

Lattice explorations of a composite Higgs

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and with M. Golterman

IFT Xmas Workshop

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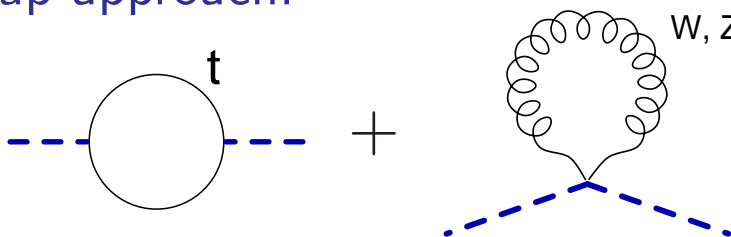
Hierarchy problem: [un]naturalness of the Higgs potential

- Higgs potential: $V(h) = -\mu^2 h^2 + \lambda h^4$

EW scale: $\sqrt{2} \langle h \rangle = \mu / \sqrt{\lambda} \simeq 246 \text{ GeV}$, Higgs mass: $m_H = \sqrt{2} \mu \simeq 125 \text{ GeV}$

- What **symmetry** stops μ^2 from becoming as large as M_{Planck}^2 ?

- Bottom-up approach:

$$-\mu^2 = \text{---} \text{---} \text{---} \text{---} + \text{---} \text{---} \text{---} \text{---} + \text{new physics} \approx g^2 \Lambda_{\text{new}}^2$$


- ★ **Supersymmetry:** (not this talk) each SM field has “super-partner” of opposite statistics; $\Lambda_{\text{new}} =$ supersymmetry breaking scale

- ★ **New strong dynamics** at the few TeV scale

Higgs is pseudo Nambu-Goldstone boson arising from

- approximate flavor symmetry of new sector (“Composite Higgs”)
- approximate scale symmetry of new sector (“Walking technicolor”)

Composite Higgs

- “Hypercolor”: new strong sector with scale $f \gg h$ ($f \sim 1$ to few TeV)
- Within hypercolor theory, Higgs is exact Nambu-Goldstone boson of (chiral) flavor symmetry breaking
- Higgs potential from coupling to SM fields: EW gauge bosons, top quark, ...

$$V_{\text{eff}}(h) = -\alpha \cos^2(h/f) + \beta \sin^2(2h/f)$$

- EW gauge bosons only: $\alpha = (3g^2 + g'^2)C_{LR} > 0$, $\beta = 0$
 - \Rightarrow vacuum alignment (see next slide)
 - \Rightarrow By itself, would lead to $h = 0$

Recall pion mass splitting: $M_{\pi^\pm}^2 - M_{\pi^0}^2 = (e^2/f^2)C_{LR} > 0$ [DGMLY 1967]

- Will need top quark contribution to $V_{\text{eff}}(h)$ to trigger EW symmetry breaking
- ★ How much tuning of α and β is needed for $h/f \ll 1$?

“UV complete” composite Higgs

[Ferretti & Karateev 2013]

- Hypercolor sector has global symmetry G spontaneously broken to H with

$$H \supset SU(3)_{\text{color}} \times SU(2)_L \times SU(2)_R \times U(1)_X$$

$$\supset SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y \qquad Y = T_{3R} + X$$

\Rightarrow EW gauge bosons couple to unbroken generators \Rightarrow vacuum alignment

- coset G/H must contain Higgs doublet: $(2, 2)$ of $SU(2)_L \times SU(2)_R$
- Unbroken custodial symmetry \Rightarrow ρ parameter $\equiv \frac{M_W^2}{M_Z^2 \cos^2(\theta_W)} \approx 1$
- partially composite top quark:
need hyperbaryons that can couple linearly to $q_L = (t_L, b_L)$ and to t_R
- SM \times hypercolor is anomaly free
- QCD & hypercolor are asymptotically free
- ★ don't worry about b and other SM fermion masses

Ferretti's model – Higgs field & EW couplings

- $SU(4)$ gauge theory with 5 sextet Majorana fermions Υ_i , $i = 1, 2, \dots, 5$
- Non-linear sigma model (sextet is real representation):

$$\Sigma_{ij} \sim \langle \Upsilon_i \Upsilon_j \rangle \sim \langle \Upsilon_j \Upsilon_i \rangle$$

$$\Sigma = \Sigma^T = \exp(2i\Pi/f) \in SU(5)/SO(5)$$

$$\Sigma \rightarrow g\Sigma g^T, \quad g \in SU(5)$$

- embedding of Higgs $SU(2)_L \times SU(2)_R \approx SO(4) \subset SO(5)$

$$\Pi = \text{singlet} + \text{Higgs} + 3 \text{ triplets} = (1, 1) + (2, 2) + (3, 3) \Rightarrow 14 \text{ NGBs}$$

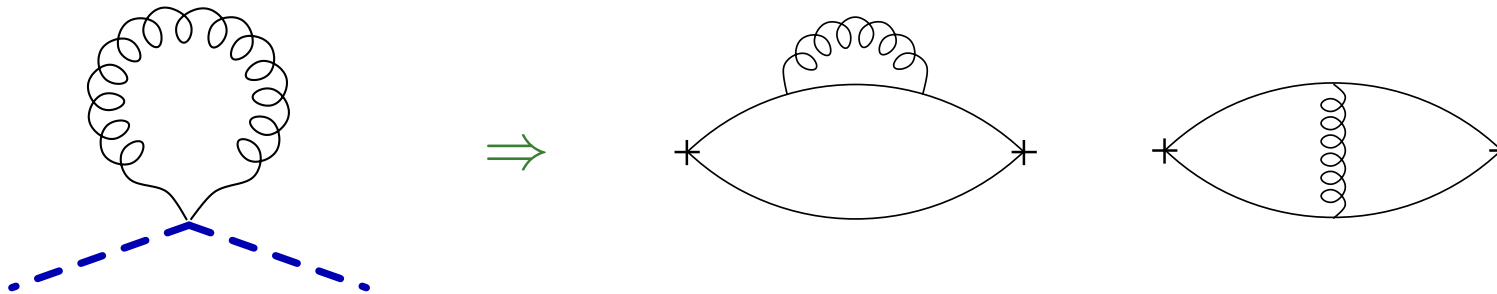
$$\text{Higgs} = \sqrt{2} \begin{pmatrix} 0 & 0 & 0 & 0 & \text{Im } H_+ \\ 0 & 0 & 0 & 0 & \text{Re } H_+ \\ 0 & 0 & 0 & 0 & -\text{Im } H_0 \\ 0 & 0 & 0 & 0 & \text{Re } H_0 \\ \text{Im } H_+ & \text{Re } H_+ & -\text{Im } H_0 & \text{Re } H_0 & 0 \end{pmatrix}$$

Gauge bosons part of Higgs potential

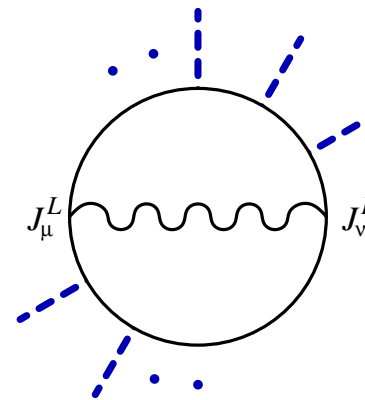
$$C_{LR} = \int_0^\infty dq^2 q^2 \Pi_{LR}(q^2)$$

$$q^2 \Pi_{LR}(q^2) = \langle \mathcal{A}_\mu \mathcal{A}_\nu - \mathcal{V}_\mu \mathcal{V}_\nu \rangle^\perp \geq 0$$

[Witten 1983]



chiral symmetry relates diagrams
with 0, 1, 2, ... NGBs



Lattice calculation of C_{LR}

- Simulating 5 Majoranas is hard; for now, 4 Majoranas = 2 Diracs
⇒ Can use standard HMC algorithm
- Coset $SU(4)/SO(4)$ not $SU(5)/SO(5)$; hopefully C_{LR} is not too different.
- Dynamical Wilson-clover fermions (no chiral symmetry for $m \rightarrow 0$)
- Calculate C_{LR} with valence overlap fermions
⇒ exact chiral symmetry for $m \rightarrow 0$
⇒ need mixed-action Chiral Perturbation Theory

In this exploratory study:

two $12^3 \times 24$ ensembles with different lattice (gauge) actions

same lattice spacing ($r_1/a \approx 3 \Rightarrow a^{-1} \approx 2$ GeV in QCD)

Direct summation

Π_{LR} is transverse part of

$$-\sum_x e^{iqx} \langle J_\mu^L(x) J_\nu^R(0) \rangle_{\text{conn}}$$

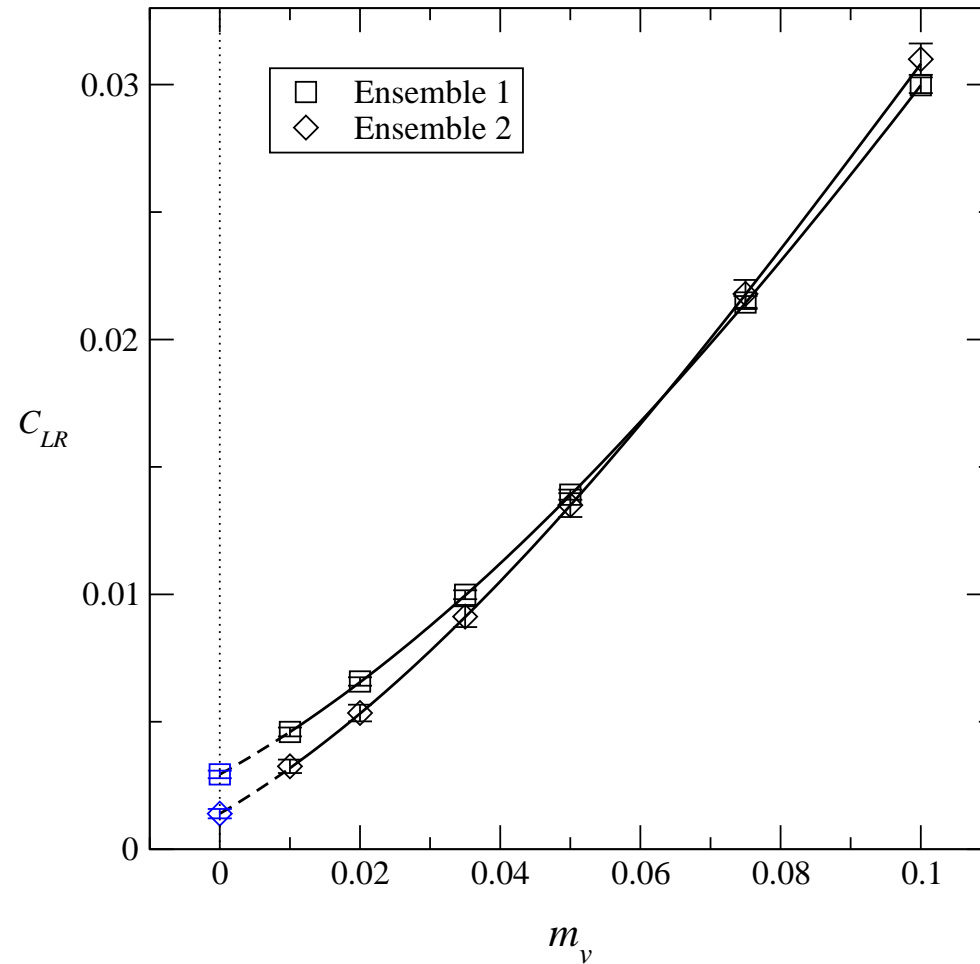
mode sum:

$$C_{LR}(m_\nu) = \frac{16\pi^2}{V} \sum_{q_\mu} \Pi_{LR}(q_\mu)$$

while modelling pole

at $q_\mu = 0$ via

$$\Pi_{LR}(q_\mu) \simeq c + \frac{f_\pi^2}{q^2}$$



Minimal Hadron Approximation

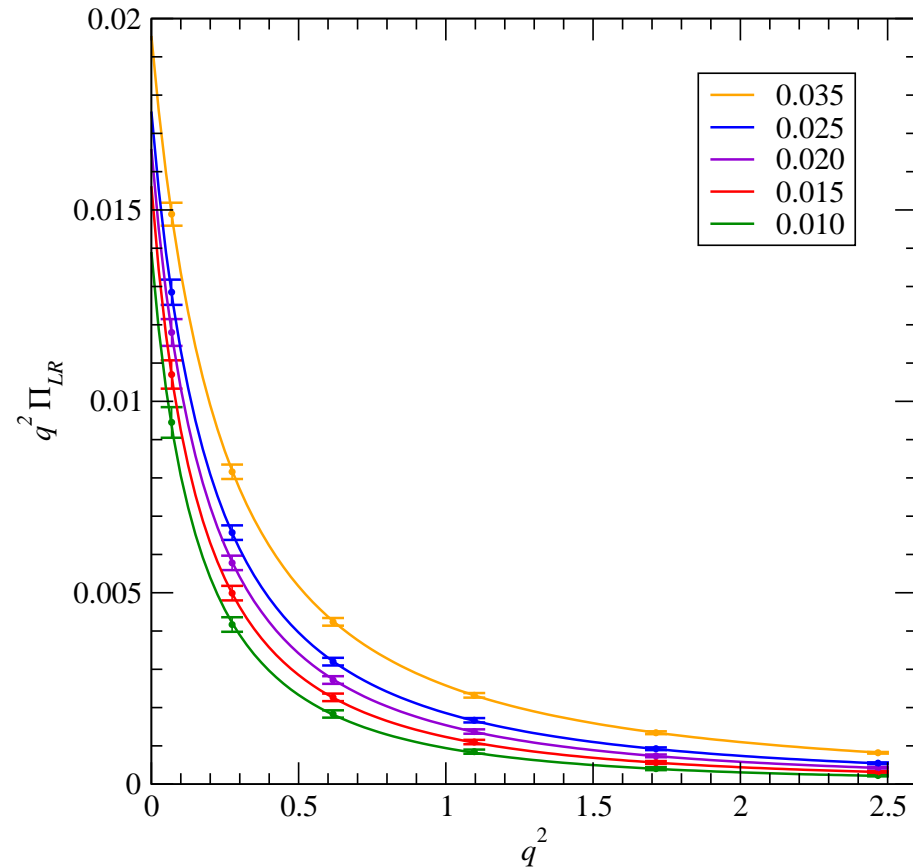
$$\Pi_{LR}(q^2) \approx \frac{f_\pi^2}{q^2} + \frac{f_{a_1}^2}{q^2 + m_{a_1}^2} - \frac{f_\rho^2}{q^2 + m_\rho^2}$$

kinematical pole at $q^2 = 0$ (any $m_\pi \geq 0$)

5 parameter fits to different rays
in momentum space

Then integrate

$$C_{LR} \approx f_\pi^2 \frac{m_{a_1}^2 m_\rho^2}{m_{a_1}^2 - m_\rho^2} \log \left(\frac{m_{a_1}^2}{m_\rho^2} \right)$$



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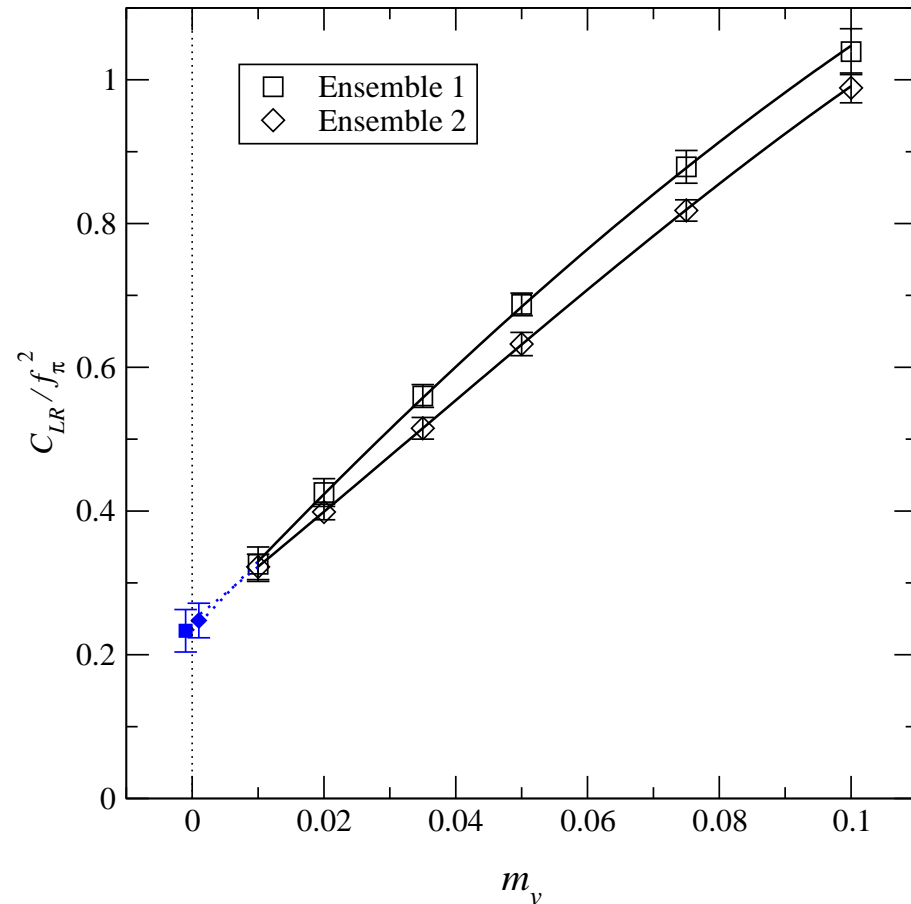
5 parameter fits to different rays
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Then integrate

$$C_{LR} \approx f_\pi^2 \frac{m_{a_1}^2 m_\rho^2}{m_{a_1}^2 - m_\rho^2} \log \left(\frac{m_{a_1}^2}{m_\rho^2} \right)$$

Look at ratio C_{LR}/f_π^2

no continuum limit yet



Ferretti's model – top quark sector

- $SU(4)$ gauge theory with 5 sextet Majorana fermions $\Upsilon_{i[AB]}$, and now adding 3 fundamental Dirac fermions $\Psi_A^a, \tilde{\Psi}_a^A$.

- Partial compositeness:

Top quark couples linearly to hypercolor baryons (“top partners”) hyperbaryons with same SM quantum numbers as top quark:

$$\Upsilon\Psi\Psi = \epsilon^{ABCD}\epsilon_{abc}\Upsilon_{i[AB]}\Psi_C^b\Psi_D^c, \quad \bar{\Upsilon}\Psi\Psi, \quad \Upsilon\tilde{\Psi}\tilde{\Psi}, \quad \bar{\Upsilon}\tilde{\Psi}\tilde{\Psi}, \quad \Upsilon\Psi\tilde{\Psi}, \quad \bar{\Upsilon}\Psi\tilde{\Psi}$$

- Symmetry breaking G/H with

$$G = SU(5) \times SU(3) \times SU(3)' \times U(1)_B \times U(1)'$$

$$H = SO(5) \times SU(3)_{\text{color}} \times U(1)_B$$

⇒ First 2-irreps lattice simulations!

(Majorana sextets, Dirac fund.): (4, 2) instead of (5, 3)

Scale separation?

[Raby, Dimopoulos, Susskind 1980]

(“Tumbling gauge theories,” “Maximally attractive channel”)

gap-equation critical coupling: $\frac{1}{g^2(\Lambda_{\chi\text{SB}})} = \frac{3C_2}{4\pi^2}$

one-loop running: $\frac{1}{g^2(\mu)} = b_1 \log\left(\frac{\mu}{\Lambda_{\text{IR}}}\right)$

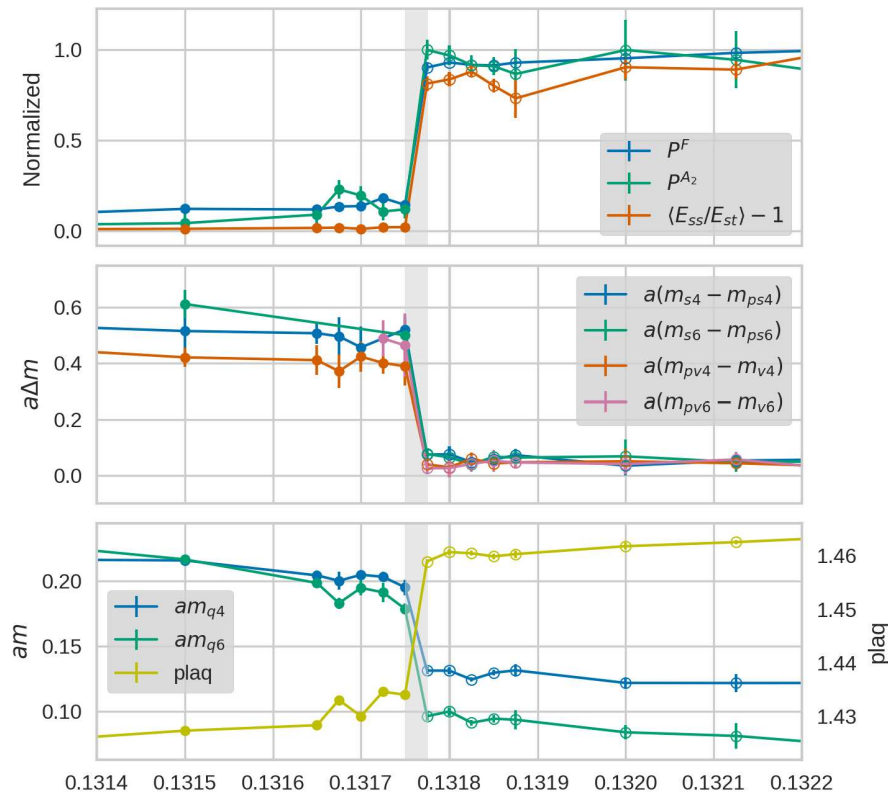
chiral symmetry breaking scale: $\frac{\Lambda_{\chi\text{SB}}}{\Lambda_{\text{IR}}} = \exp\left(\frac{3C_2}{4\pi^2 b_1}\right)$

- For irrep r , $\Lambda_{\chi\text{SB}}(r)$ depends exponentially on $C_2(r)$
- Identify IR scale Λ_{IR} with confinement scale Λ_{conf}
- Exponential behavior of $\Lambda_{\chi\text{SB}}/\Lambda_{\text{conf}}$ found in old-days quenched simulations
- What about dynamical fermions?

Finite-temperature phase transition

bare coupling: β , hopping parameters: κ_4 (fund.), κ_6 (sextet)

slice varying κ_6 at fixed β, κ_4 on $12^3 \times 6$ lattices



Polyakov loops, flow unisotropy

Parity-partners mass splitting

quark masses, plaquette (lattice units)

⇒ single (1st-order) transition!

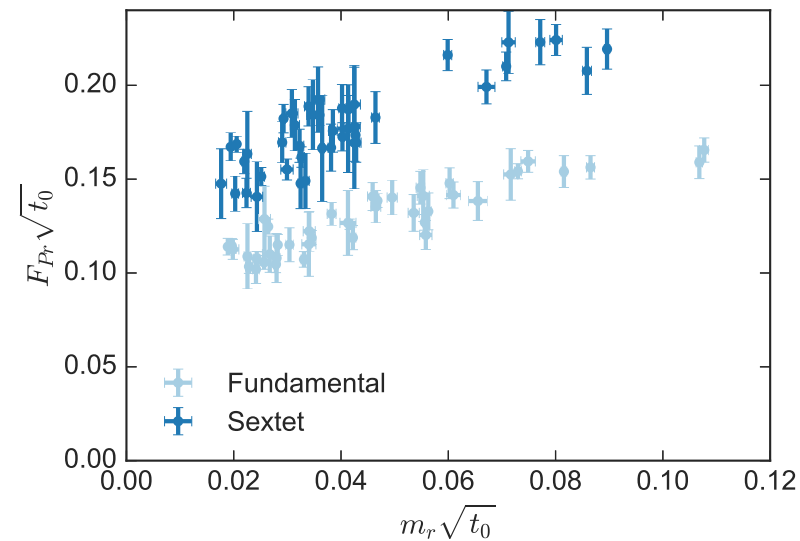
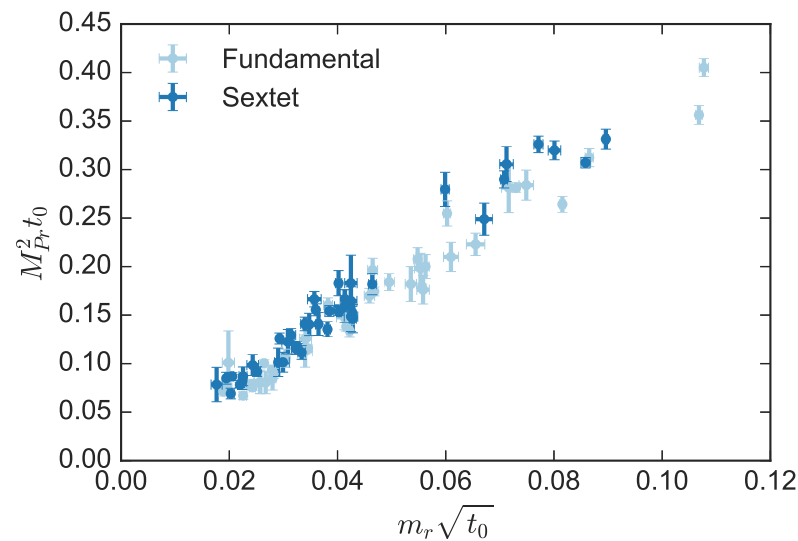
2-reps results: M_π and F_π vs. same-rep fermion mass

Wilson-clover fermions, nHYP smeared links, Dislocation-Suppressing term

Ensembles: $16^3 \times 18$, $16^3 \times 32$ (\sim twenty each), $24^3 \times 48$ (four)

physical units: flow scale t_0 on the same ensemble

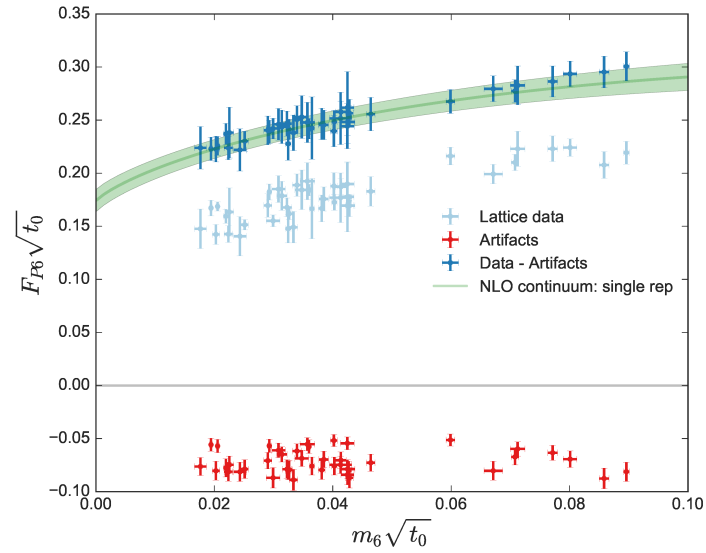
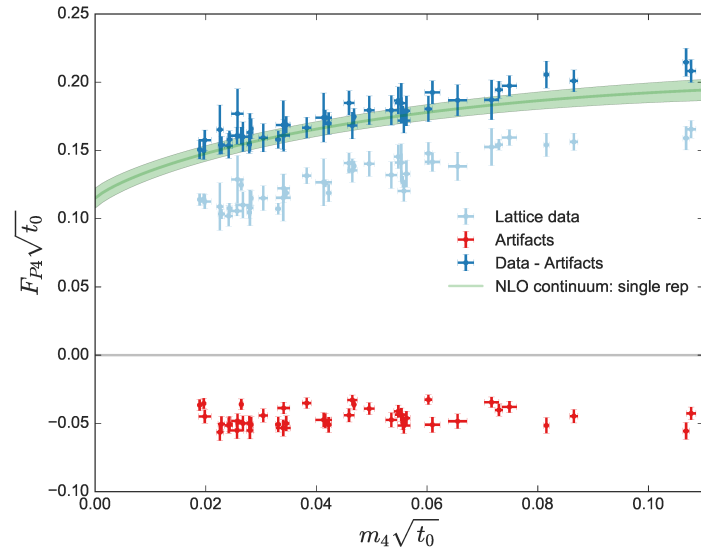
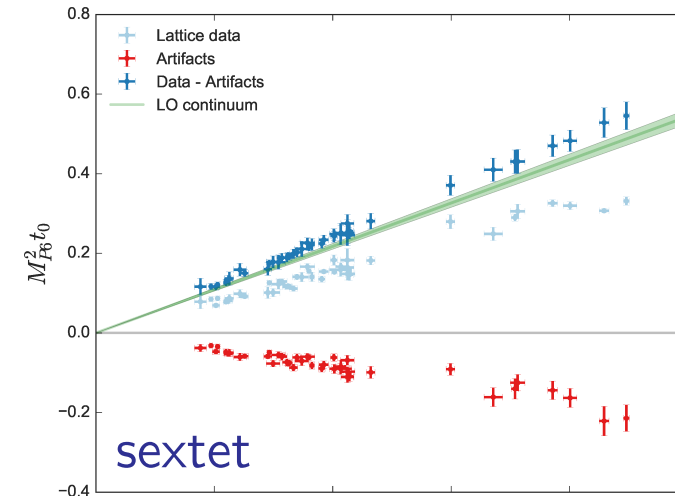
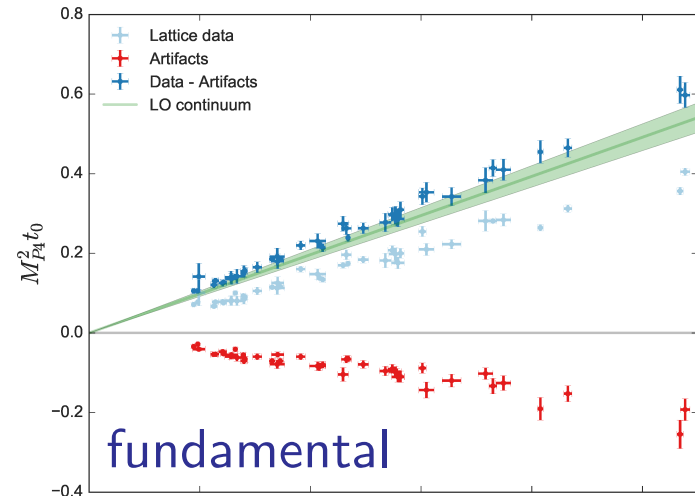
cutoff: 1.5 – 2.2 GeV pion masses: 400 – 800 MeV (“QCD equivalent”)



(results depend also on other-rep fermion mass and lattice spacing)

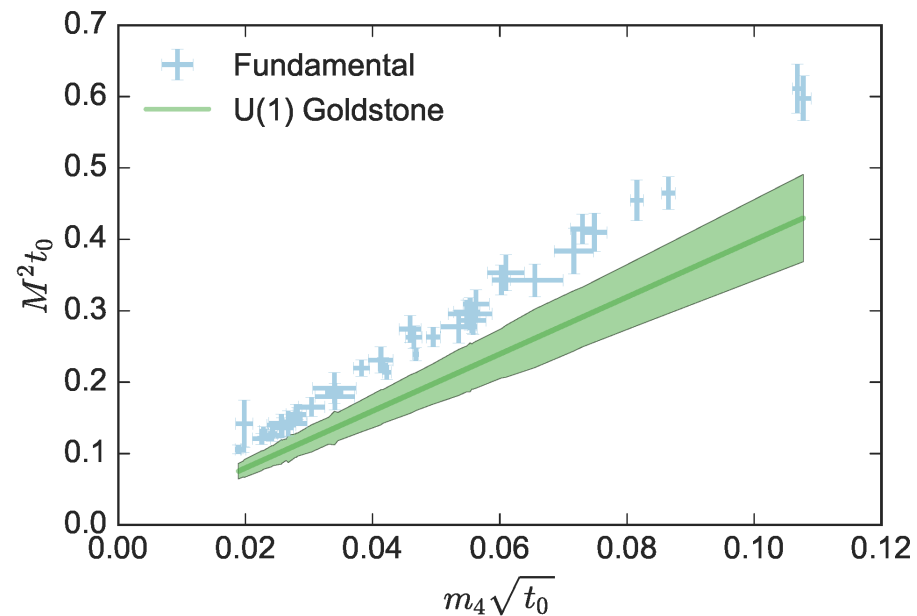
“Remnant scale separation”: $F_\pi(\text{sextet}) > F_\pi(\text{fund.})$

2-reps results: Wilson ChPT



2-reps results: singlet Nambu-Goldstone boson

Singlet mass in sextet chiral limit (extracted from fit)



Singlet somewhat lighter than fund sector pions in this limit
might be interesting for cosmology

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(Majorana sextets, Dirac fund.): (4, 2) instead of (5, 3)

4-fermi couplings and top-induced Higgs potential

- As in technicolor, need **extended hypercolor** that breaks at high scale Λ_{EHC} to generate 4-fermi couplings at Λ_{HC} scale

$$\begin{aligned} \mathcal{L}_{\text{EHC}} = & \lambda_1 \overline{T}_L^{(5,3,1)} B_R^{(5,3,1)} + \lambda_2 \overline{T}_R^{(\bar{5},3,1)} B_L^{(\bar{5},3,1)} + \lambda_3 \overline{T}_L^{(5,1,3)} B_R^{(5,1,3)} \\ & + \lambda_4 \overline{T}_R^{(\bar{5},1,3)} B_L^{(\bar{5},1,3)} + \lambda_5 \overline{T}_L^{(\bar{5},\bar{3},\bar{3})} B_R^{(\bar{5},\bar{3},\bar{3})} + \lambda_6 \overline{T}_R^{(5,\bar{3},\bar{3})} B_L^{(5,\bar{3},\bar{3})} + \text{h.c.} . \end{aligned}$$

spurions

$$T_L = \frac{1}{\sqrt{2}} \begin{pmatrix} ib_L \\ b_L \\ it_L \\ -t_L \\ 0 \end{pmatrix}, \quad T_R = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ t_R \end{pmatrix}$$

- Work out: effective top-Yukawa couplings
top-induced effective potential for Higgs (and other pNGBs)
- Depend on low-energy constants (calculable on lattice), and on λ 's

Things to worry about

- top-Yukawa $y_t \sim \lambda\lambda \sim (\Lambda_{\text{HC}}/\Lambda_{\text{EHC}})^4$ by naive counting
- need small $\Lambda_{\text{HC}}/\Lambda_{\text{EHC}}$ to keep flavor violations under control
- Way too small y_t unless (a) theory is near-conformal and (b) 4-fermi's have large anomalous dimensions $\Rightarrow y_t \sim (\Lambda_{\text{HC}}/\Lambda_{\text{EHC}})^{4-2\gamma}$ (**..examples??**)
- Instead, other fermion masses might come from yet another sector
- Calculate full effective potential; structure varies substantially for different Ferretti-Karateev models
- Study minima as function of λ' s (a multi-dimensional problem!)
- Role of other pNGBs: finding singlet VEV is fine; but triplet VEV must not break the custodial symmetry (ρ parameter)
- Difficult to make progress without some understanding of EHC theory

~~Classic technicolor~~

[Susskind 1979]

[Raby, Dimopoulos, Susskind 1980]

- New strong sector
- Elegant mechanism to generate W and Z masses:
techni-fermions condense, 3 techni-pions eaten by W and Z
- Higgs VEV $v \equiv$ technipion decay constant f
- Fermion masses?

LHC era:

- ★ Higgs-less
 - ★ Where are the techni-rho's?
- ⇒ Successful technicolor can't be upscaled version of QCD

Walking (extended) technicolor

- Small-but-still-negative beta function
- Fermion condensation break spontaneously both chiral symmetry and approximate scale symmetry
- “Dilatonic Higgs” arising as pseudo Nambu-Goldstone boson for approx. scale symmetry
- Helps with fermion masses, too (original motivation!)

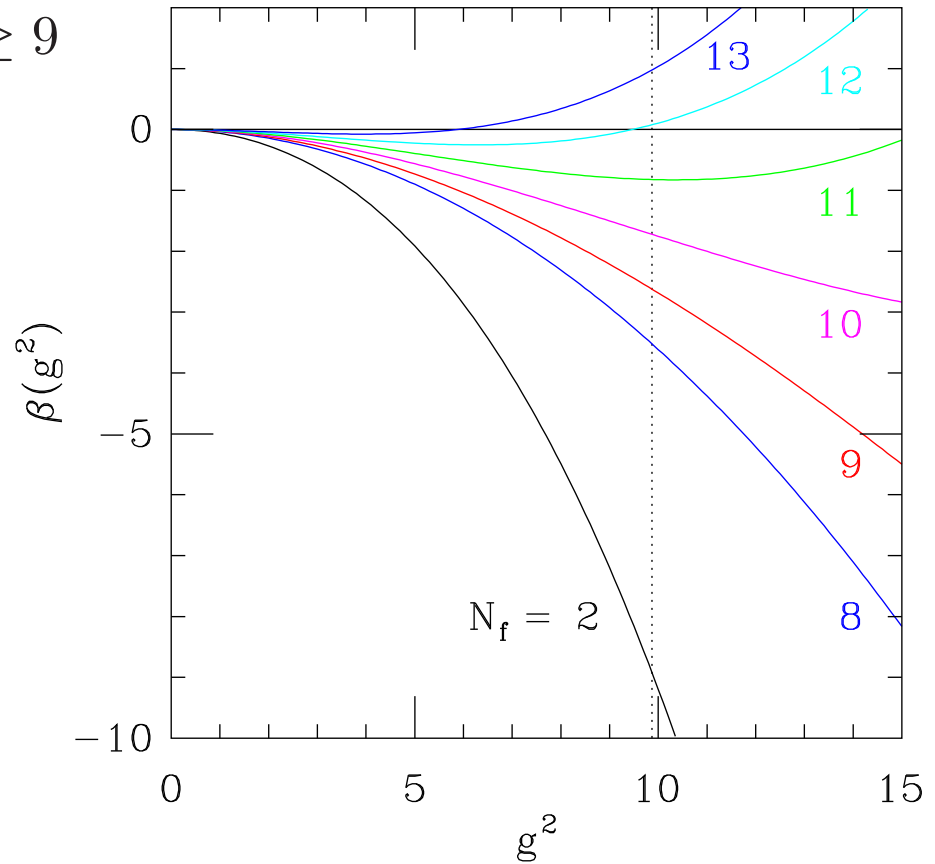
$$\text{4-fermi operators: } \frac{\bar{q}q\bar{q}q}{\Lambda_{ETC}^2}, \quad \frac{\bar{q}q\bar{Q}Q}{\Lambda_{ETC}^2}, \quad \frac{\bar{Q}Q\bar{Q}Q}{\Lambda_{ETC}^2}$$

$$\text{fermion masses: } \frac{\langle \bar{Q}Q \rangle \bar{q}q}{\Lambda_{ETC}^2} \approx \Lambda_{TC} \left(\frac{\Lambda_{TC}}{\Lambda_{ETC}} \right)^{2-\gamma_m} \bar{q}q$$

★ In case $\gamma_m \lesssim 1$, not good enough for top mass

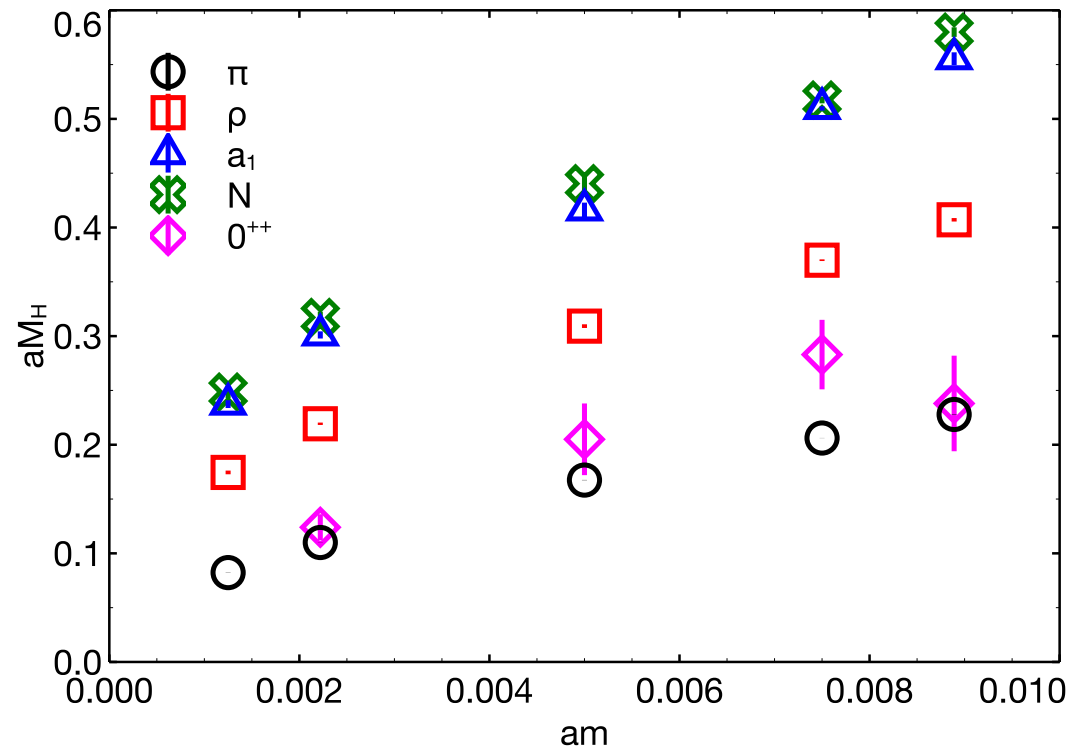
Phases of $SU(3)$ with N_f fundamental-rep Dirac fermions

- running slows with increased N_f
- two-loop IRFP $g_* = g_*(N_f)$ for $N_f \geq 9$
- ChSB when $g^2(\mu)$ reaches g_c^2
- Here $g_c^2 = \pi^2 \simeq 9.87$
- ⇒ chirally broken if $g_c < g_*$
- ⇒ IR conformal if $g_c > g_*$
- sill of conformal window: $g_* = g_c$
- For $SU(3)$ sill is $N_f^* \simeq 12$



A light flavor-singlet scalar: dilatonic Higgs?

- $SU(3)$, $N_f = 8$ fund. [LatKMI, LSD,..]



Consistent low-energy theory must contain both pions and the flavor-singlet scalar

LSD collaboration, PRD 93 (2016) 114514

- $SU(3)$, $N_f = 2$ two-index symmetric rep (sextet)

Constructing an Effective Field Theory

- symmetries
- spurions: external fields transforming under the symmetries
 - fixing “VEVs” of spurions \Rightarrow explicit breaking of symmetries
 - Low-energy constants determined by matching correlation functions defined by differentiation w.r.t. spurions

- Chiral symmetry: spurion $\langle \chi \rangle = m$, effective field $\Sigma \sim \psi_L \bar{\psi}_R$

microscopic theory: $\bar{\psi}_R \chi^\dagger \psi_L + \bar{\psi}_L \chi \psi_R \Rightarrow$ effective theory: $\text{tr}(\chi^\dagger \Sigma + \Sigma^\dagger \chi)$

- Dilatations: $\Phi_i(x) \rightarrow \lambda^{\Delta_i} \Phi_i(\lambda x)$, Δ_i scaling dimension of field $\Phi_i(x)$

$$S^{\text{MIC}}(\sigma) = \int d^d x \frac{e^{\sigma(d-4)}}{g_0^2} \left(\frac{1}{4} F^2 + \dots \right) = \int \left(\mathcal{L}^{\text{MIC}}(0) + \sigma T_{an} + \dots \right)$$

$S^{\text{MIC}}(\sigma, \chi)$ invariant in d dimensions if $\chi(x), e^{\sigma(x)}$ transform as scalar fields

Trace anomaly:

$$\partial_\mu S_\mu = T_{\mu\mu} = -T_{cl} - T_{an} = -(\beta(g^2)/4g^2) F^2 - (1 + \gamma_m) m \bar{\psi} \psi$$

Effective Field Theory with pions and dilatonic meson $\tau(x)$

- scale transformation:

source fields: $\sigma(x) \rightarrow \sigma(\lambda x) + \log \lambda$, $\chi(x) \rightarrow \lambda^{4-y}\chi(\lambda x)$

effective fields: $\tau(x) \rightarrow \tau(\lambda x) + \log \lambda$, $\Sigma(x) \rightarrow \Sigma(\lambda x)$

- invariant low-energy theory: $\tilde{\mathcal{L}}^{\text{EFT}} = \tilde{\mathcal{L}}_\pi + \tilde{\mathcal{L}}_\tau + \tilde{\mathcal{L}}_m + \tilde{\mathcal{L}}_d$ where

$$\tilde{\mathcal{L}}_\pi = V_\pi(\tau - \sigma) (f_\pi^2/4) e^{2\tau} \text{tr} (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$$

$$\tilde{\mathcal{L}}_\tau = V_\tau(\tau - \sigma) (f_\tau^2/2) e^{2\tau} (\partial_\mu \tau)^2$$

$$\tilde{\mathcal{L}}_m = -V_M(\tau - \sigma) (f_\pi^2 B_\pi/2) e^{y\tau} \text{tr} (\chi^\dagger \Sigma + \Sigma^\dagger \chi)$$

$$\tilde{\mathcal{L}}_d = V_d(\tau - \sigma) f_\tau^2 B_\tau e^{4\tau}$$

with invariant potentials: $V(\tau(x) - \sigma(x)) \rightarrow V(\tau(\lambda x) - \sigma(\lambda x))$

(Similar to large- N chiral lagrangian with $U(1)_A$ symmetry and η')

\Rightarrow No predictability without power counting

Power counting

- Veneziano limit: $N_f, N_c \rightarrow \infty$ with $n_f = N_f/N_c$ fixed
- Assume: $\beta(g^2) \sim n_f - n_f^*$ at the ChSB scale
- Prove: $V(\tau - \sigma) = \sum_{n=0}^{\infty} \sum_{k \geq n} \tilde{c}_{nk} (n_f - n_f^*)^k (\tau - \sigma)^n$
- ⇒ Systematic expansion in $p^2 \sim m \sim n_f - n_f^* \sim 1/N$
- Set $\sigma = 0$, $\chi = m$, shift τ field, obtain leading order EFT

$$\mathcal{L}_\pi = (f_\pi^2/4) e^{2\tau} \text{tr} (\partial_\mu \Sigma^\dagger \partial_\mu \Sigma)$$

$$\mathcal{L}_\tau = (f_\tau^2/2) e^{2\tau} (\partial_\mu \tau)^2$$

$$\mathcal{L}_m = -(m f_\pi^2 B_\pi / 2) e^{y\tau} \text{tr} (\Sigma + \Sigma^\dagger)$$

$$\mathcal{L}_d = \tilde{c}_{11} (n_f - n_f^*) (\tau - 1/4) f_\tau^2 B_\tau e^{4\tau}$$

here $y = 3 - \gamma_m^*$, where γ_m^* is mass anomalous dimension at sill

Some tree-level results

- $V_{\text{cl}}(\tau)$ bounded below provided $\tilde{c}_{11} < 0$ (recall $n_f < n_f^*$)
- consistent with matching of the trace anomaly

$$\begin{aligned}\partial_\mu S_\mu &= \tilde{c}_{11}(n_f - n_f^*) f_\tau^2 B_\tau e^{4\tau} + (1 + \gamma_m^*) \frac{f_\pi^2 B_\pi m}{2} e^{y\tau} \text{tr}(\Sigma + \Sigma^\dagger) \\ &= -\frac{\beta(g^2)}{4g^2} F^2(\text{EFT}) - (1 + \gamma_m^*) m \bar{\psi}\psi(\text{EFT})\end{aligned}$$

- $v = \langle \tau \rangle = 0$, for $m = 0$ (τ shift). $v(m)$ monotonically increasing with m
- pion mass: $m_\pi^2 = 2B_\pi m \times e^{(y-2)v(m)}$
- dilatonic meson mass: $m_\tau^2 = 4\tilde{c}_{11}(n_f - n_f^*) B_\tau \times e^{2v(m)} (1 + (4 - y)v(m))$
- LSD results: $y \approx 2$ [Appelquist *et al.* arXiv:1711.00067]
- Tree level scaling: e.g., $F_\pi(m) = e^{v(m)} f_\pi(0)$

... in Conclusion

Phenomenology

- Fate of Composite Higgs, Dilatonic Technicolor, remains to be seen
- Lattice provides crucial input for UV-complete composite Higgs: spectrum, low-energy constants

Field theory

- Non-perturbative physics of 2-irreps gauge theories
- Near conformal gauge theories

In particular, are $N_f = 8$ results fully described by dilaton EFT?