F-theory and AdS_3/CFT_2

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1612.05640, 1612.06393 CL, S. Schäfer-Nameki, T. Weigand
1705.04679 C. Couzens, CL, D. Martelli, S. Schäfer-Nameki, J. Wong
1712.07631 C. Couzens, D. Martelli, S. Schäfer-Nameki
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AdS Solutions of F-theory

F-theory Solutions: construct Type IIB solutions where axio-dilaton, τ^{IIB}, varies over (a part of) spacetime, including monodromies in the $SL(2,\mathbb{Z})$ duality group [Couzens, CL, Martelli, Schäfer-Nameki, Wong]

$SL(2,\mathbb{Z})$ monodromy of $\tau^{IIB} \Rightarrow$ 7-branes $\Rightarrow$ F-theory!

Consider general solutions with an AdS$_p$ factor preserving some supersymmetry – top-down approach to AdS/CFT

→ no previously known solutions with full $SL(2,\mathbb{Z})$ monodromy

→ for poles in τ^{IIB} see [Couzens], [D'Hoker, Gutperle, Uhlemann]

Dual CFTs can be difficult to understand

(p,q) 7-branes $\Rightarrow$ genuinely non-perturbative effects

In this talk: mainly AdS$_3$ $\Rightarrow$ dual to 2d SCFTs

Strings of F-theory

2d SCFTs arise in "string sector" of F-theory

In any F-theory compactification on elliptic n-fold $\pi : Y \to B$ there can exist strings in spectrum from

$$\text{D3-branes} \quad \text{on} \quad C \subset B$$

Worldvolume theory of string is SCFT with supersymmetry

n	2	3	4	5
$\mathcal{N}$	$(0,8)$	$(0,4)$	$(0,2)$	$(0,2)$

Can pinpoint loci in the moduli space with interesting physics

Strings of F-theory – $(0,4)$

Strings of 6d $\mathcal{N}=(1,0)$ SCFTs [del Zotto, Lockhart]

- tensionless strings are hallmark of superconformal symmetry in 6d
- instanton part of 6d Nekrasov PF $\leftrightarrow$ elliptic genera of strings

Strings of 6d $\mathcal{N}=(1,0)$ Supergravities [Haghighat, Murthy, Vafa, Vandoren]

- 5d BPS black holes arise from 6d BPS strings on S^1
- microstate counting of strings in 6d $\rightarrow$ macroscopic entropy

Strings of F-theory – $(0,2)$

In 4d $\mathcal{N} = 1$, strings are

$\rightarrow$ dual to instantons

$\rightarrow$ not BPS $\Rightarrow$ tension not protected $\Rightarrow$ quantum corrections [Mayr]

In 2d $\mathcal{N} = (0,2)$, strings are

$\rightarrow$ spacefilling

$\rightarrow$ required to wrap specific curve class by tadpole cancellation

[Schäfer-Nameki, Weigand], [Apruzzi, Hassler, Heckman, Melnikov]

[CL, Schäfer-Nameki, Weigand]

Roadmap

1. Single D3-brane on C with varying τ [CL, Schäfer-Nameki, Weigand]
   $\rightarrow$ study explicitly via topological duality twist
2. Multiple D3-branes on C with varying τ
   [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
   $\rightarrow$ no explicit construction
   $\rightarrow$ construct AdS_3 supergravity duals with $(0, 4)$
   $\rightarrow$ determine central charges from holography
3. Solutions with different SUSY [Couzens, Martelli, Schäfer-Nameki]
   $\rightarrow$ $(0, 2)$
   $\rightarrow$ $(2, 2)$
4. An $\mathcal{N} = 4$ SYM Anomaly Polynomial [CL, Schäfer-Nameki]
   $\rightarrow$ Moduli space of couplings terms in anomaly polynomial
   $\rightarrow$ Integration and comparison to AdS_3 central charges

Topological Duality Twist

Abelian $\mathcal{N} = 4$ SYM

$\Rightarrow$ "bonus" $U(1)_D$ symmetry [Intriligator], [Kapustin, Witten]

$$\gamma : \tau \to \frac{a\tau + b}{c\tau + d}, \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \quad \to \quad e^{i\alpha(\gamma)} \equiv \frac{c\tau + d}{|c\tau + d|} \in U(1)_D$$

Objects have charge q_D if transforms by $e^{iq_D\alpha(\gamma)}$ under γ

Object	F^+	F^-	Ψ, Q	$\widetilde{\Psi}, \widetilde{Q}$	ϕ_i
q_D	1	-1	$1/2$	$-1/2$	0

We have a $U(1)_D$ connection

$$\mathcal{A}_D = \frac{\mathrm{d}\tau_1}{2\tau_2}$$

Topological duality twist (TDT): To preserve SUSY compensate non-trivial transformation of supercharges under holonomy of C and $U(1)_D$ by R-symmetry transformation. [Martucci]

Anomaly Polynomials and Central Charges

Construct topological duality twisted dimensional reduction to 2d for strings in base of elliptic Calabi–Yau n-fold [CL, Schäfer-Nameki, Weigand]

Can compute the central charges in each case; for F-theory to 6d:

$$
\begin{aligned}
c_R &= 3C \cdot C + 3c_1(B) \cdot C \\
c_L &= 3C \cdot C + 9c_1(B) \cdot C
\end{aligned}
$$

[Haghighat, Murthy, Vafa, Vandoren], [CL, Schäfer-Nameki, Weigand]

Anomaly polynomial for $(0,4)$ theory

$$
I_4 = -\frac{1}{24} p_1(T) \left[-6c_1(B) \cdot C\right] - c_2(R) \left[\frac{1}{2} C \cdot C + \frac{1}{2} c_1(B) \cdot C\right] + \cdots
$$

Matches AP for strings in 6d [Berman, Harvey], [Shimizu, Tachikawa]

Stacks of D3-branes

How to generalize to multiple D3-branes on C?

Interaction term in fermionic variations does not respect $U(1)_D$!
$\Rightarrow$ Topological duality twist does **not** (obviously) generalize

Instead can consider M5-branes [Assel, Schäfer-Nameki]

$$\text{D3-brane on } M_4 \text{ with TDT}$$
$$\updownarrow$$
$$\text{M5-brane on Kähler elliptic threefold } \widehat{M_4} \text{ with } \underline{\text{geometric twist}}$$

Single M5 with geometric twist on $\widehat{M_4}$ and "non-abelianize" [Cordova, Jafferis] to determine non-abelian $N = 4$ SYM with varying τ

[Assel, Schäfer-Nameki]

$\rightarrow$ Subtlety: 4d theory has duality defects (3–7 strings)

Stacks of D3-branes

How to generalize to multiple D3-branes on C?

Interaction term in fermionic variations does not respect $U(1)_D$!

$\Rightarrow$ Topological duality twist does **not** (obviously) generalize

$\rightarrow$ instead can consider M5-branes [Assel, Schäfer-Nameki]

Can consider AdS/CFT

$\Rightarrow$ large N

$\Rightarrow$ large numbers of D3-branes

General Solutions for IIB with AdS_3 Factor and $(0,2)$ SUSY

IIB content:

$$F_5 \quad \longleftrightarrow \text{D3-branes}$$

$$G_3 \begin{cases} F_3 & \longleftrightarrow \text{D1/D5-branes} \\ H_3 & \longleftrightarrow \text{F-strings/NS5-branes} \end{cases}$$

$$P_1 = \frac{i}{2\tau_2} d\tau \quad \longleftrightarrow \text{7-branes}$$

Set $G_3 = 0$

General starting point:

$$ds^2 = e^{2A} ds^2(\mathrm{AdS}_3) + ds^2(M_7)$$

$$F_5 = (1 + *)\mathrm{vol}(\mathrm{AdS}_3) \wedge F^{(2)}$$

To preserve $(0,2)$ SUSY solve Killing spinor equation

$$\nabla_M \epsilon + \frac{i}{192} \Gamma^{P_1 P_2 P_3 P_4} F_{M P_1 P_2 P_3 P_4} \epsilon = 0$$

General Solutions for IIB with AdS_3 Factor and $(0,2)$ SUSY

General solution [Couzens, CL, Martelli, Schäfer-Nameki, Wong]
[Couzens, Martelli, Schäfer-Nameki]

$$
\begin{array}{ccc}
S^1 & \hookrightarrow & M_7 \\
 & & \downarrow \\
 & & M_6
\end{array}
$$

S^1 fibration provides $U(1)_r$ R-symmetry of $(0,2)$

τ variation combines into an auxilliary Kähler elliptic fibration M_8 over M_6 with non-trivial constraint

$$
\Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8ij} R_8^{ij} = 0
$$

First consider more SUSY

- $\rightarrow (0,4)$ SUSY $\Rightarrow$ dual to strings in 6d
- $\rightarrow (2,2)$ SUSY $\Rightarrow$ see later

Preserving $(0,4)$ SUSY

Requiring $(0,4)$ is highly constrained, $A = \text{const}$ and

$$\begin{array}{ccccccc} S^1 & \hookrightarrow & S^3 & & Y_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \downarrow & & \\ M_6 = & & S^2 & \times & B_2 & & \end{array}$$

Killing spinors transform in $(\mathbf{2},\mathbf{1})$ of S^3 isometry

$$SO(4) = SU(2)_r \times SU(2)_L$$

$$SU(2)_r \to \text{superconformal R-symmetry}$$
$$SU(2)_L \to \text{additional flavour symmetry}$$

Preserving $(0,4)$ SUSY

Requiring $(0,4)$ is highly constrained $A = \text{const}$ and

$$\begin{array}{ccccccc} S^1 & \hookrightarrow & S^3/\Gamma & & Y_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \downarrow & & \\ M_6 = & & S^2 & \times & B_2 & & \end{array}$$

Killing spinors transform in $(\mathbf{2},\mathbf{1})$ of S^3 isometry

$$SO(4) = SU(2)_r \times SU(2)_L$$

$SU(2)_r \to$ superconformal R-symmetry

$SU(2)_L \to$ additional flavour symmetry when $\Gamma = 1$

We preserve the same SUSY for $\Gamma \subset SU(2)_L$ finite subgroup
$\to$ consider $\Gamma = \mathbb{Z}_M$ in this talk

$(0, 4)$ Solution

General F-theory solution of Type IIB SUGRA dual to 2d $(0, 4)$ is

$$
\begin{array}{ccc}
 & \mathbb{E}_\tau \hookrightarrow & Y_3 \\
 & & \downarrow \\
\mathrm{AdS}_3 \times S^3/\Gamma \times & & B_2
\end{array}
$$

with F_5 flux

$$F_5 = (1 + *)\mathrm{vol}(\mathrm{AdS}_3) \wedge J_B$$

J_B is Kähler form on B Poincaré dual to a curve C
 $\Rightarrow C$, wrapped by D3-brane, ample in B

Generalisation of previously known solutions:

$$\mathrm{AdS}_3 \times S^3 \times T^4 \quad \text{and } \mathrm{AdS}_3 \times S^3 \times K3$$

Brown–Henneaux

$$c_{\text{SUGRA}}^{\text{IIB}} = \frac{3R_{\text{AdS}}}{2G_N^{(3)}} = N^2 \frac{3\text{vol}(S^3/\mathbb{Z}_M)\text{vol}(B)32\pi^2}{\text{vol}(S^3/\mathbb{Z}_M)} = 6N^2 M \text{vol}(B)$$

Further

$$\text{vol}(B) = \frac{1}{2}\int_B J_B \wedge J_B = \frac{1}{2} C \cdot C$$

So

$$c_{\text{SUGRA}}^{\text{IIB}} = 3N^2 M C \cdot C$$

is the leading order contribution to the (left and right) central charge

Gravitational Chern–Simons couplings from 7-branes bulk

$$S_{CS}(\Gamma_{\mathrm{AdS}_3}) = \frac{c_L - c_R}{96\pi} \int_{\mathrm{AdS}_3} \omega_{CS}(\Gamma_{\mathrm{AdS}_3})$$

$\Rightarrow$

$$c_L - c_R = 6Nc_1(B) \cdot C$$

Gauging $SO(4)$ isometry of S^3

$\Rightarrow$

$$k_r^{(1)} = \frac{1}{2}Nc_1(B) \cdot C$$

Central Charges from Type IIB SUGRA

Leading and subleading central charges

$$
\begin{aligned}
c_R^{\mathrm{IIB}} &= 3N^2 C \cdot C + 3N c_1(B) \cdot C \\
c_L^{\mathrm{IIB}} &= 3N^2 C \cdot C + 9N c_1(B) \cdot C
\end{aligned}
$$

Matches with spectrum computation for $N = 1$:

$$
\begin{aligned}
c_R^{\mathrm{spectrum}} &= 3C \cdot C + 3c_1(B) \cdot C \\
c_L^{\mathrm{spectrum}} &= 3C \cdot C + 9c_1(B) \cdot C
\end{aligned}
$$

Only for $M = 1$

$\Rightarrow$ subleading contributions for $M > 1$ tricky

$\Rightarrow$ look at T-duality to M-theory

Recap – The Story So Far

1. Constructed general solution of Type IIB supergravity with
   $\rightarrow$ $(0,4)$ SUSY in dual SCFT
   $\rightarrow$ $G_3 = 0$ and arbitrary τ
2. Geometry:
$$\mathrm{AdS}_3 \times S^3/\Gamma \times B_2$$
3. Flux through (<u>ample</u>) curve in $B_2 \Rightarrow N$ D3-branes on C
4. Dual SCFT
   $\rightarrow$ worldvolume theory of string in 6d F-theory compactification

T-duality to M-theory

F-theory on Y_3 T-dual to M-theory on Y_3

General solution:

$$\mathrm{AdS}_3 \times S^2 \times Y_3$$

with flux

$$G_4 = \mathrm{dvol}(S^2) \wedge J_{Y_3}$$

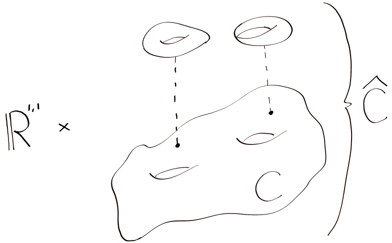
(See [Colgain, Wu, Yavartanoo])

J_{Y_3} is Kahler form on Y_3 Poincaré dual to divisor

$$MB + N\widehat{C}$$

T-duality to M-theory and MSW Strings

M5-branes on $\mathbb{R}^{1,1} \times P$
$P \in |MB + N\widehat{C}|$



$$N \text{ D3-branes on } C \longleftrightarrow N \text{ M5-branes on } \widehat{C}$$
$$M \text{ KK monopoles } \longleftrightarrow M \text{ M5-branes on } B$$

See also [Bena, Diaconescu, Florea]

$\widehat{C} \cdot \widehat{C} \cdot \widehat{C} = 0$
$\Rightarrow$ divisor $\widehat{C}$ not ample, not Poincaré dual to Kähler form
$\Rightarrow$ no AdS dual to string from M5-branes wrapping $\widehat{C}$

Central Charges from M-theory

KK monopoles now M5-branes on B

$\rightarrow$ Brown–Henneaux for holographic central charges for all M

$$c_R^{\text{M-th}} = 3N^2 M C \cdot C + 3N(2 - M^2) c_1(B) \cdot C$$
$$c_L^{\text{M-th}} = 3N^2 M C \cdot C + 3N(4 - M^2) c_1(B) \cdot C$$

Matches $c_{R,L}^{\text{IIB}}$ for $M = 1$

Includes both leading and subleading orders in N

$\rightarrow$ also subsubleading $\rightarrow$ center of mass contributions

$\rightarrow$ not discussed today (but agrees with microscopic constructions)

Summary of $(0, 4)$ Solution

Construct general AdS_3 solution of IIB SUGRA with dual $(0, 4)$ SCFT

Computed holographic central charges ($M = 1$)

$$c_R = 3N^2 C \cdot C + 3N c_1(B) \cdot C$$
$$c_L = 3N^2 C \cdot C + 9N c_1(B) \cdot C$$

Agrees with central charge computation from

1. 11d supergravity
2. Self-dual strings in 6d
3. M5-brane anomaly inflow
4. Spectrum (for $N = 1$)

AdS_3 Solutions Preserving SUSY $\neq (0,4)$

Recall that the general solution preserving $(0,2)$ was

$$AdS_3 \times (S^1 \to Y_4)$$

with $\pi : Y_4 \to B_3$ is an elliptically fibered fourfold satisfying

$$\Box_8 R_8 - \frac{1}{2} R_8^2 + R_{8ij} R_8^{ij} = 0$$

Recall: Y_4 is not necessarily Calabi–Yau

Formidable to solve in general

$\to$ look at special solutions, eg, where [Couzens, Martelli, Schäfer-Nameki]

$$B_3 = \Sigma_1 \times M_2$$

(0, 2) Universal Twist Solutions and (2, 2) Solutions

"Universal twist solutions" [Couzens, Martelli, Schäfer-Nameki]

$$
Y_4 = \begin{cases} \begin{array}{ccccc} & & \mathcal{T}_3 & \hookleftarrow & \mathbb{E}_\tau \\ & & \downarrow & & \\ \Sigma_1 & \times & M_2 & & \end{array} \quad \text{or} \quad \begin{array}{ccccc} \mathbb{E}_\tau & \hookrightarrow & S_2 & & \\ & & \downarrow & & \\ & & \Sigma_1 & \times & M_2 \end{array} \end{cases}
$$

Neither elliptic surface S_2 nor elliptic threefold $\mathcal{T}_3$ are Calabi–Yau.

$\rightarrow$ Dual SCFTs

$\rightarrow$ topological duality twisted compactifications of 4d $\mathcal{N} = 1$ SCFTs

If $(2, 2)$ SUSY is imposed and $S^1 \rightarrow Y_4$ is compact

$\Rightarrow Y_4$ is trivial elliptic fibration

$\Rightarrow$ axio-dilaton does not vary

Moduli Space of Couplings and the Anomaly Polynomial

AP can contain terms $\propto$ forms on moduli space of couplings
Example: 4d $\mathcal{N} = 2$ class $\mathcal{S}$ theories [Tachikawa, Yonekura], [Bah, Nardoni]

Theories of class $\mathcal{S}$ arise from M5-brane on C_g
$\rightarrow$ integrate M5 AP over C_g to get class $\mathcal{S}$ AP

$$I_6 = \int_C I_8 = -\left(\frac{h_G^\vee d_G}{6} c_2(R) + \frac{r_G}{12}\left(c_2(R) + \frac{1}{4} p_1(T_4)\right)\right) \int_{C_g} t^2 + \cdots$$

t is c_1 of relative tangent bundle of C over $\mathcal{M}_g$, and [Wolpert]

$$\int_C t^2 \propto \omega^{WP}$$

where ω^{WP} is Weil–Petersson metric on $\mathcal{M}_g$

$\mathcal{N} = 4$ SYM Anomaly Polynomial

Add coupling terms to $\mathcal{N} = 4$ SYM AP [CL, Schäfer-Nameki]

$$I_6 = \frac{1}{2}N^2 c_3(S_6^+) - \frac{1}{2}N c_2(S_6^+) c_1(\mathcal{L}_D) + \frac{1}{12}N c_1(\mathcal{L}_D)^3 - \frac{1}{12}N c_1(\mathcal{L}_D) p_1(T_4)$$

Topological twist along C:

$$I_4 = \int_C I_6 = c_2(R)\left[-\frac{1}{2}N^2 C\cdot C - \frac{1}{2}N c_1(B)\cdot C\right] + \cdots$$

Central charge:

$$c_R = 6k_R = 3N^2 C\cdot C + 3N c_1(B)\cdot C$$

<u>Matches AdS_3 supergravity dual</u>

Conclusions and Future Directions

- Started systematically exploring holographic constructions in F-theory – varying axio-dilaton.
- Constructed AdS_3 solutions preserving $(0,2)$, $(0,4)$, and $(2,2)$ SUSY in dual CFT_2
- For $(0,4)$ we obtained a microscopic understanding of the holographic constructions
  $\to$ what about $G_3 \neq 0 \to$ all AdS_3 solutions dual to $(0,4)$
- AdS duals to strings of minimial 6d SCFTs [del Zotto, Lockhart]
  $\to$ curve wrapped by D3-branes not ample
- Anomaly polynomials with $U(1)_D$ related terms
  $\Rightarrow$ information about field theories with varying coupling
  $\to$ extension to 4d $\mathcal{N} = 1$ SCFTs with F-theory duals
  [Couzens, Martelli, Schäfer-Nameki], [CL, Schäfer-Nameki]