the case that large Yukawa couplings are involved the following two conditions are proposed

[Nilles,Srednicki,Wyler],[Alvarez-Gaume,Polchinski],[Derendinger,Savoy],[Claudson,Hall,Hinchliff]

- Stau VEVs:

$$A_\tau^2 < 3(m_{H_d}^2 + |\mu|^2 + m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2) \tag{8}$$

- Stop VEVs:

$$A_t^2 < 3(m_{H_u}^2 + |\mu|^2 + m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2)\,. \tag{9}$$

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A algorithm to constrain stop vacua in the limit $\tan\beta \to \infty$ was proposed in `hep-ph/0103341`, it results in

[C. Le Mouel]

$$A_t^2 < (0.65 - 0.85)^2(3(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2 + 2m_t^2)) \quad (10)$$

Summary of conditions proposed in literature II

For first and second generation of squarks the conditions are changed to [Casas, Lleyda, Munoz,hep-ph/9507294]

$$A_u^2 < 3(m_{H_u}^2 + m_{\tilde{u}_L}^2 + m_{\tilde{u}_R}^2)\,. \tag{11}$$

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This can be translated into a condition for the GUT parameters in the C(N)MSSM [Ellwanger, Hugonie,hep-ph/9902401]

$$(A_0 - 0.5M_{1/2})^2 < 9M_0^2 + 2.67M_{1/2}^2 \tag{12}$$

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Validity

These two rules are derived under the assumption that no large Yukawa coupling is involved.

$\rightarrow$ They were not supposed to be used for stops

$\rightarrow$ They should be over-constraining if applied for stops

Summary of conditions proposed in literature III

For light staus additional conditions are proposed:

[Hisano,Sugiyama,1011.0260],[Kitahara,Yoshinaga,1303.0461]

$$\mu \tan\beta < 213.5\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} - 1.30 \times 10^4 \text{ GeV}$$
$$-17.0(m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}) + 4.52 \times 10^{-2} \text{ GeV}^{-1} (m_{\tilde{\tau}_L} - m_{\tilde{\tau}_R})^2 \quad (13)$$

$$|(Y_\tau v_u \mu)/(\sqrt{2} m_\tau)| < 56.9\sqrt{m_{\tilde{\tau}_L} m_{\tilde{\tau}_R}} + 57.1(m_{\tilde{\tau}_L} + 1.03 m_{\tilde{\tau}_R}) - 1.28 \times 10^4 \text{ GeV}$$
$$+\frac{1.67 \times 10^6 \text{ GeV}^2}{m_{\tilde{\tau}_L} + m_{\tilde{\tau}_R}} - 6.41 \times 10^6 \text{ GeV}^3 \left(\frac{1}{m^2_{\tilde{\tau}_L}} + \frac{0.983}{m^2_{\tilde{\tau}_R}}\right) \quad (14)$$

Assumptions

These conditions are derived for the large $\tan\beta$ limit in the pMSSM and neglect A-terms.

Vevacious

Vevacious ... [Camargo-Molina,O'Leary, Porod, FS,1307.1477]

...is a tool to find the global minimum of the 1-loop effective potential and checks the stability of the 'correct' vacuum.

- Written in Python and C++; includes LHPC [O'Leary]
- Makes use of HOM4PS, pyminuit and CosmoTransitions

[Lee, Yi, Tsai,Computing, 83, pp109-133],[Wainwright,1109.4189]]

vevacious.hepforge.org

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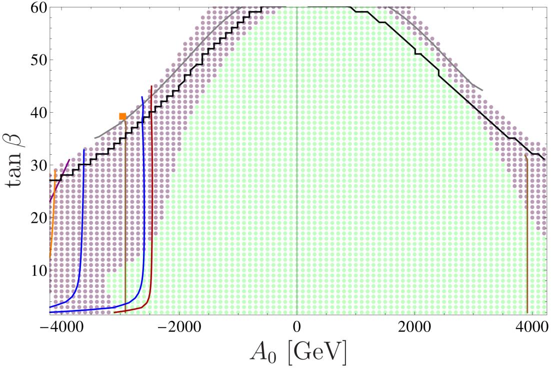
vevacious.hepforge.org

If you use it, you'll see that thumb rules like

$$A_t^2 < 3(m_{H_u}^2 + m_{\tilde{t}_l}^2 + m_{\tilde{t}_R}^2)$$

are not sufficient to identify CCB vacua in the MSSM! [1309.7212,1405.7376]

Vevacious vs. analytical conditions



[Camargo-Molina, O'Leary,Porod,FS,1309.7212]

$$m_0 = 1\text{TeV}$$
$$M_{1/2} = 1 \text{ TeV}$$
$$\mu > 0$$

- (1): $[A_t]$
- (2): $[A_\tau]$
- (3): $[c \cdot A_t]$
- (4): $[A_u]$
- (5): $[A_0]$
- (6): $[\mu \tan\beta]$
- (7): $[Y_\tau]$
- $\tilde{\chi}^0$ LSP
- BFP of 1204.4199