Form Factors in N=4 SYM & Higgs+Gluon Amplitudes

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with

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(and earlier work with E. Hughes, R. Panerai, B. Spence, C. Wen, G. Yang & D. Young)

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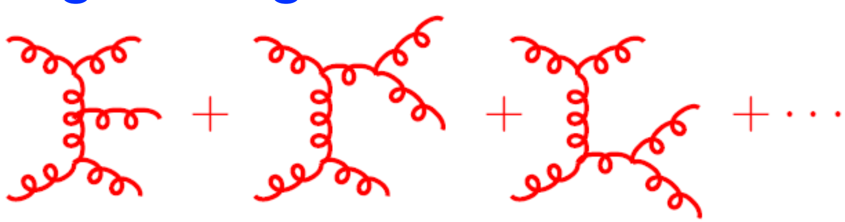
Why calculate amplitudes & form factors?

- Strong Physics motivations
  - LHC physics
  - precision perturbative QCD
- Mathematical physics motivations
  - AdS/CFT
  - New dualities
  - Hidden symmetries
  - Links to integrability

Hard calculations lead to…

- Calculations with standard Feynman rules are cumbersome
  - Gauge dependence, off-shell
  - Many diagrams
  - Huge cancellations
- E.g.: for 5 gluons



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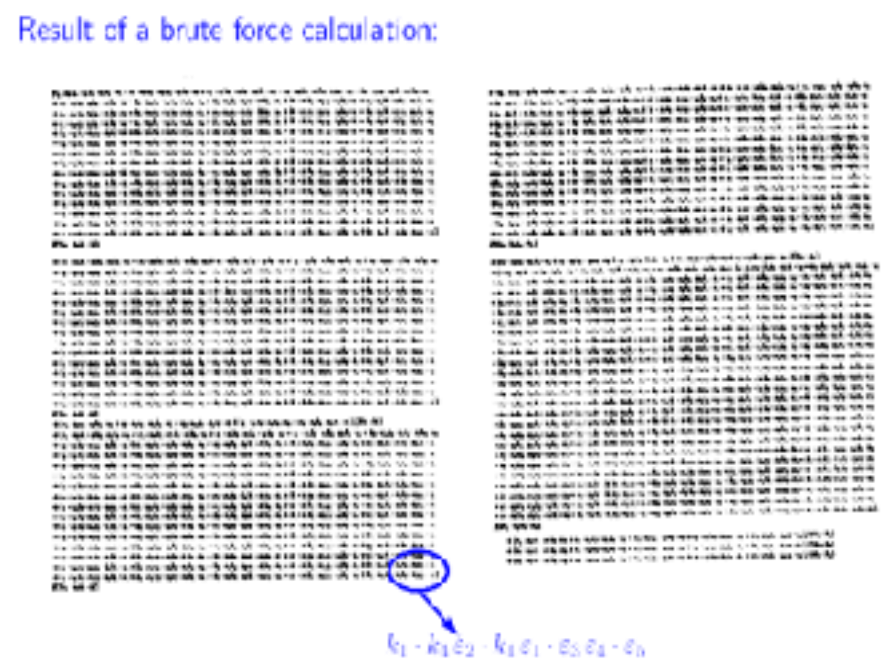


- 2 gluons → n gluons at tree-level

gg => n g	n=7	n=8	n=9
Diagrams	559405	10525900	224449225

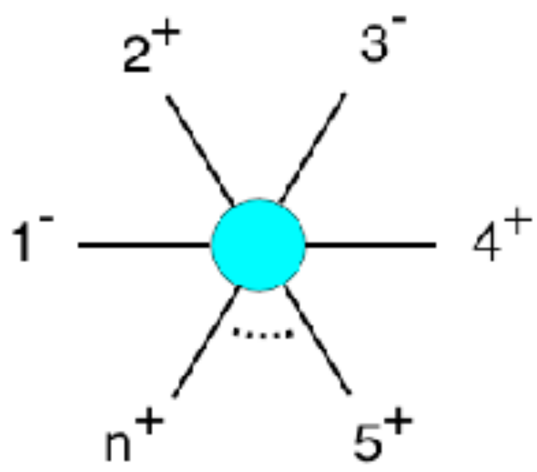
...surprising simplicity

- All-plus/single-minus helicity tree amplitudes

$$A(1^\pm, 2^+, 3^+, \ldots, n^+) = 0$$

- Maximally Helicity Violating (MHV) tree amplitudes with 2 negative helicity and n-2 positive helicity gluons

$$A_{\rm MHV}(1^+, 2^+, \ldots, i^-, \ldots, j^-, \ldots, n^+) = \frac{\langle ij\rangle^4}{\langle 12\rangle\langle 23\rangle \ldots \langle n1\rangle}$$



i=1, j=3

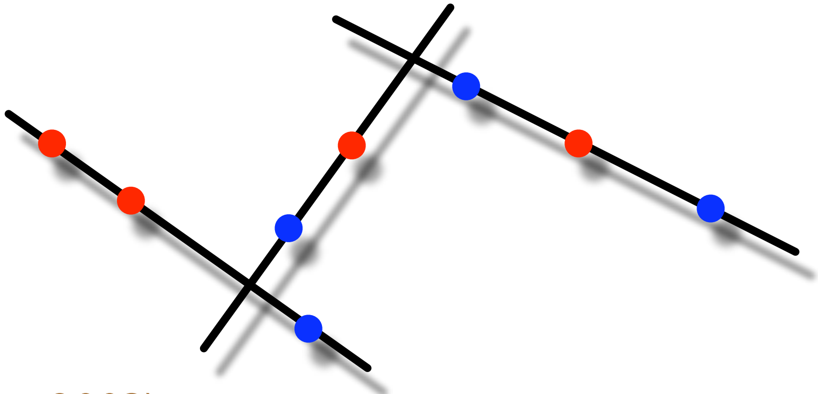
Parke-Taylor/Mangano-Parke

$$\langle ij\rangle \sim \sqrt{s_{ij}} e^{i\phi_{ij}}$$

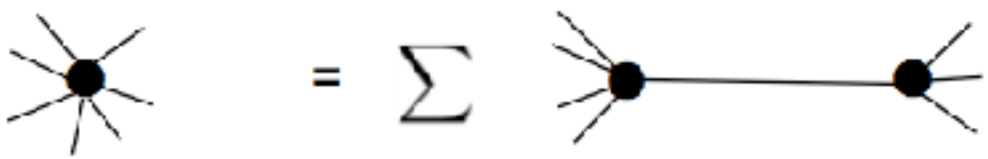
$$s_{ij} = (p_i + p_j)^2$$

Traditional methods obscure simplicities and hidden symmetries

Hidden Simplicities in Amplitudes…



- Why are amplitudes so simple? Can we make use of this fact?
  - Geometry in twistor space (Witten 2003)
  - Iterative structures of S-matrix of gauge theory & gravity



Only 3-point Amplitudes needed as Input!!

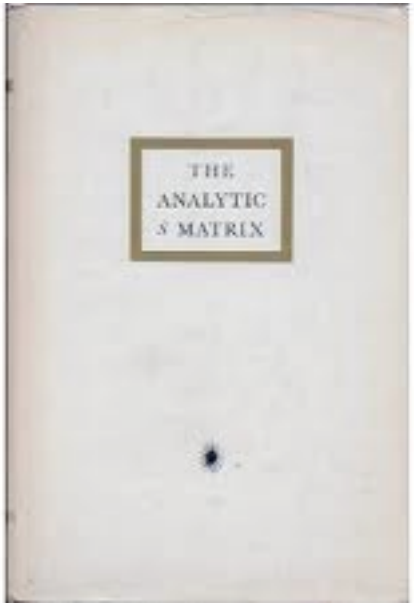
- Avoid problems of standard Feynman rules
- gauge dependence, ghosts
- off-shell
- large number of diagrams

... Inspire New Methods

- Novel Methods
  - MHV Diagrams (Cachazo-Svrcek-Witten; AB, Spence Travaglini)
  - Generalised Unitarity algebraic; no phase space/dispersion integrals!
    (Bern, Dixon, Dunbar, Kosower,... Britto, Cachazo, Feng,...)

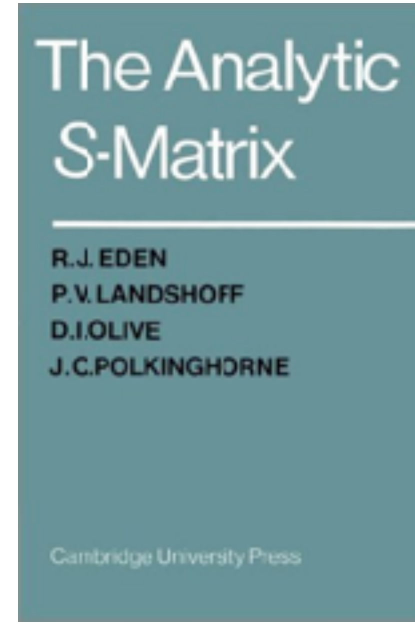
  - On-shell Recursion Relations (Britto-Cachazo-Feng-Witten)
- Important common feature
  - Only on-shell quantities needed
    e.g. MHV rules need only

$$\mathcal{A}_{n,\mathrm{MHV}} = \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$



Geoffrey Chew

Return of the...



- Key ideas go back to pre-history old S-matrix approach of 60's !

- On-shell ...fields themselves are of little interest. Deal with physical, on-shell S-matrix elements directly...

- Unitarity ...use the analytic structure of the scattering amplitudes (poles, branch cuts, factorisation), whether or not some underlying Lagrangian theory exists...

- Complexify ...One of the most remarkable discoveries in elementary particle physics has been that of the complex plane

- The S-Matrix is unitary

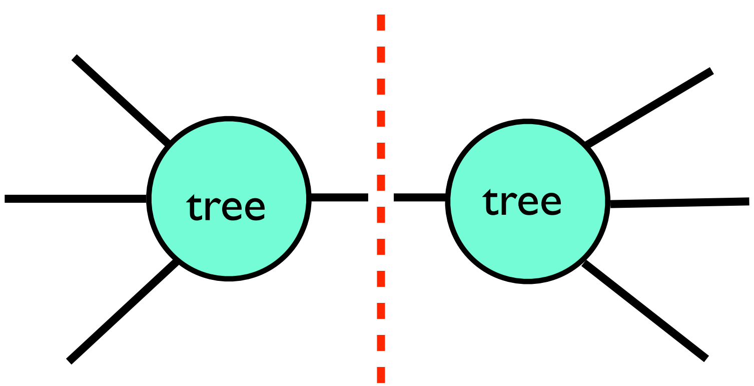
$$S = 1 + iT \quad \rightarrow \quad -i(T - T^\dagger) = T^\dagger T$$

BCFW recursion relations

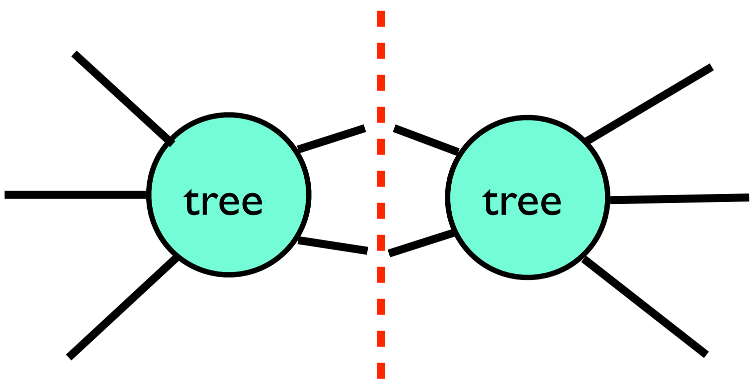
Trees from Trees

Generalised Unitarity Method

Loops from Trees



$$\frac{1}{l_i^2 + i\epsilon} \rightarrow -i\pi\delta(l_i^2)$$



$$\frac{1}{l_i^2 + i\epsilon} \rightarrow -i\pi\delta(l_i^2)$$

N=4 Super Yang-Mills

- Gluons, 4 Weyl fermions, 3 complex scalars X, Y, Z all in adjoint representation
- Simplest interacting gauge theory in 4D: superconformal, beta function = 0
- New symmetries of amplitudes: Dual Conformal Symmetry => Yangian Symmetry => Integrability
- Playground to test and refine new methods: generalised unitarity, BCFW recursion relations, symbol of functions
- Clearly very different from QCD: BUT
  - Gluon tree amplitudes in QCD same as N=4 SYM
  - N=4 often captures large chunks of full QCD computations

Beyond amplitudes

- Long-term goal: extend success of on-shell methods to "partially or fully off-shell" quantities
  - Partially off-shell: form factors (main focus today)
    - MHV diagrams, BCFW, generalised unitarity, remainder functions, symbols, scattering equations (CHY)… (AB, Hughes, Panerai, Penante, Spence, Travaglini, Wen, Yang, Young; Bork, Kazakov, Vartanov; Loebbert, Nandan, Sieg, Wilhelm, Yang; Gehrmann, Henn…)
    - Remarkable simplicities/regularities but no dual conformal symmetry
  - Fully off-shell: correlation functions (Engelund-Roiban; AB, Penante, Travaglini, Young)

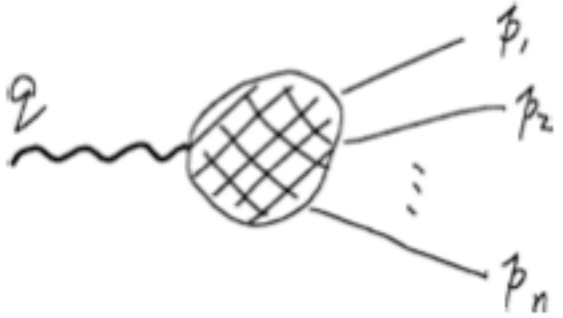
$$\langle 0|\, \mathcal{O}(x_1)\bar{\mathcal{O}}(x_2)\,|0\rangle \sim \frac{1}{((x_1-x_2)^2)^{\Delta_0+\gamma}}$$

- Anomalous dimensions γ = eigenvalues of hamiltonian ${H^A}_B$ (dilatation operator) of an integrable spin-chain in N=4!

Form Factors: "going partially off-shell"

- More general objects than correlation functions, Wilson loops, amplitudes: e.g. Wilson loops with operator insertions, correlators of Wilson loops ...
- Form Factors: interpolate between correlators and amplitudes, partially off-shell

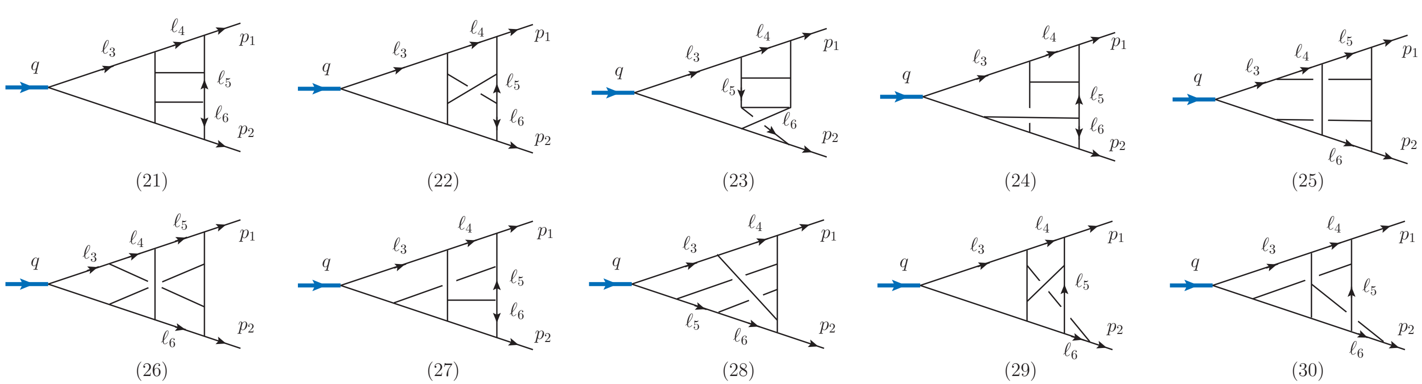
$$\int d^4x \, e^{-iqx} \langle 1 \cdots n | \mathcal{O}(x) | 0 \rangle \; = \; \delta^{(4)}(q - \sum_{i=1}^{n} p_i) \, \langle 1 \cdots n | \mathcal{O}(0) | 0 \rangle$$



$$q = \sum_{i=1}^{n} p_i$$

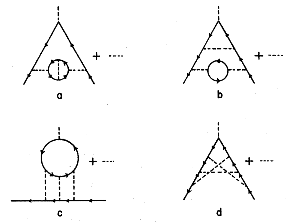
$$q^2 \neq 0 \quad , \quad \text{off} - \text{shell!}$$

- Simplest case (QCD) Sudakov FF (n=2): IR divergences
  - In N=4 2-Loop Sudakov FF first studied by Van Neerven
  - 3 Loops: (Gehrmann, Henn, Huber)
  - 4 & 5 Loops (Boels, Huber, Yang):
    - Color-Kinematics duality (Bern-Carrasco-Johansson)
    - Cusp anomalous dim, Casimir scaling violated at four loops



FFs appear in many physics contexts

- Three-loop correction to electron g–2

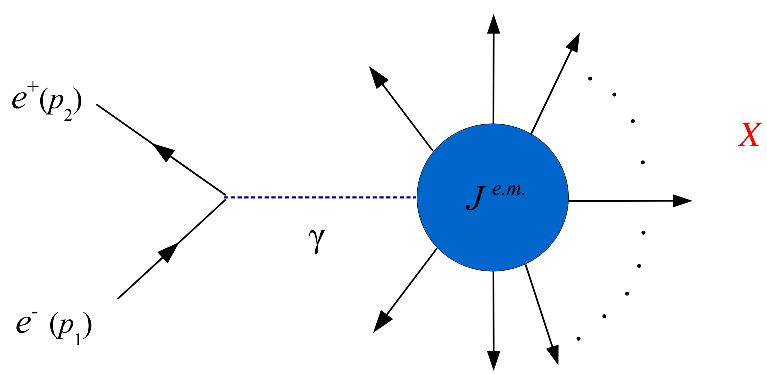
72 diagrams like  $= (1.181241456...)\,(\alpha_{\rm e.m.}/\pi)^3$

(Cvitanovic & Kinoshita '74)

(Laporta & Remiddi '96)

  - wild oscillations between individual diagram
  - result is O(1) => mysterious cancellations

- $e^+ e^- \to$ hadrons (LEP):



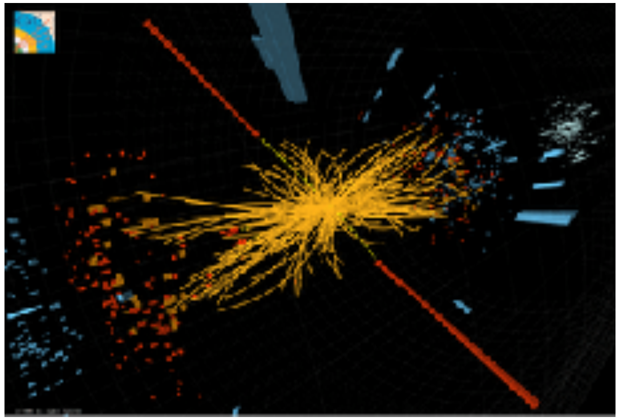
$$e\,\bar{v}(p_2)\gamma_\mu u(p_1)\;\frac{\eta^{\mu\nu}}{(p_1+p_2)^2}\;(-e)\langle X|\,J_\nu^{e.m.}(0)\,|0\rangle$$

hadronic electromagnetic current

$e^+ e^- \to$ hadrons (X)

all orders in $\alpha_{\rm strong}$, first order in $\alpha_{\rm e.m.}$

Effective Lagrangians



- Higgs + multi-gluon amplitudes
  - at low M_H , dominant Higgs production at the LHC through gluon fusion
  - coupling to gluons through a quark loop
  - for $M_H < 2\, m_t$ integrate out top quark
- Effective Lagrangian description: leading

$$\mathcal{L}_{\rm eff} \sim H\, \mathrm{Tr}\, F^2 \qquad \mathrm{Tr} F^2 = \mathrm{Tr} F^2_{\rm SD} + \mathrm{Tr} F^2_{\rm ASD}$$

  - coupling $\dfrac{\alpha_S}{12\pi v}$, $v = 246 GeV$ independent of m_t
  - subleading: $$\mathcal{L}_{\rm sub} \sim \frac{C_1}{v m_t^2} H \mathrm{tr} F^3 + \frac{C_2}{v m_t^2} H \mathrm{tr} DFDF + \ldots$$

FFs = amplitudes in effective theories

- Higgs amplitudes are form factors of $\mathrm{Tr}\, F^2$
  - bring down one interaction, and Wick-contract the Higgs field

$$F_{F^2_{\mathrm{ASD}}} = \int d^4x\, e^{-iqx} \langle state | \mathrm{Tr}\, F^2_{\mathrm{ASD}}(x) | 0 \rangle \quad \text{with} \quad q^2 = M_{\mathrm{H}}^2$$

- Can we look at the same quantity, but in N=4 SYM?
  - Highly symmetric theory, easier to identify any structure
  - Find an appropriate translation of the matrix element to N=4 SYM
  - What operator? What state? Key question: can we use supersymmetry to simplify/organise the calculation?

Higgs + gluon amplitudes

- Leading order $\mathcal{L}_{\rm eff} \sim H \, {\rm Tr}\, F^2$
  - Early application of on-shell techniques to tree- and one-loop amplitudes (Badger, Dixon, Glover, Khoze; Badger, Glover, Risager, Mastrolia, Williams)

$$F^{\rm tree}_{{\rm tr}F^2}(1^-,2^-,3^+) = \frac{\langle 12\rangle^3}{\langle 23\rangle\langle 31\rangle} \quad , \quad F^{\rm tree}_{{\rm tr}F^2}(1^+,2^+,3^+) = \frac{q^4}{[12][23][31]} \quad , \quad q^2 = m_H^2$$

  - This has been pushed in QCD to 2 & 3-loop order for 2 gluons (Anastasiou, Melnikov; Harlander, Kilgore; Anastasiou, Duhr, Buehler, Herzog, Dulat, Furlan, Mistlberger),

    and to 2 loops for 3 partons (Glover, Gehrmann, Jaquier & Koukoutsakis)
- Subleading, finite top-mass corrections have been studied as well (e.g. Neill; Dawson, Lewis, Zeng….)

Higgs + gluon amplitudes: from QCD to N=4

- In N=4 SYM operators are organised in multiplets and are related by SUSY transformations

- A) Protected operators (BPS): eg. stress tensor multiplet

$$\mathrm{tr}(X^2) = \mathrm{tr}(\phi_{12}^2) \xrightarrow{Q^4} \mathcal{L}_{\text{on-shell}} \sim \mathrm{tr}(F_{\mathrm{SD}}^2) + \ldots$$

- B) Non-protected: $\mathrm{tr}(F^3)$, $\mathrm{tr}(DFDF)$, $\ldots$
  - In N=4 related to Konishi operator, $K \sim \mathrm{tr}(\bar{X}X + \bar{Y}Y + \bar{Z}Z)$

- Question: are there any similarities between QCD & N=4?

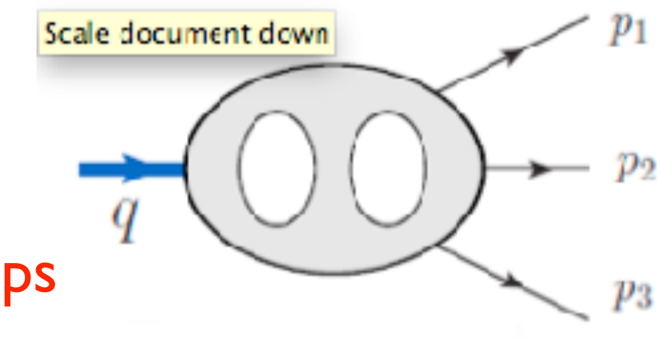
Higgs + gluons amplitude translated to N=4

- Need precise translation of operator $\mathrm{Tr}\,(F_{\mathrm{ASD}})^2$ in QCD to N=4 SYM:

  $\mathrm{Tr}\,(F_{\mathrm{ASD}})^2$ is contained in the on-shell Lagrangian $\mathcal{L}_{\mathrm{on-shell}}$

- $\mathcal{L}_{\mathrm{on-shell}}$ belongs to a special half-BPS supermultiplet of operators:
- The stress-tensor multiplet:
  - Not renormalised
  - zero anomalous dimension
  - no operator mixing
- correspondingly, there is a supersymmetric form factor of the chiral part of the stress tensor multiplet (AB, Gurdogan, Mooney, Travaglini, Yang)
- use equivalently the operator $\mathrm{Tr}\,X^2$, with X = one of the complex scalars (simpler!)

3-point 2-loop MHV FF in N=4



- Start with 3-point FF at 2-loops

$$F_3(1,2,3) \;=\; \langle X(p_1)\, X(p_2)\, g^+(p_3)\, |\mathrm{Tr} X^2\, |0\,\rangle$$

- This is how we mimic $\langle g^\pm(p_1)\, g^\pm(p_2)\, g^+(p_3)\, |\mathrm{Tr} F^2_{\mathrm{ASD}}\, |0\,\rangle$ in QCD (Higgs into 3 gluons)

- At loop level tree FF (and color factor) can be stripped off

$$F_3^{(L)} \;=\; F_3^{\mathrm{tree}}\, \mathcal{G}_3^{(L)}(1,2,3)$$

- $\mathcal{G}_3^{(2)}$ is helicity-blind, scalar function, permutation symmetric
  - UV finite in N=4
  - IR divergences exponentiate

Finite remainders

(Catani; Anastasiou, Bern, Dixon, Kosower; Bern, Dixon, Smirnov)

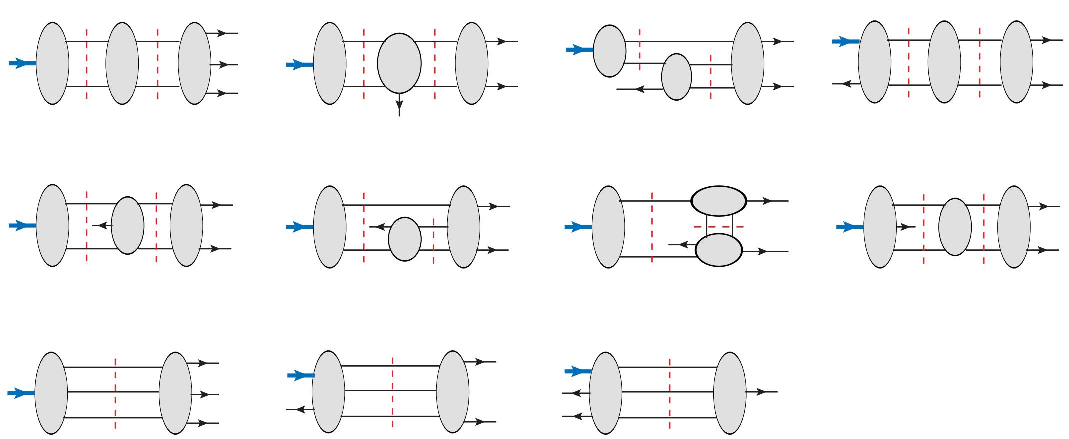
- Subtract off universal IR divergences from the (renormalised) L-loop answer

- All loops: $\mathcal{A}_{n,\mathrm{MHV}} = \mathcal{A}^{\mathrm{tree}}_{n,\mathrm{MHV}} \, \mathcal{M}_n$

$$\mathcal{M}_n := 1 + \sum_{L=1}^{\infty} a^L \mathcal{M}_n^{(L)} \sim \exp\left[\mathrm{BDS} + \mathcal{R}\right] \qquad a \sim g^2 N/(8\pi^2)$$

  - $\mathrm{BDS} \sim \mathrm{div} + \gamma_K \, \mathrm{Finite}^{(1)}(p_1, \ldots, p_n)$ BDS Ansatz, completely known
    - div = universal infrared-divergent part, exponentiation is expected
    - $\mathrm{Finite}^{(1)}(p_1, \ldots, p_n)$ = finite part of one-loop amplitude
    - γ_K = cusp anomalous dimension $\rightarrow$ integrability
    - R is the so-called remainder function the most interesting part!

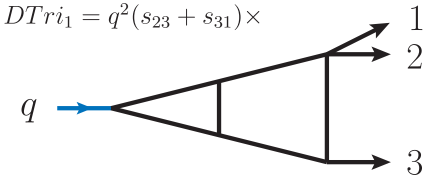
- Exponentiation of finite parts for one-loop amplitude due to dual conformal symmetry (Drummond, Henn, Korchemsky, Sokatchev)
    - Four- and five-point amplitudes: $\mathrm{R} = 0$
    - Non-trivial remainder appears from six points on (Drummond, Henn, Korchemsky, Sokatchev; Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich)
- No dual conformal symmetry for form factors
    - Still, exponentiating finite parts leads to a very simple remainder
- Compute using modern unitarity methods (Bern, Dixon, Dunbar, Kosower; BDK; Britto, Cachazo, Feng)
    - Construct amplitude with two- and multi-particle cuts



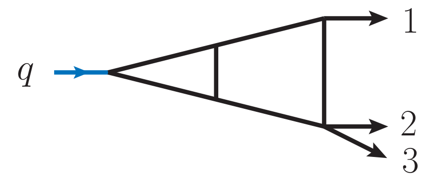
• Result of 2-loop calculation: (AB, Travaglini, Yang)

$$\frac{F_3^{(2)}}{F_3^{\rm tree}} = \sum_{i=1}^{2}(DTri_i + DBox_i) + TriPent + NBox + NTri + \text{cyclic}$$

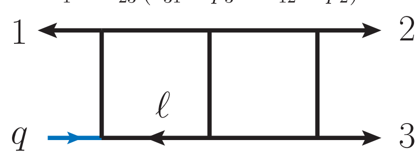
$DTri_1 = q^2(s_{23}+s_{31})\times$



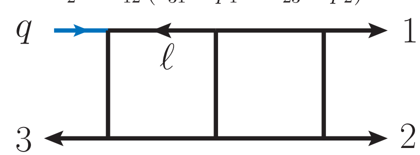
$DTri_2 = q^2(s_{12}+s_{31})\times$



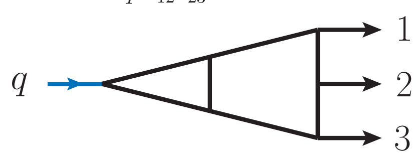
$DBox_1 = s_{23}\left(s_{31}\ell\cdot p_3 - s_{12}\ell\cdot p_2\right)\times$



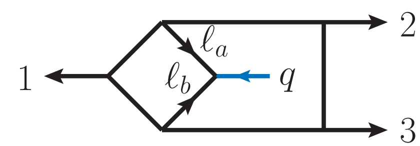
$DBox_2 = s_{12}\left(s_{31}\ell\cdot p_1 - s_{23}\ell\cdot p_2\right)\times$



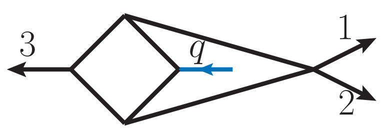
$TriPent = q^2 s_{12}s_{23}\times$



$NBox = s_{23}\left(\frac{1}{2}s_{12}s_{31} - s_{12}\ell_a\cdot p_2 - s_{31}\ell_b\cdot p_3\right)\times$



$NTri = \frac{1}{2}q^2(s_{23}+s_{31})\times$



result expressed as rational coefficients × two-loop planar and non-planar integrals

- Main feature of two-loop remainder:
  - sum of functions has homogeneous degree of transcendentality = 4
- Transcendentality
  - "constants" have transcendentality 0
  - π, $\log$ transcendentality 1
  - π^2 ; $\log^2$, Li_2 transcendentality 2
  - ... ζ_n , Li_n , $\log \times \mathrm{Li}_{n-1}$ ... transcendentality n
  - at L loops: expect transcendentality equal to $2L$
  - Goncharov polylogarithms:
    degree-k Goncharov polylog = k-fold iterated integral:

$$G(a_k, a_{k-1}, \ldots, a_1; z) = \int_0^z G(a_{k-1}, \ldots, a_1; t) \frac{dt}{t - a_k}, \quad G(z) = 1$$

- Final answer (with the help of the symbol)

(AB, Travaglini, Yang)

$$\mathcal{R}_3^{(2)} = -2\left[J_4\left(-\frac{uv}{w}\right) + J_4\left(-\frac{vw}{u}\right) + J_4\left(-\frac{wu}{v}\right)\right] - 8\sum_{i=1}^{3}\left[\mathrm{Li}_4\left(1-u_i^{-1}\right) + \frac{\log^4 u_i}{4!}\right]$$
$$-2\left[\sum_{i=1}^{3}\mathrm{Li}_2(1-u_i^{-1})\right]^2 + \frac{1}{2}\left[\sum_{i=1}^{3}\log^2 u_i\right]^2 - \frac{\log^4(uvw)}{4!} - \frac{23}{2}\zeta_4$$

- $u_1 = u = s_{12}\,/\,q^2,\ u_2 = v = s_{23}\,/\,q^2,\ u_3 = w = s_{31}\,/\,q^2$ kinematic invariants
- $J_4(z) := \mathrm{Li}_4(z) - \log(-z)\mathrm{Li}_3(z) + \frac{\log^2(-z)}{2!}\mathrm{Li}_2(z) - \frac{\log^3(-z)}{3!}\mathrm{Li}_1(z) - \frac{\log^4(-z)}{48}\ .$
- Bloch-Wigner-Ramakrishnan(-Zagier) polylogarithmic function
- Result:
  - only classical polylogarithms - Goncharov polylogarithms “cancel”
  - condenses several pages of more complicated functions!

Next: QCD

Higgs + parton amplitudes in QCD

- Higgs + 3 partons (Koukoutsakis 2003; Gehrmann, Glover, Jaquier & Koukoutsakis 2011)

  - $H\, g^+\, g^-\, g^-$ MHV
  - $H\, g^+\, g^+\, g^+$ maximally non-MHV
  - $H\, q\;\; \bar{q}\;\; g$ fundamental quarks

$$F^{\rm tree}(H, g_1^-, g_2^-, g_3^+) \;=\; \frac{\langle 1\,2\rangle^2}{\langle 2\,3\rangle\,\langle 3\,1\rangle}$$

$$F^{\rm tree}(H, g_1^+, g_2^+, g_3^+) \;=\; \frac{q^4}{[1\,2]\,[2\,3]\,[3\,1]}$$

$$q^2 \;=\; M_H^2$$

- In N=4 SYM:

  - $(H\, g^+\, g^-\, g^-)$ and $(H\, g^+\, g^+\, g^+)$ both derived from super form factor
  - from supersymmetric Ward identities:

$$\frac{F^{(L)}(g_1^-, g_2^-, g_3^+)}{F^{\rm tree}(g_1^-, g_2^-, g_3^+)} \;=\; \frac{F^{(L)}(g_1^+, g_2^+, g_3^+)}{F^{\rm tree}(g_1^+, g_2^+, g_3^+)} \;=\; \mathcal{G}^{(L)}(u, v, w)$$

$\leftarrow$ what we computed

- QCD answer from Gehrmann, Glover, Jaquier & Koukoutsakis
  - very different looking than N=4 SYM result!
  - transcendentality 4,3,2,1 and 0 (rational). In N=4, only degree 4
  - expressed in terms of several pages of Goncharov polylogarithms
  - expected because of expansion as $\sum$ (coefficient x integral) !
    - each integral is separately quite complicated
- Comparing the two quantities…

…we find a surprising relation

$$\mathcal{R}^{(2)}_{H\, g^- g^- g^+}\Big|_{\mathrm{MAX\ TRANS}} = \mathcal{R}^{(2)}_{H\, g^+ g^+ g^+}\Big|_{\mathrm{MAX\ TRANS}} = \mathcal{R}^{(2)}_{\mathcal{N}=4\,\mathrm{SYM}}$$

- N=4 SYM answer is the maximally transcendental part of the QCD amplitudes, and is helicity-independent
- not a "perturbative" expansion
- different calculations !
  - different operators $(\mathrm{Tr}\,(F_{\mathrm{ASD}})^2$ vs $\mathrm{L}_{\text{on-shell}})$
  - different theories (N=4 SYM vs QCD)
- Goncharov polylogarithms in QCD results eliminated in favour of classical polylogarithms (Duhr)

- Nothing similar holds for the $(H, q, \bar{q}, g)$ form factor

- Principle of maximal transcendentality:
  - results in N=4 SYM obtained from results in QCD by deleting all terms with less-than-maximal transcendentality
    - an experimental rule, valid only in some cases
    - discovered by Kotikov, Lipatov, Onischchenko and Velizhanin in the context of anomalous dimensions of twist-2 operators
    - so far seen only in kinematic-independent quantities
    - several counter-examples in amplitudes, e.g. broken for one-loop amplitudes in pure Yang-Mills for n>4
- Supersymmetry as an organisational principle…
  - …even if supersymmetry is not realised in nature
- Next testing ground: form factors of higher-dimensional operators describing Higgs + multigluon scattering

From N=4 to QCD

(AB, Kostacińska, Penante, Travaglini, Young '16; AB, Kostacińska, Penante, Travaglini '17 +in progress)

- Effective field theory description for finite $m_{\rm top}$ corrections
  - Beyond leading-order term $\mathcal{L}_{\rm eff}^{(0)} \sim H\, {\rm Tr} F^2$ (infinite $m_{\rm top}$)
  - Next corrections: 4 dimension-7 operators in QCD
  - Two particular operators also present in N=4 SYM:

$$\mathcal{L}_{\rm eff}^{(1)} \sim H\, {\rm Tr} F^3 \qquad \mathcal{L}_{\rm eff}^{(2)} \sim H\, {\rm Tr}(D_\mu F_{\rho\sigma})(D^\mu F^{\rho\sigma})$$

  - Goal: compute in N=4 SYM and compare to QCD (result not yet available in literature)
  - previous work at one loop: Dawson, Lewis & Zeng; Neill; Harlander & Neumann…
  - higher-dimensional operators also studied as corrections to the Standard Model (Buchmuller & Wyler '85 and MANY more!)

Questions & Conjectures

- Does the maximal-transcendentality connection still hold?
- Do protected and non-protected operators in N=4 SYM play a special role for Higgs+gluon amplitudes?
- Can we identify universal building blocks in the results?
- Connections to the dilatation operator of N=4 SYM?

- Answer is YES for all questions!

Two-loop results

- Compare remainders for the two form factors:

$\langle g^+ g^+ g^+ |\, \mathrm{Tr} F^3_{\mathrm{ASD}} \,| 0 \rangle$ in any theory (even without supersymmetry)

$\langle XXX |\, \mathrm{Tr} X^3 \,| 0 \rangle$ in N=4 SYM

  - $\mathrm{Tr}\,(X^3)$ is protected "half-BPS", X = one of the three complex N=4 scalars
  - $\mathrm{Tr}\,(F_{\mathrm{ASD}})^3$ is NOT protected — definitely not part of the same multiplet

- Maximally transcendental parts identical !!

$$\mathcal{R}^{(2)}_{F^3_{\mathrm{ASD}}}\Big|_{\mathrm{MAX\,TRANS}} = \mathcal{R}^{(2)}_{\mathrm{BPS}} = -\frac{3}{2}\,\mathrm{Li}_4(u) + \frac{3}{4}\,\mathrm{Li}_4\left(-\frac{uv}{w}\right) - \frac{3}{2}\log(w)\,\mathrm{Li}_3\left(-\frac{u}{v}\right) + \frac{1}{16}\log^2(u)\log^2(v)$$
$$+\frac{\log^2(u)}{32}\Big[\log^2(u) - 4\log(v)\log(w)\Big] + \frac{\zeta_2}{8}\log(u)\Big[5\log(u) - 2\log(v)\Big]$$
$$+\frac{\zeta_3}{2}\log(u) + \frac{7}{16}\,\zeta_4 + \mathrm{perms}\,(u,v,w)\,.$$

$$u := \frac{s_{12}}{q^2} \qquad v := \frac{s_{23}}{q^2} \qquad w := \frac{s_{31}}{q^2}$$

- Same as earlier conclusion for leading-order coupling
  - $\mathrm{Tr}\,(X)^2$ also half-BPS, evaluated in N=4 SYM
  - $\mathrm{Tr}\,(F_{\mathrm{ASD}})^2$ form factor evaluated in N=0

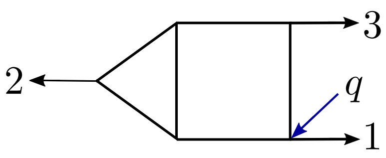
BPS operators in N=4 SYM compute (parts of) phenomenologically relevant quantities in QCD!

- Onto subleading in transcendentality terms
  - Need to identify the "appropriate translation"…
  - …or we can just compute with the "component" operator $\mathrm{Tr}\,(F_{\mathrm{ASD}})^3$

- Translating the operator $\mathrm{Tr}\,(F_{\mathrm{ASD}})^3$ to N=4 language leads to the Konishi supermultiplet
    - simplest non-protected operator multiplet in the theory
    - tree-level super form factor in N=4 computed recently using LHC superspace ! (Chicherin & Sokatchev)
    - generalisation to N=2 and N=1 supersymmetry straightforward

- Remainder contains two types of terms:
    - purely transcendental: 4 (already discussed), 3, 2, 1 and 0
    - new: multiplied by a rational prefactor, e.g. $u/v,\ u/w,\ v/w$

- Calculation in N=4 done, N=2,1, 0 almost ready
    - maximally transcendental part is universal as new integrals have lower transcendentality



- complete N=4 result extremely simple
  - intriguing relations to the finite remainders that have emerged in the calculation of the dilatation operator of the theory.
    - UV: Reproduce expected 2-loop anomalous dimension of Konishi operator
    - IR-divergences exponentiate as expected
- N=2,1,0
  - Calculation considerably more involved.
    Still: remainders R differ only slightly
  - Running coupling, need to renormalise form factors…
  - … and compute Catani's remainder to remove UV and IR divergences
    - New predictions for anomalous dimensions in N=2,1,0 (S)YM

- Transcendentality $3,2,1,0$ parts of the N=4 SYM result for the Konishi supermultiplet:

$$\mathcal{R}^{(2)}_{\mathcal{K},3}\Big|_{\text{pure}} = \text{Li}_3(u) + \text{Li}_3(1-u) - \frac{1}{4}\log^2(u)\log\left(\frac{vw}{(1-u)^2}\right) + \frac{1}{3}\log(u)\log(v)\log(w) + \zeta_2\log(u) - \frac{5}{3}\zeta_3 \; + \; \text{perms}\,(u,v,w)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};3}\Big|_{u/w} = \Big[-\text{Li}_3\left(-\frac{u}{w}\right) + \log(u)\text{Li}_2\left(\frac{v}{1-u}\right) - \frac{1}{2}\log(1-u)\log(u)\log\left(\frac{w^2}{1-u}\right) + \frac{1}{2}\text{Li}_3\left(-\frac{uv}{w}\right) + \frac{1}{2}\log(u)\log(v)\log(w) + \frac{1}{12}\log^3(w) + (u\leftrightarrow v)\Big] + \text{Li}_3(1-v) - \text{Li}_3(u) + \frac{1}{2}\log^2(v)\log\left(\frac{1-v}{u}\right) \; - \; \zeta_2\log\left(\frac{uv}{w}\right)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};2}\Big|_{\text{pure}} = -\text{Li}_2(1-u) - \log^2(u) + \frac{1}{2}\log(u)\log(v) - \frac{13}{2}\zeta_2 \; + \; \text{perms}\,(u,v,w)$$

$$\mathcal{R}^{(2)}_{\mathcal{K};2}\Big|_{u^2/w^2} = \text{Li}_2(1-u) + \text{Li}_2(1-v) + \log(u)\log(v) - \zeta_2$$

$$\mathcal{R}^{(2)}_{\mathcal{K};1} = \left(-4 + \frac{v}{w} + \frac{u^2}{2vw}\right)\log(u) \; + \; \text{perms}\,(u,v,w)\,, \qquad \mathcal{R}^{(2)}_{\mathcal{K};0} = 7\left(12 + \frac{1}{uvw}\right)$$

More surprises…

- result for Konishi almost identical to result for $\mathrm{Tr}\,(X[Y,Z])$
- Same building blocks have already appeared in the computation of the form factor/dilatation operator of the theory in the $SU(2)$ sector (operators built out of X and Y)
    - surprising! [technical comment: $\mathrm{Tr}\,(X[Y,Z])$ belongs to the $SU(2|3)$ sector > $SU(2)$]
    - results much more structured than expected. Connections to integrability?
- Hints at universal building blocks?
- Results for N<4, and for pure Yang-Mills on the way.
  Stay tuned!

Summary

- Form factors in N=4 SYM
  - share simplicity of amplitudes
  - compute Higgs amplitudes in QCD in an effective Lagrangian approach
  - remarkable simplicity of the remainders
  - N=4 SYM computes the most complicated part of the remainder

- Systematise (understand!) the connection between Higgs amplitudes in QCD and form factors in N=4 SYM

Further open questions

- Reinforce links with integrability
  - Dual conformal symmetry of amplitudes implies Yangian symmetry of dilatation operator *D*
  - Can extract *D* from form factors, e.g. SU(2|3)/SU(2) sector at 2 loops; complete 2-loop dilatation operator?
- Hidden symmetries responsible for simple results?
  How is dual conformal symmetry of amplitudes realised?
- Form factors link completely on-shell and off-shell worlds.
- More applications/connections/similiarities to/with phenomenologically interesting theories to be explored