# CAMB/MGCAMB/EFTCAMB

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IFT, Madrid
14 March 2017

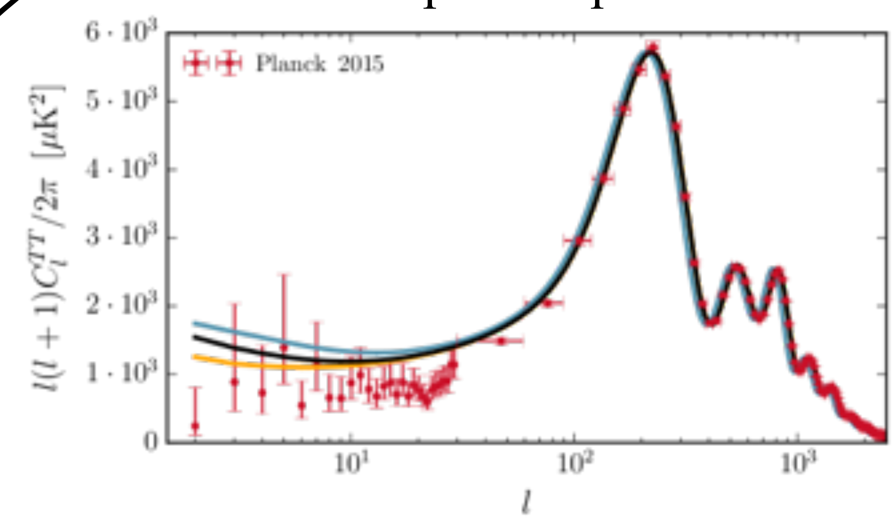
The 2nd School on
Cosmology Tools

# CAMB by A. Lewis & A. Challinor

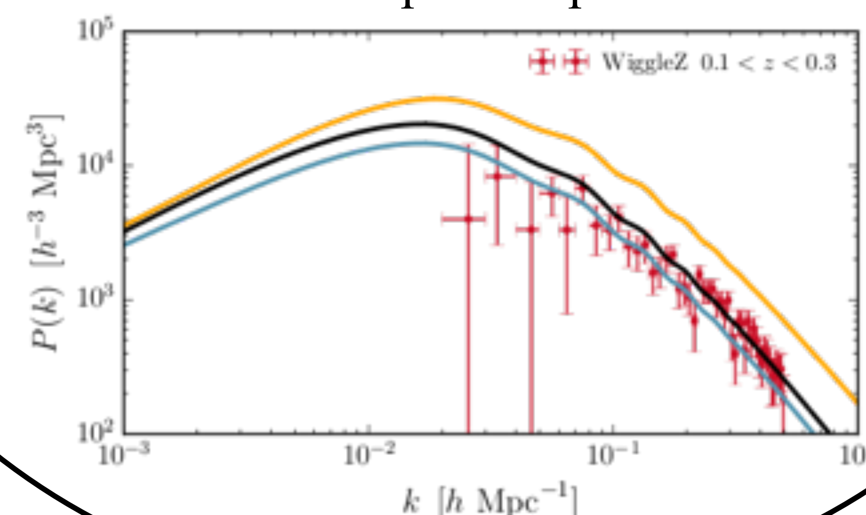
ICs for matter and metric perturbations
+
theory

EINSTEIN-BOLTZMANN SOLVER

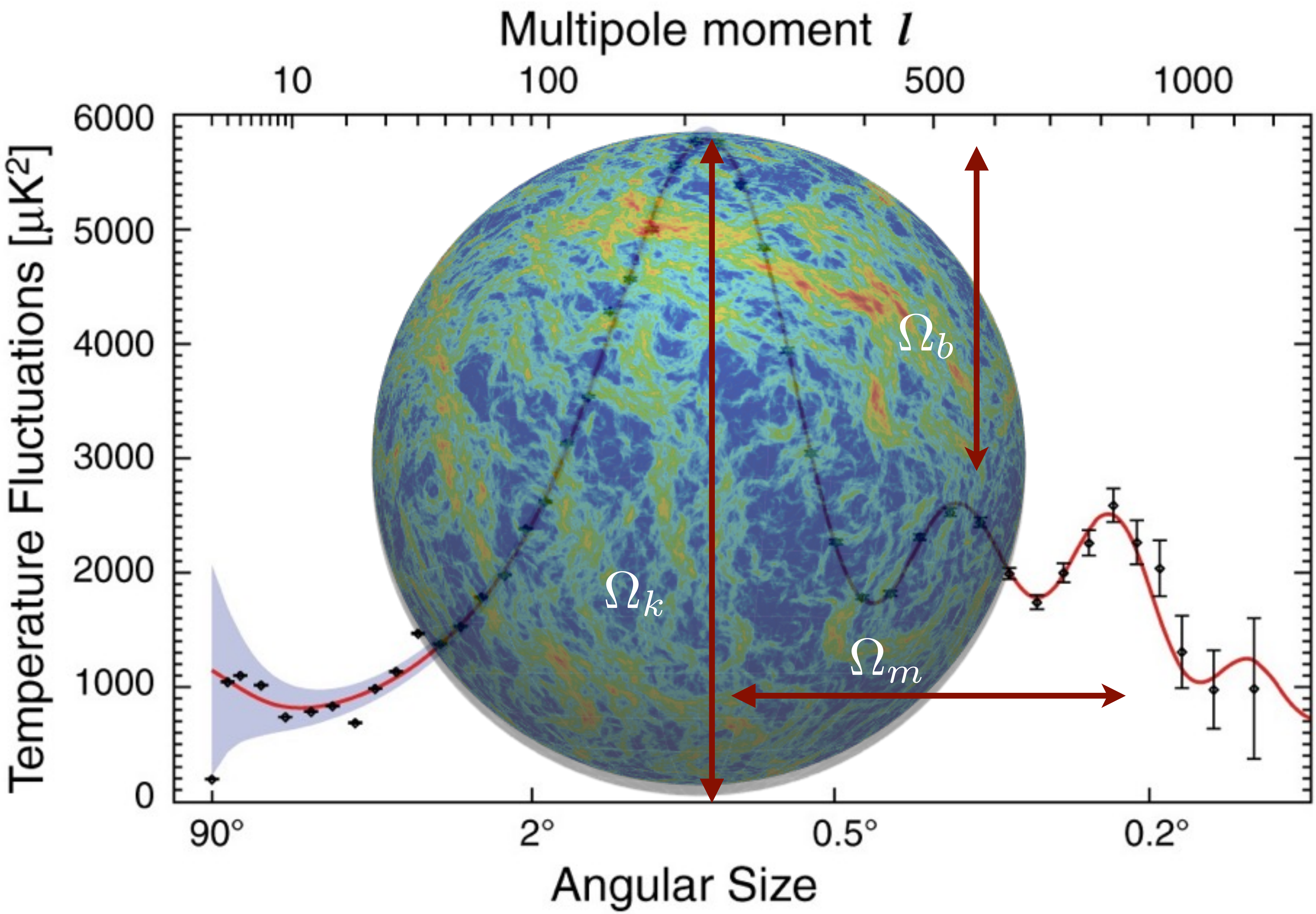
CMB power spectrum




matter power spectrum



# The success story of the CMB



# Outlook

1. CAMB

   1.1 General Introduction

2. MGCAMB

   2.1 Structure & Features

   2.2 Examples

3. EFTCAMB

   3.1 Structure & Features

   3.2 Examples

# CAMB

**C**ode for **A**nisotropies in the **M**icrowave **B**ackground

by A. Lewis & A. Challinor (November 1999)

camb.info

# CAMB Facts

Introduced in 1999, originally based on CMBFAST by Seljak & Zaldarriaga, itself based on a Boltzmann code by Bertschinger, Ma and Bode.

Ma&Bertschinger ‘Cosmological Perturbations in Synchronous and Newtonian Gauge’ is a very useful guide. (ApJ 455, 1995).

CAMB Notes: http://cosmologist.info/notes/CAMB.pdf

In continuous evolution, latest version from January 2017.

Written in Fortran 90, needs gfortran or fort Intel compiler.

Equations are in *synchronous* gauge (with zero CDM velocity), and conformal time

# MAIN VARIABLES & EQUATIONS

METRIC VARIABLES & EINSTEIN EQS.:

main variable, evolved:

$$etak = k\eta$$

additional variables: $2k\mathcal{Z} = \dot{h}$ perturbation to expansion rate

$$\sigma = \frac{\dot{h} + 6\dot{\eta}}{2k}$$ shear

Equations:

$$k^2\eta = k\mathcal{H}\mathcal{Z} - \frac{1}{2}\kappa\rho\Delta\,,$$

$$\frac{2}{3}k^2\left(\sigma - \mathcal{Z}\right) = \kappa\rho q$$

where $\rho q = \left(\rho + P\right)v$

# WHERE IS THE PHYSICS

Angular power spectra, e.g. scalar contribution to the TT spectrum:

$$C_\ell^{TT} = (4\pi)^2 \int k^2 dk\, P(k) |\Delta_\ell^T(k)|^2$$

where the angular transfer function is:

physics goes in here!

$$\Delta_\ell^T(k) = \int_0^{\tau_0} d\tau\, S_T(k,\tau) j_\ell(k\tau)$$

and the source term is:

$$S_T(k,\tau) = g\left(\Delta_{T0} + 2\frac{\dot\sigma}{k} + \frac{\dot v_b}{k} + \frac{\Pi}{4} + \frac{3\ddot\Pi}{4k^2}\right) + \frac{e^{-\kappa}}{k}(k\dot\eta + \ddot\sigma) + \dot g\left(\frac{\sigma}{k} + \frac{v_b}{k} + \frac{3\dot\Pi}{2k^2}\right) + \frac{3\ddot g \Pi}{4k^2}$$

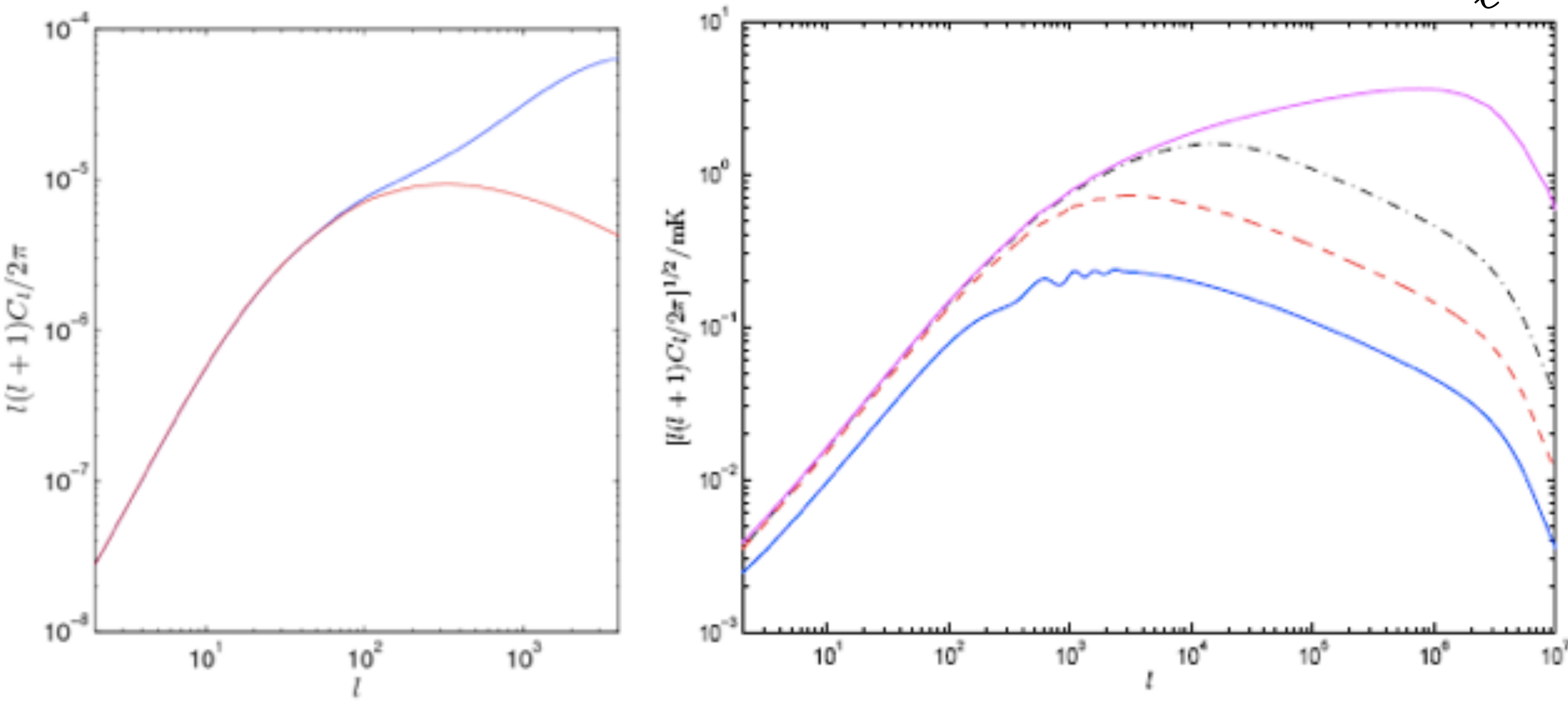
this is the ISW contribution,
and it is where MG will matter most!

$\Pi = \Delta_{T2} + \Delta_{P2} + \Delta_{P0}$ polarization tensor

# CAMB Sources, CosmoMC and Halofit

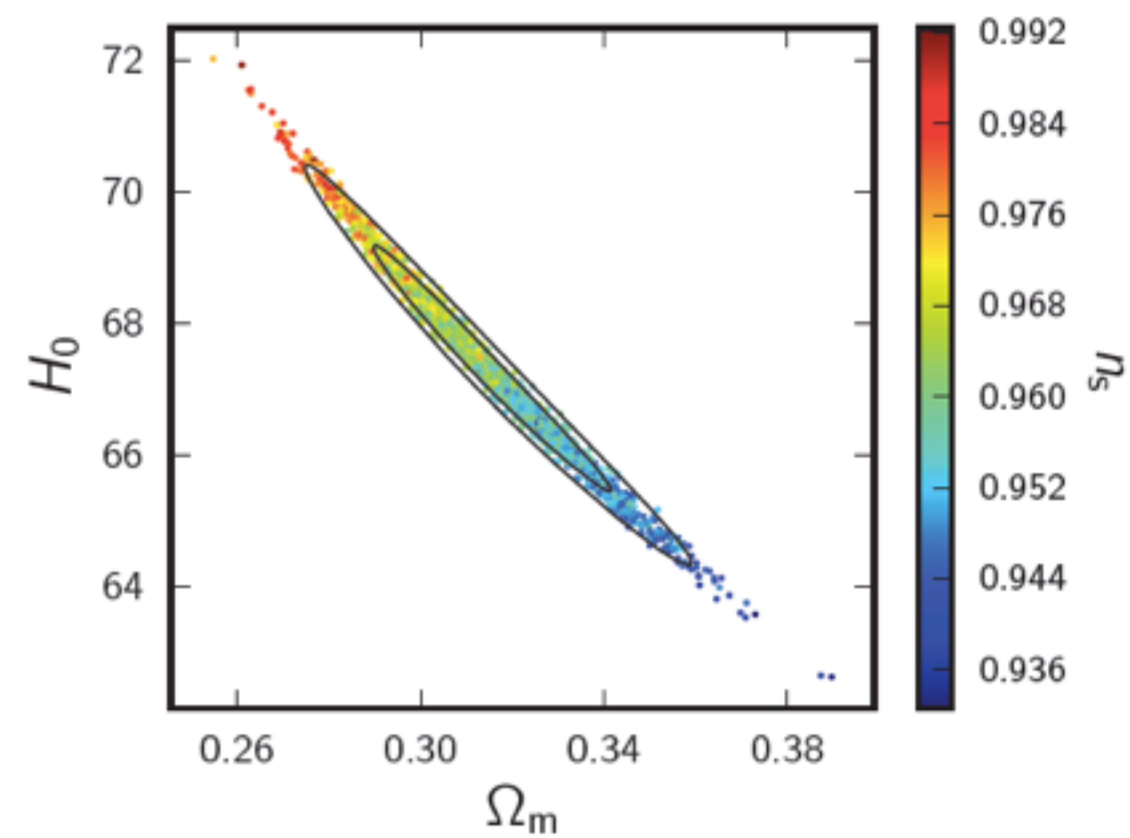
**CAMB sources: for number counts, lensing and dark-age 21cm power spectra**
plus thermal history, perturbed recombination, CMB cross-correlations

$C_\ell^{XY}$




non-linear corrections from Halofit model (LCDM based). (arXiv:1208.2701)

**Cosmological MonteCarlo**




Samples from Planck likelihood, with contours for Planck+WMAP polarization

# MGCAMB

**M**odification of **G**rowth with CAMB

by A. Hojjati, G. Zhao, L. Pogosian, A.Silvestri (April 2009)

& A. Zucca

http://aliojjati.github.io/MGCAMB/mgcamb.html


Designed to explore the phenomenology of linear *scalar* perturbations in Modified Gravity.

Framework naturally defined in Newtonian gauge, then translated into synchronous gauge for implementation in CAMB.

Involves two generic functions of time and scale, to model departures from LCDM at the level of LSS.

# The framework: (μ,γ)

$$\boxed{ds^2 = -a^2(\tau)\left[(1+2\Psi(\tau,\vec{x}))\,d\tau^2 - (1-2\Phi(\tau,\vec{x}))\,d\vec{x}^2\right]}$$

Continuity and Euler eqs.

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \begin{aligned} \delta' + \frac{k}{aH}v - 3\Phi' &= 0 \\ v' + v - \frac{k}{aH}\Psi &= 0 \end{aligned}$$

Einstein eqs.

Poisson: $$k^2\Psi = -\mu(a,k)\frac{a^2}{2M_P^2}\rho\Delta$$

anisotropy: $$\frac{\Phi}{\Psi} = \gamma(a,k)$$

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Poisson:

$$k^2\Psi = -\mu(a,k)\frac{a^2}{2M_P^2}\left\{\rho\Delta + 3\left(\rho+P\right)\sigma\right\}$$

anisotropy:

$$k^2\left[\Phi - \gamma(a,k)\Psi\right] = \mu(a,k)\frac{3a^2}{2M_P^2}\left(\rho+P\right)\sigma$$

Hojjati, Pogosian, Zhao, JCAP 1108:005 (2011)

# The framework: ($\mu$,$\gamma$)

* This is a consistent set of equations for the evolution of perturbations that can be incorporated into std Boltzmann codes, like CAMB.

* Solutions of linear cosmological perturbations in any particular theory can be expressed in terms of $\mu$ and $\gamma$; moreover, on sub-horizon scales they can have particularly simple forms.

* Everything that observations can tell us about the growth of structure can be stored as a measurement of $\mu$ and $\gamma$ (and projected onto solutions of specific models if needed).

* They allow us to perform consistency tests of GR *as well as* exploring allowed parameter space of alternative models.

# Alternative functions

lensing: $k^2\left(\Phi+\Psi\right)=-\Sigma(a,k)\dfrac{a^2}{M_P^2}\rho\Delta$ $\qquad (\mu,\gamma)\rightarrow(\mu,\Sigma)$

$$\mu \leftrightarrow G_{\mathrm{matter}}$$

$$\Sigma \leftrightarrow G_{\mathrm{light}}$$

$$\gamma \leftrightarrow \varpi = \frac{1}{\gamma}-1$$

$$\gamma \leftrightarrow \eta$$

many references ...

Pogosian et al., PRD (2010)

# Implementation in CAMB

In synchronous gauge, the framework reads: *

$$k^2(\dot{\alpha} + \mathcal{H}\alpha) = -\frac{\kappa}{2}\mu(k,a)\{\rho\Delta + 3(\rho+P)\sigma\}\,,$$
$$\eta - \mathcal{H}\alpha - \gamma(\dot{\alpha} + \mathcal{H}\alpha) = \frac{3\kappa}{2k^2}\mu(\rho+P)\sigma\,,$$

One can manipulate the equations to get:

$$\alpha = \left\{\eta + \frac{\mu\kappa}{2k^2}\left[\gamma\rho\Delta + 3(\gamma-1)(\rho+P)\sigma\right]\right\}/\mathcal{H}\ ,$$

$$\begin{aligned}\dot{\eta} \;=\; & \frac{\kappa\rho}{2\mathcal{D}}\{(1+w)\left[\mu\gamma\theta\left(1+\frac{3\kappa\rho}{2k^2}(1+w)\right) + k^2\alpha(\mu\gamma-1)\right] + \Delta\left[\mu(\gamma-1)\mathcal{H} - \dot{\mu}\gamma - \dot{\gamma}\mu\right] \\ & + 3\dot{\sigma}(1+w)(1-\gamma)\mu + 3\sigma(1+w)\left[3w\mu(\gamma-1)\mathcal{H} - (\gamma-1)\dot{\mu} - \mu\dot{\gamma}\right]\}\ ,\end{aligned}$$

where $\mathcal{D}$ is

$$\mathcal{D} = k^2 + \frac{3\kappa}{2}\gamma\mu\rho(1+w)\ .$$

And then Z can be worked out easily.

* using notation of Hojjati et al., JCAP 1108 (2011), slightly different from CAMB one: $k\alpha = \sigma$

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**ISW**

$$\dot{\eta}+\ddot{\alpha} = \frac{\kappa}{2k^2}\left\{-\left[(\gamma+1)(\dot{\rho}\Delta+\rho\dot{\Delta}) + \gamma\frac{3}{2}(\rho+P)\dot{\sigma} + \gamma\frac{3}{2}(\dot{\rho}+\dot{P})\sigma\right] + \dot{\gamma}\mu\left[(\rho\Delta)+\frac{3}{2}(\rho+P)\sigma\right]\right\}\ ,$$

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# Choices for (μ,γ)

What to do with μ and γ themselves?

- pick a specific functional form

$$\mu = \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$

CFHTLenS:F. Simpson et al., arXiv: 1212.3339

$$\mu = \mu_0 + \frac{1-\mu_0}{2}\left(1 + \tanh\frac{z - z_s}{\Delta z}\right)$$

Zhao et al., Phys. Rev. D 81, 103510 (2010)

- QSA:

$$\mu = \frac{1 + \beta_1 \lambda_1^2 k^2 a^s}{1 + \lambda_1^2 k^2 a^s}$$

Bertschinger & Zukin, Phys. Rev. D 78, 024015(2008)

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$$\Phi_{\rm Yuk} \sim \frac{1}{r}\left[1 + (\beta_1 - 1)\, e^{-r/\lambda_1}\right]$$

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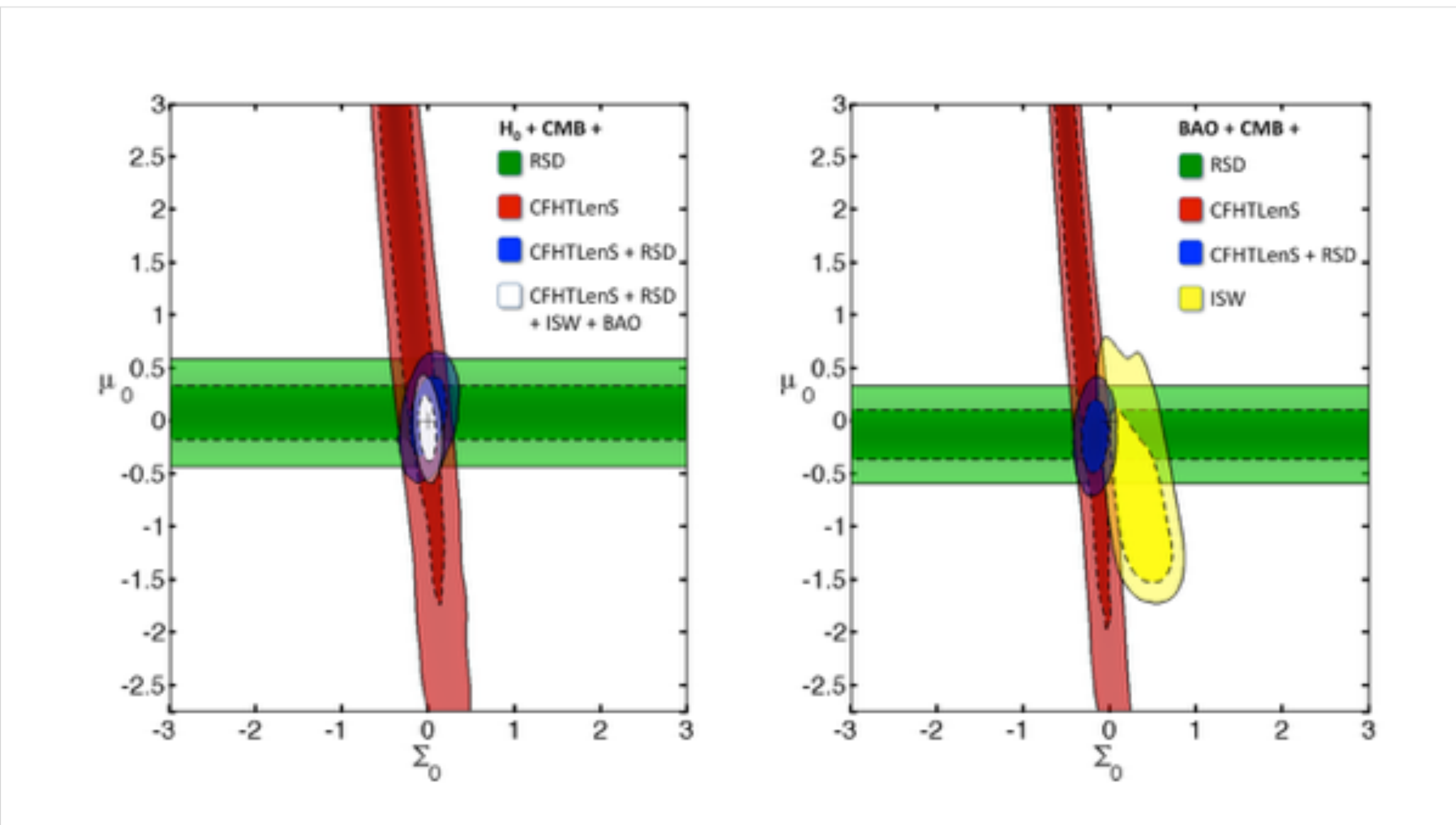
- bin them in time and space and constrain directly the resulting parameters or perform a 2D PCA (which is a very useful forecast tool)

- QSA: fix their scale-dependence, according to general arguments of locality and then perform a 1D PCA on the time-dependence

# Choices for (μ,γ): time-dependent

$$\Sigma(a) = \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_\Lambda}$$

$$\Sigma(a) = \Sigma_1 \cdot a$$




'CFHTLenS: Testing the Laws of Gravity with Tomographic Weak Lensing and Redshift Space Distortions'
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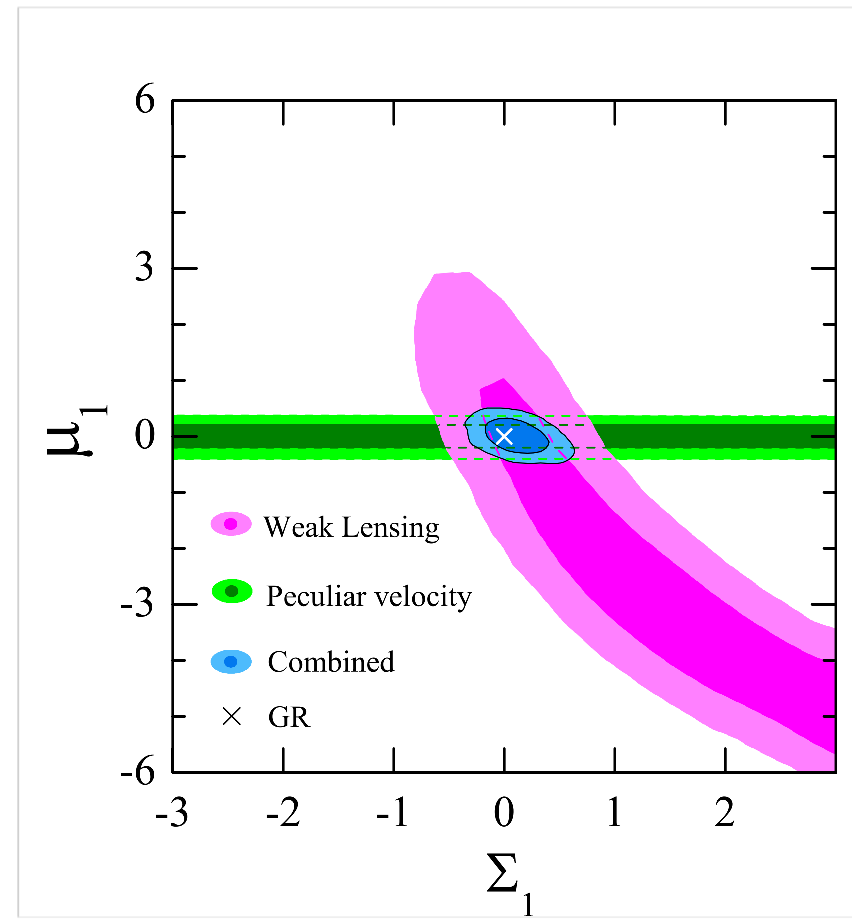


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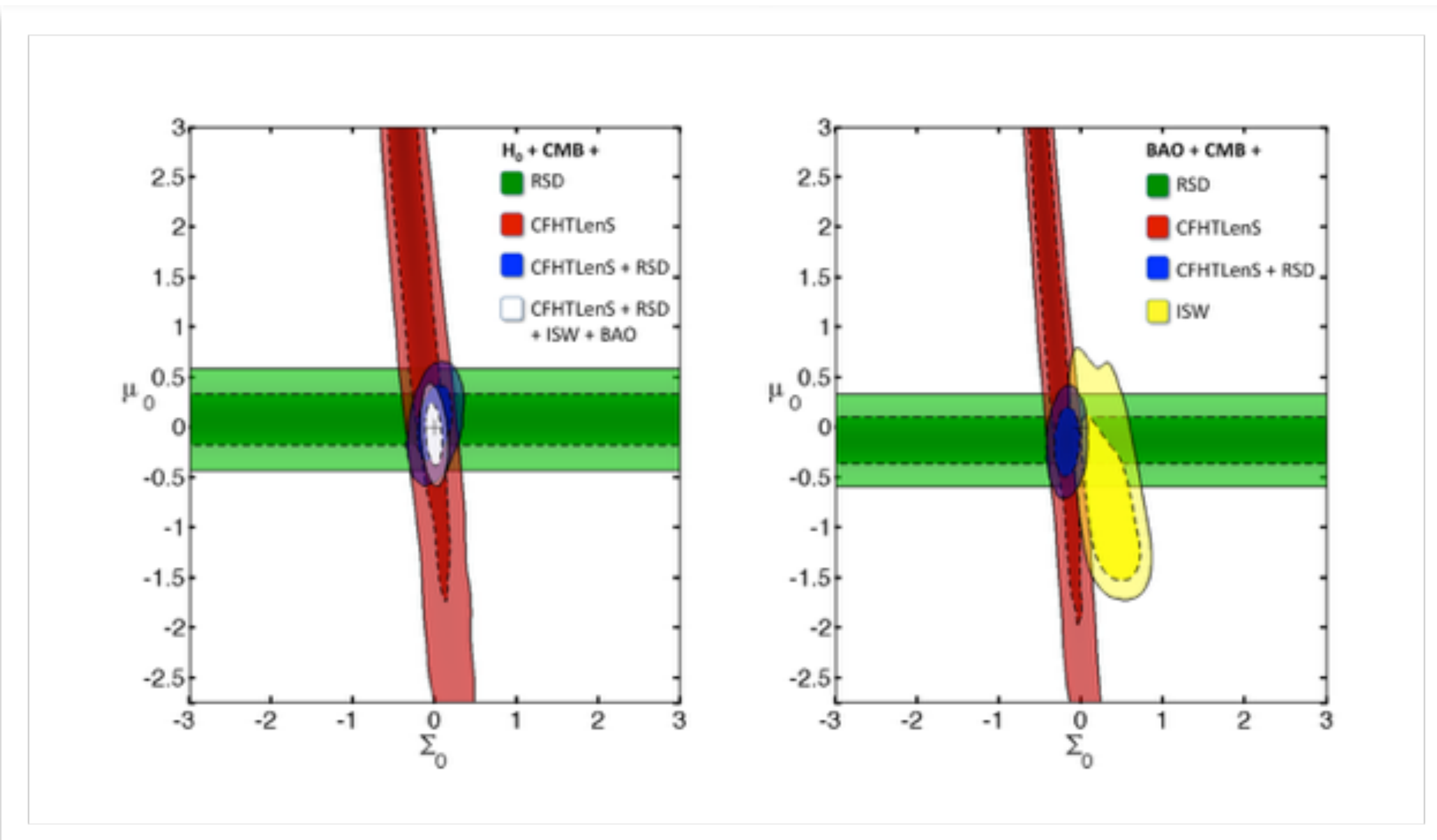


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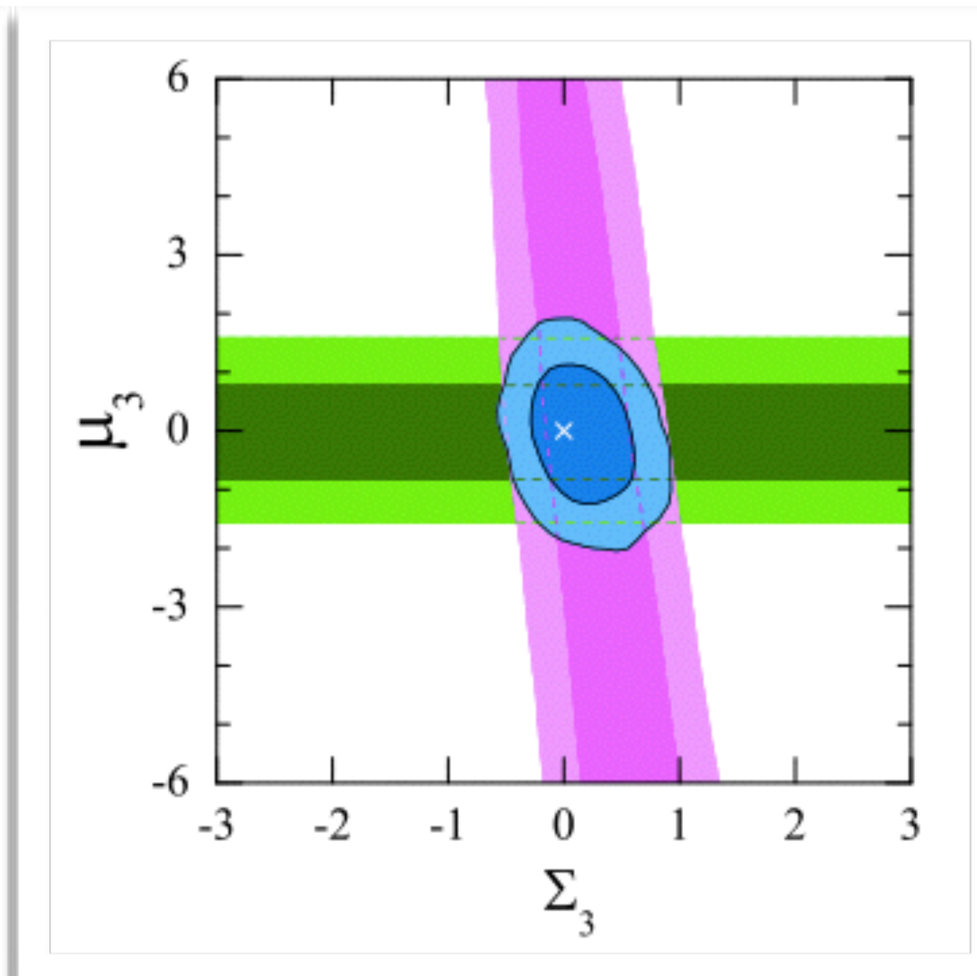


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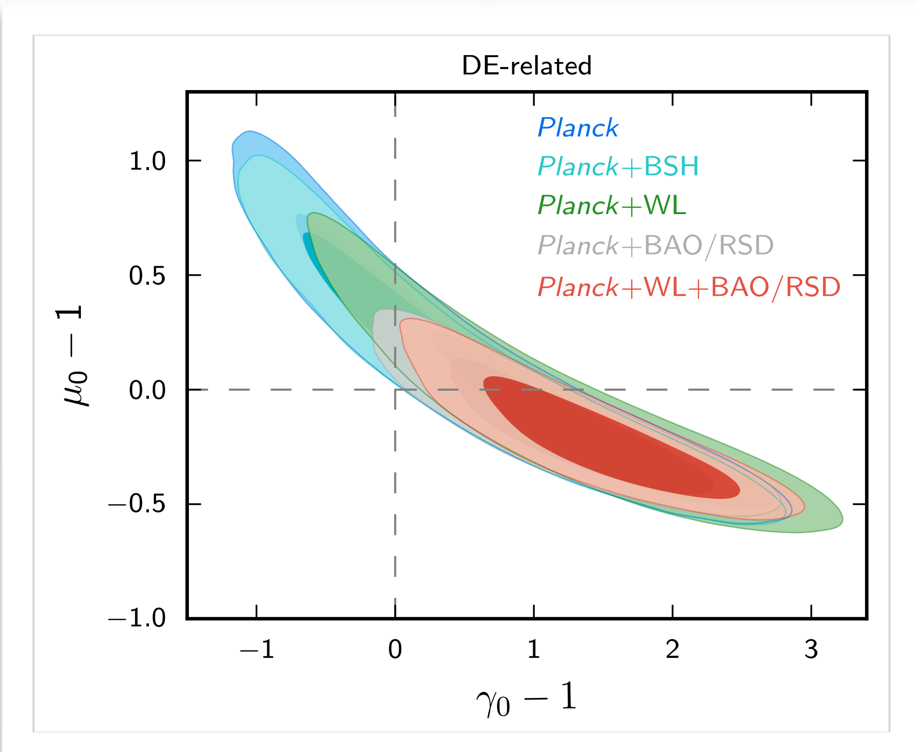
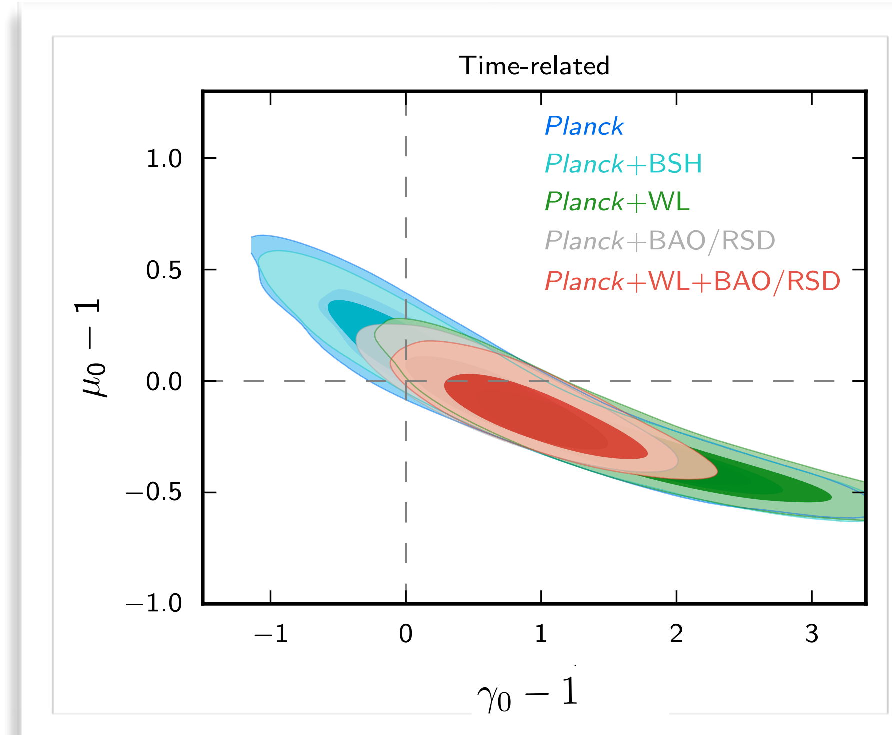
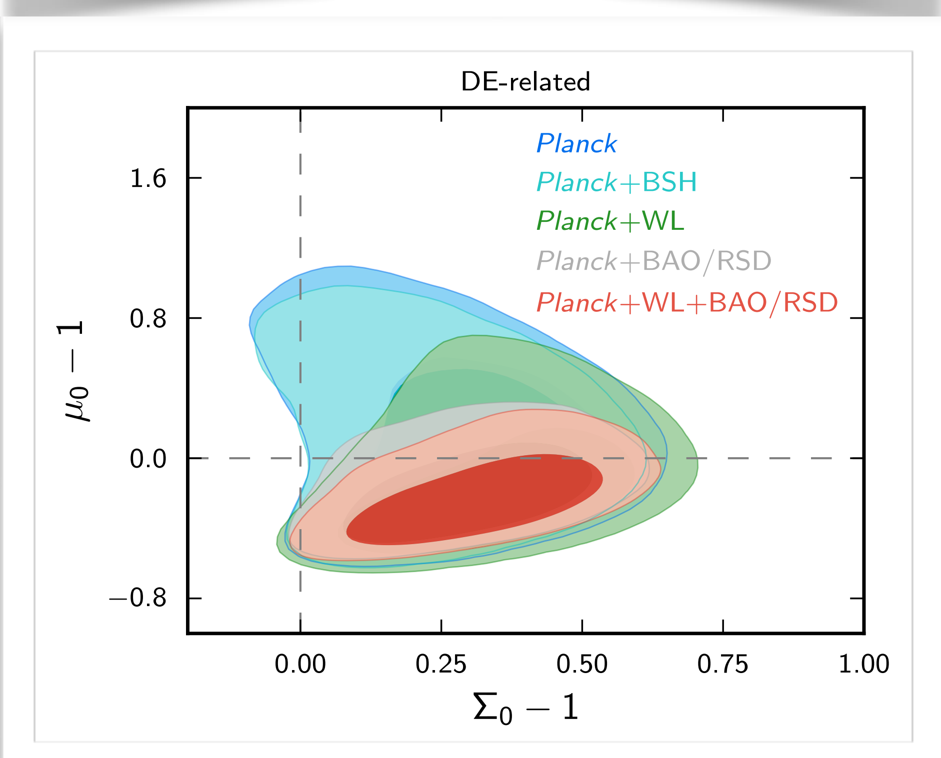
$$\mu(a) - 1 = (\mu_0 - 1)\Omega_{\rm DE}(a)$$




$$\Omega_{\rm DE}(a) = \Omega^0_{\rm DE}\frac{H_0^2}{H^2}$$

$$\mu(a) - 1 = \mu_0 + \mu_1(1-a)$$

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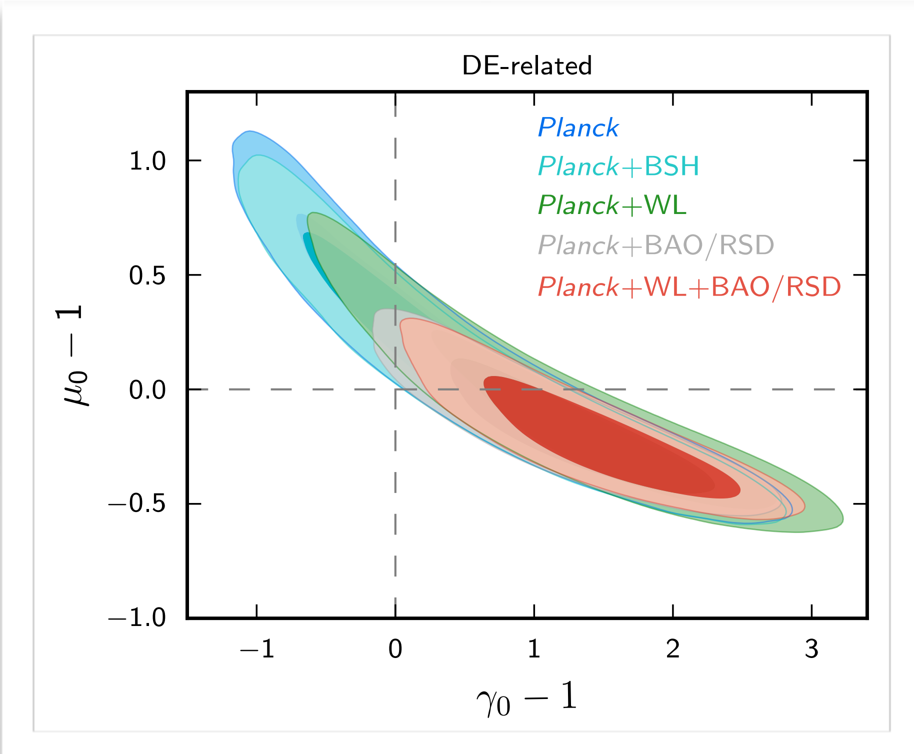


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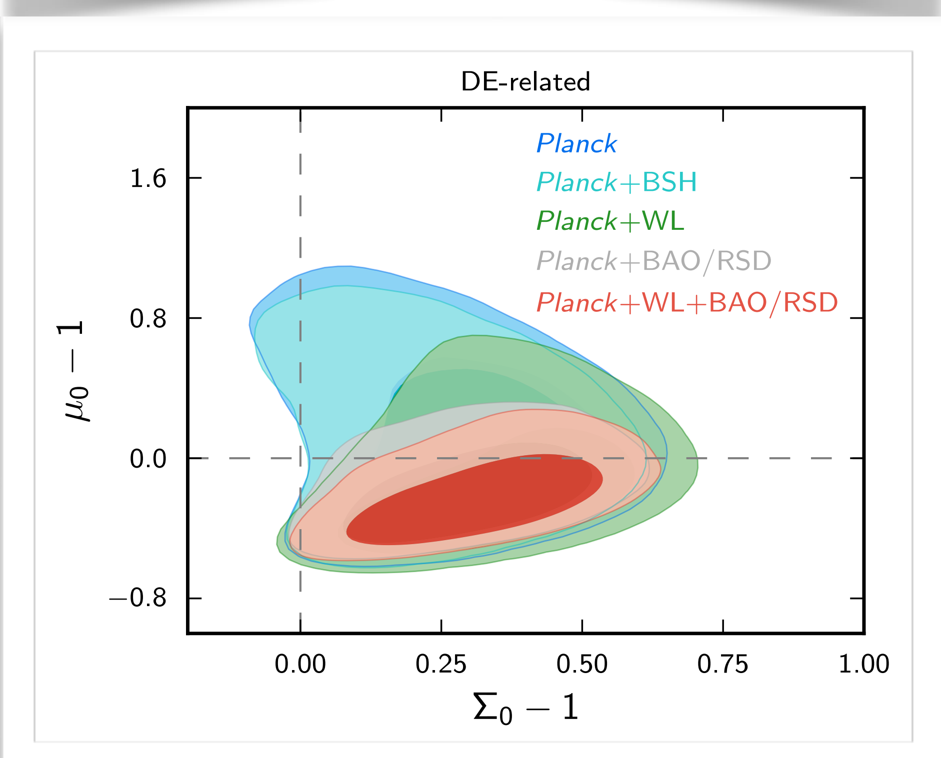
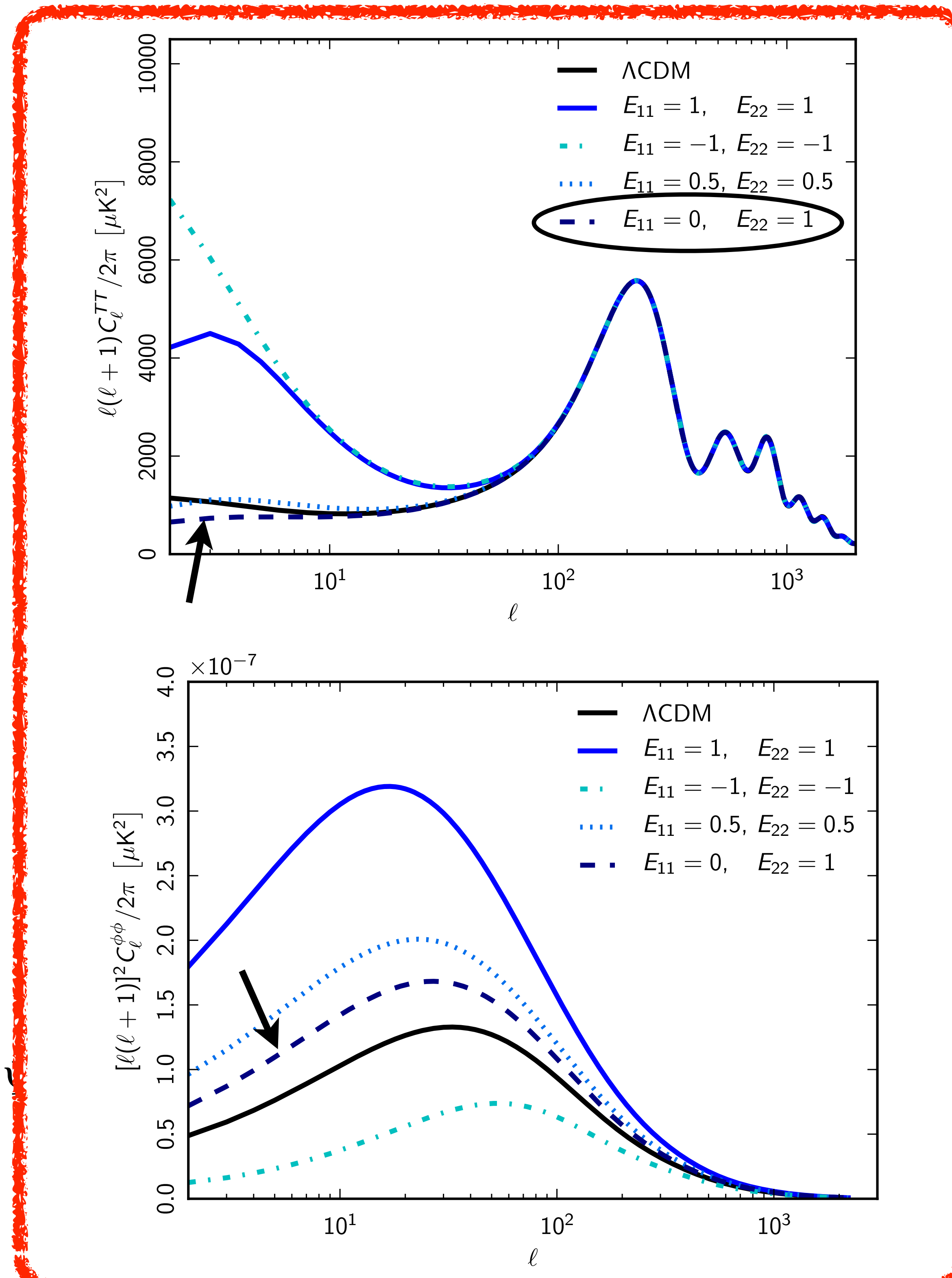



$$k^2\,(\Phi +$$



# Choices for (μ,γ): QSA

The way in which (μ,γ) are defined, they are very general, valid on all linear scales, and they do not imply any approximation.

However, if you want to specialize to a given model, you need either to work with a numerical reconstruction of (μ,γ), which implies first solving for the perturbations; or you can assume the *Quasi-Static* regime and find analytical expressions for (μ,γ), before solving anything.

Well-known examples are:

f(R)

$$\mu = \frac{1}{1+f_R}\,\frac{1+4\frac{f_{RR}}{1+f_R}\frac{k^2}{a^2}}{1+3\frac{f_{RR}}{1+f_R}\frac{k^2}{a^2}} \qquad \gamma = \frac{1+2\frac{f_{RR}}{1+f_R}\frac{k^2}{a^2}}{1+4\frac{f_{RR}}{1+f_R}\frac{k^2}{a^2}}$$

Pogosian & Silvestri, PRD 77 (2008)

Chameleon-type

$$\mu = e^{-\kappa\alpha}\,\frac{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\frac{k^2}{a^2m^2}} \qquad \gamma = \frac{1+\left(1-\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}{1+\left(1+\frac{1}{2}\alpha'^2\right)\frac{k^2}{a^2m^2}}$$

Zhao et al., PRD 79 (2009)

Alternatively, the BZ parametrization:

$$\mu = \frac{1+\beta_1\lambda_1^2k^2a^s}{1+\lambda_1^2k^2a^s} \qquad \gamma = \frac{1+\beta_2\lambda_2^2k^2a^s}{1+\lambda_2^2k^2a^s}$$

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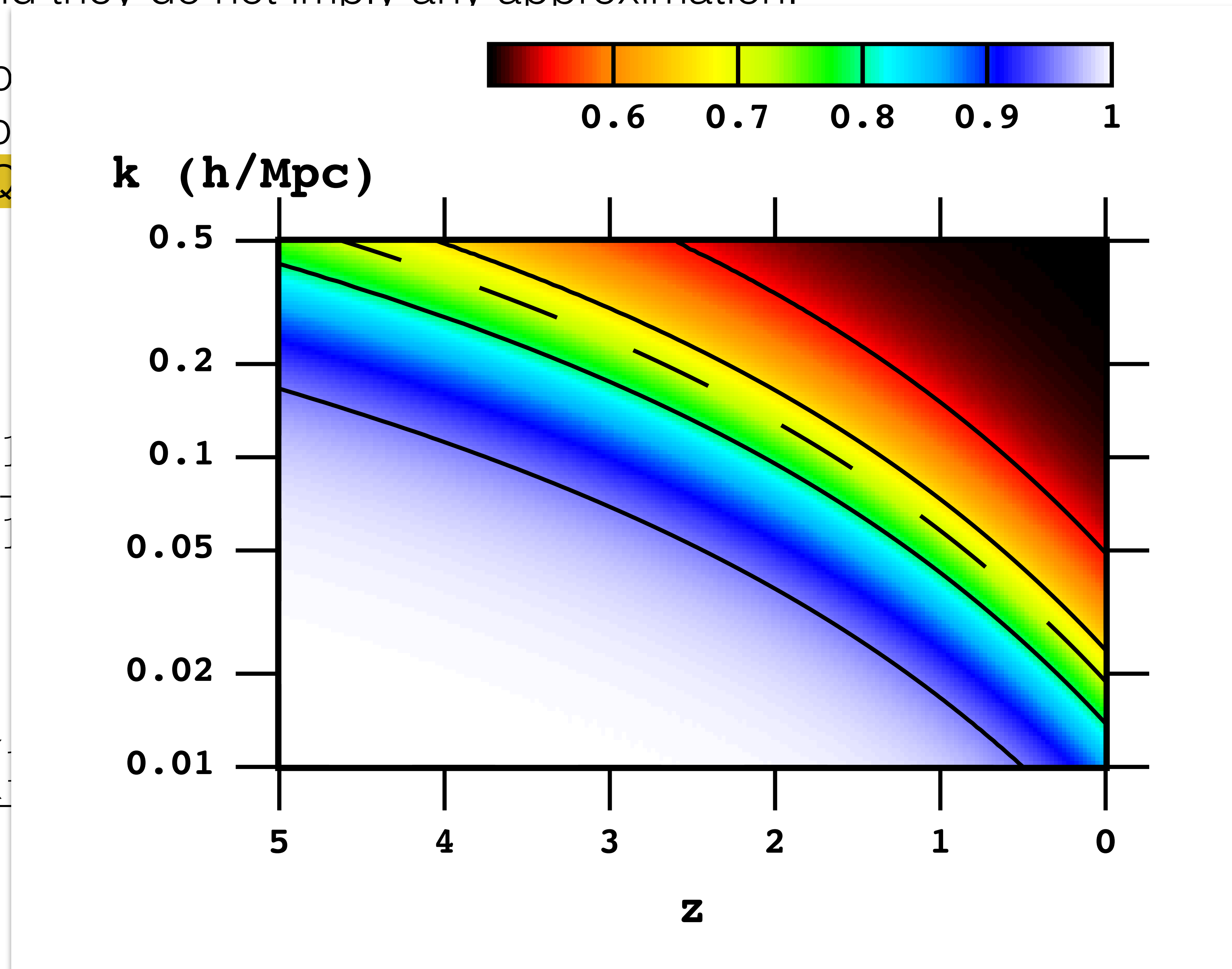


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Bertschinger & Zukin, Phys. Rev. D 78, 024015(2008)

# Choices for (μ,γ): QSA

The k-dependence of these functions should not be completely arbitrary if we wish to consider local covariant theories with equations of motion derived from a variational principle.

For models with one scalar obeying 2nd order eoms, (this includes non-minimal coupling and theories with functions of Lovelock invariants), μ and γ reduce to:

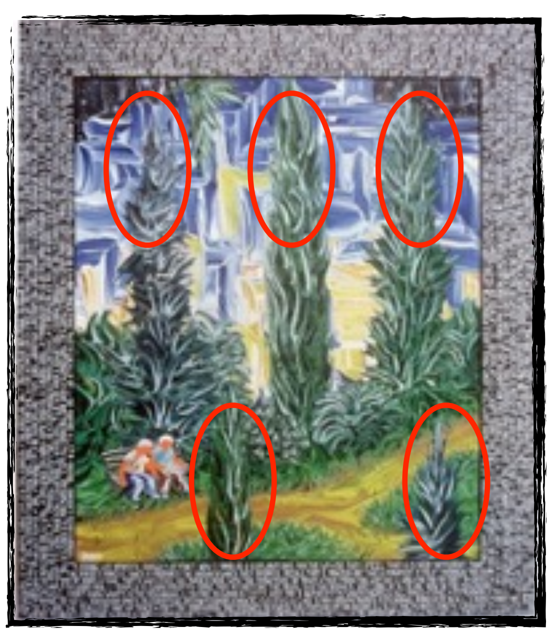
$$\gamma = \frac{p_1(t) + p_2(t)k^2}{1 + p_3(t)k^2}$$

it allows for some near and super-horizon modifications

Horndeski Theories

$$\mu = \frac{1 + p_3(t)k^2}{p_4(t) + p_5(t)k^2}$$

$$\{p_1(t), p_2(t), p_3(t), p_4(t), p_5(t)\}$$



*Talking about gravity*, G. Horndeski


De Felice, Kobayashi,Tsujikawa, Phys.Lett.B 706 (2011)    Gleyzes, Langlois, Piazza, Vernizzi, JCAP 1308(2013)026    Bloomfield, JCAP 12(2013)044

Amendola, Kunz, Motta, Saltas, Sawicki, Phys.Rev. D87, 023501 (2013)    Baker, Ferreira , Skordis, Phys.Rev. D87, 024015 (2013)

# Built-in choices

general functions of time/binning

growth index $\Omega_m(a)^\gamma$

Bertschinger-Zukin (BZ)

f(R) a la' BZ*

Hu-Sawicki f(R)

Chameleon

Dilaton

Generalized Dilaton

Symmetron

# Built-in choices

general functions of ti

growth index

Bertschinger-Zukin (B

$$\mu^{\mathrm{BZ}}(a,k) = \frac{1}{1 - B_0 a^{s-1}/6}\left[\frac{1+(2/3)B_0\bar{k}^2 a^s}{1+(1/2)B_0\bar{k}^2 a^s}\right]$$

$$\gamma^{\mathrm{BZ}}(a,k) = \frac{1+(1/3)B_0\bar{k}^2 a^s}{1+(2/3)B_0\bar{k}^2 a^s}\,,$$

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on any w(a)! → general functions of ti

growth index

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on LCDM →

Bertschinger-Zukin (B

f(R) a la' BZ*

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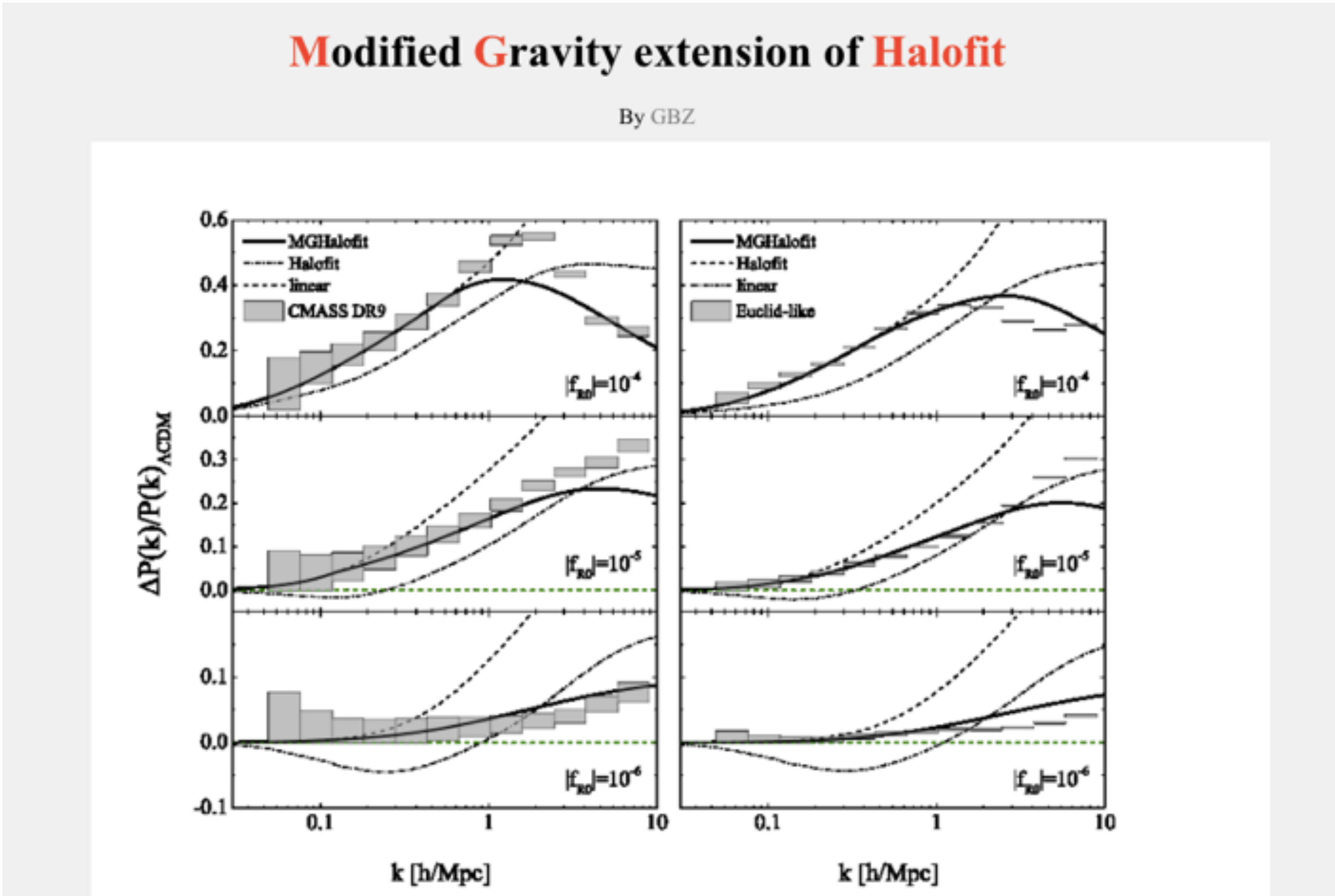
# MGHalofit

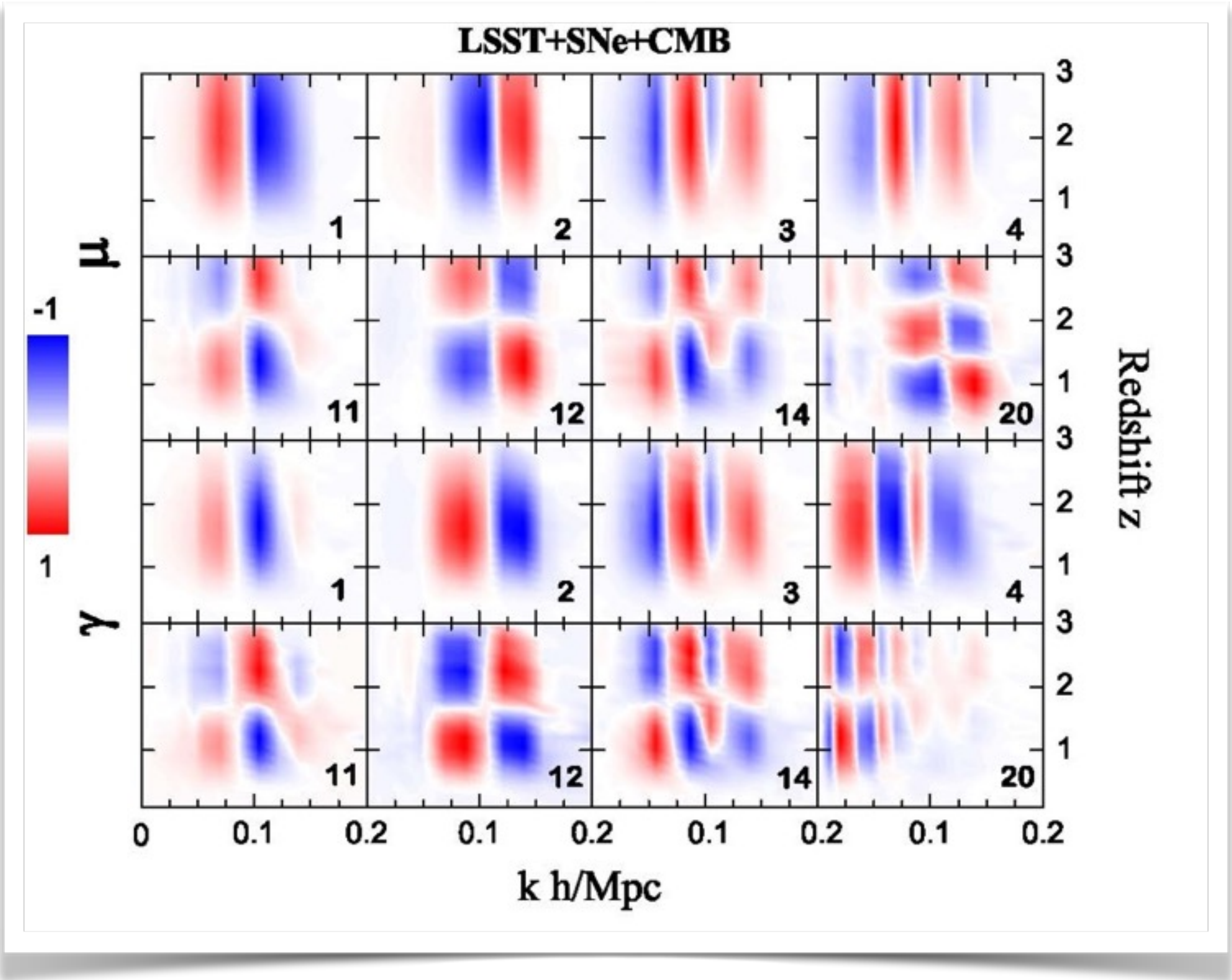
by G.B. Zhao, ApJS, 211, 23 (2014)




Extension of Halofit to apply it to MG. Particularly designed for Hu-Sawicki f(R).

http://www.icosmology.info/index.php?m=content&c=index&a=show&catid=2&id=10

# PRINCIPAL COMPONENT ANALYSIS



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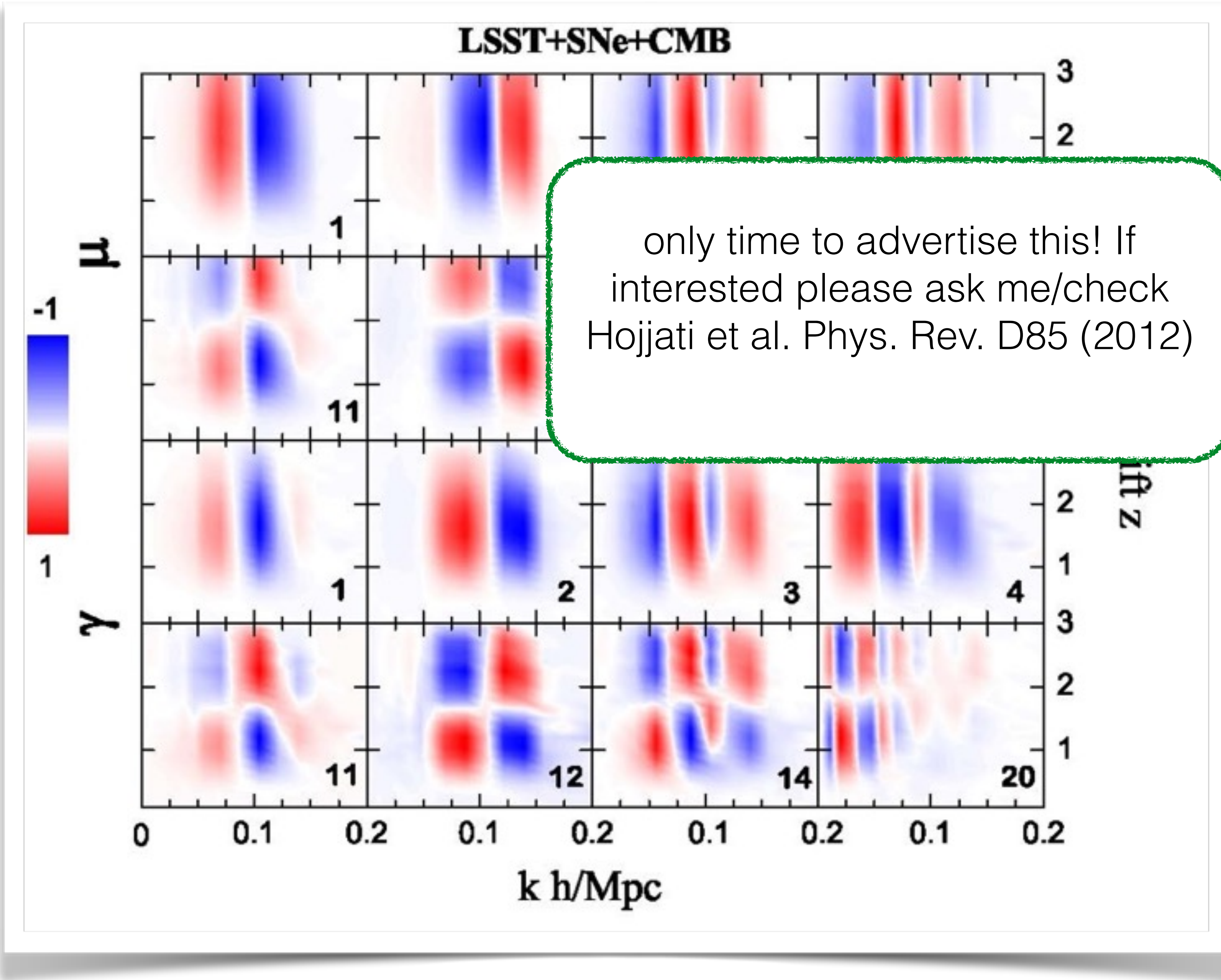


only time to advertise this! If interested please ask me/check Hojjati et al. Phys. Rev. D85 (2012)

# EFTCAMB

**E**ffective **F**ield **T**heory CAMB

by M.Raveri, B.Hu, N.Frusciante, A.Silvestri (December 2013)

http://eftcamb.org/

# EffectiveFieldTheory CAMB

Designed to explore the phenomenology of linear perturbations in Single Field Modified Gravity.

Framework naturally defined in unitary gauge, then translated into synchronous gauge for implementation in CAMB.

Involves a handful of functions of time, to model departures from LCDM at the level of linear perturbations.

# EFT of Dark Energy

$$S = \int d^4x \sqrt{-g}\, \frac{m_0^2}{2} R \;+\; S_m[g_{\mu\nu}]$$

IN UNITARY GAUGE

Creminelli et al., JCAP 0902 (2009) 018 ; Gubitosi et al., JCAP 1302 (2013) 032 ; Bloomfield et al., JCAP 1308 (2013) 010

# EFT of Dark Energy

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} \left[1 + \Omega(\tau)\right] R + \Lambda(\tau) - a^2 c(\tau) \delta g^{00} \right\}$$
$$+ S_m[g_{\mu\nu}]$$

IN UNITARY GAUGE

Creminelli et al., JCAP 0902 (2009) 018 ; Gubitosi et al., JCAP 1302 (2013) 032 ; Bloomfield et al., JCAP 1308 (2013) 010

# EFT of Dark Energy

$$S = \int d^4x \sqrt{-g} \left\{ \frac{m_0^2}{2} \left[1 + \Omega(\tau)\right] R + \Lambda(\tau) - a^2 c(\tau) \delta g^{00} + \frac{M_2^4(\tau)}{2} (a^2 \delta g^{00})^2 - \frac{\bar{M}_1^3(\tau)}{2} a^2 \delta g^{00} \delta K^\mu_\mu - \frac{\bar{M}_2^2(\tau)}{2} (\delta K^\mu_\mu)^2 - \frac{\bar{M}_3^2(\tau)}{2} \delta K^\mu_\nu \delta K^\nu_\mu + \frac{a^2 \hat{M}^2(\tau)}{2} \delta g^{00} \delta R^{(3)} + \ldots \right\} + S_m[g_{\mu\nu}]$$

IN UNITARY GAUGE

Creminelli et al., JCAP 0902 (2009) 018 ; Gubitosi et al., JCAP 1302 (2013) 032 ; Bloomfield et al., JCAP 1308 (2013) 010

# EFT of Dark Energy

it is an interesting framework that offers both a model-independent parametrization of alternatives to LCDM and a unifying language to analyze specific DE/MG models.

pure EFT:

$$\{\Omega(\tau), c(\tau), \Lambda(\tau), M_2(\tau), \bar{M}_1(\tau), \bar{M}_2(\tau), \bar{M}_3(\tau), \hat{M}(\tau), m_2(\tau)\}$$

mapping EFT:

f(R) $$\Omega = f_R; \quad \Lambda = \frac{m_0^2}{2}\left[f - R f_R\right]; \quad c = 0$$

minimally coupled quintessence $$\Omega = 0; \quad c - \Lambda = V(\phi); \quad c = \frac{\dot{\phi}^2}{2}$$

Gubitosi, Piazza, Vernizzi, JCAP 1302 (2013) 032

# EFT of Dark Energy

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unifying language

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minimally coupled quintessence $$\Omega = 0; \quad c - \Lambda = V(\phi); \quad c = \frac{\dot{\phi}^2}{2}$$

Gubitosi, Piazza, Vernizzi, JCAP 1302 (2013) 032

# EFT of Dark Energy

it is an interesting framework that offers both a model-independent
LCDM and a unifying language to analyze
c DE/MG models.

all single-field scalar DE/MG models
for which there exists a well defined
Jordan frame

f(R)

f(R,G)

quintessence
(minimally and non-minimally coupled)

k-essence

kinetic braiding

galileon

Horndeski

Hořava-Lifshitz

model-independent

$$, \bar{M}_1(\tau), \bar{M}_2(\tau), \bar{M}_3(\tau), \hat{M}(\tau), m_2(\tau)\}$$

unifying language

$$- Rf_R]\,; \quad c = 0$$

$$\Omega = 0; \quad c - \Lambda = V(\phi); \quad c = \frac{\dot{\phi}^2}{2}$$

Gubitosi, Piazza, Vernizzi, JCAP 1302 (2013) 032

# Correspondence to models

| Operator | $\Omega$ | $\Lambda$ | $c$ | $M_2^4$ | $\bar{M}_1^3$ | $\bar{M}_2^2$ | $\bar{M}_3^2$ | $\hat{M}^2$ | $m_2^2$ |
|---|---|---|---|---|---|---|---|---|---|
| **Model** | R | | $\delta g^{00}$ | $(\delta g^{00})^2$ | $\delta g^{00}\delta K^{\mu}{}_{\mu}$ | $(\delta K^{\mu}{}_{\mu})^2$ | $\delta K^{\mu}{}_{\nu} K^{\nu}{}_{\mu}$ | $\delta g^{00}\delta R^{(3)}$ | $\frac{\bar{g}^{ij}}{a^2}\partial_i g^{00}\partial_j g^{00}$ |
| $\Lambda$CDM | 1 | ✓ | 0 | - | - | - | - | - | - |
| Quintessence | 1/✓ | ✓ | ✓ | - | - | - | - | - | - |
| $F(R)$ | ✓ | ✓ | 0 | - | - | - | - | - | - |
| $k$-essence | 1/✓ | ✓ | ✓ | ✓ | - | - | - | - | - |
| Galileon [39]<br>Kinetic Braiding [40] | 1/✓ | ✓ | ✓ | ✓ | ✓ | - | - | - | - |
| DGP [41] | ✓ | ✓† | ✓† | ✓† | ✓ | - | - | - | - |
| Ghost Condensate [42] | 1/✓ | ✓ | 0 | - | - | ✓ | ✓ | - | - |
| Horndeski [26] | ✓ | ✓ | ✓ | ✓ | ✓ | ✓† | ✓† | ✓† | - |

Bloomfield, Flanagan, Park, Watson JCAP 1308 (2013) 010

# Solving the background

pure EFT

$$\mathcal{H}^2 = \frac{a^2}{3m_0^2(1+\Omega)}(\rho_m + 2c - \Lambda) - \mathcal{H}\frac{\dot{\Omega}}{1+\Omega},$$

$$\dot{\mathcal{H}} = -\frac{a^2}{6m_0^2(1+\Omega)}\left(\rho_m + 3P_m\right) - \frac{a^2(c+\Lambda)}{3m_0^2(1+\Omega)} - \frac{\ddot{\Omega}}{2(1+\Omega)}$$

# Solving the background

pure EFT

$$\mathcal{H}^2 = \frac{a^2}{3m_0^2(1+\Omega)}(\rho_m + 2c - \Lambda) - \mathcal{H}\frac{\dot\Omega}{1+\Omega},$$
$$\dot{\mathcal{H}} = -\frac{a^2}{6m_0^2(1+\Omega)}(\rho_m + 3P_m) - \frac{a^2(c+\Lambda)}{3m_0^2(1+\Omega)} - \frac{\ddot\Omega}{2(1+\Omega)}$$

designer approach: fix the expansion history and choose $\Omega(\tau)$.

$$c = -\frac{m_0^2\ddot\Omega}{2a^2} + \frac{m_0^2\mathcal{H}\dot\Omega}{a^2} + \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 - \dot{\mathcal{H}}) - \frac{1}{2}(\rho_m + P_m),$$
$$\Lambda = -\frac{m_0^2\ddot\Omega}{a^2} - \frac{m_0^2\mathcal{H}\dot\Omega}{a^2} - \frac{m_0^2(1+\Omega)}{a^2}(\mathcal{H}^2 + 2\dot{\mathcal{H}}) - P_m.$$

# Solving the background

mapping EFT: choose a DE or MG model, and solve for its background.
Then map the solution into the EFT functions.

# Solving the background

mapping EFT: choose a DE or MG model, and solve for its background.
Then map the solution into the EFT functions.

There are two ways in which you can solve the background.

1. designer mapping EFT

You fix an expansion history, and solve the background equations of the given model accordingly. You can then reconstruct numerically all functions of interest needed to build the EFT functions.

2. pure mapping EFT

You have the functional form for the given model, and you solve for the background corresponding to this form. In other words you find $\phi(a)$ and H(a). Then you can extract the time dependence of all the quantities needed for the EFT functions.

# Equations for perturbations

At this point, in the mapping case we have all is needed to determine all EFT functions, also those affecting only perturbations. In the pure EFT case, we need to make a choice for the EFT functions multiplying the higher order operators (i.e. those that affect only perturbations). After this choice is made, we can move on and study the dynamics of perturbations.

*energy-momentum equations:* standard ones since we are in the Jordan frame

*Einstein equations:* messy equations involving contributions from 'all' EFT functions

*$\pi$ field equation:*

$$\boxed{A\ddot{\pi} + B\dot{\pi} + \left(C + k^2 D\right)\pi + E = 0}$$

$$A = A[c, \Lambda, \Omega, \ldots](\tau, k)$$

*tensors eq.:* $$A_T \ddot{h}_{ij} + B_T \dot{h}_{ij} + D_T k^2\, h_{ij} + E_{Tij} = 0$$

# Implementation in CAMB

The dynamical equations that EFTCAMB evolves can be written[1] as:

$$A(\tau,k)\,\ddot{\pi} + B(\tau,k)\,\dot{\pi} + C(\tau)\,\pi + k^2\,D(\tau,k)\,\pi + H_0E(\tau,k) = 0\,, \tag{20}$$

$$k\dot{\eta} = \frac{1}{X}\left[\frac{1}{1+\Omega}\frac{a^2(\rho_{m,\nu}+P_{m,\nu})}{m_0^2}\frac{v_{m,\nu}}{2} + \frac{k^2}{3H_0}F + (U-X)\frac{\mathcal{Z}k^2}{3}\right]\,, \tag{21}$$

while constraint equations take the form:

$$\sigma = \frac{1}{X}\left[\mathcal{Z}U + \frac{1}{1+\Omega}\frac{3}{2k^2}\frac{a^2(\rho_{m,\nu}+P_{m,\nu})}{m_0^2}v_{m,\nu} + \frac{F}{H_0}\right]\,, \tag{22}$$

$$\dot{\sigma} = \frac{1}{X}\left[-2\mathcal{H}\,[1+V]\,\sigma + k\eta - \frac{1}{k}\frac{a^2P_{m,\nu}}{m_0^2}\frac{\Pi_{m,\nu}}{1+\Omega} + \frac{N}{H_0}\right]\,, \tag{23}$$

$$\ddot{\sigma} = \frac{1}{X}\left[-2\,(1+V)\left(\dot{\mathcal{H}}\sigma + \mathcal{H}\dot{\sigma}\right) - 2\mathcal{H}\sigma\dot{V} + k\dot{\eta} + \frac{1}{k}\frac{a\mathcal{H}\Omega'}{(1+\Omega)^2}\frac{a^2P_{m,\nu}}{m_0^2}\Pi_{m,\nu} - \frac{1}{k(1+\Omega)}\frac{d}{d\tau}\left(\frac{a^2P_{m,\nu}}{m_0^2}\Pi_{m,\nu}\right) - \dot{X}\dot{\sigma} + \frac{\dot{N}}{H_0}\right]\,, \tag{24}$$

$$\mathcal{Z} = \frac{1}{G}\left[\frac{k\eta}{\mathcal{H}} + \frac{1}{2\mathcal{H}(1+\Omega)k}\frac{a^2\delta\rho_{m,\nu}}{m_0^2} + \frac{L}{kH_0}\right]\,, \tag{25}$$

$$\begin{aligned}\dot{\mathcal{Z}} &= \frac{1}{U}\left[-2\mathcal{H}\,[1+Y]\,\mathcal{Z} + k\eta - \frac{1}{1+\Omega}\frac{3}{2k}\frac{a^2\delta P_{m,\nu}}{m_0^2} - \frac{3}{2k(1+\Omega)}\frac{M}{H_0}\right]\,,\\ &= \frac{1}{U}\left[-2\mathcal{H}\mathcal{Z}\left(1+Y-\frac{G}{2}\right) - \frac{1}{2(1+\Omega)k}\frac{a^2\delta\rho_{m,\nu}}{m_0^2} - \frac{3}{2(1+\Omega)k}\frac{a^2\delta P_{m,\nu}}{m_0^2} - \frac{\mathcal{H}L}{H_0k} - \frac{3}{2(1+\Omega)k}\frac{M}{H_0}\right]\,,\end{aligned} \tag{26}$$

$$\begin{aligned}\texttt{EFTISW} = \ddot{\sigma} + k\dot{\eta} = \frac{1}{X}\Bigg[&-2\,[1+V]\left(\dot{\mathcal{H}}\sigma + \mathcal{H}\dot{\sigma}\right) - 2\mathcal{H}\sigma\dot{V} + \frac{(1+X)}{2(1+\Omega)X}\frac{a^2(\rho_{m,\nu}+P_{m,\nu})}{m_0^2}v_{m,\nu}\\ &+ \frac{1}{k}\frac{a\mathcal{H}\Omega'}{(1+\Omega)^2}\frac{a^2P_{m,\nu}}{m_0^2}\Pi - \frac{1}{k(1+\Omega)}\frac{d}{d\tau}\left(\frac{a^2P_{m,\nu}}{m_0^2}\Pi_{m,\nu}\right) + \frac{(1+X)k^2}{3H_0X}F\\ &+ \frac{(1+X)k^2}{3X}\mathcal{Z}(U-X) - \dot{X}\dot{\sigma} + \frac{\dot{N}}{H_0}\Bigg]\,,\end{aligned}$$

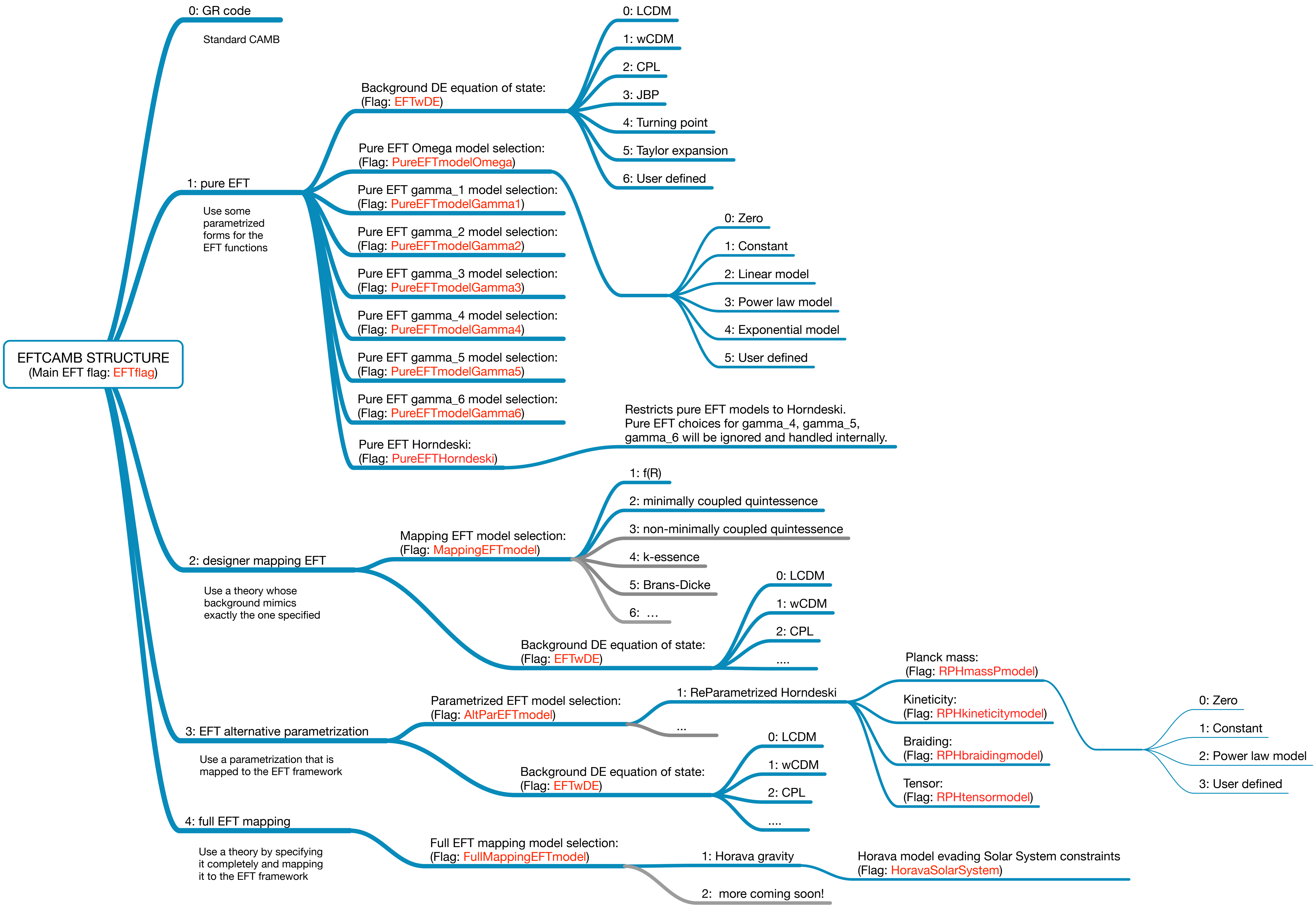
$$\texttt{EFTLensing} = \dot{\sigma} + k\eta = \frac{1}{X}\left[-2\mathcal{H}\,(1+V)\,\sigma + (1+X)k\eta - \frac{1}{k(1+\Omega)}\frac{a^2P_{m,\nu}}{m_0^2}\Pi_{m,\nu} + \frac{N}{H_0}\right]\,.$$

# Implementation in CAMB

The modification to the CAMB code is mainly implemented by modifying or adding the following files:

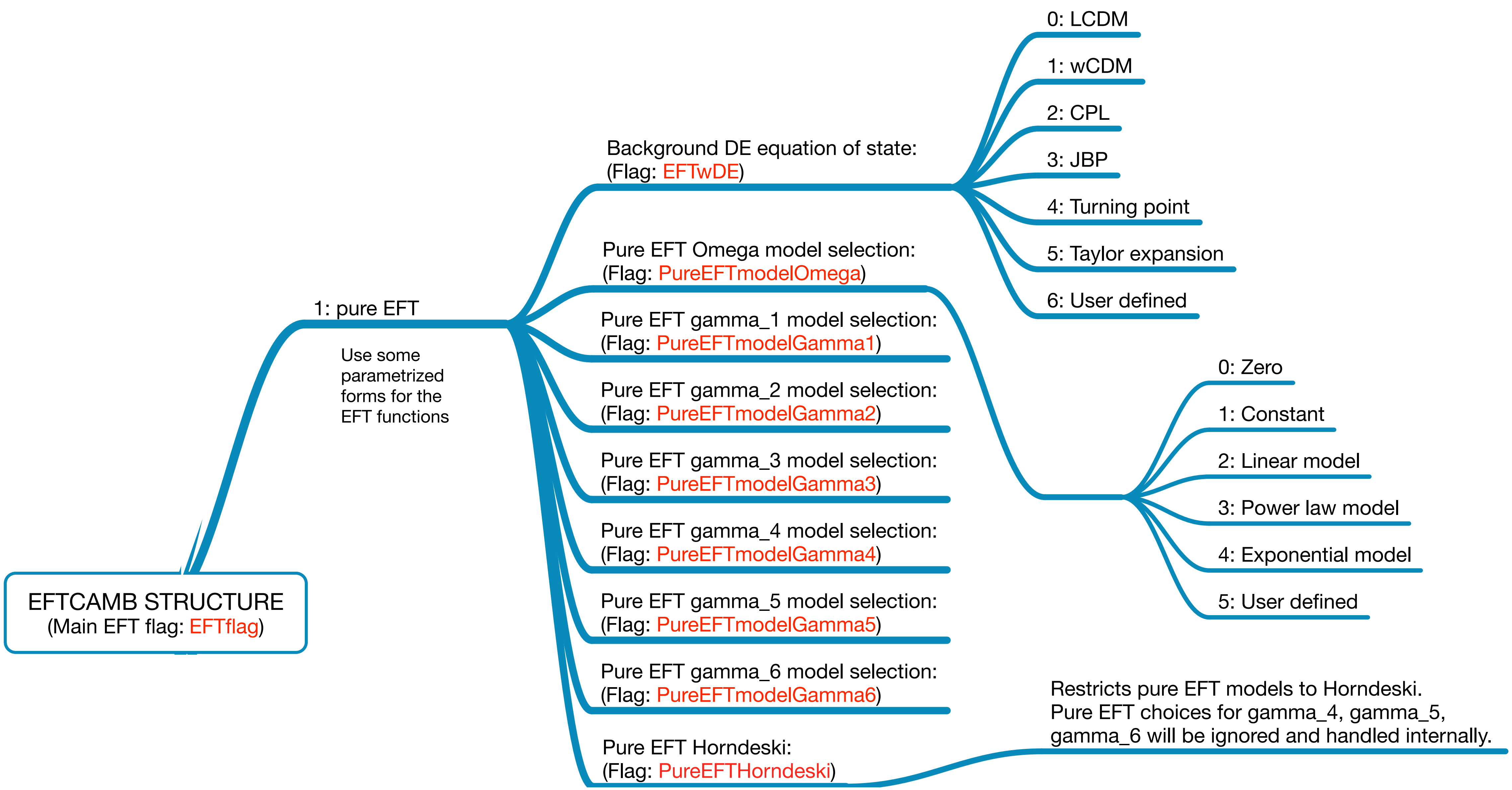
- `EFT_def.f90`: this file contains all the compile time EFTCAMB options and the definition of the parametrizations for the dark energy equation of state.
- `EFT_designer.f90`: this file contains the module for designer models.
- `EFT_Horndeski.f90`: this file contains all the code regarding implementation of Horndeski models.
- `EFT_functions.f90`: this file contains the definition of the EFT functions for the chosen model, in particular it contains the definitions of the *pure* EFT parametrizations.
- `EFT_main.f90`: this file contains a module that evaluates whether the considered DE/MG model is stable or not and a module that finds the time (if it exists) at which the considered model is so close to GR that it is pointless to evolve the DE equation.
- `EFTstabilitySpace.f90`: this file contains a program that can be compiled with the directive **`make eftstability`** to serve the purpose of making simple explorations of the stability region in parameter space for the theory of interest. This proves extremely helpful to understand and visualize the shape of the parameter space of a theory before exploring it with CosmoMC. For a direct application of this see Figure (1) in [6].
- `equations_EFT.f90`: this is a modified version of the standard CAMB equations file. It contains all the equations that the code needs to solve to get the full behaviour of perturbations in DE/MG models. These equations are reported in Section IV and being written in terms of the EFT functions there is no need for the user to modify this file to include new DE/MG models in CAMB.
- `cmbmain.f90`: it includes modification of the standard CAMB file to run the designer code, the stability check and the return to GR detection just after EFTCAMB is launched. It also contains an optional code that will print the behaviour of perturbations in the DE/MG model that is considered. The latter part of code is not controlled by the parameter file so the user has to search for it in the code and manually activate it. It is also possible to use it at debug purposes.

# STRUCTURE



EFTCAMB STRUCTURE
(Main EFT flag: EFTflag)
0: GR code
Standard CAMB
1: pure EFT
Use some parametrized forms for the EFT functions
Background DE equation of state:
(Flag: EFTwDE)
0: LCDM
1: wCDM
2: CPL
3: JBP
4: Turning point
5: Taylor expansion
6: User defined
Pure EFT Omega model selection:
(Flag: PureEFTmodelOmega)
Pure EFT gamma_1 model selection:
(Flag: PureEFTmodelGamma1)
Pure EFT gamma_2 model selection:
(Flag: PureEFTmodelGamma2)
Pure EFT gamma_3 model selection:
(Flag: PureEFTmodelGamma3)
Pure EFT gamma_4 model selection:
(Flag: PureEFTmodelGamma4)
Pure EFT gamma_5 model selection:
(Flag: PureEFTmodelGamma5)
Pure EFT gamma_6 model selection:
(Flag: PureEFTmodelGamma6)
0: Zero
1: Constant
2: Linear model
3: Power law model
4: Exponential model
5: User defined
Pure EFT Horndeski:
(Flag: PureEFTHorndeski)
Restricts pure EFT models to Horndeski.
Pure EFT choices for gamma_4, gamma_5, gamma_6 will be ignored and handled internally.
2: designer mapping EFT
Use a theory whose background mimics exactly the one specified
Mapping EFT model selection:
(Flag: MappingEFTmodel)
1: f(R)
2: minimally coupled quintessence
3: non-minimally coupled quintessence
4: k-essence
5: Brans-Dicke
6: …
Background DE equation of state:
(Flag: EFTwDE)
0: LCDM
1: wCDM
2: CPL
....
3: EFT alternative parametrization
Use a parametrization that is mapped to the EFT framework
Parametrized EFT model selection:
(Flag: AltParEFTmodel)
1: ReParametrized Horndeski
...
Planck mass:
(Flag: RPHmassPmodel)
Kineticity:
(Flag: RPHkineticitymodel)
Braiding:
(Flag: RPHbraidingmodel)
Tensor:
(Flag: RPHtensormodel)
0: Zero
1: Constant
2: Power law model
3: User defined
Background DE equation of state:
(Flag: EFTwDE)
0: LCDM
1: wCDM
2: CPL
....
4: full EFT mapping
Use a theory by specifying it completely and mapping it to the EFT framework
Full EFT mapping model selection:
(Flag: FullMappingEFTmodel)
1: Horava gravity
Horava model evading Solar System constraints
(Flag: HoravaSolarSystem)
2: more coming soon!

# STRUCTURE



EFTCAMB STRUCTURE
(Main EFT flag: EFTflag)
1: pure EFT
Use some parametrized forms for the EFT functions
Background DE equation of state:
(Flag: EFTwDE)
0: LCDM
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4: Turning point
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6: User defined
Pure EFT Omega model selection:
(Flag: PureEFTmodelOmega)
Pure EFT gamma_1 model selection:
(Flag: PureEFTmodelGamma1)
Pure EFT gamma_2 model selection:
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Pure EFT gamma_5 model selection:
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Pure EFT gamma_6 model selection:
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Pure EFT Horndeski:
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Restricts pure EFT models to Horndeski.
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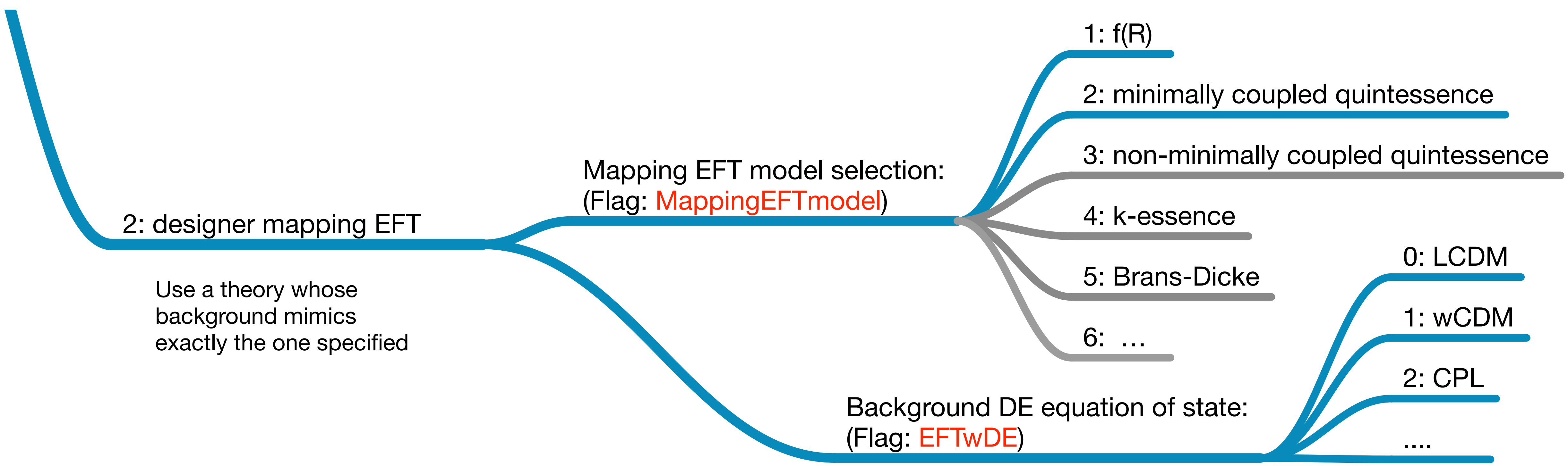
# STRUCTURE


2: designer mapping EFT
Use a theory whose background mimics exactly the one specified
Mapping EFT model selection:
(Flag: MappingEFTmodel)
1: f(R)
2: minimally coupled quintessence
3: non-minimally coupled quintessence
4: k-essence
5: Brans-Dicke
6: …
Background DE equation of state:
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0: LCDM
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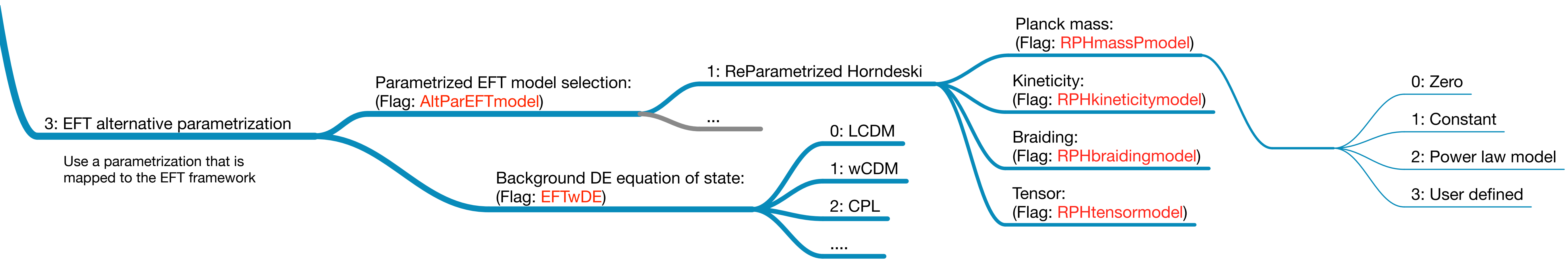
# STRUCTURE


3: EFT alternative parametrization
Use a parametrization that is mapped to the EFT framework
Parametrized EFT model selection:
(Flag: AltParEFTmodel)
1: ReParametrized Horndeski
...
Background DE equation of state:
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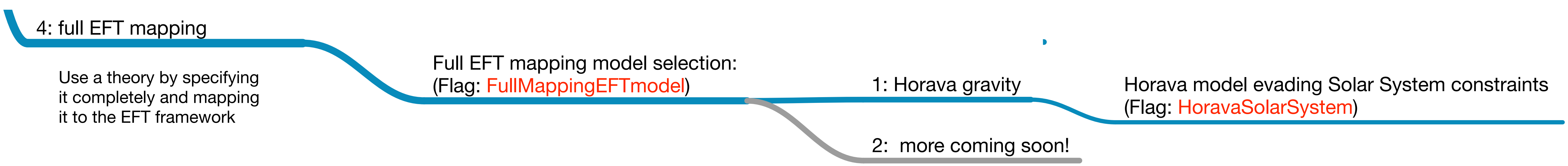
2: Power law model
3: User defined

# STRUCTURE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Full EFT mapping model selection:
(Flag: FullMappingEFTmodel)
1: Horava gravity
2: more coming soon!
Horava model evading Solar System constraints
(Flag: HoravaSolarSystem)

# Initial Conditions

Generically, at early times, as a model goes back to GR, it displays highly oscillatory modes in the perturbations of the extra scalar d.o.f..

This can be seen nicely in f(R) theories. Let us look at the trace equation in the super-horizon, heavy scalaron (i.e. high curvature) limit:

$$\delta\ddot{f}_R + 2\mathcal{H}\,\delta\dot{f}_R + a^2 m^2_{f_R}\delta f_R = \frac{a^2}{3M_P^2}\left(\frac{3\delta P}{\delta\rho} - 1\right)\rho\delta$$

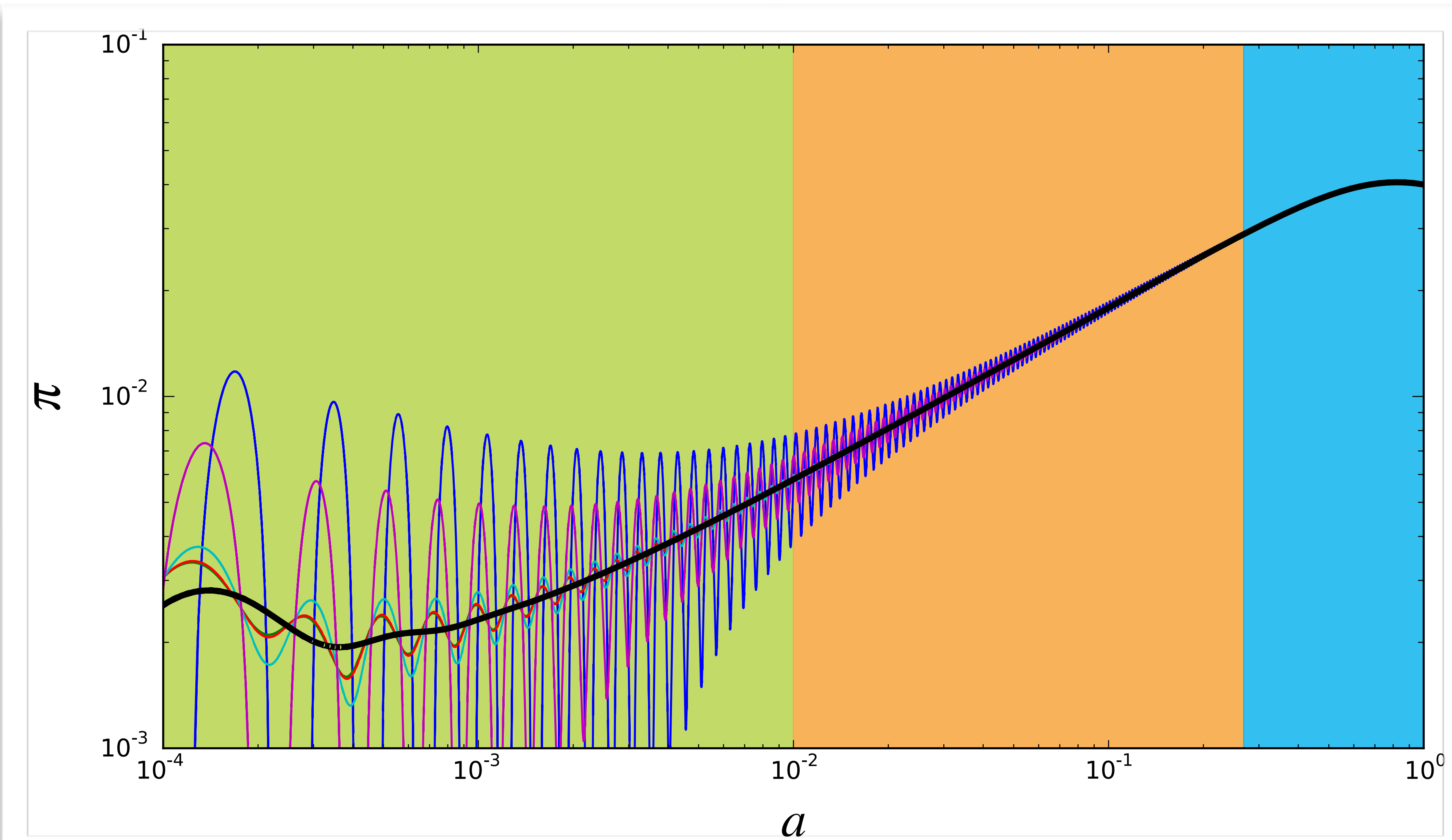
It is a damped inhomogeneous harmonic equation for the scalaron. As long as $m^2_{f_R} \propto 1/f_{RR} > 0$ the solutions of this equation are damped oscillations around the particular solution set by the source, with frequency and amplitude proportional to the mass of the scalaron, hence decreasing in time. This is related to the requirement of $f_{RR} > 0$ in order for the theory to be stable.

Noticeably, no oscillations are present in the lensing potential, which means no observational imprints of these oscillations. Indeed in the super-horizon, heavy scalaron limit, the equations for the scalaron and for the lensing potential decouple and the lensing potential follows closely the evolution it would have in the standard case:

$$(\Phi \dot{+} \Psi) + \mathcal{H}\,(\Phi + \Psi) = \frac{a^2(\rho + P)}{2M_P^2}\frac{v}{kF}$$

# Initial Conditions

This is a quite generic feature of beyond $\Lambda$CDM models, related to their higher order nature. Consequently, *when dealing with evolution at early times and initial conditions, we need a strategy to cope with these oscillations.* One strategy we use in EFTCAMB is to sit the perturbations of the scalar field on the source and start evolving its dynamics only when the model starts departing from GR. As the following picture shows, the source term represents an attractor solution.




This of course is not applicable to models of early dark energy, which give significant departures at early times.

# EFT meets CosmoMC: viability priors

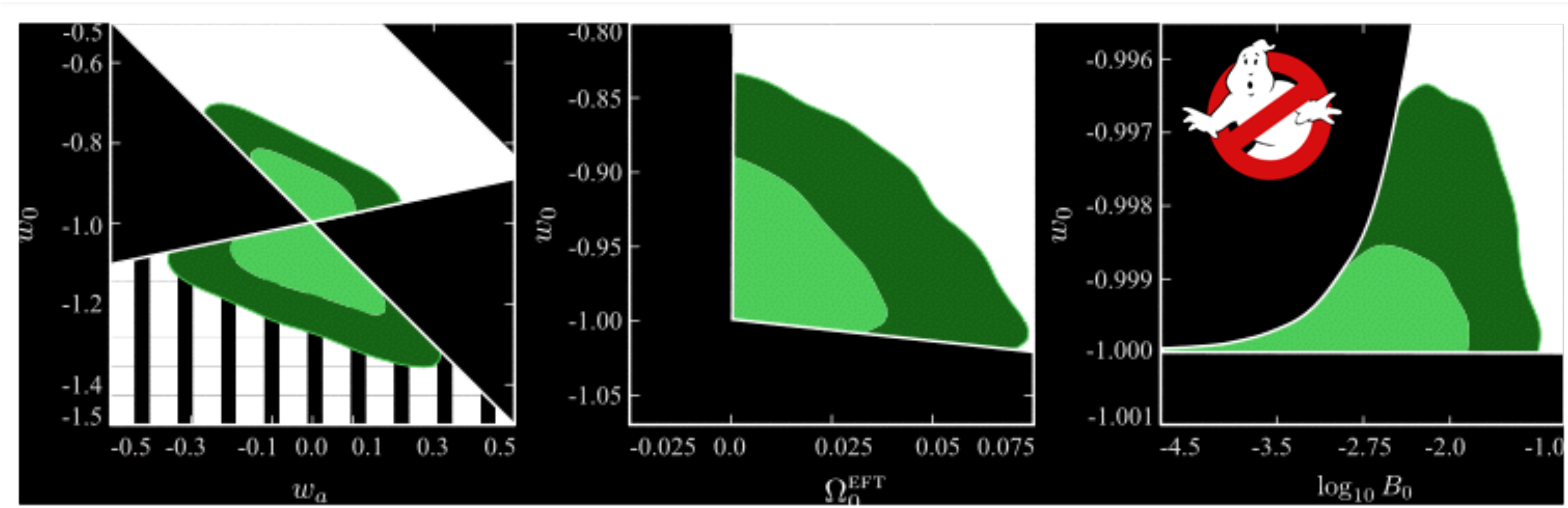
Since we start from a parametrized action, that we can expand to quadratic order in perturbations, we can introduce general, model-independent viability conditions that are well motivated theoretically (e.g. no ghosts, no laplacian instabilities, etc.) and ensure also numerical stability; when exploring the parameter space we impose them in the form of <u>viability priors</u>. In some cases they dominate over the constraining power of data.

Viability priors are a powerful tool for the advocated open-minded approach to cosmological tests of GR. They provide theoretically motivated yet model-independent conditions to impose in order to ensure the investigation of physically viable models. More work is needed, and ongoing, to determine the full, general, correct set of conditions to impose.

# EFT meets CosmoMC: viability priors

## VIABILITY PRIORS

Physics remains the guiding principle in the agnostic exploration of dark energy!




Marginalized joint likelihoods for parameters of interest in three different dark energy models: quintessence, non-minimally coupled quintessence and f(R) gravity.

# EFT meets CosmoMC: viability priors

## VIABILITY PRIORS

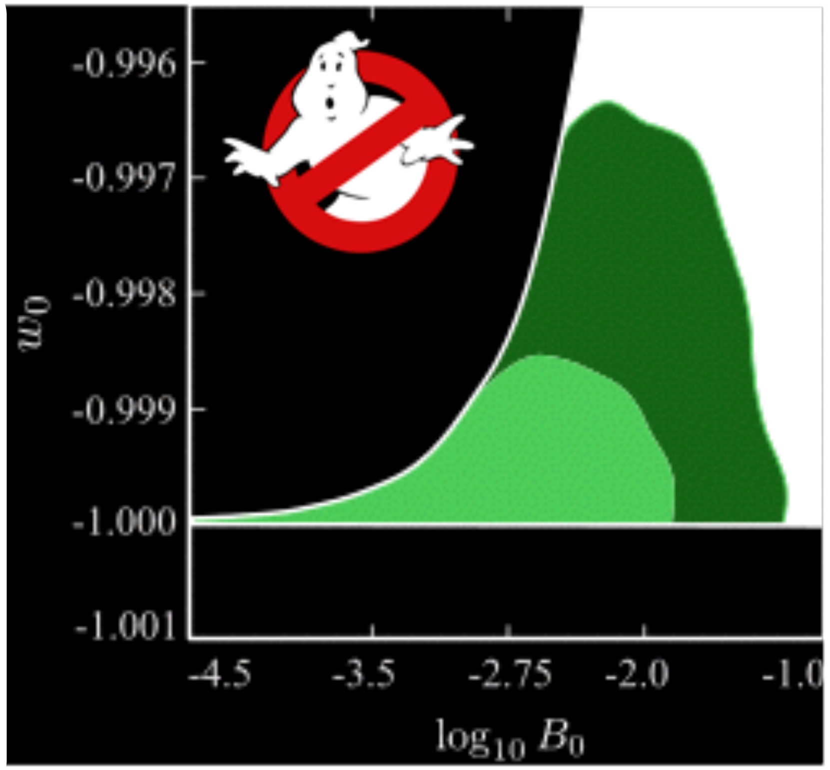
Physics remains the guiding principle in the agnostic exploration of dark energy!

designer f(R) on wCDM background:

$$w_0 \ \epsilon \ (-1, -0.9997) \ \ (95\%\mathrm{C.L.})$$

with Planck, lensing, WP, BAO data

$B_0$ and $w_0$ are strongly correlated via a theoretical prior

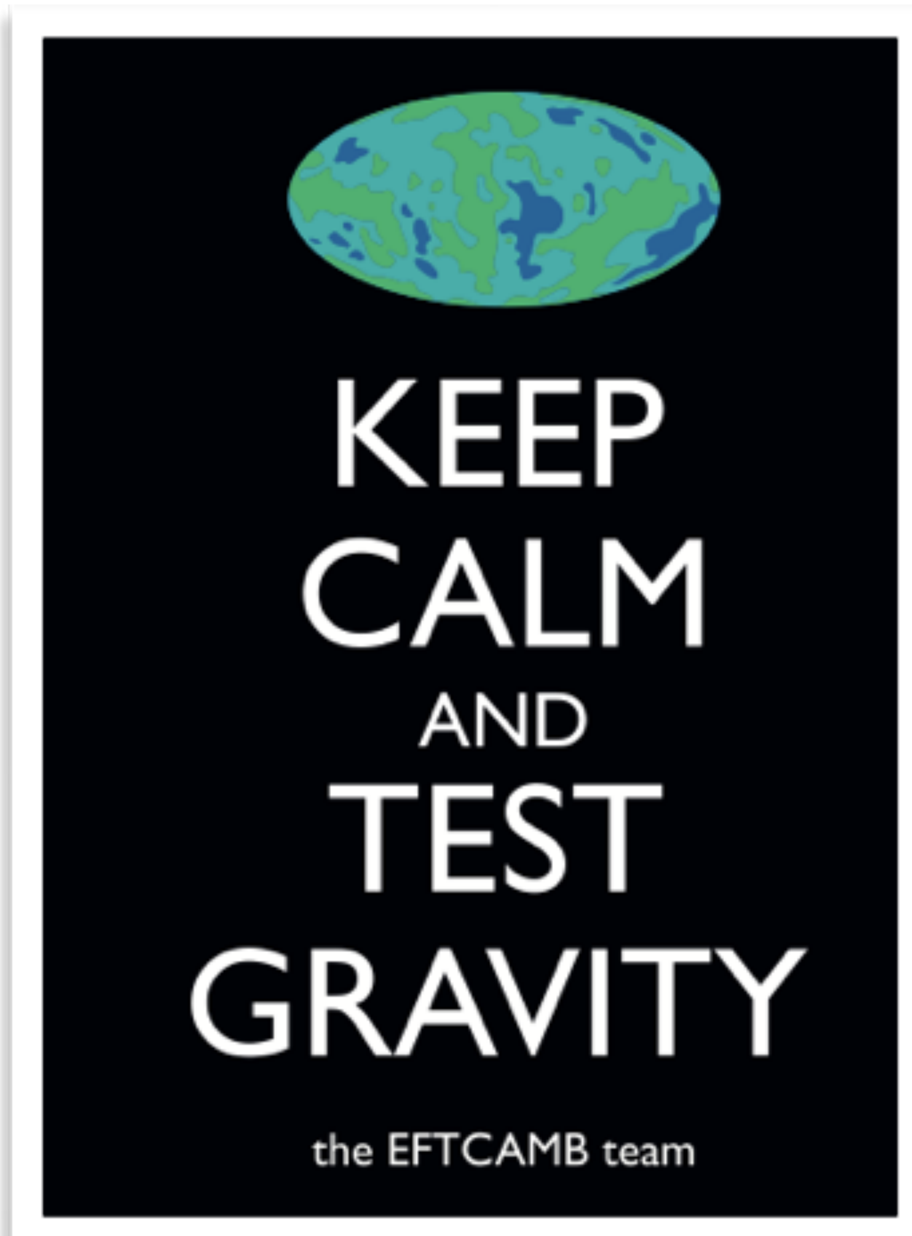



Marginalized joint likelihoods for parameters of interest in three different dark energy models: quintessence, non-minimally coupled quintessence and f(R) gravity.

# EFTCAMB vs. MGCAMB

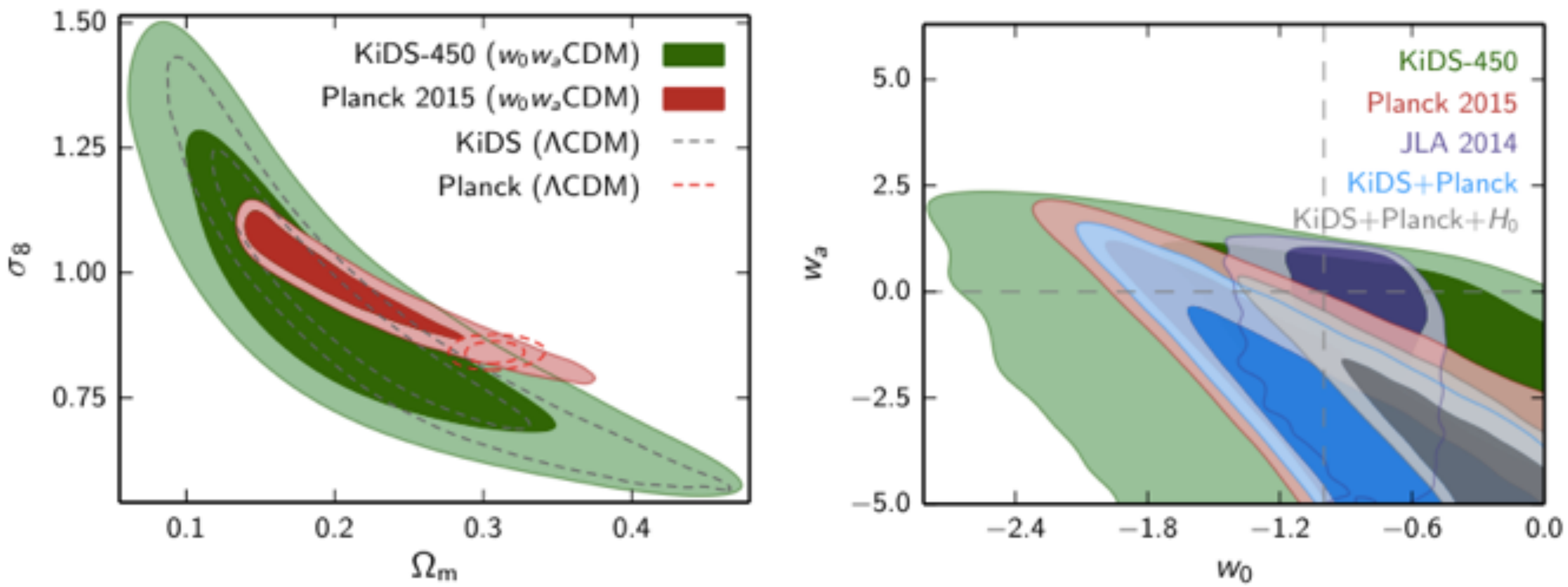
## EFT of DE vs. $(\mu,\gamma)$

| $(\mu,\gamma)$ | EFT |
|---|---|
| * very general, captures basically any deviation from LCDM | * ~ single scalar field models allowing a well defined Jordan frame |
| * needs QSA to treat specific models | * NO QSA when specializing to models |
| * quite distant from theoretical details. Might end up exploring physically unviable options…?! | * closer link to theory; allows implementation of theoretical viability priors to make exploration much more efficient…too strong?! |


KEEP
CALM
AND
TEST
GRAVITY
the EFTCAMB team

# Exploring constraints under stability conditions

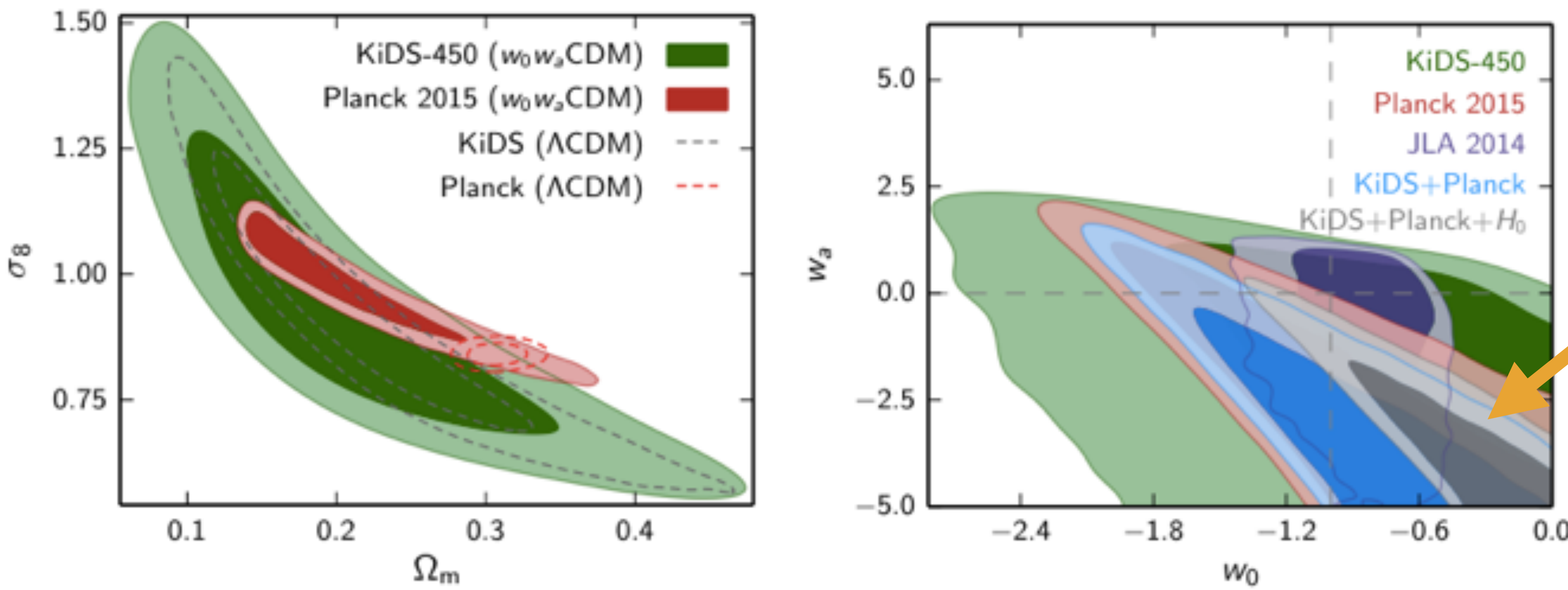
CPL DE: $w = w_0 + w_a(1-a)$




Joudaki *et al.*, arXiv:1610.04606

# Exploring constraints under stability conditions

CPL DE: $w = w_0 + w_a(1-a)$

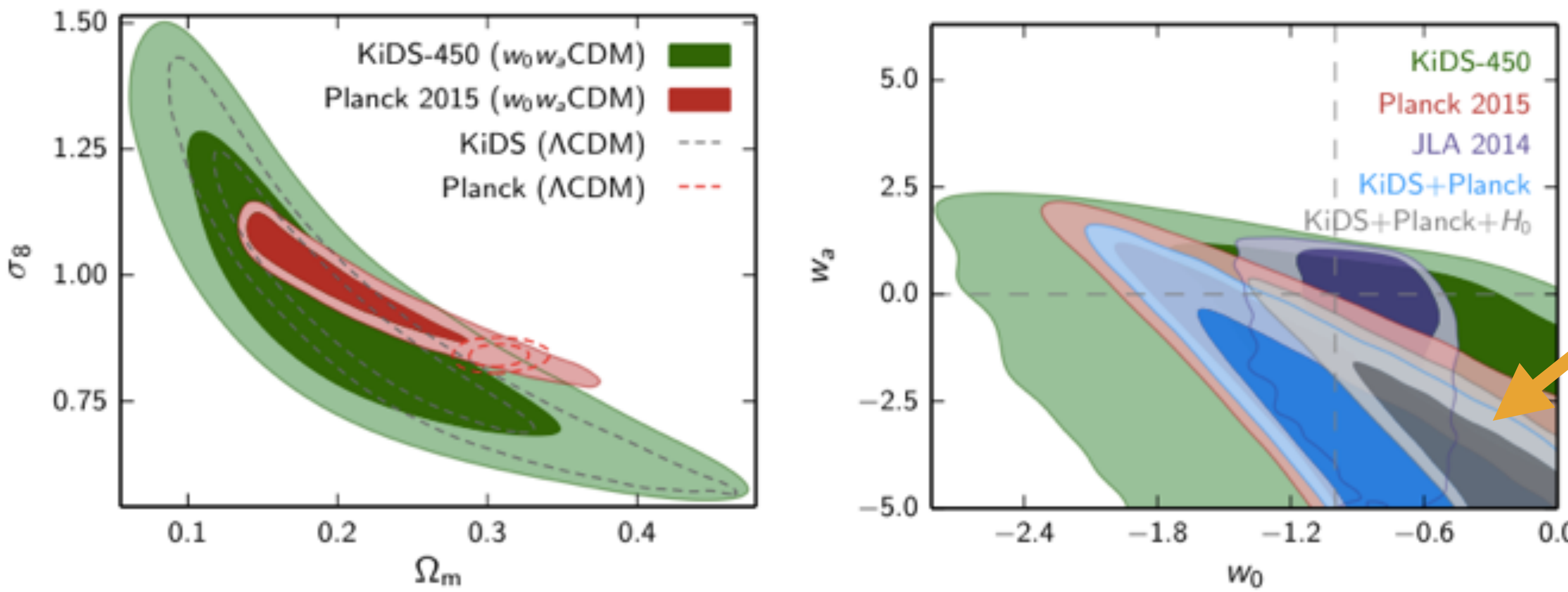



Joudaki *et al.*, arXiv:1610.04606

WHICH DE IS IT?

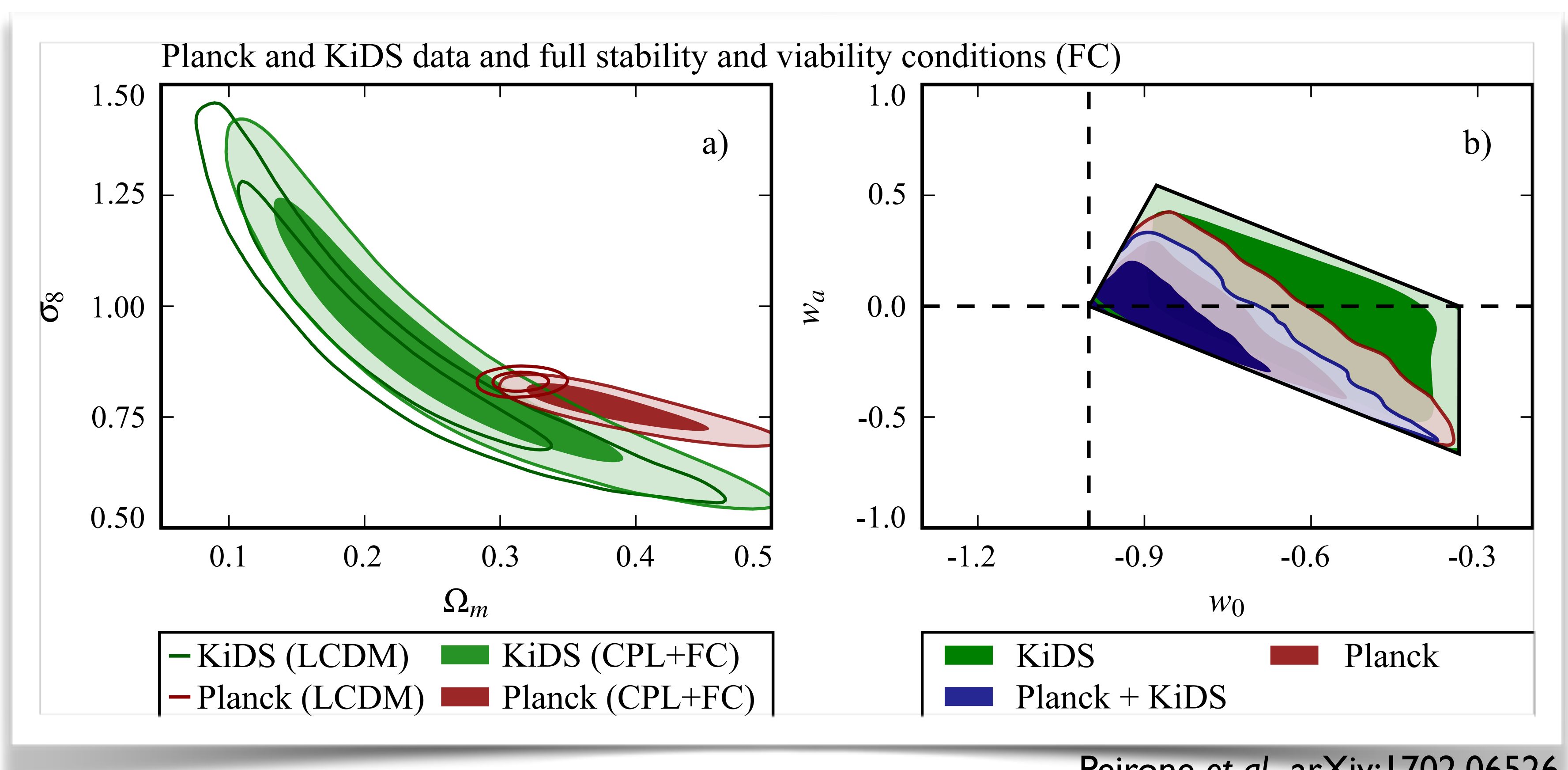
# Exploring constraints under stability conditions

CPL DE: $w = w_0 + w_a(1 - a)$




WHICH DE IS IT?

Joudaki *et al.*, arXiv:1610.04606




Peirone *et al.*, arXiv:1702.06526