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# Global F-theory Constraints on Gauge Symmetry and Matter Representations

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# Outline/Topics

Focus on F-theory implications of Mordell-Weil Group of Ellipticallly Fibered Calabi-Yau manifolds

i. Free part, associated with U(1) gauge symmetries:
   new insights for global symmetry constrains
   in the presence of non-Abelian gauge symmetries;
   implications for F-theory `swampland'

ii. Torsion part, associated with modding-out non-Abelian factors by a discrete symmetry:
   novel non-Abelian enhancements & matter;
   open issues

## Mordell-Weil and global constrains on gauge symmetry

M.C. and Ling Lin,
"The Global Gauge Group Structure of F-theory Compactification with U(1)s,"
arXiv:1706.08521 [hep-th], JHEP

## Mordell-Weil torsion and novel gauge symmetry enhancements

Florent Baume, M.C., Craig Lawrie, Ling Lin,
"When Rational Sections Become Cyclic: Gauge Enhancement in F-theory via Mordell-Weil Torsion,"
arXiv:1709.07453 [hep-th], JHEP

# I. U(1)-symmetries in F-theory

# Abelian Gauge Symmetries

Different: (1,1) forms $\omega_m$, supporting U(1) gauge bosons, isolated & associated with $I_1$-fibers, only

[Morrison,Vafa'96]

$\boxed{\text{(1,1) - form } \omega_m \Leftrightarrow \text{rational section of elliptic fibration}}$

# Abelian Gauge Symmetry & Mordell-Weil Group

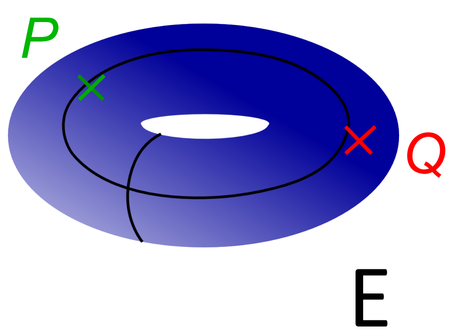
rational sections of elliptic fibr. $\Leftrightarrow$ rational points of elliptic curve

Rational point Q on elliptic curve E with zero point P

- is solution $(x_Q, y_Q, z_Q)$ in a field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

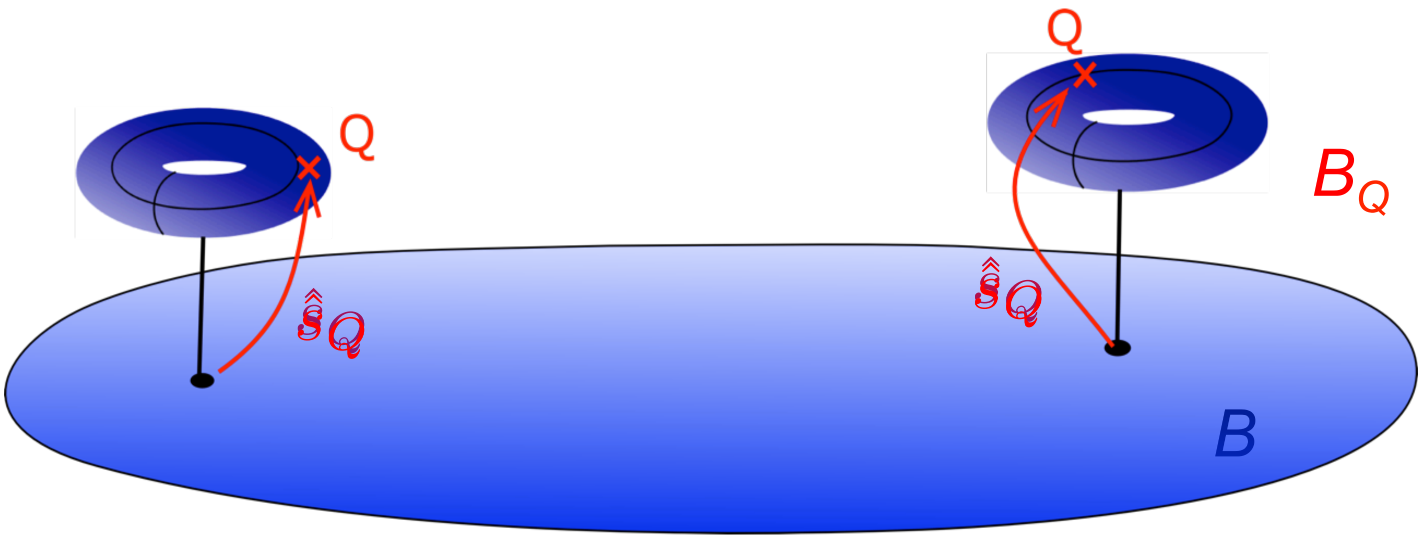
- Rational points form group (addition) on E




$\Rightarrow$ Mordell-Weil group of rational points

# U(1)'s-Abelian Symmetry &Mordell-Weil Group

Point **Q** induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration




➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor $B_Q$ in *X*

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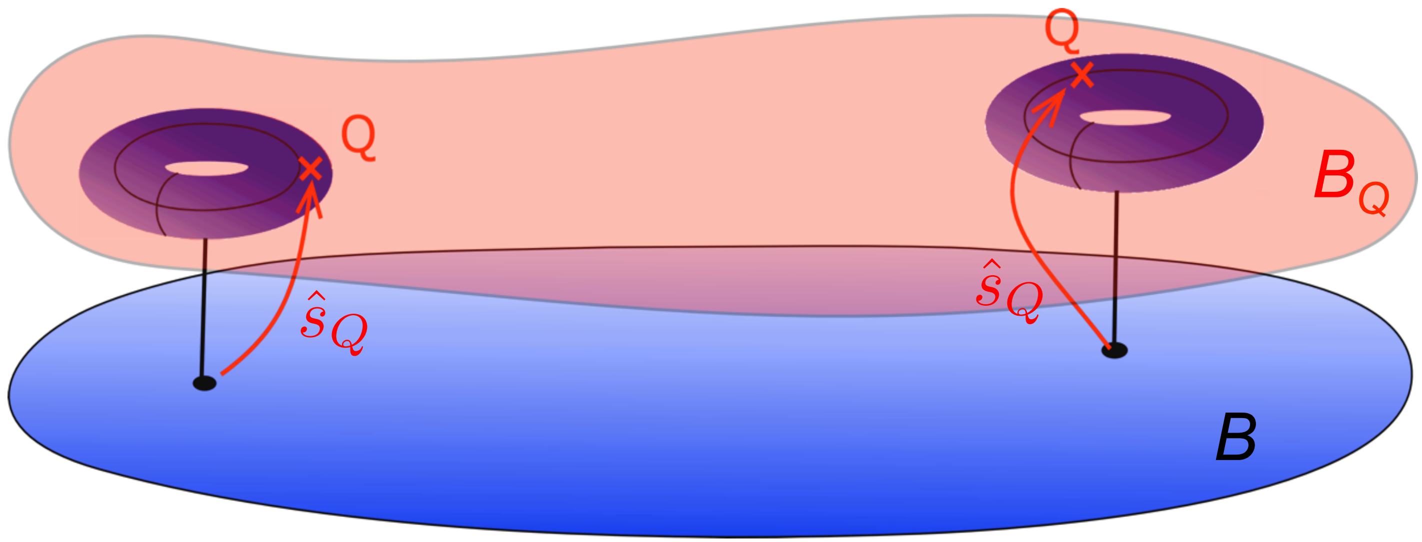



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor $B_Q$ in *X*

➡ (1,1)-form $\omega_m$ constructed from divisor $B_Q$ (Shioda map)

indeed (1,1) - form $\omega_m$ ⟷ rational section

# III. Implications of Mordell-Weil

# Shioda map & Non-Abelian Gauge symmetry

[M.C. and Ling Lin 1706.08521]

## Shioda map of section $\hat{s}_Q$ more involved than $\mathcal{B}_Q$:

a map onto divisor complementary to $\mathcal{B}_\mathcal{P}$ divisor of zero section $\hat{s}_P$

& $\mathcal{E}_i$ – resolution (Cartan) divisors of non-Abelian gauge symmetry

$$\sigma\left(\hat{s}_Q\right) = B_Q - B_P - \sum_i l_i E_i + \cdots$$

Ensures proper F-theory interpretation of U(1)
(via M-theory/F-theory duality)

$l_i = C^{-1}_{ij}(B_Q - B_P) \,.\, \mathbb{P}^1_j$ - fractional #

always an integer $\kappa$ s.t. $\forall i : \kappa\, l_i \in \mathbb{Z}$

Cartan matrix

Fiber of divisor $E_j$

# Consequences:

- U(1) matter charges q(w) = $\sigma\left(\hat{s}_Q\right).\mathbb{P}^1$

  intersection of Shioda divisor $\sigma\left(\hat{s}_Q\right)$ with matter curve $\mathbb{P}^1$

All matter charges are integral multiples of 1/$\kappa$, w/ $q_{\mathfrak{u}(1)} = \frac{n}{\kappa}\,,\, n \in \mathbb{Z}$

$\left(B_Q - B_P\right).\mathbb{P}^1 \in \mathbb{Z}$

- In the presence of non-Abelian gauge symmetry $\mathfrak{g}$

  Two weights $\mathbf{w}, \mathbf{v}$ in the same $\mathfrak{g}$-rep $\mathcal{R}_{\mathfrak{g}}$: $l_i \mathbf{w}_i = l_i \mathbf{v}_i \mod \mathbb{Z} := L(\mathcal{R}_{\mathfrak{g}}^{(i)})$

  $\mathcal{R}^{(i)} = (q^{(i)}, \mathcal{R}_{\mathfrak{g}}^{(i)})$ we have $q^{(i)} = L(\mathcal{R}_{\mathfrak{g}}^{(i)}) \mod \mathbb{Z}$

For $\mathfrak{g}$ = SU(5) [Braun, Grimm, Keitel '13; Lawrie, Schäfer-Nameki, Wong '15]
some aspects via KK-reduction [Grimm, Kapfer, Klevers '15]

# Construct non-trivial central element of U(1) × $\mathcal{G}$:

Employing (a) $q_{\mathfrak{u}(1)} = \frac{n}{\kappa}\,, n \in \mathbb{Z}$ & (b) $l_i \mathbf{w}_i = l_i \mathbf{v}_i \mod \mathbb{Z}$ **:=** $L(\mathcal{R}_{\mathfrak{g}}^{(i)})$

$C(\mathbf{w}) := [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i\, l_i\, \mathbf{w}_i} \times \mathbb{1})]\,\mathbf{w} \overset{(b)}{=} [e^{2\pi i q(\mathbf{w})} \otimes (e^{-2\pi i\, L(\mathcal{R}_{\mathfrak{g}})} \times \mathbb{1})]\,\mathbf{w}$
defines element in centre of $U(1) \times G$; (a) $\Rightarrow C^{\kappa} = 1$.

& $C(\mathbf{w}) = \exp(2\pi i\, \xi(\mathbf{w}))\,\mathbf{w} = \mathbf{w}$.

$\xi(w) = (B_Q - B_P)\,.\,\mathbb{P}^1 \in \mathbb{Z}$

⬇

$$G_{\text{global}} = \frac{U(1) \times G}{\langle C \rangle} \cong \frac{U(1) \times G}{\mathbb{Z}_{\kappa}}$$

# Global Constraint on Gauge Symmetry:

$$G_{\text{global}} = \frac{U(1)\times G}{\langle C\rangle} \cong \frac{U(1)\times G}{\mathbb{Z}_\kappa}$$

Exemplify for SU(5) GUT's and Standard Model constructions

Including for globally consistent three family SM

[M.C., Klevers, Peña, Oehlmann, Reuter 1503.02068]

Toric construction with gauge algebra $\mathfrak{su}(3)\oplus\mathfrak{su}(2)\oplus\mathfrak{u}(1)$

$$\varphi(\sigma) = S - S_0 + \tfrac{1}{2}\,E_1^{\mathfrak{su}(2)} + \tfrac{1}{3}(2\,E_1^{\mathfrak{su}(3)} + E_2^{\mathfrak{su}(3)}) \Rightarrow C^6 = 1,$$

so $G_{\text{global}} = [SU(3)\times SU(2)\times U(1)]/\langle C\rangle \cong [SU(3)\times SU(2)\times U(1)]/\mathbb{Z}_6.$

Indeed, geometrically realized (chiral) matter representations:

$$(\mathbf{3},\mathbf{2})_{1/6}\,,\quad (\mathbf{1},\mathbf{2})_{-1/2}\,,\quad (\mathbf{3},\mathbf{1})_{2/3}\,,\quad (\mathbf{3},\mathbf{1})_{-1/3}\,,\quad (\mathbf{1},\mathbf{1})_{1}$$

# Implication for F-theory `Swampland' Criterion

With the choice of Shioda map scaling →
∃ singlet field under $\mathcal{G}$, with U(1) charge $Q_{min}=1$
`Measure stick'

A necessary condition for a field theory to be in F-theory requires U(1) Charge Constraint on non-Abelian Mater:

(1) If $\mathcal{R}^{(1)} = (q^{(1)}, \mathcal{R}_{\mathfrak{g}})$ and $\mathcal{R}^{(2)} = (q^{(2)}, \mathcal{R}_{\mathfrak{g}})$, then $q^{(1)} - q^{(2)} \in \mathbb{Z}$.
(2) If $\bigotimes_{i=1}^{n} \mathcal{R}_{\mathfrak{g}}^{(i)} = \mathbf{1}_{\mathfrak{g}} \oplus \ldots$, then $\sum_{i=1}^{n} q^{(i)} \in \mathbb{Z}$.

Caveat: Non-Higgsable U(1)'s? [Morrison,Taylor'16], [Wang'17]
In the presence of non-Abelian matter, expect to have singlet representation(s) → probably O.K.

Further comments:
studied unHiggsing; some models with non-minimal codim. 2 loci
→ strongly coupled CFT's [further studies]

# Inclusion of Fluxes and Massive U(1)'s

- Multiple U(1)'s: singlet fields w/ co-prime U(1) charges (measured with Shioda map $\omega_k = \varphi(\sigma_k)$, k=1,…m) MW spans full integer lattice
- Each U(1) has its associated charge constraint for non-Abelian matter

- Adding fluxes $G_4$ can break certain combinations of U(1)'s via Stückelberg mechanism with mass matrix:

$$M_{kl} = \sum_\alpha \xi_{k,\alpha}\,\xi_{l,\alpha} \quad \text{with} \quad \xi_{k,\alpha} = \int_Y G_4 \wedge \omega_k \wedge \pi^* J_\alpha$$ -INTEGERS

$J_\alpha$ - Kähler generator of the base

$\lambda_k^s$ -INTEGERS

- Massless linear combination: $\tilde{\omega}_s = \sum_k \lambda_k^s\,\omega_k$

w/ $\sum_i \xi_{k,\alpha}\,\lambda_k^s = 0 \quad \forall\alpha$

Sublattice of MW- group

Geometric properties leading to charge constraints still hold!

# ii. Mordell-Weil torsion & Gauge enhancement

[Baume, M.C., Lawrie, Lin 1709.07453]

Mordell-Weil: $\mathrm{MW}(Y) = \mathbb{Z}^m \oplus \bigoplus_k \mathbb{Z}_{n_k}$

$\uparrow$ rational sections $\quad$ $\uparrow$ torsional sections

[Aspinwall, Morrison '98], [Mayrhofer, Morrison, Till, Weigand '14]

Shioda-map for torsion: $\sigma(\hat{s}_Q) = B_Q - B_P - \sum_i l_i E_i + \cdots$ = 0 - no U(1)

$$l_i \in \frac{1}{n_k}\mathbb{Z}\,.$$

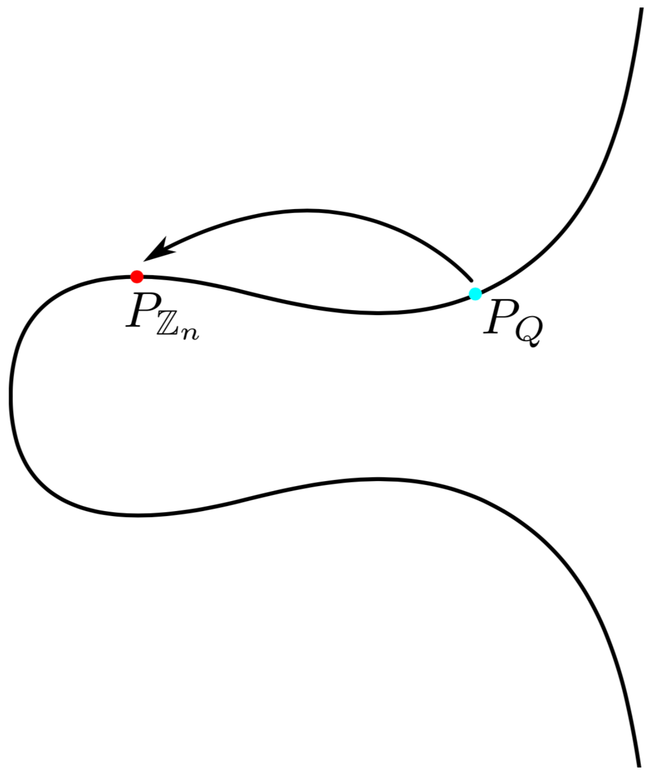
As with U(1): integer condition on Cartan charges: $\sum_i l_i \mathbf{w}_i \in \mathbb{Z}\,.$

$\Downarrow$

Results in the global gauge group: $G \supset \dfrac{G_k}{\mathbb{Z}_{n_k}}$

# Gauge enhancement via Mordell-Weil torsion

Gauge enhancement when a section becomes torsional:




Tuning a free section to a torsional one of order n → expect to enhance U(1) to $\frac{G}{\mathbb{Z}_n} \times \ldots$

# Gauge enhancement via Mordell-Weil torsion

Expect U(1) to unHiggs to non-Abelian $\mathcal{G}$ with $\pi_1(\mathcal{G}) = Z_n$

- Similar to unHiggsing through colliding free sections:

[Morrsion, Park '12]

U(1)xU(1) w/ **(2,2)** charge matter →SU(3) w/ symm. **6** rep.

[M.C., Klevers, Piragua, Taylor '15]

U(1)-model w/ charge **3** matter→SU(2) w/ three index symm. **4** rep.

[Klevers, Taylor '16]

- Torsional unHiggsing (to $Z_2$ torsion-prototype):

U(1) w/ charge **1** matter→SU(2)/$Z_2$ w/adj. **3** rep.(`Cartan ch.'**2**

[Mayrhofer, Morrison, Till, Weigand '14]

U(1) w/ charge **2** matter → Enhanced gauge symmmetry?
Matter representation?

Spoiler alert: NOT **5**-rep. ('Cartan charge' **4**)

→ possible ties to (other) `swampland' conjectures

[Klevers, Morrison, Raghuram, Taylor, '17],
c.f., Taylor's,, Valandro's talks

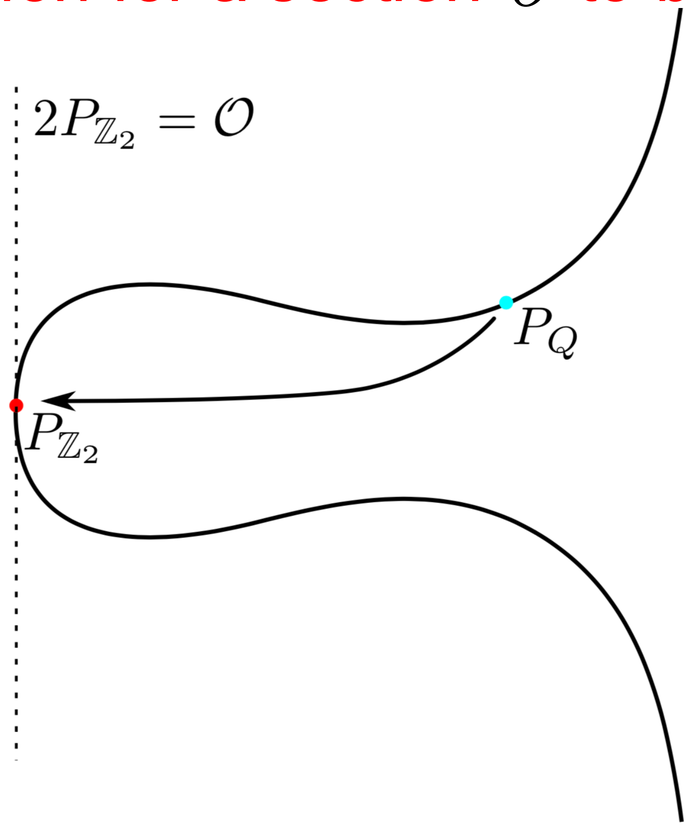
# Gauge enhancement via Mordell-Weil torsion

Explicit model: rank- one MW-group $\mathrm{Bl}_1\mathbb{P}_{112}$ [Morrison, Park `12]

→ U(1) with matter charges **1** & **2**

Implement condition for a section $\sigma$ to become 2-torsional: $y_\sigma = 0$

(Weierstrass)




→ Elliptic fibration (by construction) with MW-group torsion $\mathbb{Z}_2$

# Gauge enhancement via Mordell-Weil torsion

Resulting in Gauge group: $\frac{SU(2)\times SU(4)}{\mathbb{Z}_2}\times SU(2)$

Novel features: explicit global model with

- gauge factor [SU(2)] not affected by torsional section
- resolution of singular co-dim 2 fiber:

  new matter rep.: $(\mathbf{3},\mathbf{1},\mathbf{2})$ [no **(5,1,1)**]

# Gauge enhancement via Mordell-Weil torsion

Another example (Higgsed version of the previous one):

Construct a fibration giving rise to gauge symmetry

$$\frac{SU(2)}{\mathbb{Z}_2} \times \mathbb{Z}_2$$

torsion bisection

Bisection due to discrete symmetry (related to Tate-Shafarevich)

another topic - no time

- Construction involves genus-one fibration Y' with bisection

- There is also Jacobian map of Y'- elliptic fibration Y:

  has resolvable $I_2$-singularity (in codim 1) & $Z_2$ - MW torsion

→ Signifies $SU(2)/Z_2 \sim SO(3)$ gauge symmetry

# Gauge enhancement via Mordell-Weil torsion

Another example:

Puzzles:

- For genus-one fibration Y' with bisection
  monodromy exchanges $I_2$ components in codim 1
  → no exceptional divisor [in M-theory missing Cartan U(1)]

- Field theory (Higgsing chain) analysis: expect
  discretely charged adj. **3** of SO(3), but no apparent
  localized (codim 2) states in Y' or Jacobian Y

↓

Need to sharpen/augment the definition of F-theory on genus-one fibrations and their Jacobians.

[work in progess w/Lin, Lawrie & Weigand]

# Summary

Novel F-theory implications of Mordell-Weil Group

Encountered subtle issues:

i. Free part: presence of U(1) → global constraints on gauge symmetry and on U(1) charges of non-Abelian matter (`swampland' conjecture)

ii. Torsion part: novel gauge symmetry enhancements and representations

Even more obscure: better understanding of F-theory on torus-fibrations without sections

→ Further Studies

*Thank you!*