Massive Gravity

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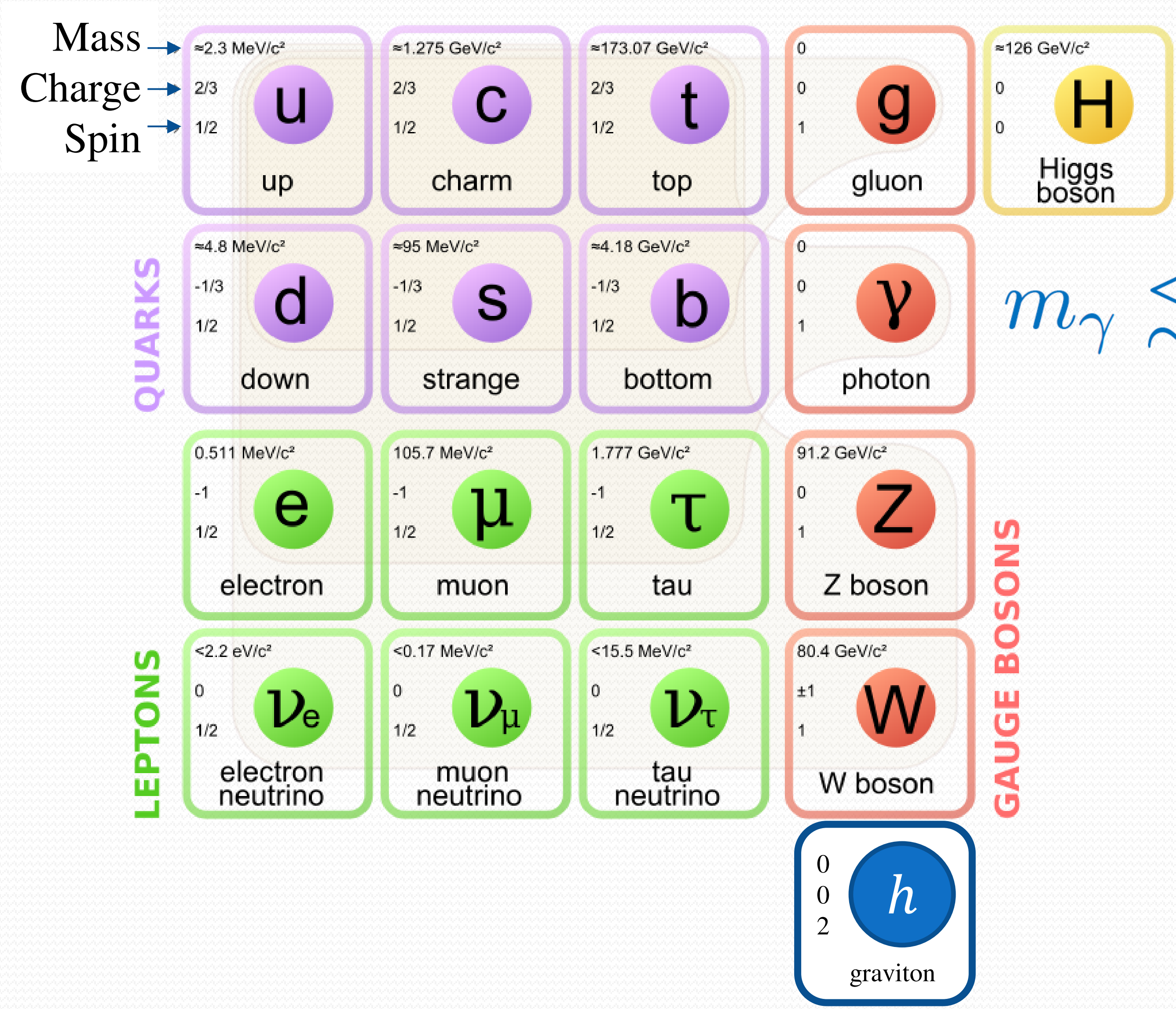
Imperial College London

XXIII Christmas workshop
IFT
December 15th 2017

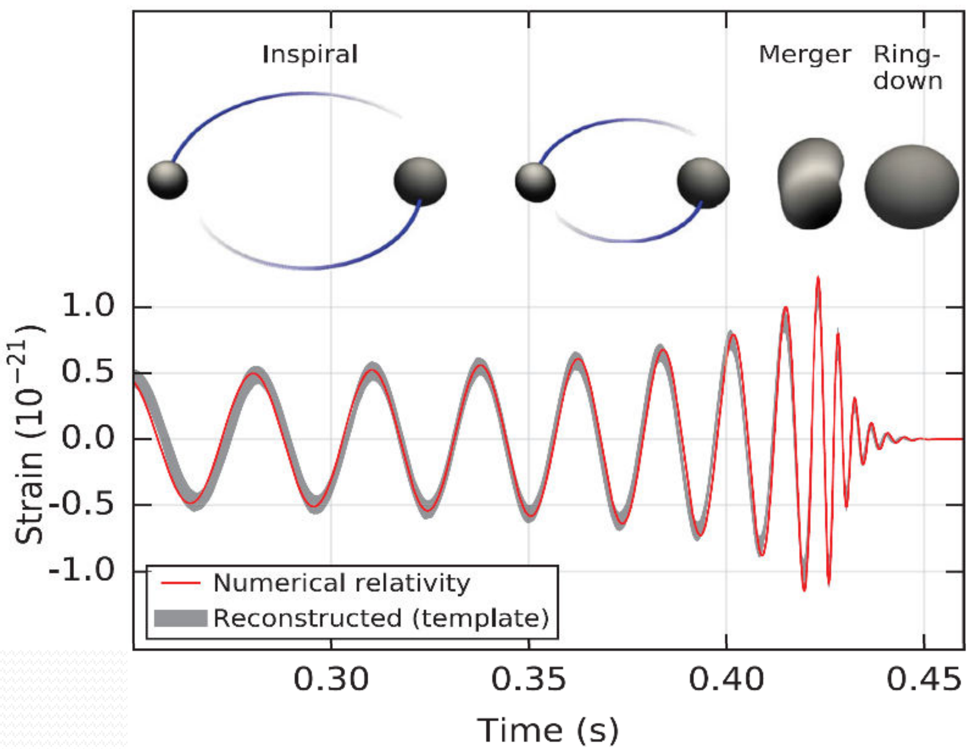
How light is gravity ???



$$m_\gamma \lesssim 10^{-20}\text{eV}$$

How light is gravity ???

GW150914



LIGO & VIRGO, Phys. Rev. Lett. 116, 221101 (2016):

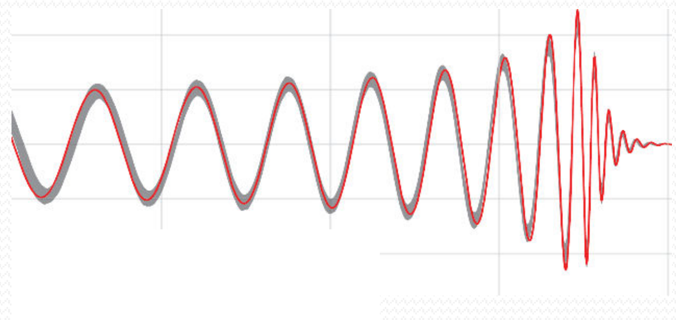
"We constrain the graviton Compton wavelength (…), obtaining a 90%-confidence lower bound of 10^{13} km"

$$m_{graviton} < 10^{-22}\text{eV}$$

How light is gravity ???

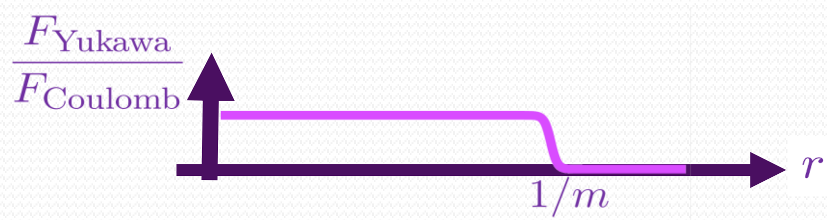
Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters

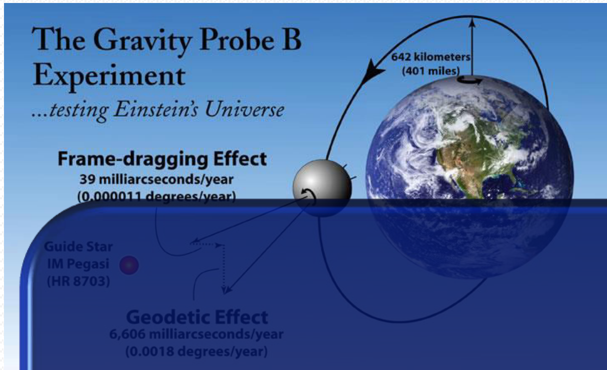


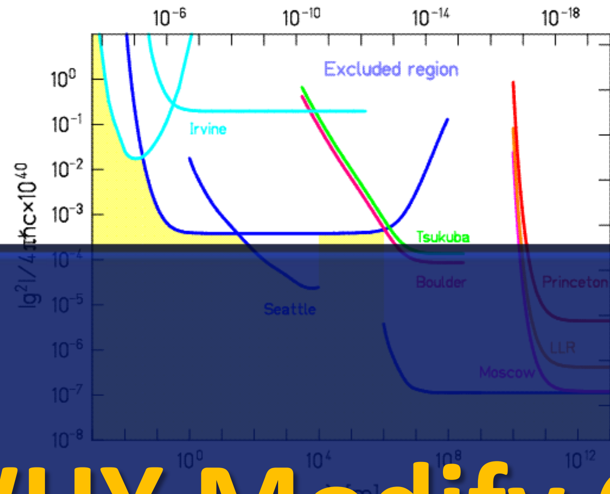
Fifth Force

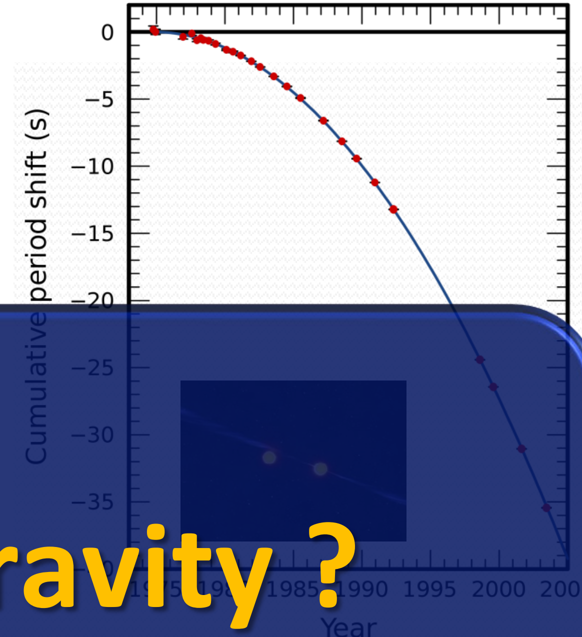
m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

CdR, Deskins, Tolley, Zhou, 1606.08462

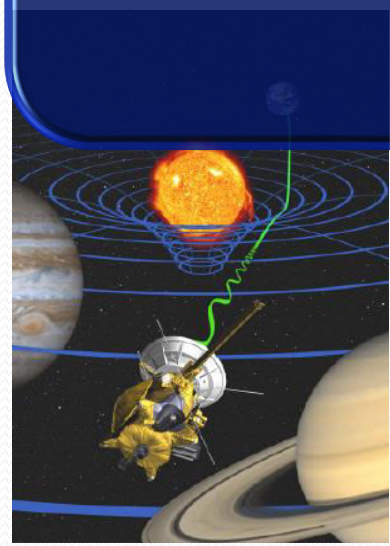
GR has been a successful theory from mm length scales to Cosmological scales

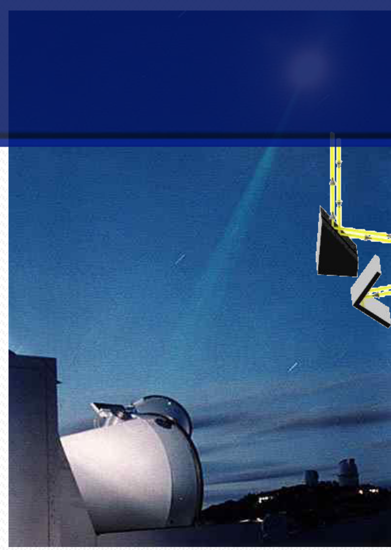


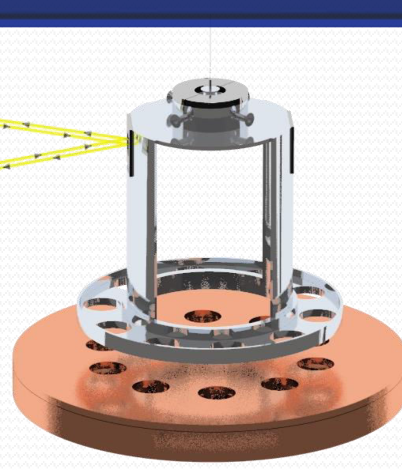


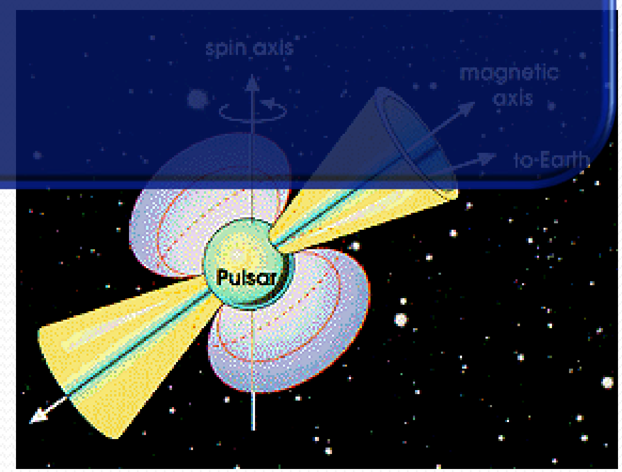


Then WHY Modify Gravity ?

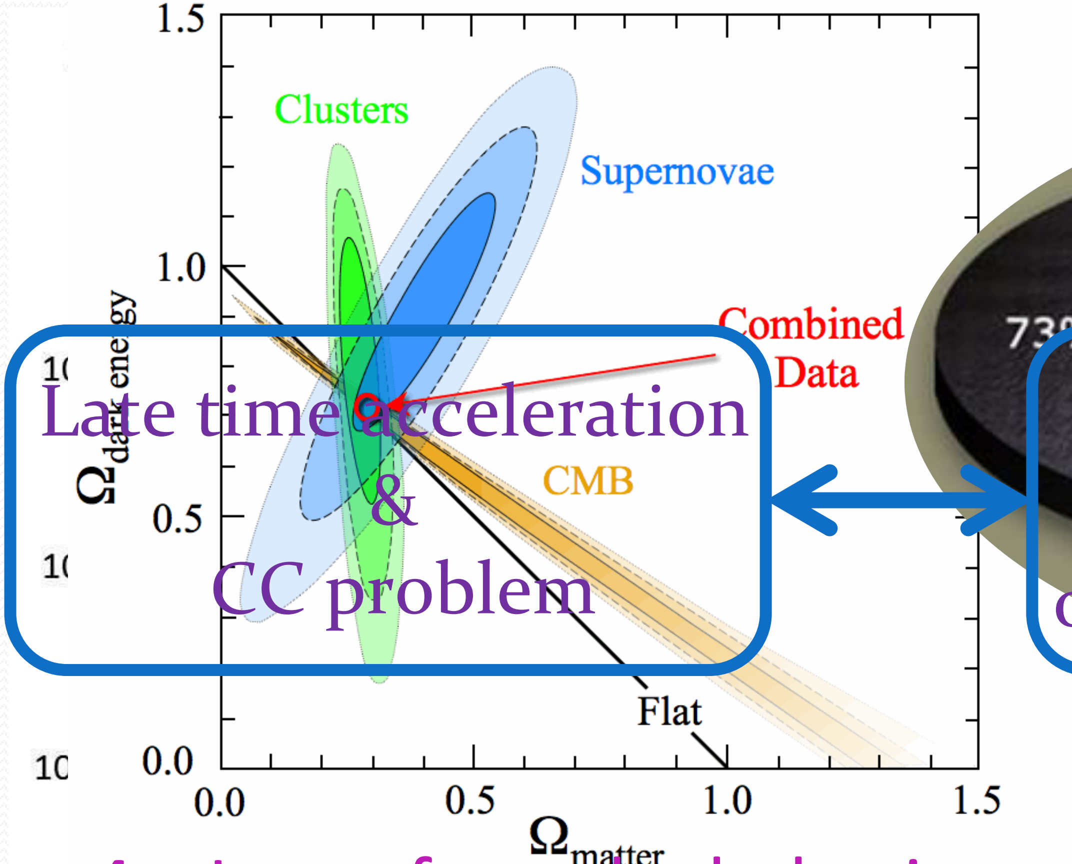




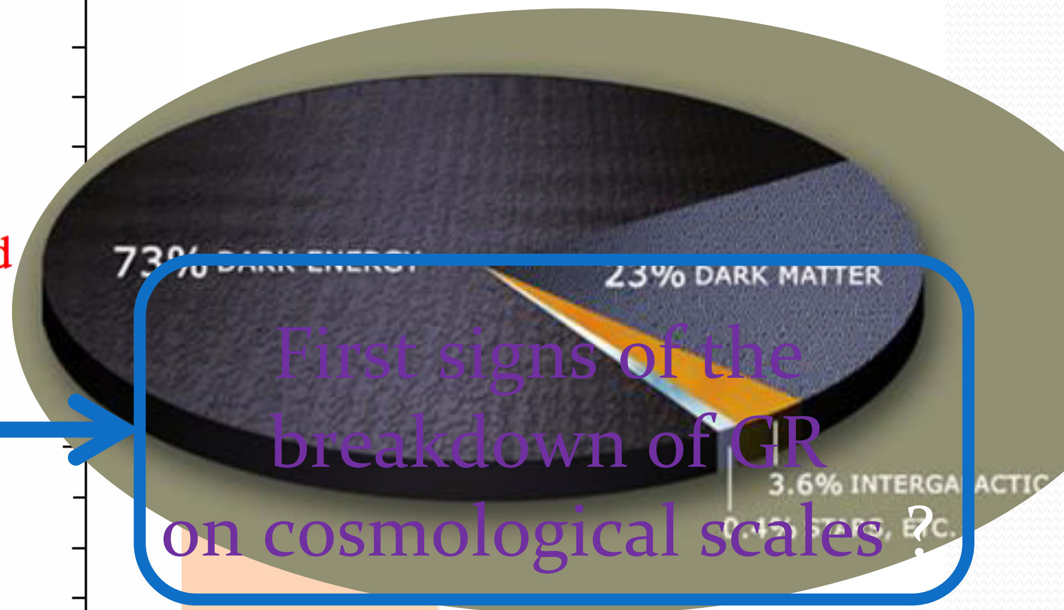




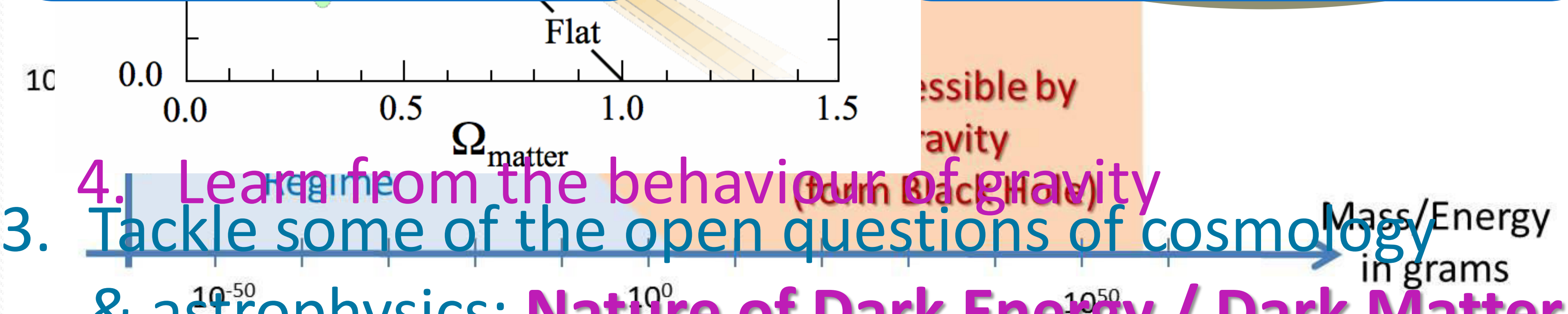
1. Tests specific *characteristics* in specific *environments*
2. Assume gravity is correct for other measurements



Late time acceleration & CC problem



First signs of the breakdown of GR on cosmological scales ?



4. Learn from the behaviour of gravity

3. Tackle some of the open questions of cosmology & astrophysics: **Nature of Dark Energy / Dark Matter**

Gravity

So far, the best model for Gravity is GR

GR is the *unique* consistent theory of a *massless spin-2 field*

Assuming Global Lorentz invariance, locality & Stability

Giving the graviton a **mass** is the most direct way to modify gravity at large distances

1960's: Feynman, Weinberg, Deser,...

Massive Gravity

- In GR (or Newtonian gravity), Gravity has an infinite range

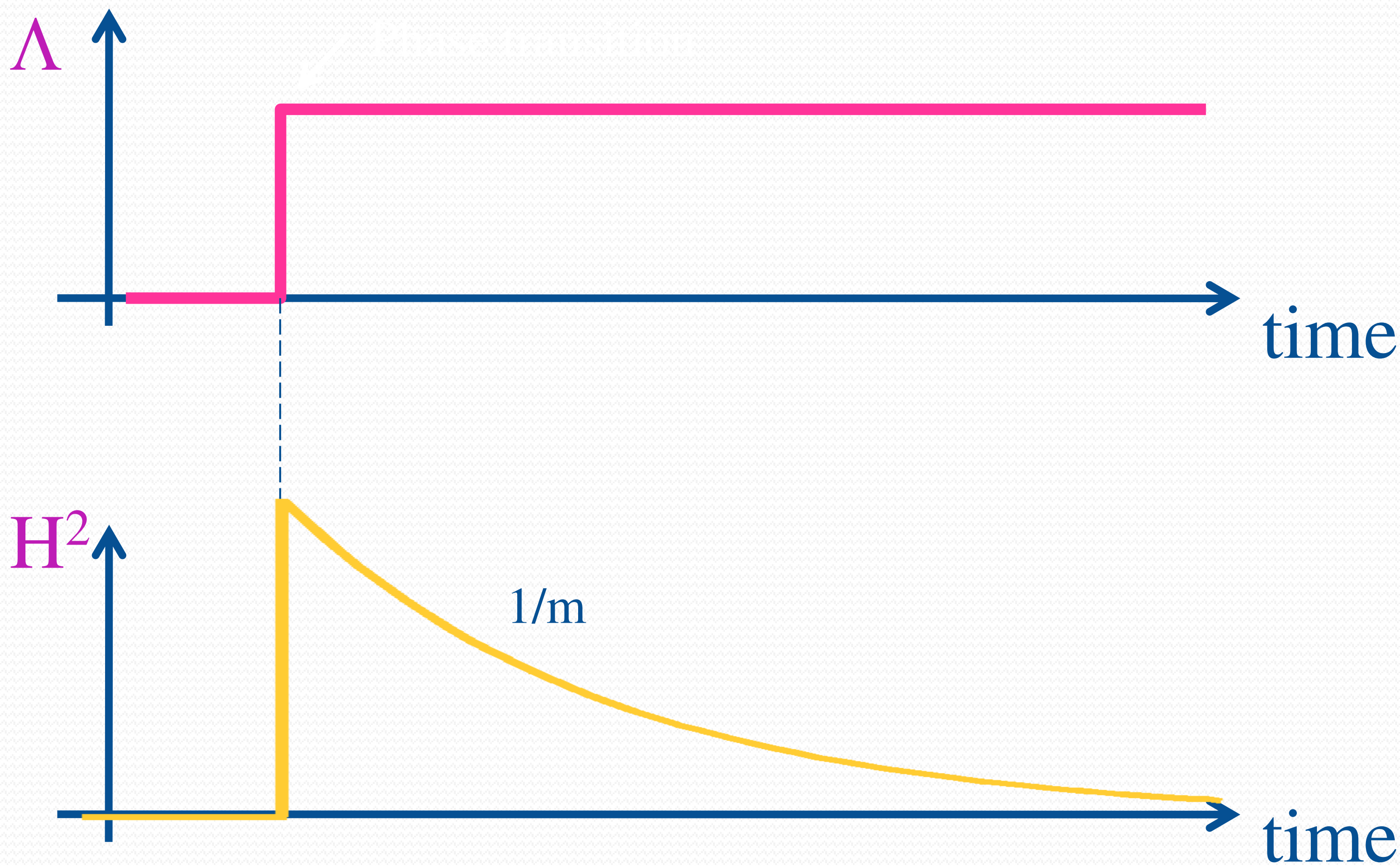
$$F = \frac{\partial}{\partial r}\left(\frac{G\ M_1 M_2}{r}\right)$$

- If gravity was mediated by a *massive particle*, the Newton's law would "shut down" at some distance $\lambda \sim m^{-1}$

$$F = \frac{\partial}{\partial r}\left(\frac{G e^{-mr} M_1 M_2}{r}\right)$$

Relaxation mechanism

- Massive gravity could lead to the degravitation of a CC



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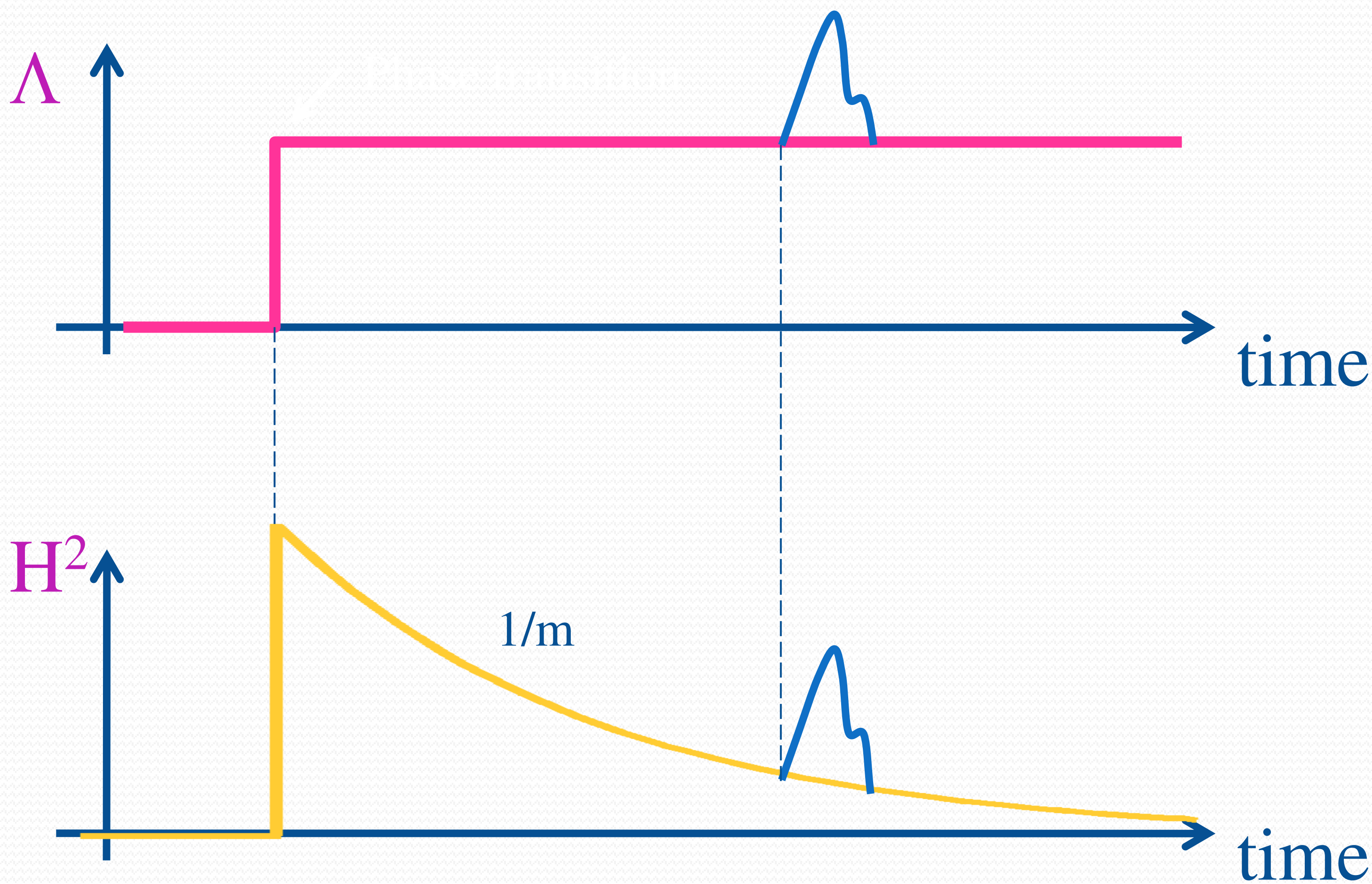


Tuning / Fine-tuning

- *From naturalness considerations*, we expect a vacuum energy of the order of the cutoff scale (Planck scale).
- But observations tell us $$\frac{\Lambda}{M_{\rm Pl}^4} \sim 10^{-120}$$
- For the degravitation mechanism to work, the mass of the graviton should be

$$\frac{m^2}{M_{\rm Pl}^2} \sim 10^{-120}$$

Tuning / Fine-tuning

- *The amount of tuning is the same*

$$\frac{m^2}{M_{\rm Pl}^2} \sim 10^{-120}$$

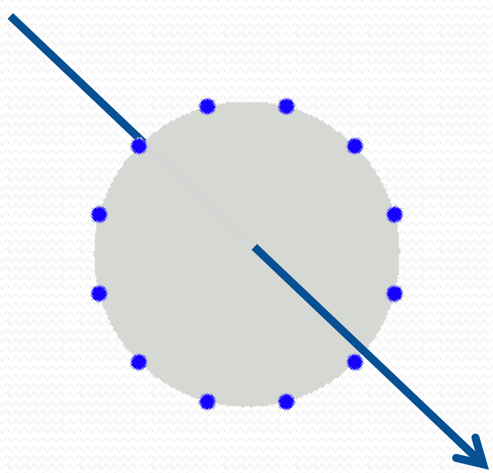
- But the graviton mass remains stable against quantum corrections
- we recover a symmetry in the limit $m \to 0$

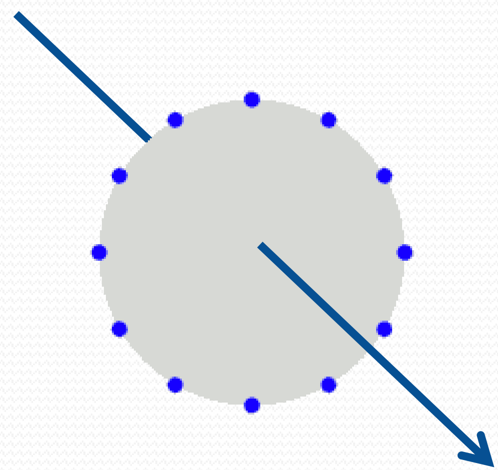
The theory is tuned
but technically natural

General Relativity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2} \ R$$

- **GR: 2** polarizations





Massive Gravity

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

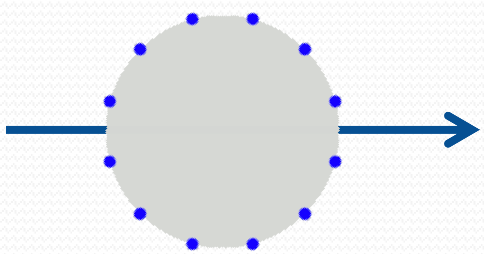
- The notion of mass requires a ***reference***
- The loss in symmetry generates new dof

$$2 + 4 = 6$$

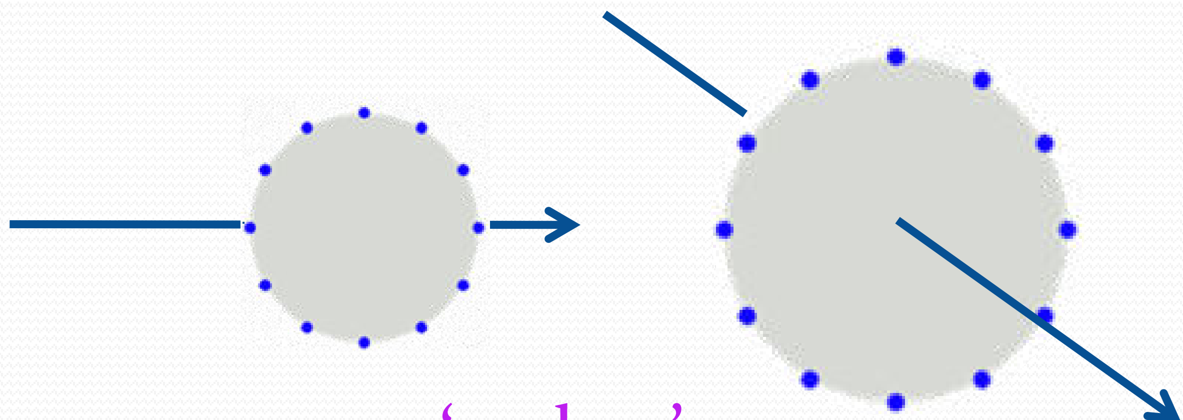
GR ← 2; 4 → Loss of 4 sym

Gravitational Waves

In principle GW could have 4 other polarizations



2 ‘vectors’



2 ‘scalars’

Potential `new degrees of freedom’

Massive Gravity

$$S=\int\sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R-\text{Mass Term}\right)$$

- The notion of mass requires a ***reference***
- The loss in symmetry generates new dof

$$2+4=6=5+1$$

Boulware & Deser, PRD6, 3368 (1972)

Fierz-Pauli Massive Gravity

$$\mathcal{U}_{\text{FP}} = H_{\mu\nu}^2 - H^2$$

- H being the 'Stückelbergized' metric fluctuation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad H_{\mu\nu} = h_{\mu\nu} + \partial_\mu \partial_\nu \pi$$

- So as to restore diff invariance

$$x^\mu \to x^\mu + \partial^\mu \xi$$

$$h_{\mu\nu} \to h_{\mu\nu} + 2\partial_\mu \partial_\nu \xi$$

$$\pi \to \pi - \xi$$

Deffayet & Rombouts, gr-qc/0505134

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Typically involves some higher derivatives which leads to a ghost

- So as to restore diff invariance

$$x^\mu \rightarrow x^\mu + \partial^\mu\xi$$

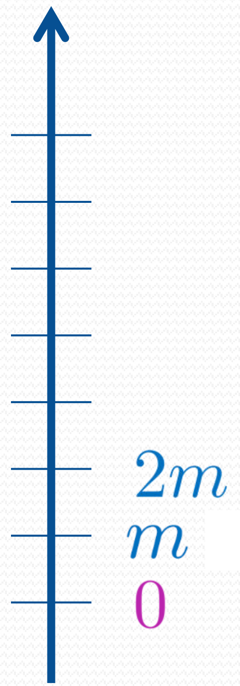
$$h_{\mu\nu} \rightarrow h_{\mu\nu} + 2\partial_\mu\partial_\nu\xi + \partial_\mu\partial_\alpha\xi\partial_\nu\partial^\alpha\xi$$

$$\pi \rightarrow \pi - \xi$$

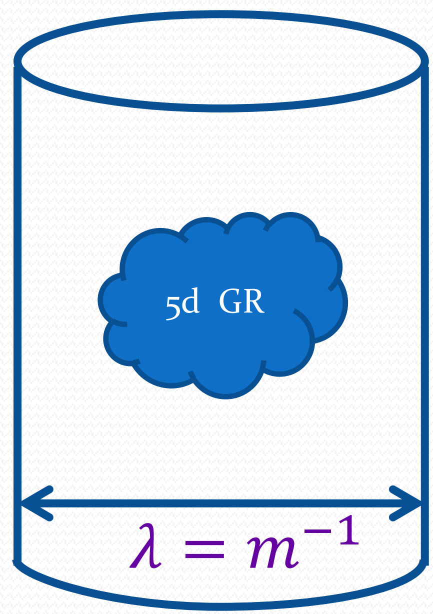
Deffayet & Rombouts, gr-qc/0505134

Mass from extra dimensions

Tower of equally spaced massive modes

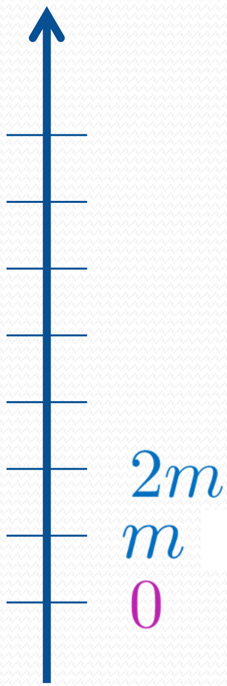


Kaluza-Klein compactification



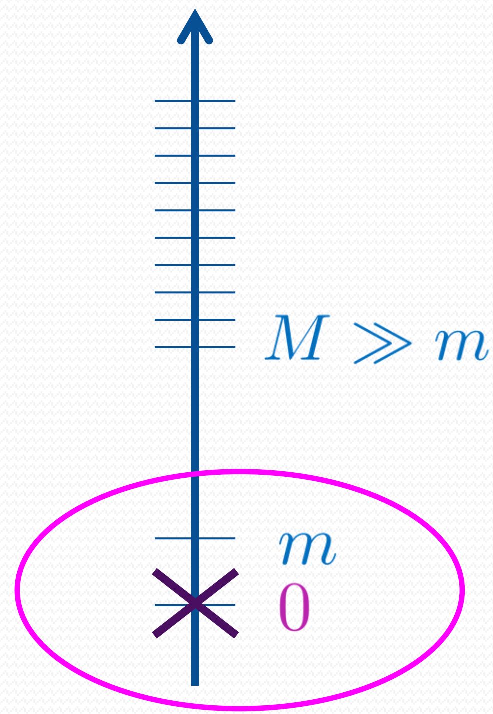
Mass from extra dimensions

Tower of equally spaced
massive modes



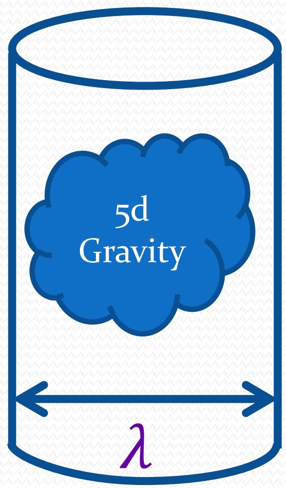
Need to modify the setup to
Remove the zero mode

& to create a
different mass gap
(hierarchy)

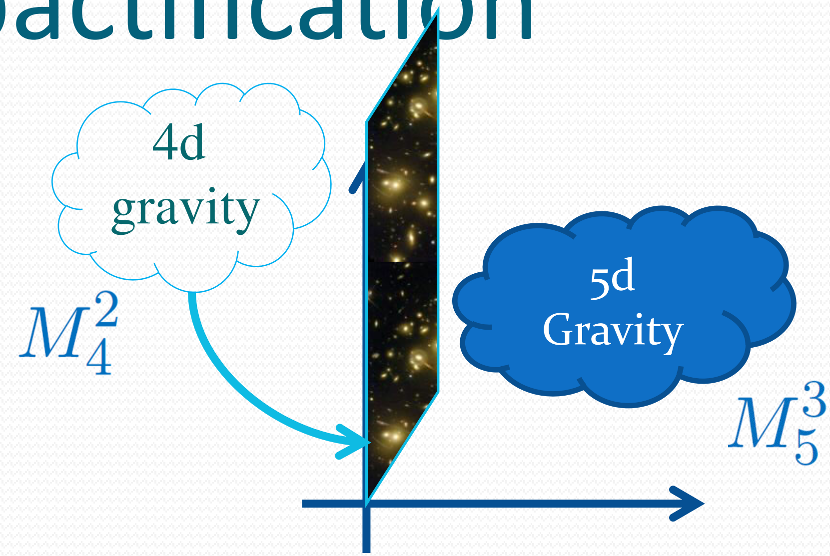


Still serves as a proof of principle

Alternative to compactification

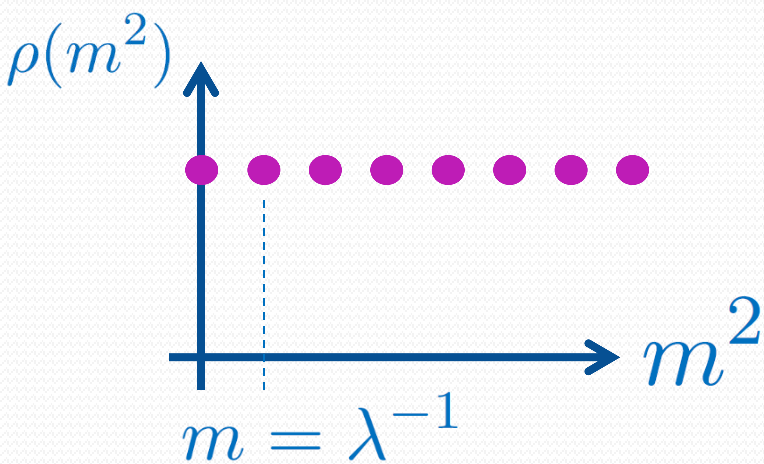


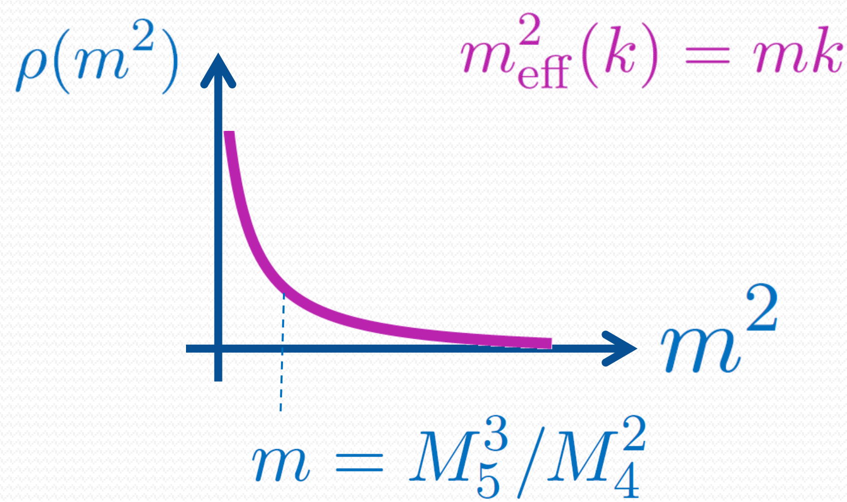
Finite-size extra dimension



Infinite extra dimension
Confine Gravity on a brane

Dvali Gabadadze Porrati, 2000





Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \left(H^2_{\mu\nu} - H^2\right)$$

$$\mathcal{U}_{\rm GF} = \left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right)$$

- *H* being the 'Stückelbergized' metric fluctuation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad H_{\mu\nu} = h_{\mu\nu} + \partial_\mu\partial_\nu\pi + \partial_\mu\partial_\alpha\pi\partial_\nu\partial^\alpha\pi$$

Typically involves some higher derivatives which leads to a ghost

$$\mathcal{K}_{\mu\nu} = \eta_{\mu\nu} - \left(\sqrt{1-H}\right)_{\mu\nu} \sim \partial_\mu\partial_\nu\pi$$

Evades any higher order derivatives

CdR & Gabadadze, PRD 82, 044020 (2010)
CdR, Gabadadze & Tolley, PRL 106, 231101 (2011)

Obstructions to graviton mass

Challenge 1: **Unitarity** – Taking care of the ghost
rely on very careful tuning of the interactions
(can get inspired from extra dimensions)

✓

Challenge 2: **Analyticity** (obstruction to UV completion)

Low strong scale (cutoff) $\Lambda = (M_{\rm Pl} m^2)^{1/3}$
Essence of the Vainshtein mechanism
Restore continuity with massless limit

Interactions at that scale don't satisfy
the usual analyticity properties

Causality, analyticity and an IR obstruction to UV completion, JHEP 0610 (2006) 014

Obstructions to graviton mass

Challenge 1: **Unitarity** – Taking care of the ghost
rely on very careful tuning of the interactions
(can get inspired from extra dimensions)

Challenge 2: **Analyticity** (obstruction to UV completion)

Could a UV completion exist for massive gravity???

Schwinger Mechanism

- Mass without violating gauge invariance

Introduce and integrate out the Stückelberg field

Proca field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 A_\mu A^\mu$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 (A_\mu - \partial_\mu \phi)^2$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2 F_{\mu\nu}\frac{1}{\Box}F^{\mu\nu}$$

- This non-local mass term can arise from loops effects after integrating out a light field

2d Schwinger model

Schwinger Mechanism for gravity

- Mass without violating gauge invariance

Introduce and integrate out the Stückelberg field

Ghost-free Massive Gravity

$$\mathcal{L} = \frac{M_{\rm Pl}^2}{2}\sqrt{-g}\left(R - \frac{1}{2}m^2 R^{\mu\nu}\frac{1}{\Box^2}G_{\mu\nu} + \cdots\right)$$

- This non-local mass term could be seen as arising from loops after integrating out light fields

Higgs mechanism for Gravity ?

- Gravity in AdS coupled to a CFT with *non-standard "transparent" boundary conditions*

- Integrating out the CFT leads to non-local contributions to the action

- At leading order these contributions are precisely equivalent to the operators we obtain from integrating out the Stuckelberg in massive gravity

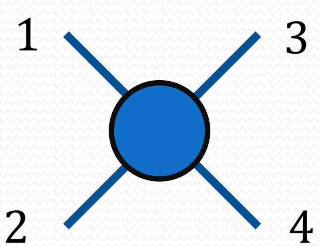
Local, causal unitary, UV completion of a single massive graviton in AdS

Porrati, 2001

Obstructions to graviton mass

Challenge 1: **Unitarity** – Taking care of the ghost
rely on very careful tuning of the interactions
(can get inspired from extra dimensions)

Challenge 2: **Analyticity** (obstruction to UV completion)



When computing Scattering amplitude, **unitarity** dictates that $\mathrm{Im}\,\mathcal{A} > 0$ (optical theorem)

Then analyticity and locality implies that

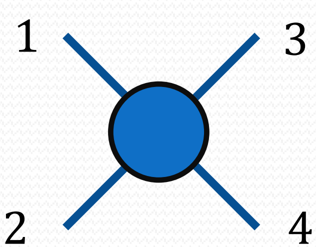
$$\tilde{A}''(s)\Big|_{s=0} > 0$$

Causality, analyticity and an IR obstruction to UV completion, JHEP 0610 (2006) 014

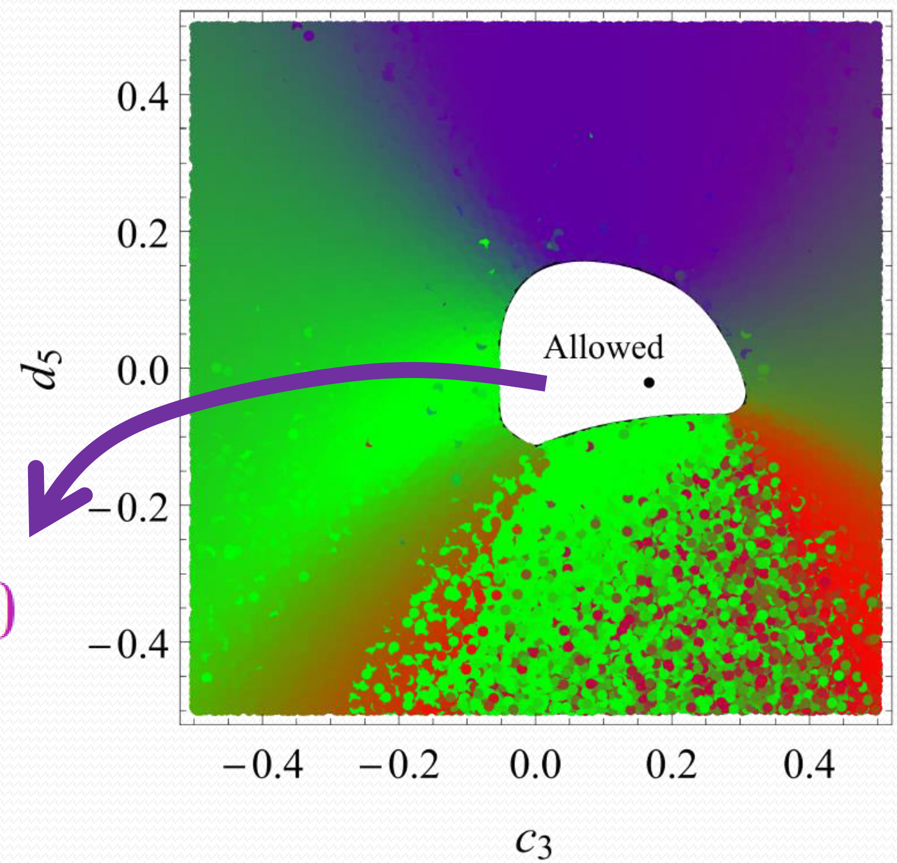
Higgs mechanism for Gravity ?

- No (known) Higgs mechanism for gravity on Minkowski

- However, obstructions to 'standard UV completion' are bypassed for some parameters of the model



$$\tilde{A}''(s)\Big|_{s=0} > 0$$



Positive Signs in Massive Gravity, Cheung & Remmen, JHEP 1604 (2016) 002

Obstructions to graviton mass

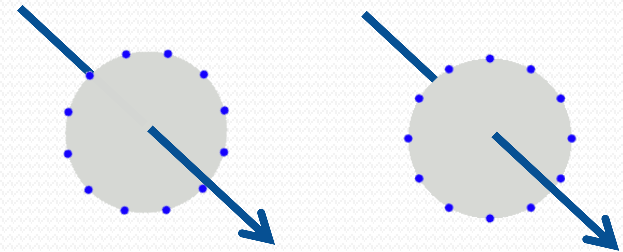
Challenge 1: **Unitarity** – (ghosts) ✓

Challenge 2: **Analyticity** (obstruction to UV completion) ✓

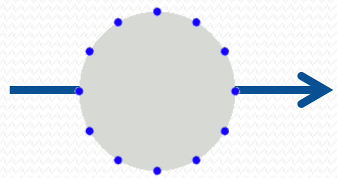
Challenge 3: **Observational constraints**

Gravitational Waves in Massive Gravity

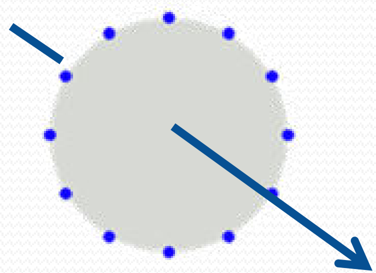
±2 tensors



±1 vectors



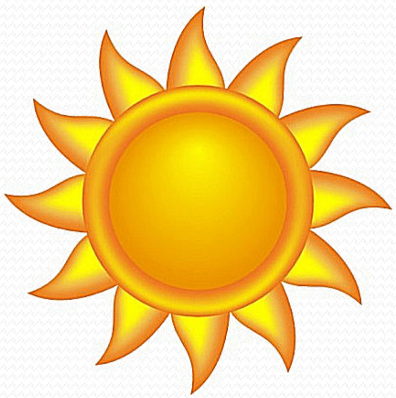
0 scalar



Additional degrees of freedom

"*screened*" via a Vainshtein mechanism

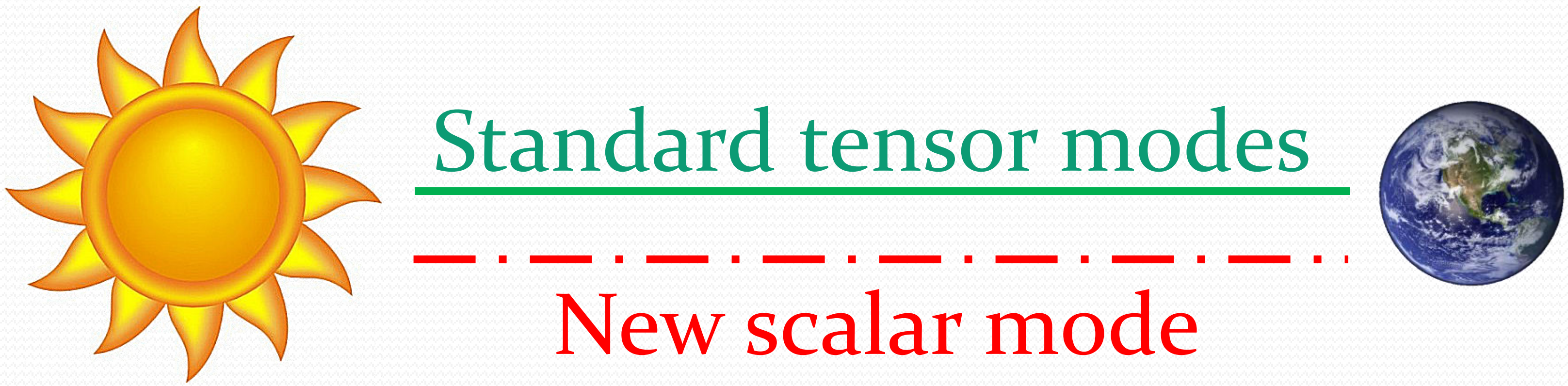
Gravitational force



Standard tensor modes

$$F_{\mathrm{GR}} = F_{\mathrm{tensor}}$$

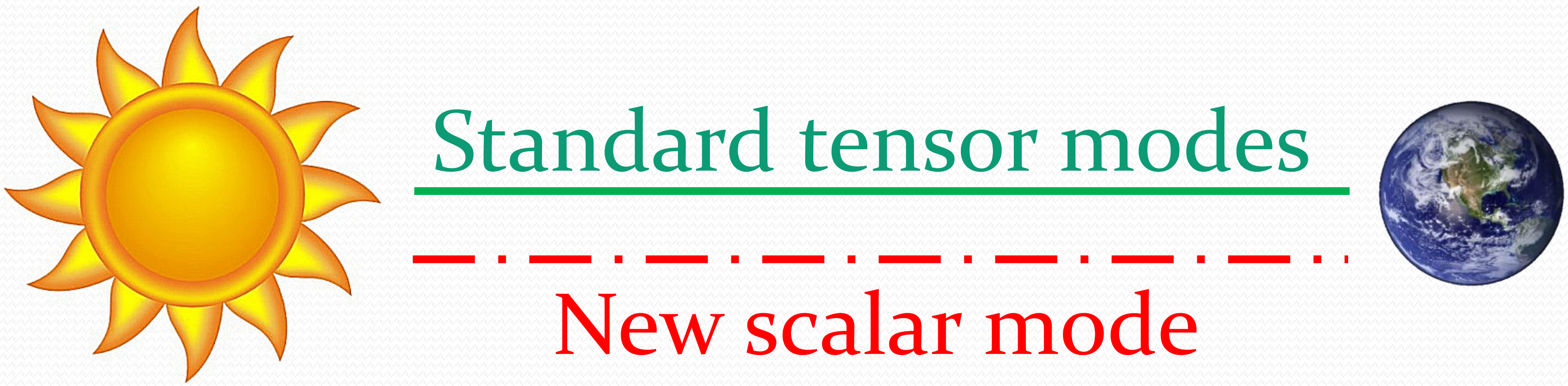
Gravitational force



$$F_{\mathrm{GR}} = F_{\mathrm{tensor}}$$

$$F_{\mathrm{massive}} = F_{\mathrm{tensor}} + F_{\mathrm{scalar}} = \frac{4}{3} F_{\mathrm{GR}}$$

Vainshtein mechanism



$$F_{\mathrm{GR}} = F_{\mathrm{tensor}}$$

$$F_{\mathrm{massive}} = F_{\mathrm{tensor}} + F_{\mathrm{scalar}} = F_{\mathrm{GR}}$$

$= 0$ in massless limit

Vainshtein mechanism

Force mediated by the scalar mode $\phi'(r)$

$$\frac{\phi'(r)}{r} + \frac{1}{M_{\rm Pl} m^2} \left(\frac{\phi'(r)}{r} \right)^2 = \frac{M_\oplus}{4\pi M_{\rm Pl} r^3}$$

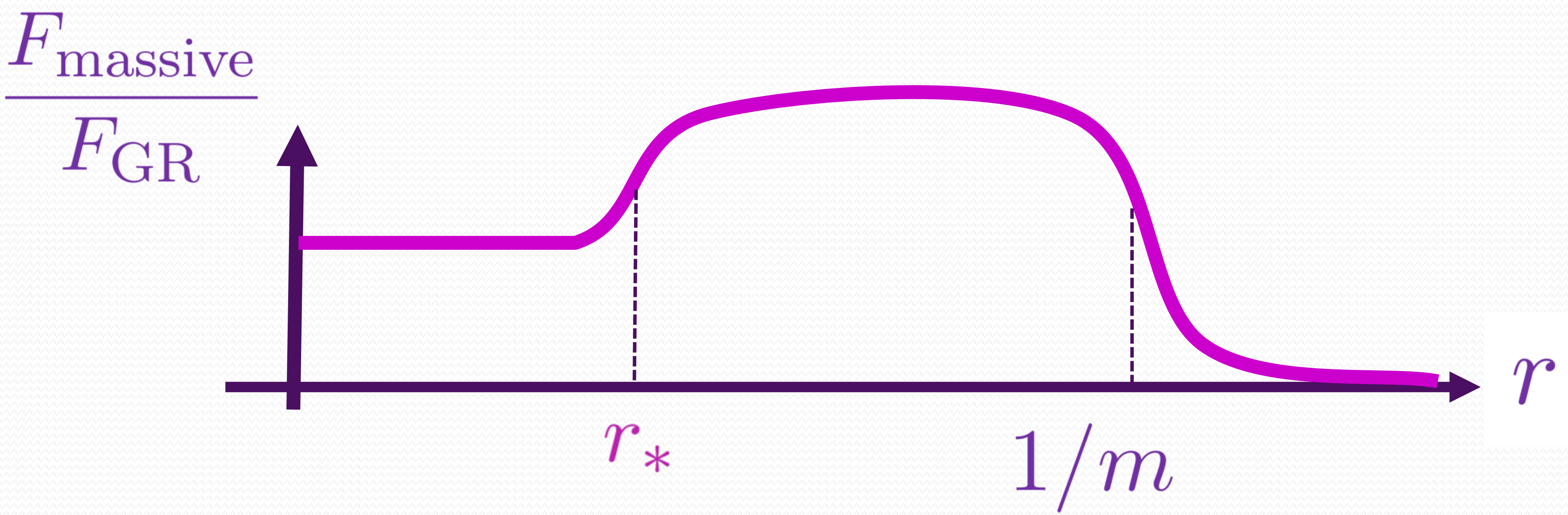
Vainshtein radius:

$$r_*^3 = \frac{1}{M_{\rm Pl} m^2} \frac{M_\oplus}{M_{\rm Pl}}$$

$$\text{for} \quad r \gg r_* \,, \quad \phi'(r) \sim \frac{M_\oplus}{M_{\rm Pl}} \frac{1}{r^2}$$

$$\text{for} \quad r \ll r_* \,, \quad \phi'(r) \sim \frac{M_\oplus}{M_{\rm Pl}} \frac{1}{r_*^{3/2} \sqrt{r}}$$

Gravitational force in MG



Vainshtein radius:

$$r_*^3 = \frac{1}{M_{\text{Pl}} m^2} \frac{M_\oplus}{M_{\text{Pl}}}$$

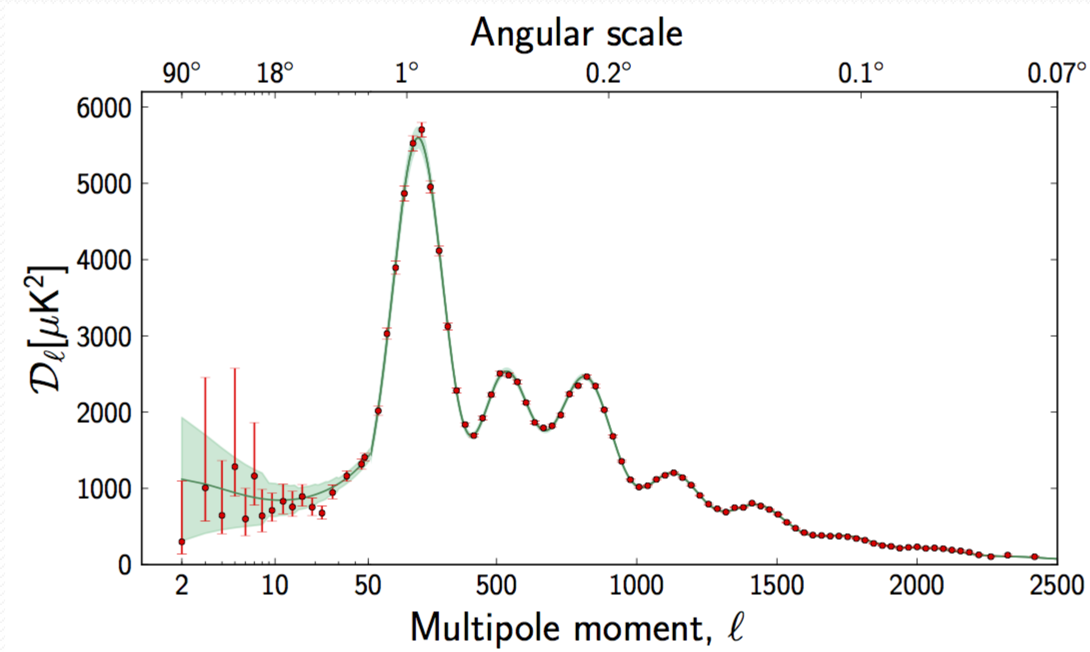
m: graviton mass

Evolution of the Universe

- At high densities, Vainshtein mechanism at work to "hide" the additional scalar mode

Evolution of the very early Universe is similar to GR (expect it to be undistinguishable)

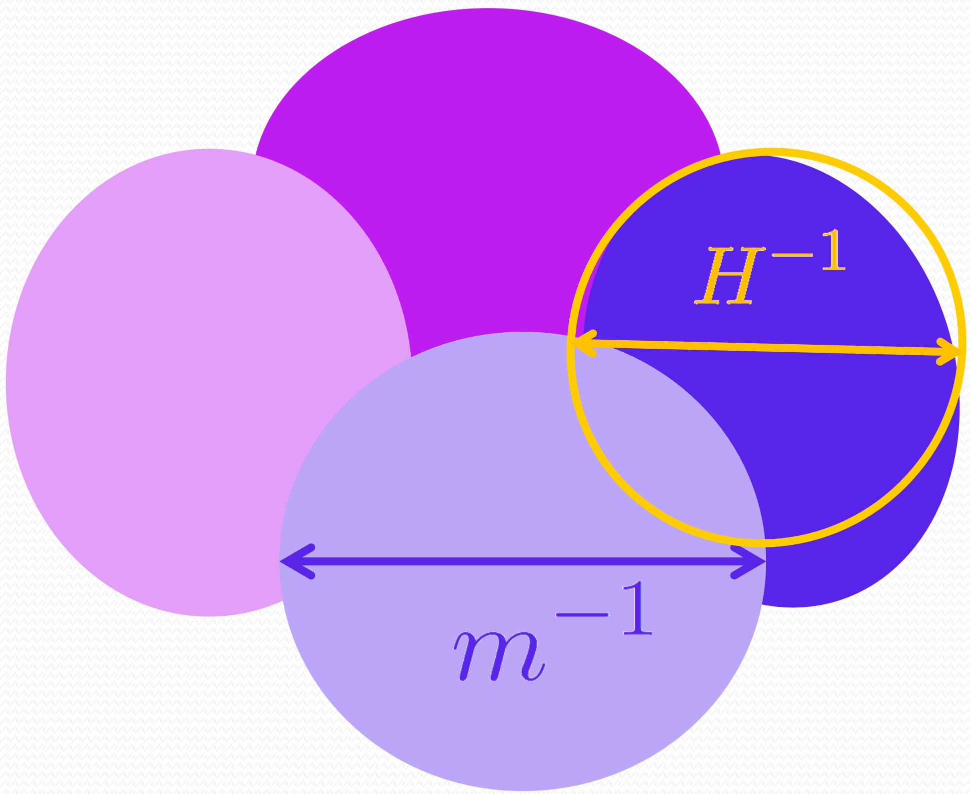
→ *no features at high l in CMB*



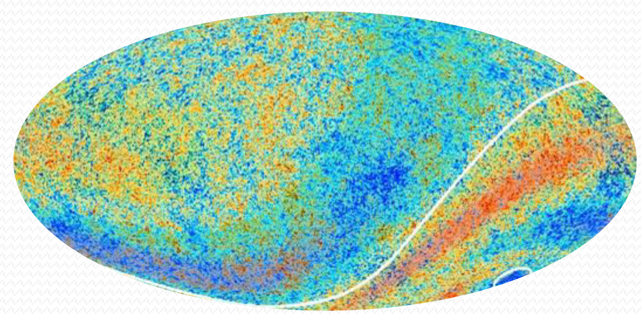
Evolution of the Universe

- At low densities, the Vainshtein mechanism stops being efficient

Large scale inhomogeneities



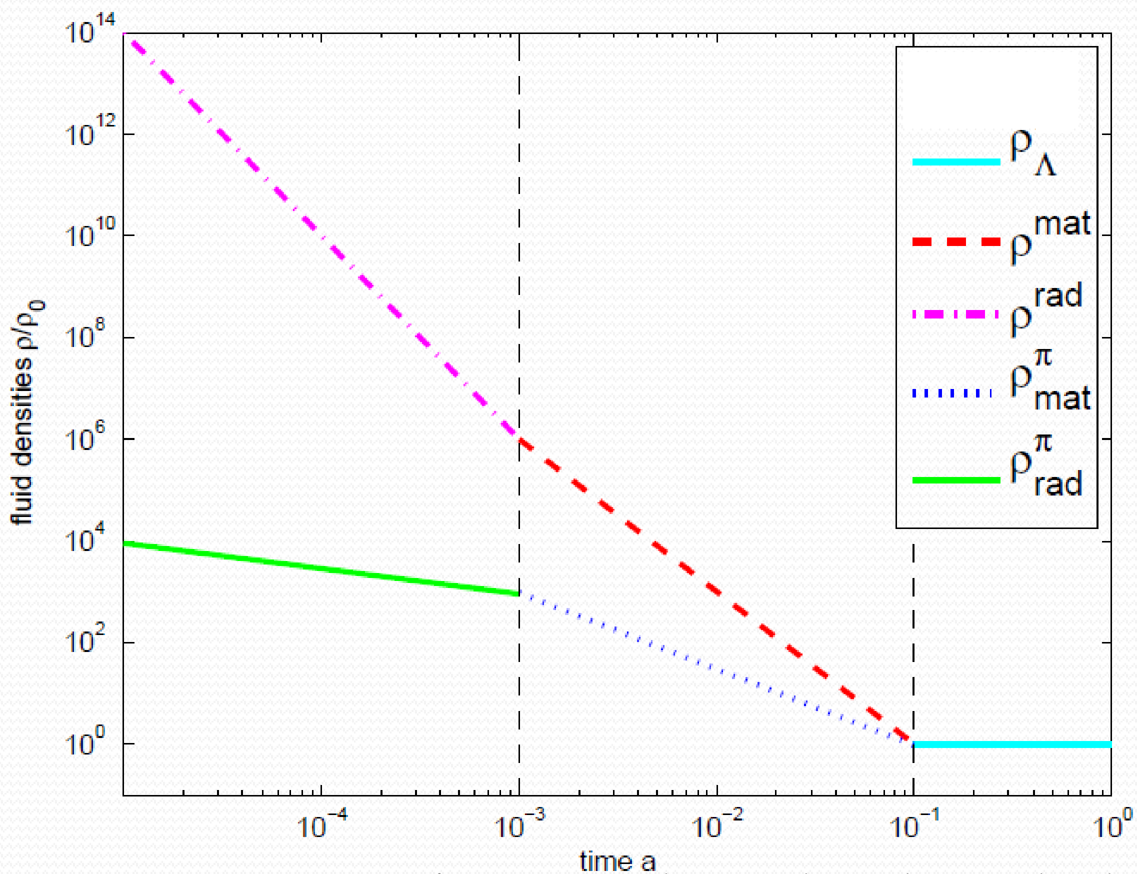
→ *expect low-ℓ CMB anomalies*…
& suppression of power…



D'Amico, CdR, Dubovsky, Gabadadze, Pirtskhalava & Tolley, 1108.5231

Evolution of the Universe

- At low densities, the Vainshtein mechanism stops being efficient.



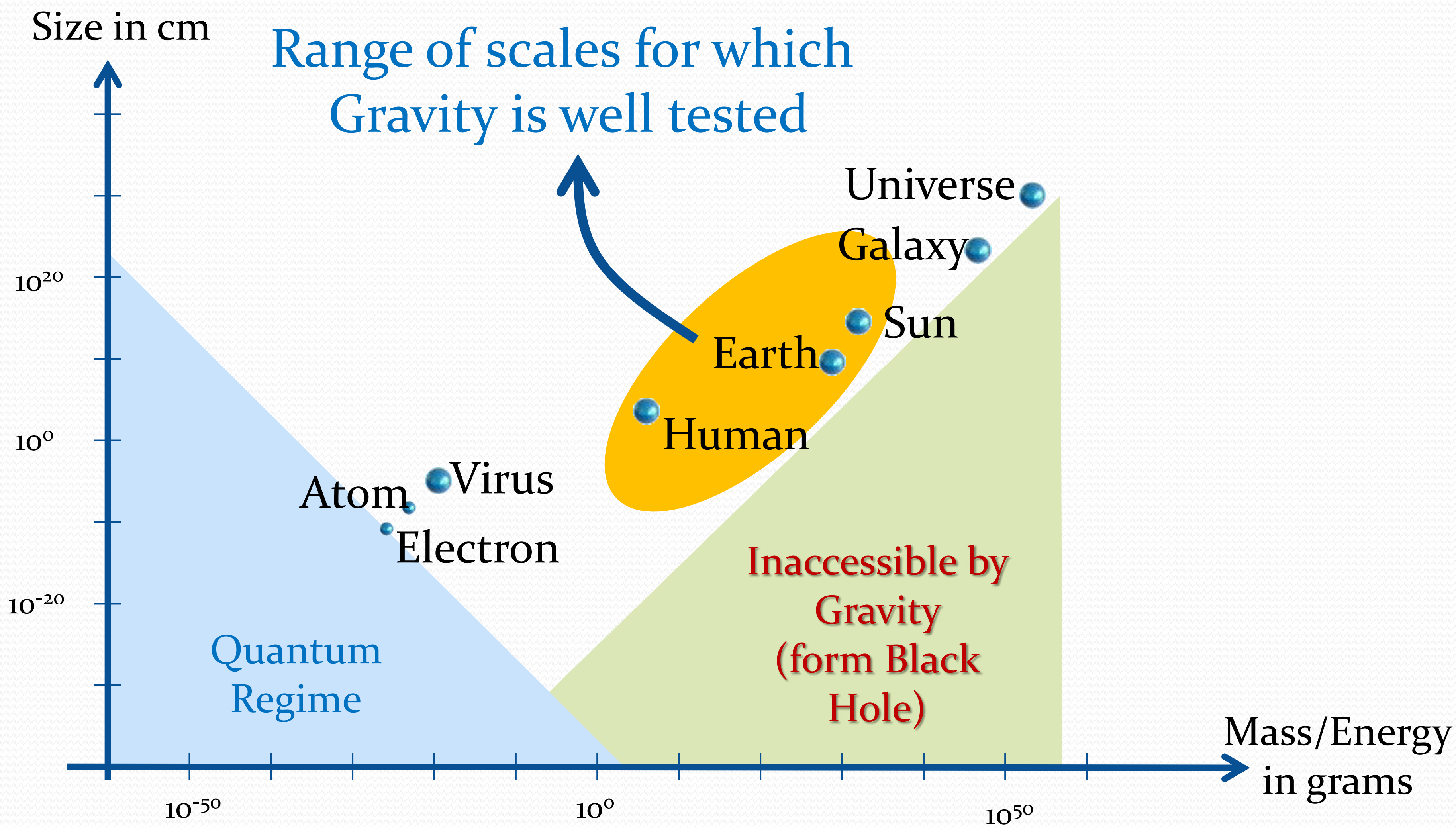
↓

leads to self-acceleration

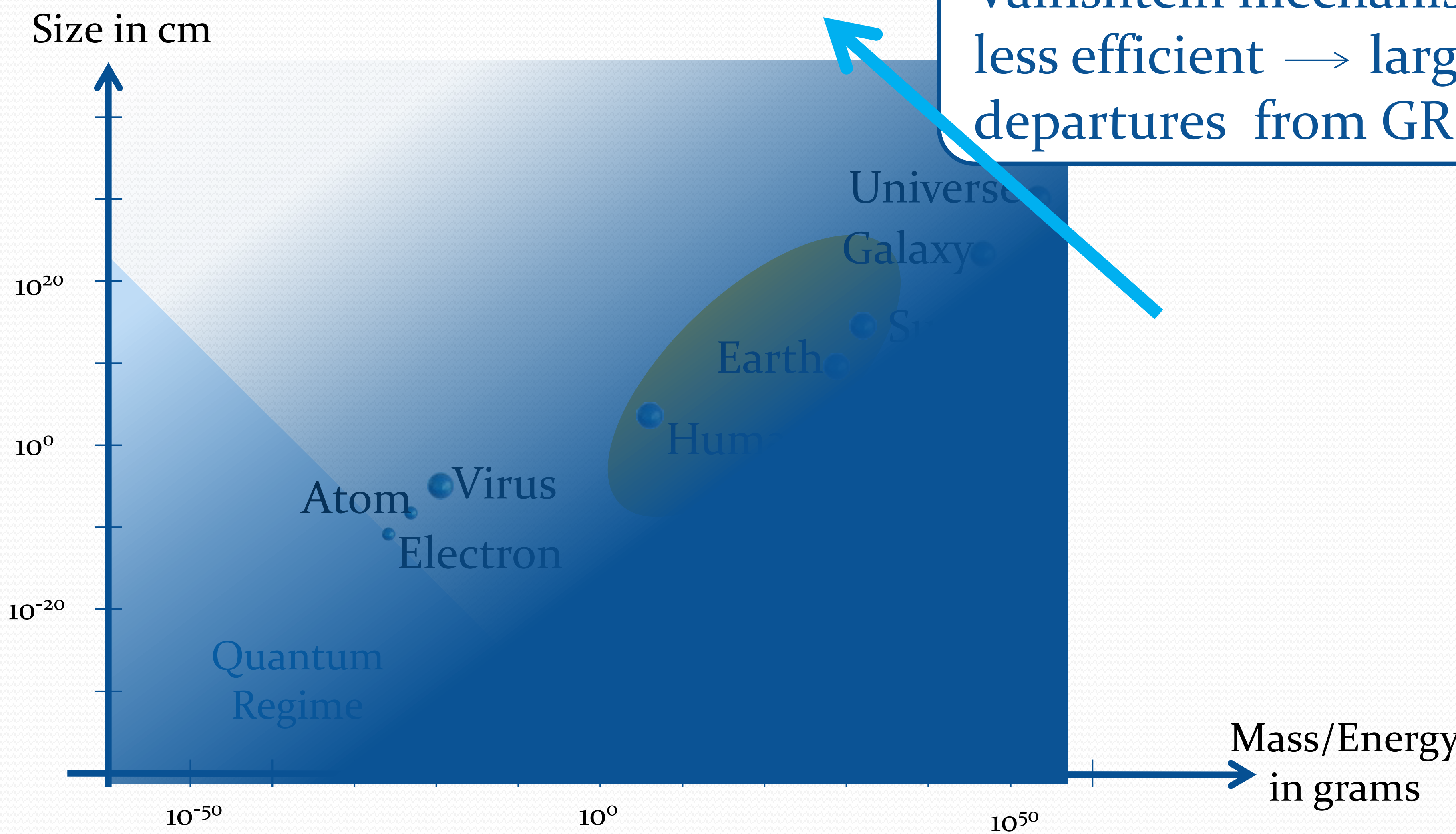
CdR, Heisenberg, 2011

D'Amico, CdR, Dubovsky, Gabadadze, Pirtskhalava & Tolley, 1108.5231

Observational Tests



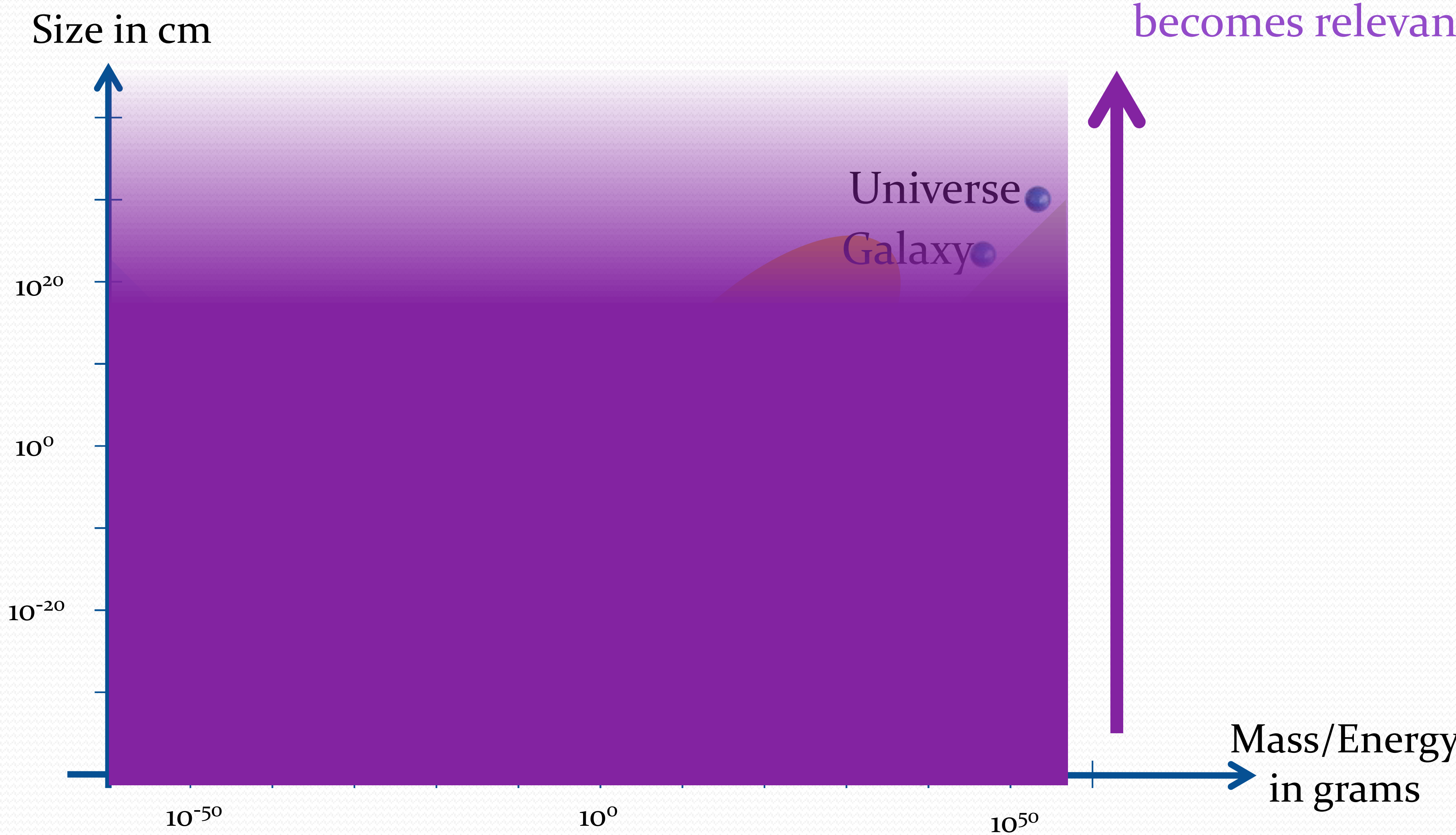
Observational Tests



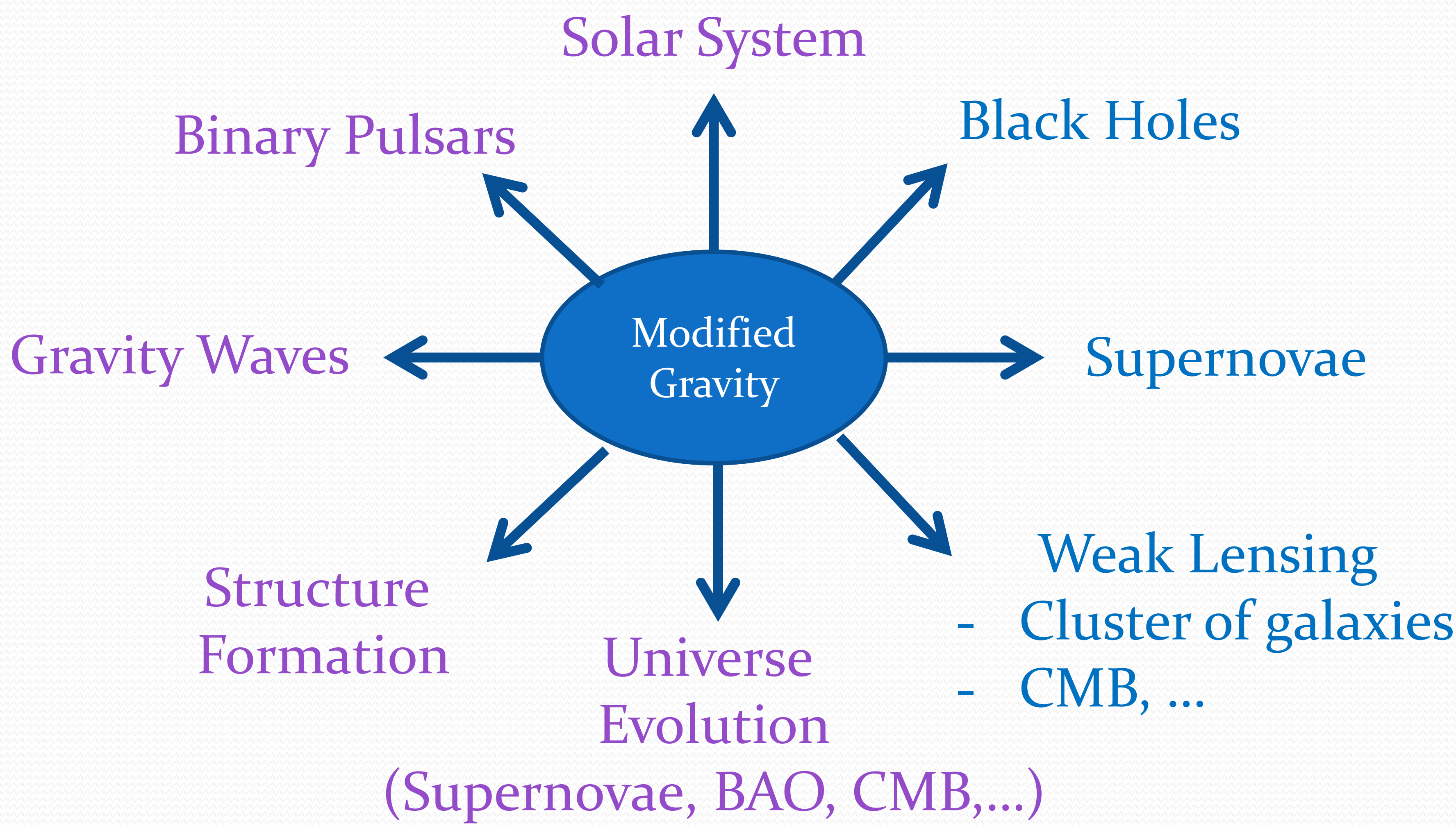
Vainshtein mechanism less efficient ⟶ larger departures from GR

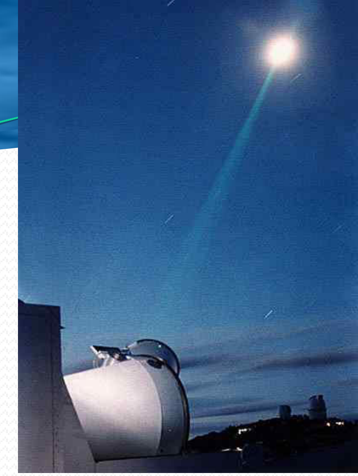
Observational Tests

Effect of mass on tensor mode becomes relevant



Observational Tests





Vainshtein mechanism

- Well understood for Static & Spherically Symmetric configurations
- Force mediated by the scalar mode is suppressed

$$\frac{F_{\rm scalar}}{F_{\rm Newton}} \sim \left(\frac{r}{r_*}\right)^{3/2}$$

LLR constraints: $\delta\Delta\varphi \lesssim 10^{-11}$

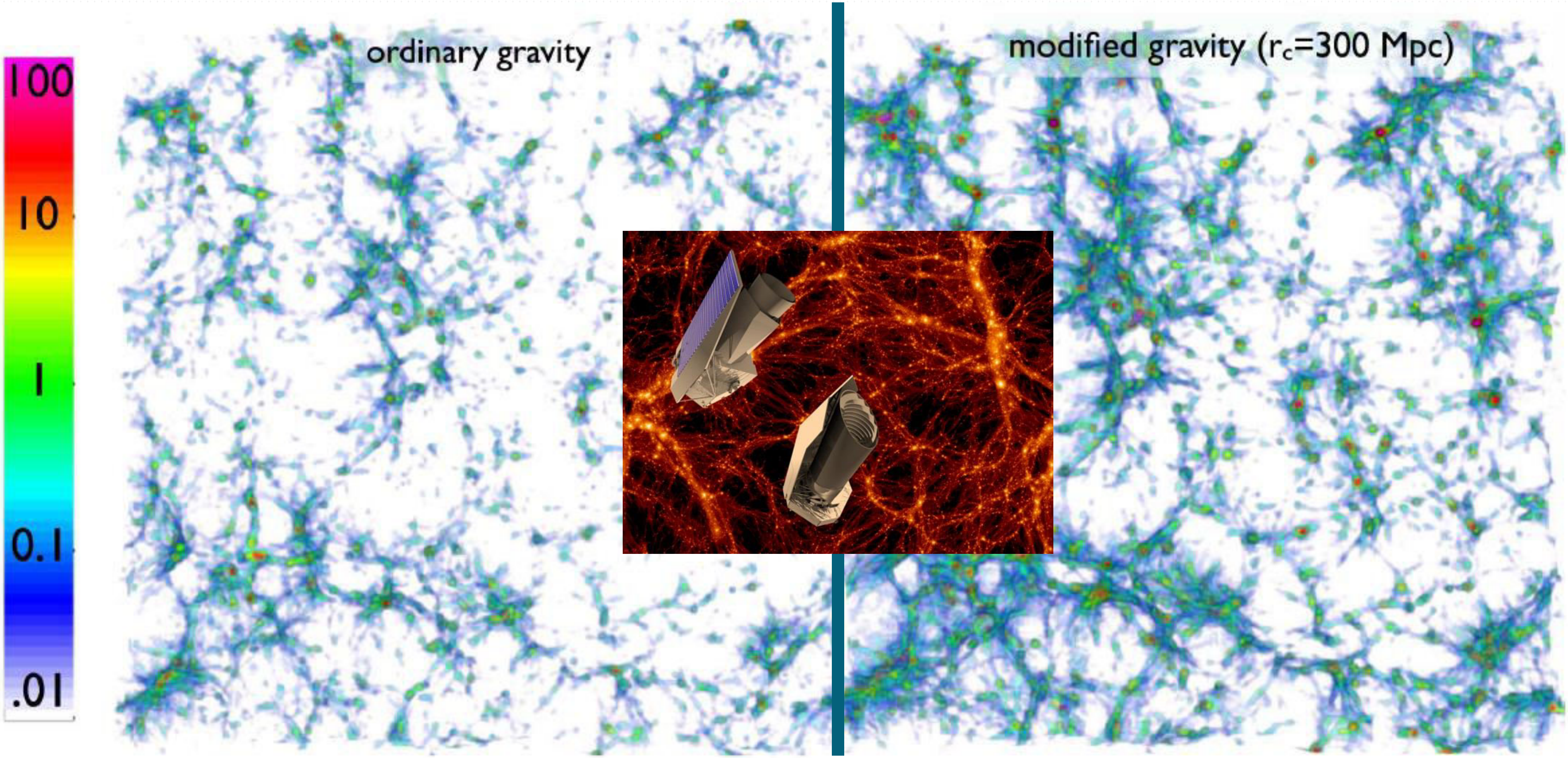
Vainshtein radius:

$$r_*^3 = \frac{1}{M_{\rm Pl} m^2}\frac{M_\oplus}{M_{\rm Pl}}$$

Constraints the mass

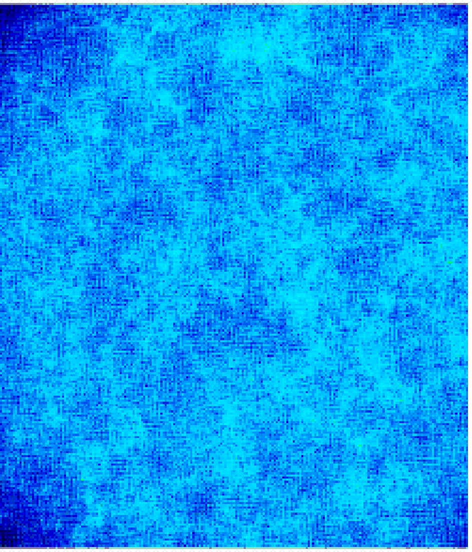
$$m_{\rm eff}^{\rm (late-time)} \lesssim 10^{-31}{\rm eV}$$

Effects on structure formation



From Khoury & Wyman, 0903.1292 for a toy model (cubic Galileon)
For massive gravity see CdR & Heisenberg, 2011

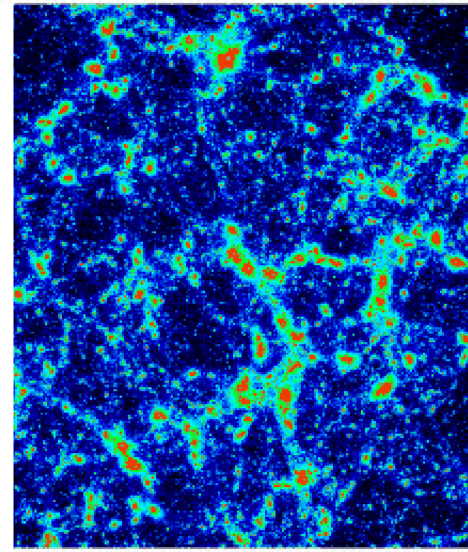
Effects on structure formation



$$\ddot{\delta}_m + 2H\dot{\delta}_m = G_{\rm eff}(t, x^i)\, \rho_m \delta_m$$

$$G_{\rm eff} \to G_N \quad \text{at high density}$$

$$G_{\rm eff} \to \frac{4}{3} G_N \quad \text{at low density}$$

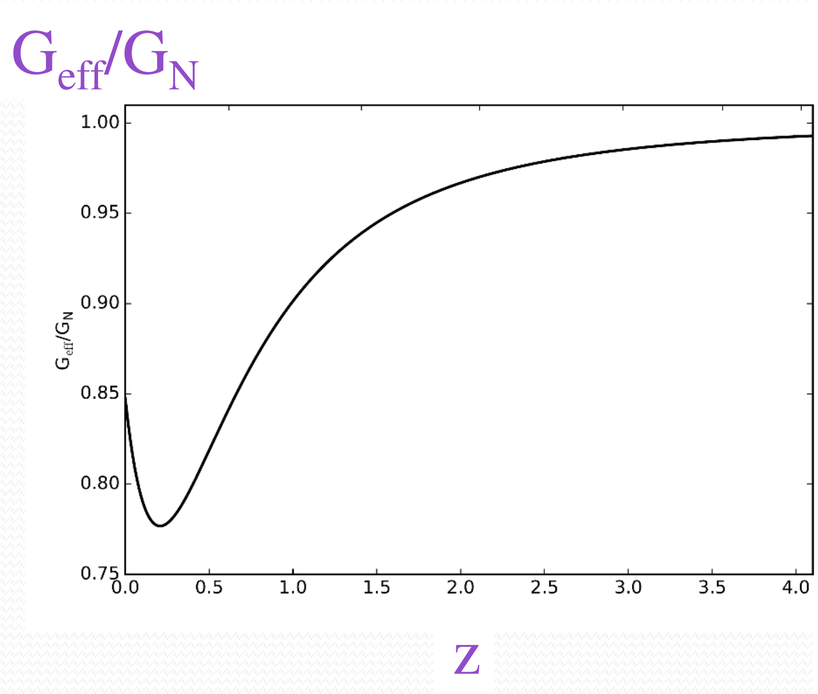


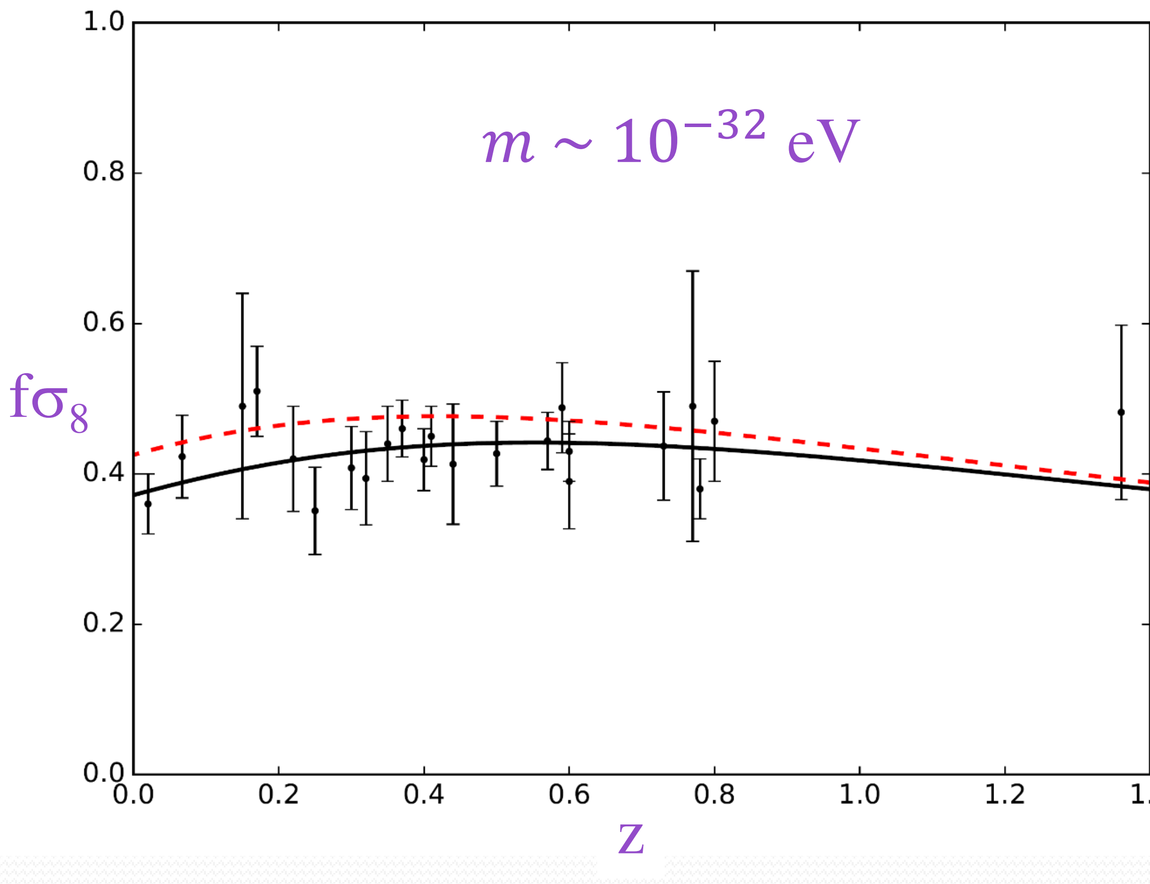
background effect	structure
$G_{\rm eff}(t) \sim G_N$ at early time	$G_{\rm eff}(x^i) \sim G_N$ for large dense structures
$G_{\rm eff}(t) \neq G_N$ at late time ✓	$G_{\rm eff}(x^i) \neq G_N$ for small structures
	Little analytical insight beyond SSS approximation

Effects on structure formation

Change in background evolution could help alleviate tension between:

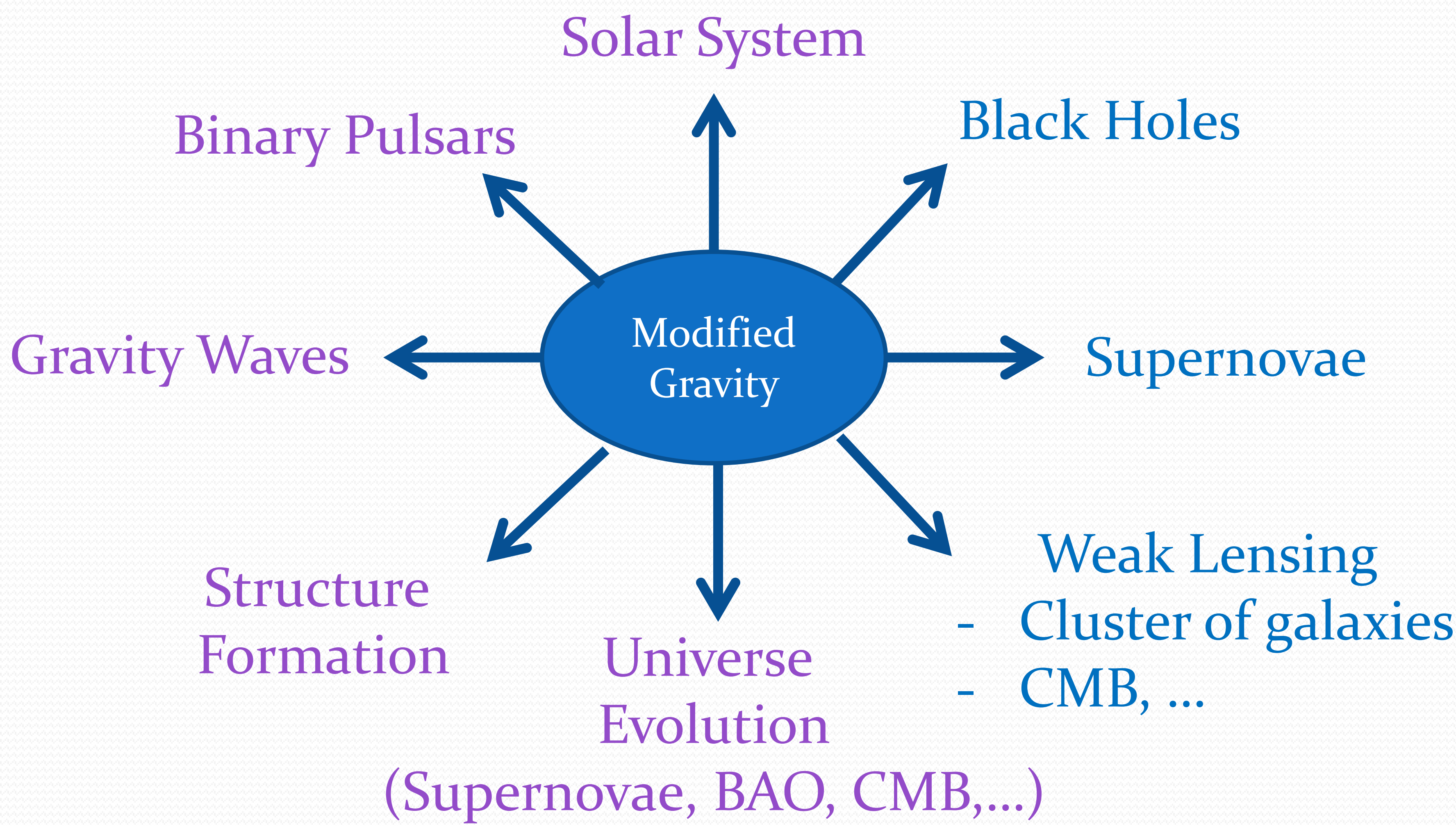
- standard Λ-CDM growth of structure
- and redshift space distortion measurements



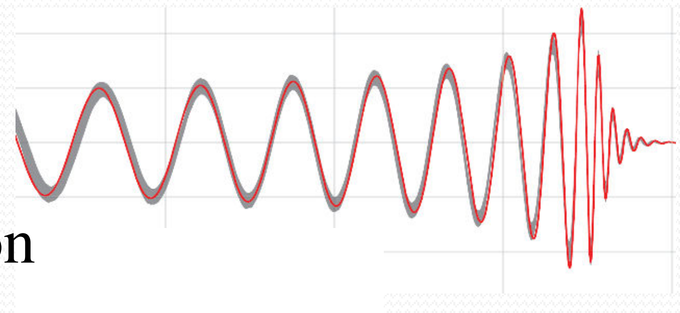


De Felice & Mukohyama, arXiv:1607.03368

Observational Tests



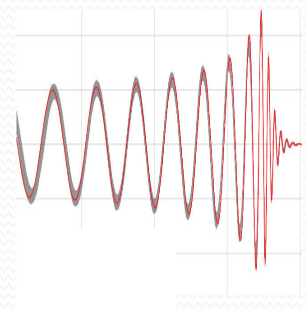
Direct detection of GWs



Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity (including DGP at the level of spectral representation)



GW signal would be more squeezed than in GR

matched filtering technique allows to determine the signal duration when emitted $\Delta\tau_e$ very accurately which can be compared with the signal duration when observed $\Delta\tau_a$.

$$\Delta t = \Delta\tau_a - \Delta\tau_e(1+z)$$

Will 1998

Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

$$m_g \lesssim 4 \times 10^{-22}\mathrm{eV} \left(f\Delta t \frac{f}{100\mathrm{Hz}} \frac{200\mathrm{Mpc}}{D} \right)^{1/2}$$

Phase distortion $f\Delta t$ can be measured up to $1/\rho$ (ρ: the signal to noise ratio)

For GW150914,

$$D \sim 400\mathrm{Mpc},\ f \sim 100\mathrm{Hz},\ \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\mathrm{eV}$$

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For GW150914,

$$D \sim 400\mathrm{Mpc}, \;\; f \sim 100\mathrm{Hz}, \;\; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\mathrm{eV}$$

For GW151226, ρ is smaller and the BHs are lighter so f is larger $\longrightarrow$ not as competitive

For GW170817, $\Delta c^2 < 10^{-15} \longrightarrow m^2 < 10^{-15}\, k^2 \sim (10^{-22} eV)^2$

Direct detection of GWs

modifications of the dispersion relation put a bound on the graviton mass

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Phase distortion $f\Delta t$ can be measured up to $1/\rho$ (ρ: the signal to noise ratio)

For eLISA, could have

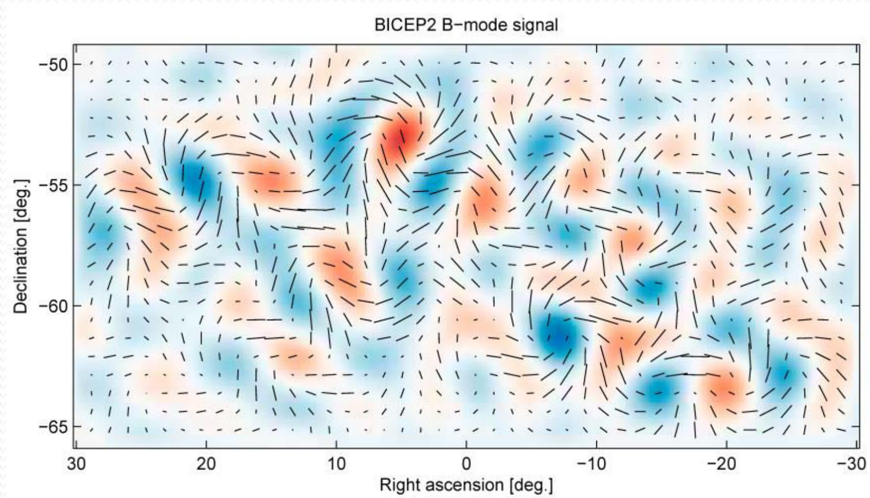
$$\rho \sim 10^3$$

$$D \sim 3\mathrm{Gpc} \quad \Longrightarrow \quad m_g \lesssim 10^{-26}\mathrm{eV}$$

$$f \sim 10^{-3}\mathrm{Hz}$$

Will 1998

Bounds from Primordial Gravitational Waves



if ever detected...

would imply the graviton is effectively massless at the time of recombination

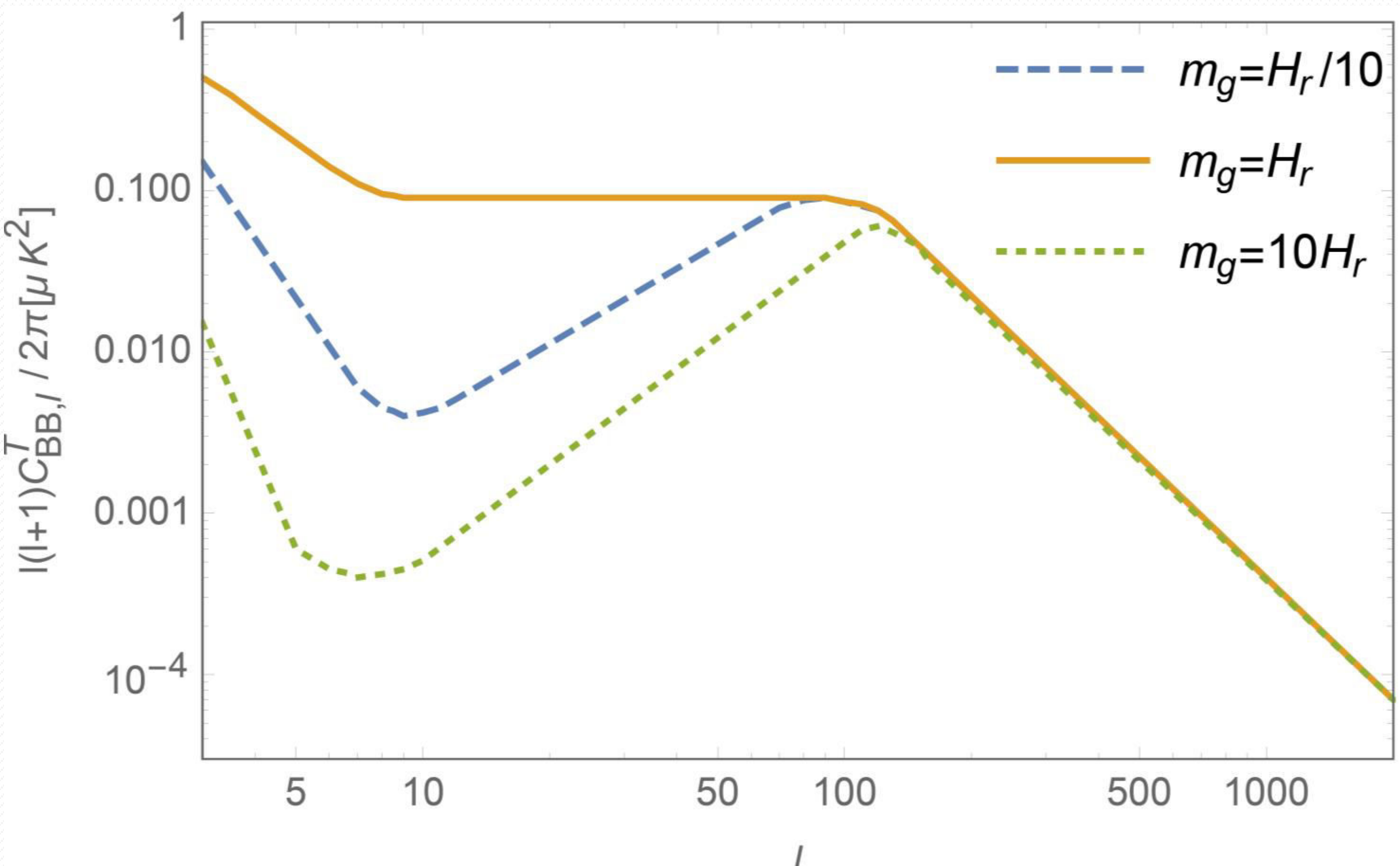
$$m_{\rm eff} \ll 10^{-29}{\rm eV}$$

Dubovsky, Flauger, Starobinsky & Tkachev, 2010 (for Lorentz-breaking MG)
Fasiello & Ribeiro, 2015, (for bi-gravity)
Lin&Ishak, 2016 (Testing gravity using tensor perturbations)

Bounds from Primordial Gravitational Waves

Modification to the tensor mode evolution

$$\mathcal{D}_q''(\tau) + 2\frac{a'}{a}\mathcal{D}_q'(\tau) + \left(q^2 + a^2 m_g^2\right)\mathcal{D}_q(\tau) = J_q(\tau)$$



Dubovsky, Flauger, Starobinsky & Tkachev, 2010 (for Lorentz-breaking MG)
Fasiello & Ribeiro, 2015, (for bi-gravity) Lin&Ishak, 2016

Indirect Gravitational Wave Detection

Pulsar Timing Arrays could in principle detect ηHZ GWs

would put a bound $m_g \lesssim f \sim 10^{-23}\text{eV}$

Lee et al., 2010

Binary Pulsar Radiation

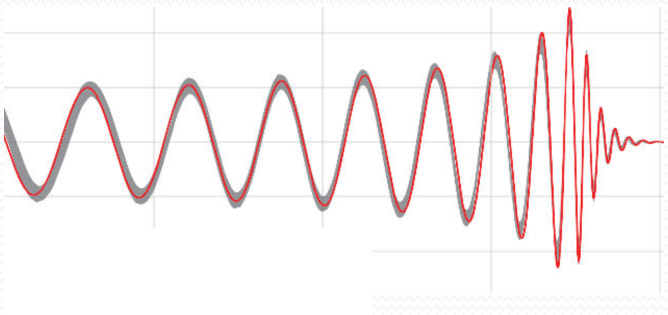
expect a correction of order m^2/f^2 to the power emitted by the tensor modes

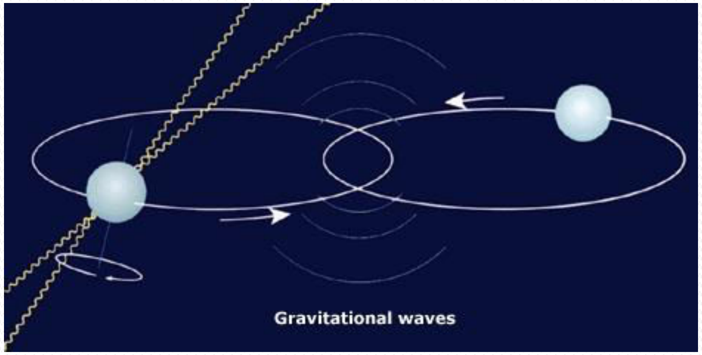
$$m_g \lesssim \frac{10^{-1}}{(\text{few hours})} \sim 10^{-20}\text{eV}$$

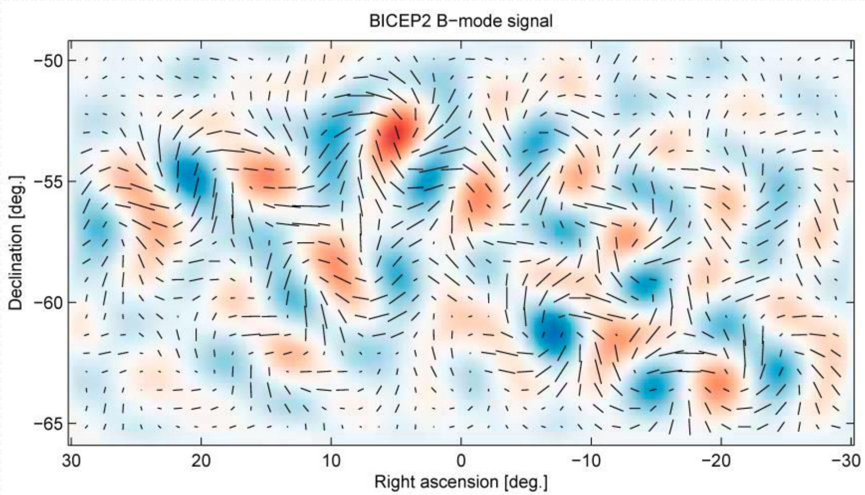
Finn and Sutton, 2002

Dispersion Relation

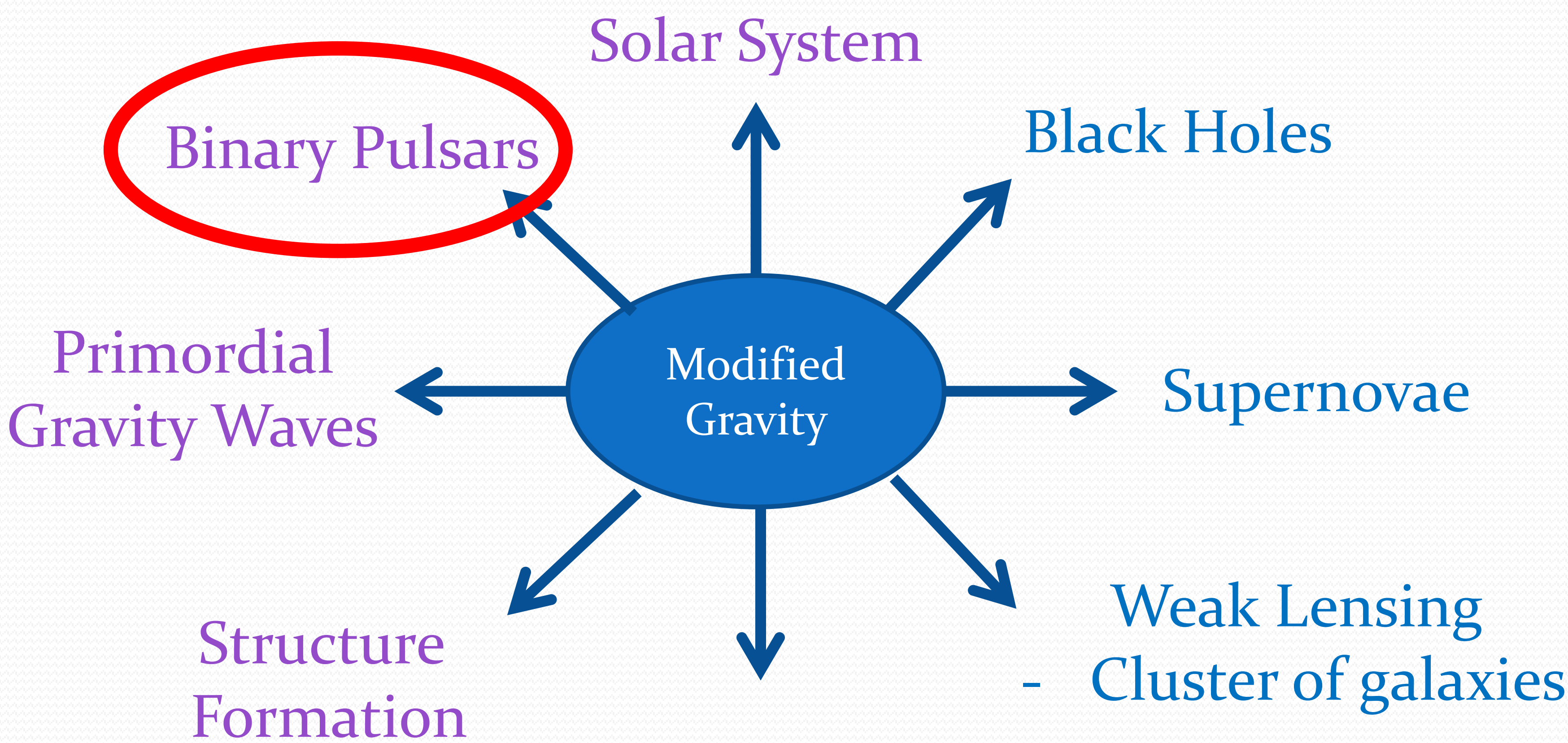
m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



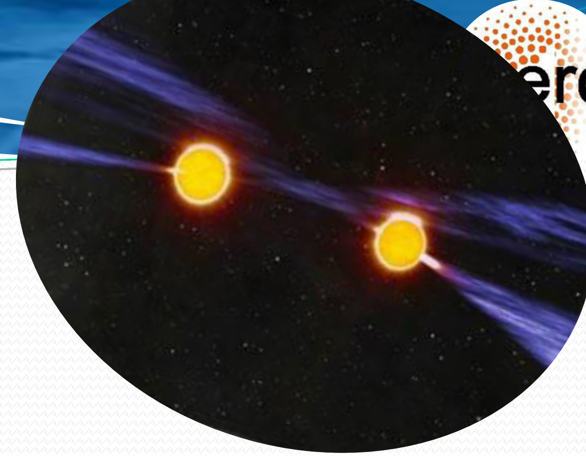


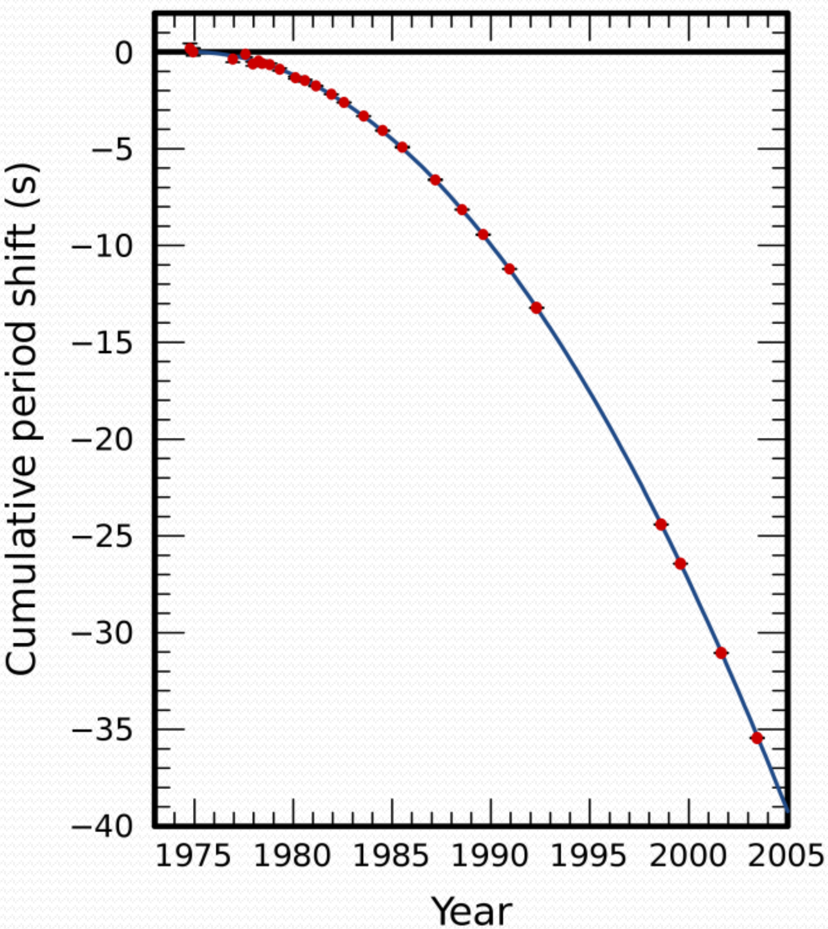


Observational Tests



Binary Pulsar System



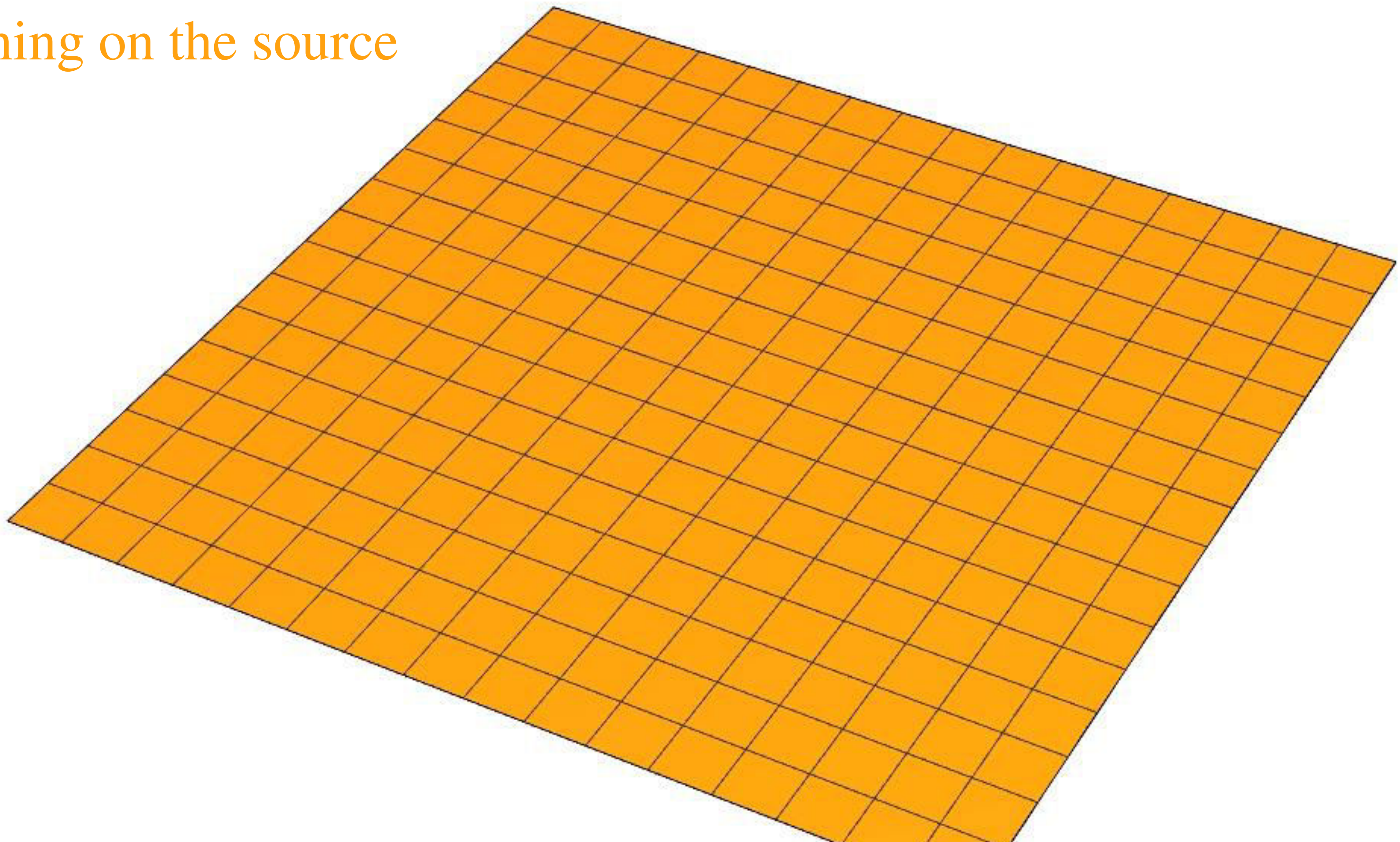


- The existence of a **scalar mode** leads to new monopole, dipole and quadrupole channels of radiation.
- They are suppressed compared to GR thanks to **Vainshtein mechanism.**
- Monopole & dipole suppressed due to conservation of energy & momentum.

Work with Furqan Dar, Tate Deskins, John Tom Giblin & Andrew Tolley

Numerical Checks

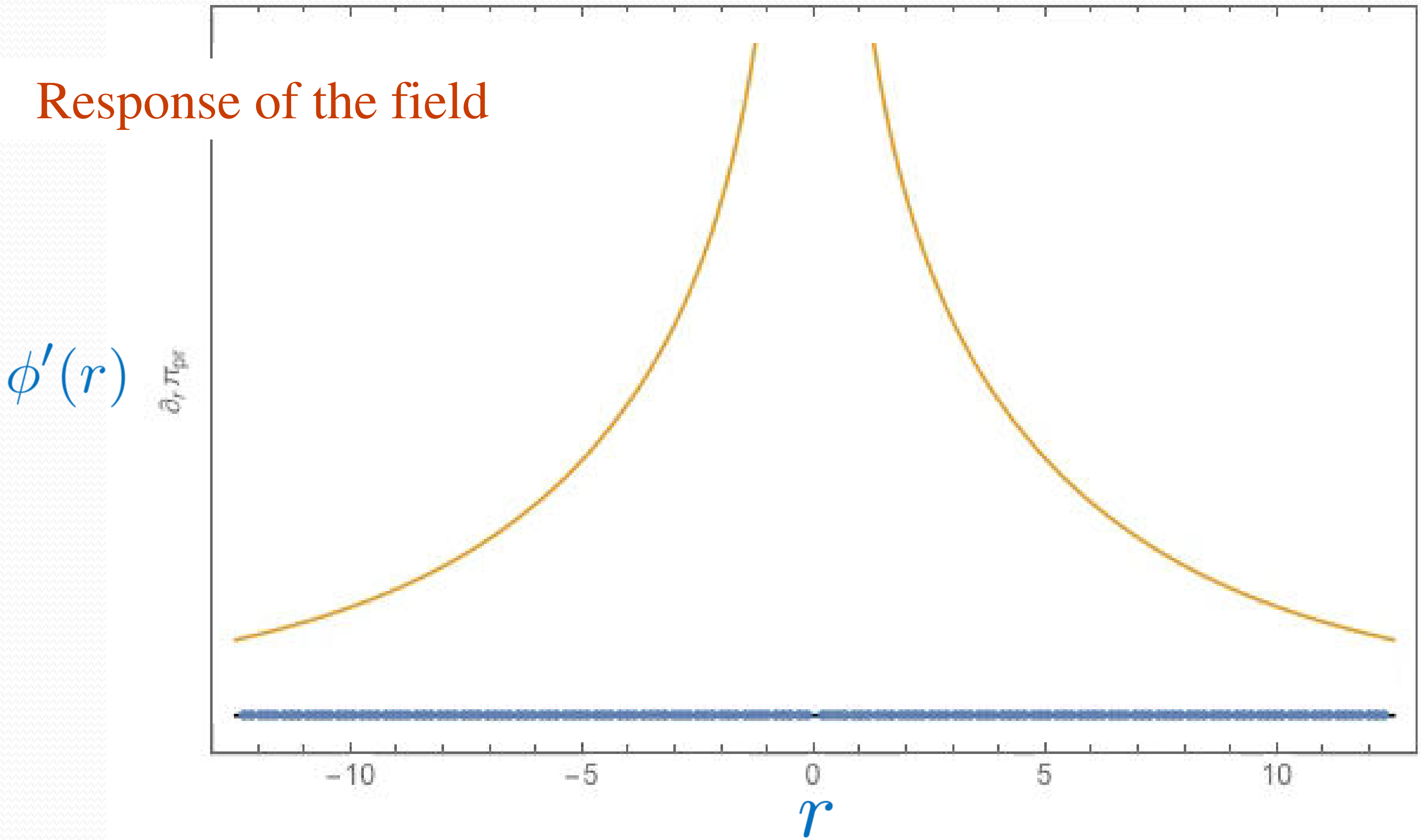
Switching on the source



Work with Furqan Dar, Tate Deskins,
John Tom Giblin & Andrew Tolley

Numerical Checks

Response of the field



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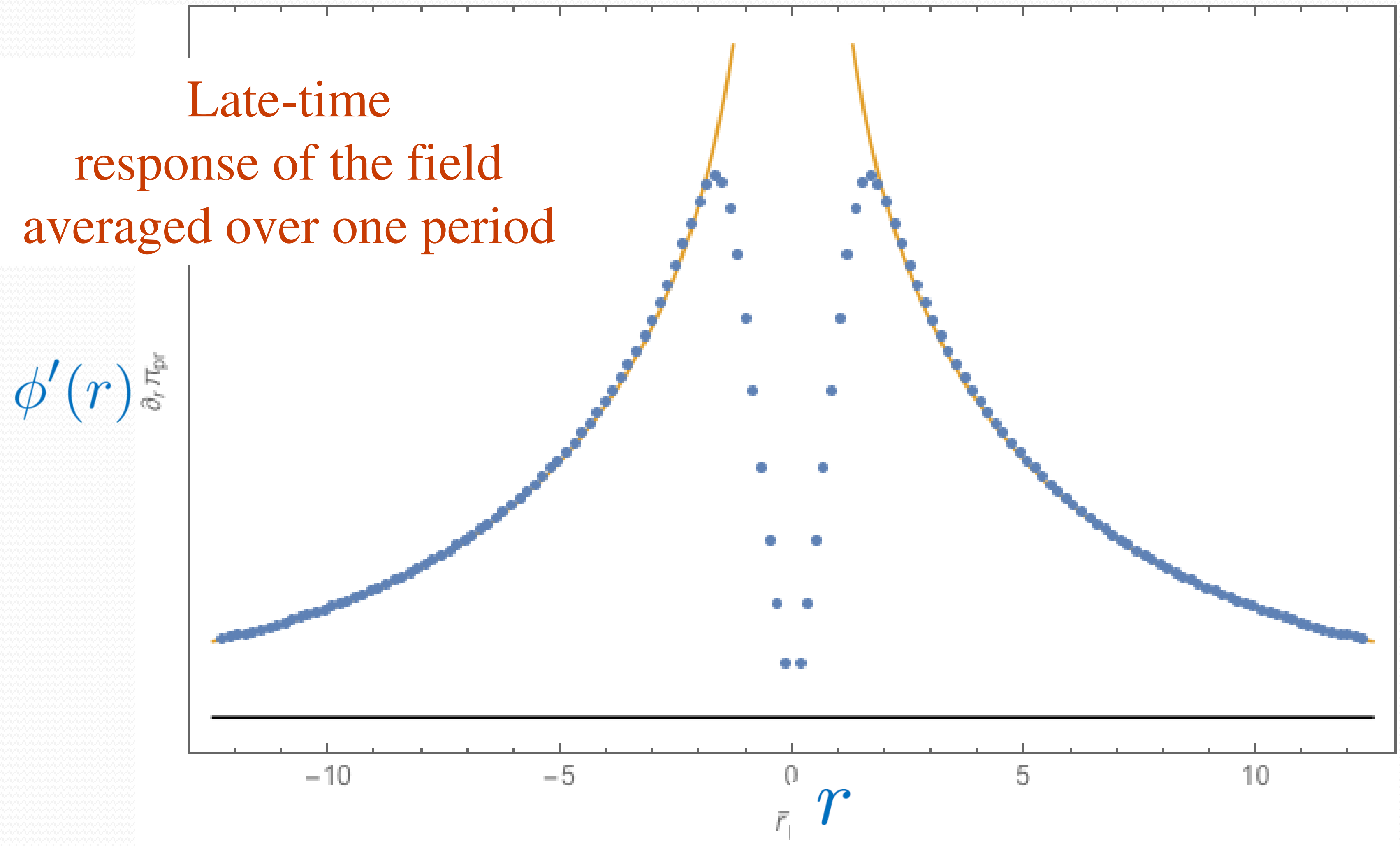
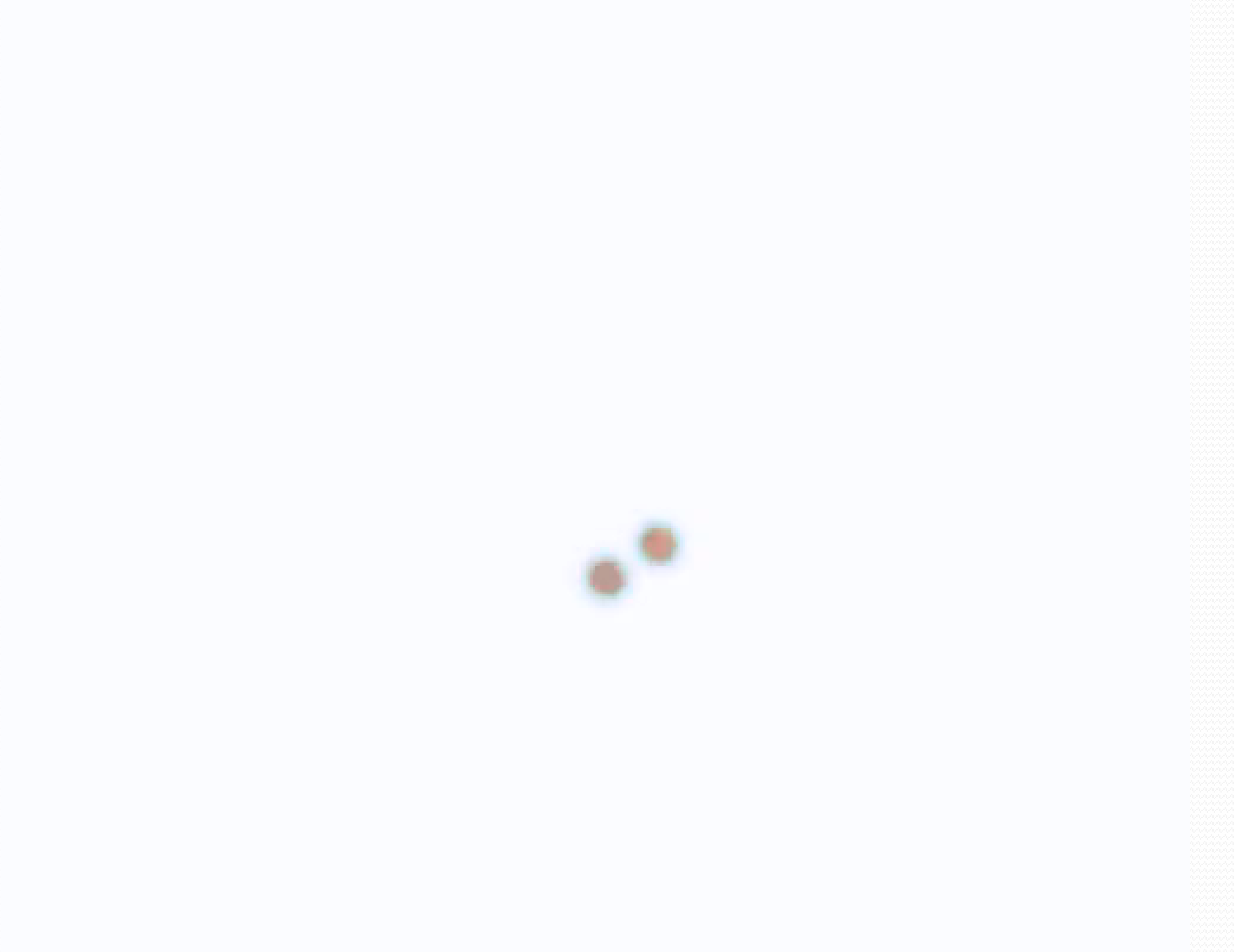
Numerical Checks

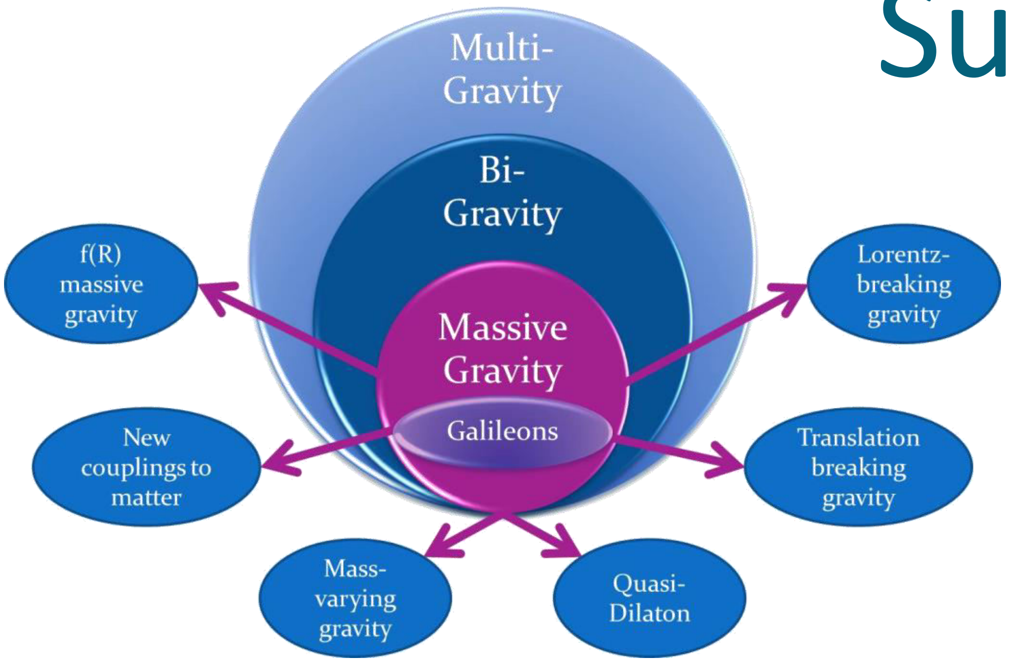




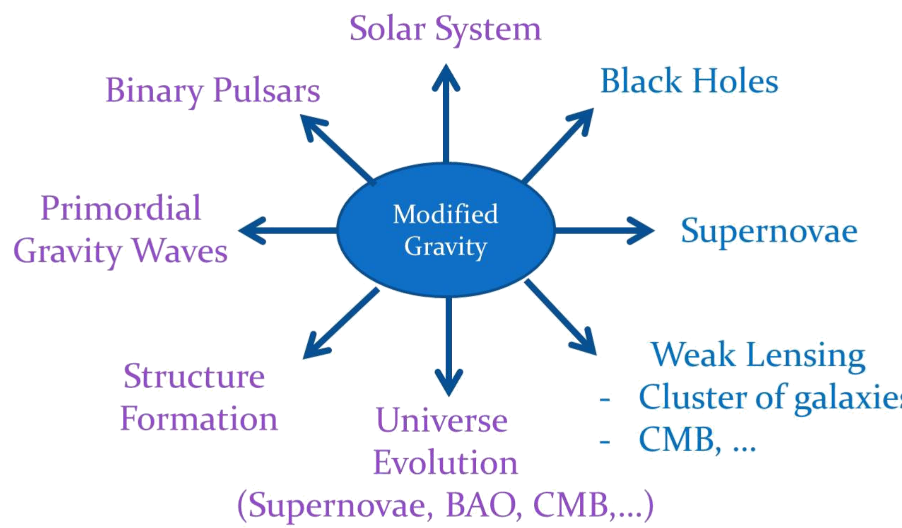
Contours of $\dot{\phi}^2$

For the cubic Galileon (toy model):
Power still in the quadrupole as in GR
Corrections to GR are very suppressed

Summary



Recent developments in Massive Gravity have opened a wealth of new models for gravity

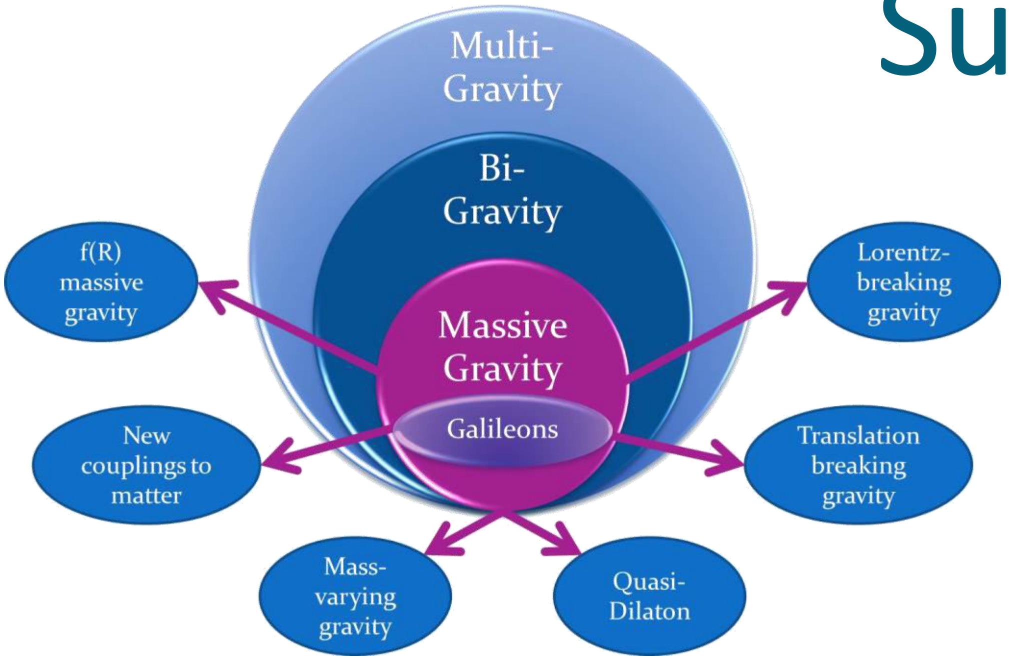


These models could play a crucial role for our understanding of our Universe

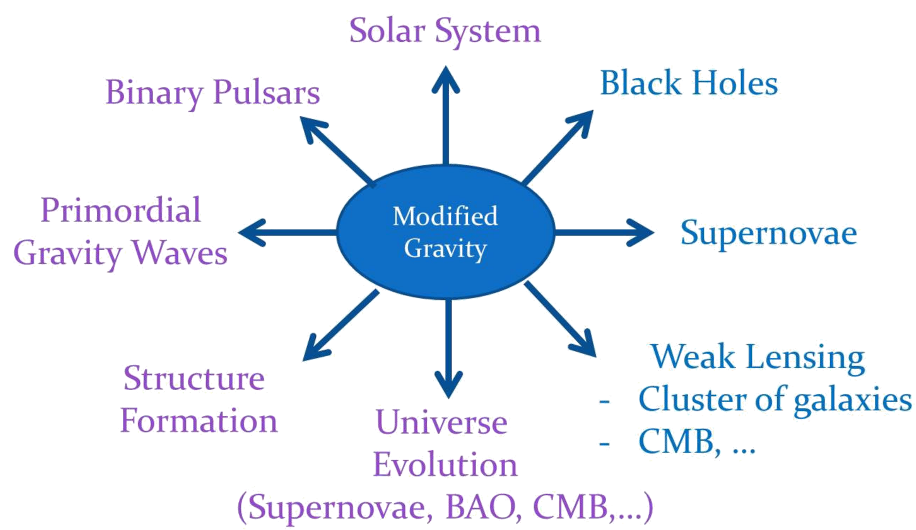
- Dark energy
- Degravitation
- Inflation
- Pre Big-Bang, bounces
- ...

Come with a multitude of observational signatures on different systems & scales

Summary



Recent developments in Massive Gravity have opened a wealth of new models for gravity



These models could play a crucial role for our understanding of our Universe

- Dark energy
- Degravitation
- Inflation
- Pre Big-Bang, bounces
- ...

Come with a multitude of observational signatures on different systems & scales

How light is gravity ???

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters

Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Massive gravity from an EFT viewpoint

$$\mathcal{L} = \mathcal{L}_{\text{Ghost-free MG}}(c_3, d_5) + \Delta c\,[h^3] + \Delta d\,[h^4] + \cdots$$

has no ghost and a strong coupling scale
$\Lambda^3 = M_{Pl} m^2$

A priori, from a naïve EFT point of view, there is "*nothing wrong*" with considering other operators that would lead to "ghost" at a scale $\Lambda^5 = M_{Pl} m^4$ (for Δc) or $\Lambda^4 = M_{Pl} m^3$ (for Δd)

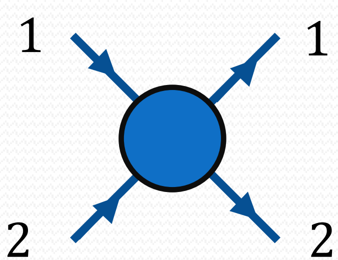
It just means that the cutoff of the EFT is lower

Are these parameters constrained by the positivity bounds ?

Massive gravity from an EFT viewpoint

$$\mathcal{L} = \mathcal{L}_{\text{Ghost-free MG}}(c_3, d_5) + \Delta c\,[h^3] + \Delta d\,[h^4] \quad + \cdots$$

Consider the positivity bound in the forward limit for the particular normalized states:



$$|1\rangle = |2\rangle = \alpha_{-2}|-2\rangle + \alpha_{-1}|-1\rangle + \varepsilon|0\rangle + \alpha_{+1}|+1\rangle + \alpha_{+2}|+2\rangle$$

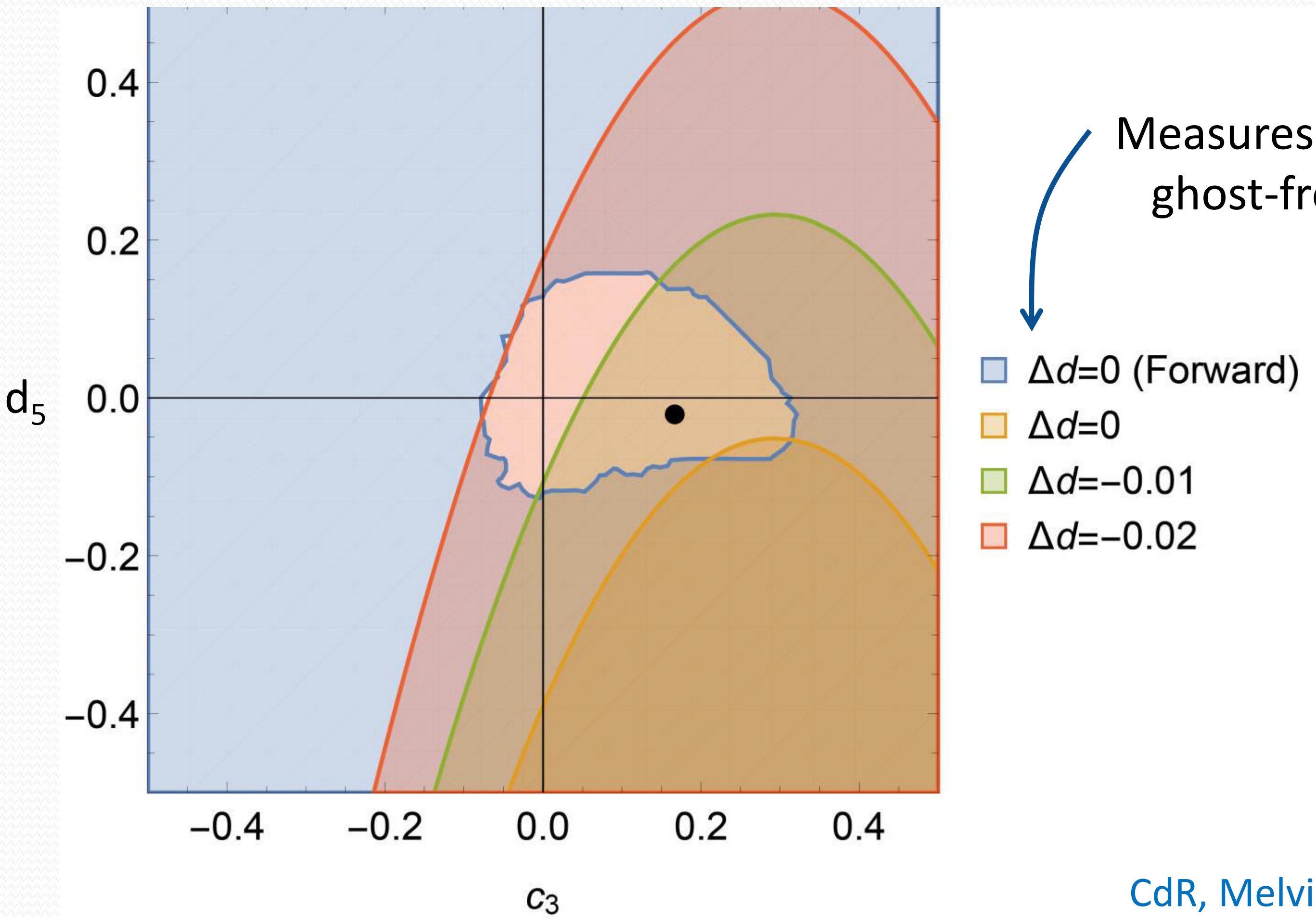
$$\frac{\partial^2}{\partial s^2} f_{\alpha\alpha}\Big|_{t=0} \propto \underbrace{(\alpha_1^2 - \alpha_{-1}^2)\Delta c\left(\varepsilon^2 + \mathcal{O}\varepsilon^4\right)}$$

Should be positive for any choice of $\alpha_{\pm 1}$

$\longrightarrow$ $\Delta c = 0$

Constraints beyond forward limit

$$\mathcal{L} = \mathcal{L}_{\text{Ghost-free MG}}(c_3, d_5) + \Delta c\,[h^3] + \Delta d\,[h^4] + \cdots$$

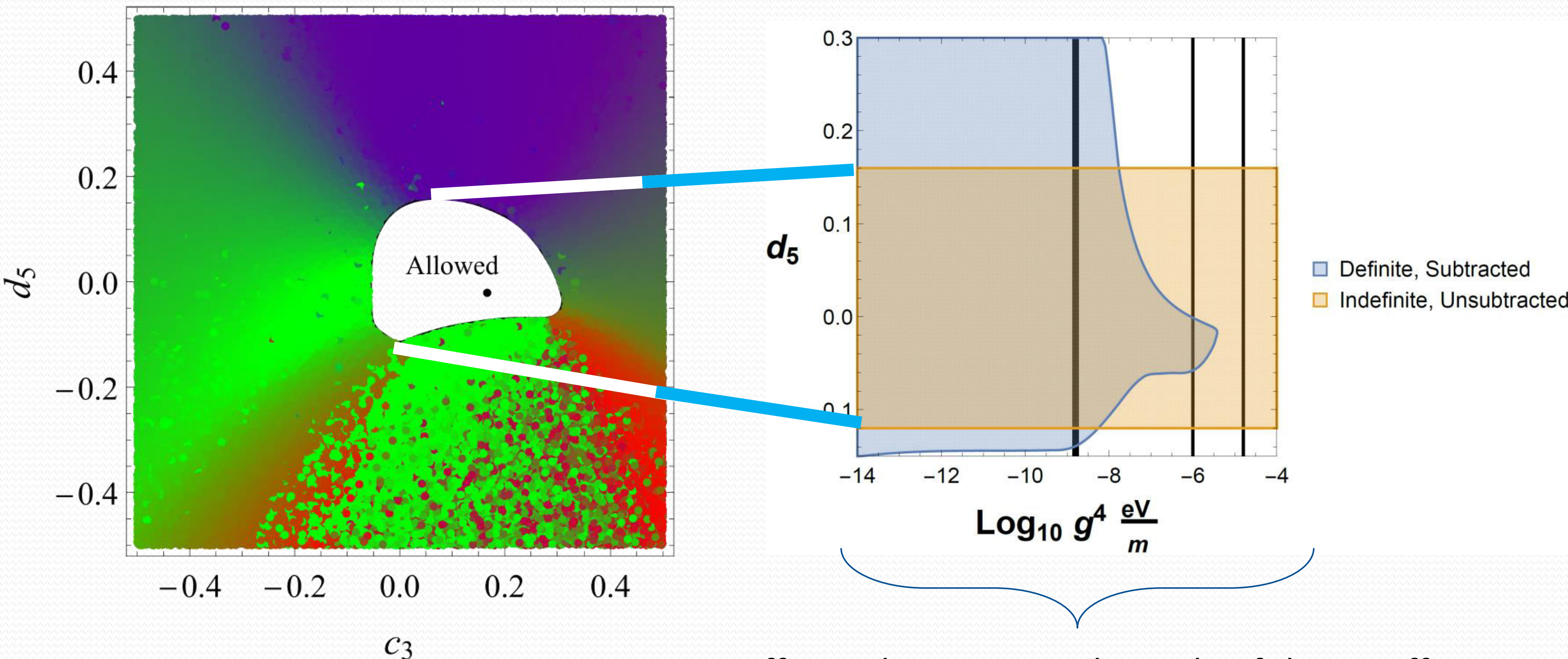


Measures the departure from ghost-free massive gravity

CdR, Melville, Tolley, Zhou, to appear

Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\,A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \sqrt{1-\frac{4m^2}{\mu^2}}\frac{\mu\sigma_{\mathrm{total}}(\mu)}{(\mu-2m^2)^3}\,.$$

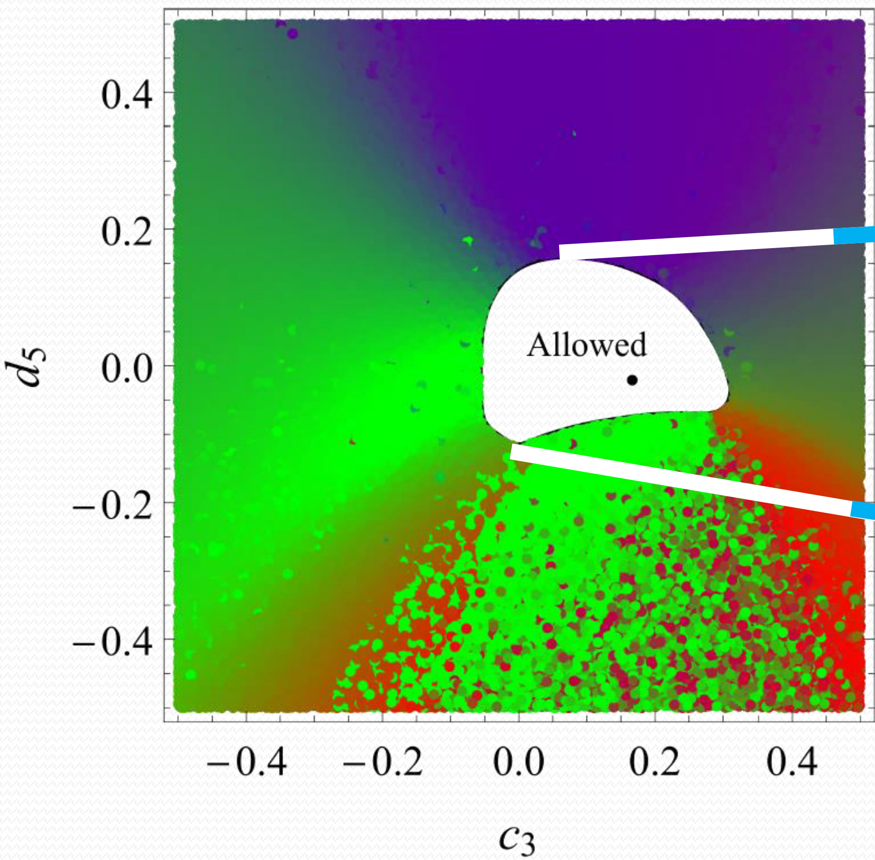


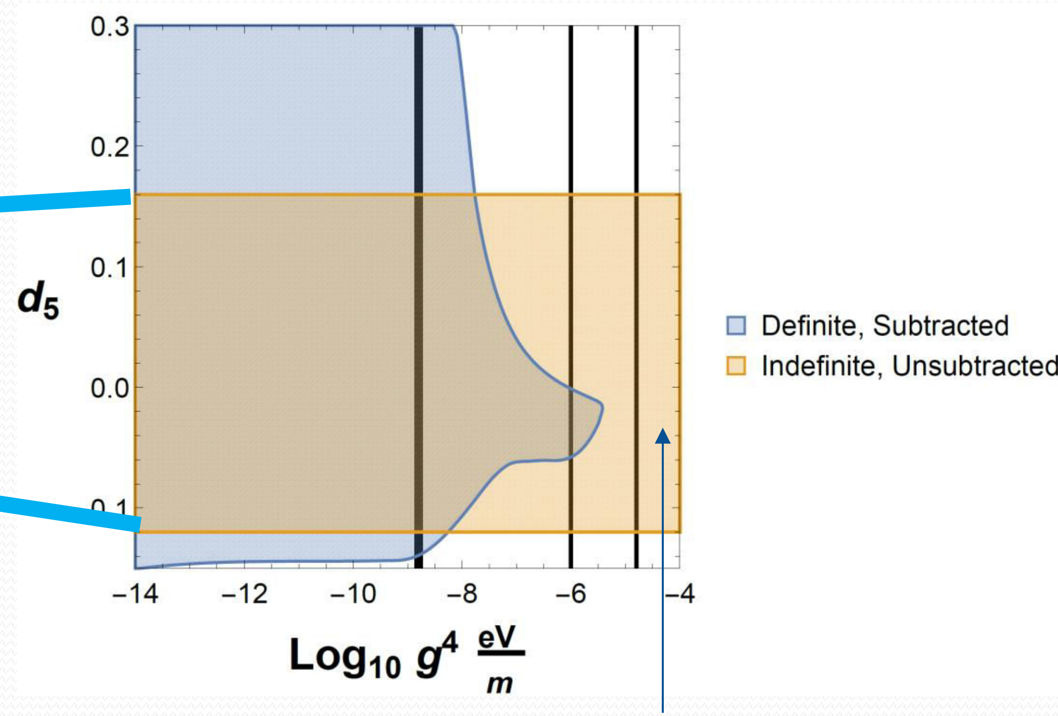
Effectively measures the scale of the cutoff

CdR, Melville, Tolley, 1710.09611

Improved positivity bounds

$$B^{(2,0)}(0) > \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu \frac{\mathrm{Im}\,A(\mu,t)}{(\mu-2m^2)^3} = \frac{4}{\pi}\int_{4m^2}^{\epsilon^2\Lambda^2} \mathrm{d}\mu\,\sqrt{1-\frac{4m^2}{\mu^2}}\,\frac{\mu\sigma_{\text{total}}(\mu)}{(\mu-2m^2)^3}\,.$$





Bellazzini, Riva, Serra, Sgarlata 1710.0253
Assuming a large enough g, ***the improved positivity*** bounds can rule out the allowed parameter space

CdR, Melville, Tolley, 1710.09611: the **improved positivity bounds** should be seen as a **constrain on the value of the cutoff !**