All 6D F-theory SCFTs from Group Theory

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IAS

Based On

- 1502.05405/hep-th
  - with Jonathan Heckman, David Morrison, and Cumrun Vafa
- 1506.06753/hep-th
  - with Jonathan Heckman
- 1601.04078/hep-th
  - with Jonathan Heckman, Alessandro Tomasiello
- 1612.06399/hep-th
  - with Noppadol Mekareeya, Alessandro Tomasiello
- work in progress
  - with Jonathan Heckman, Alessandro Tomasiello
- work in progress
  - with Darrin Frey

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Outline

I. Atomic Classification of 6D SCFTs

II. All 6D SCFTs from Group Theory

III. Summary and Future Research

Atomic Classification of 6D SCFTs

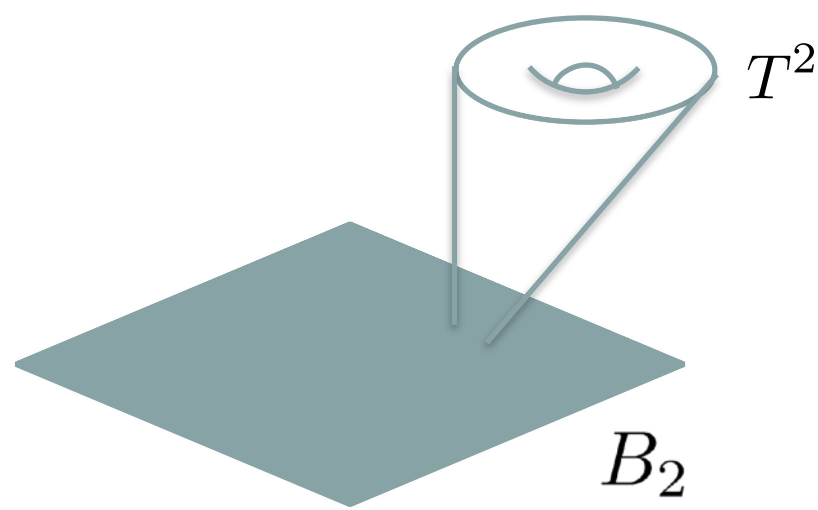
6D Theories and F-theory

Vafa '96, Vafa Morrison, I/II '96

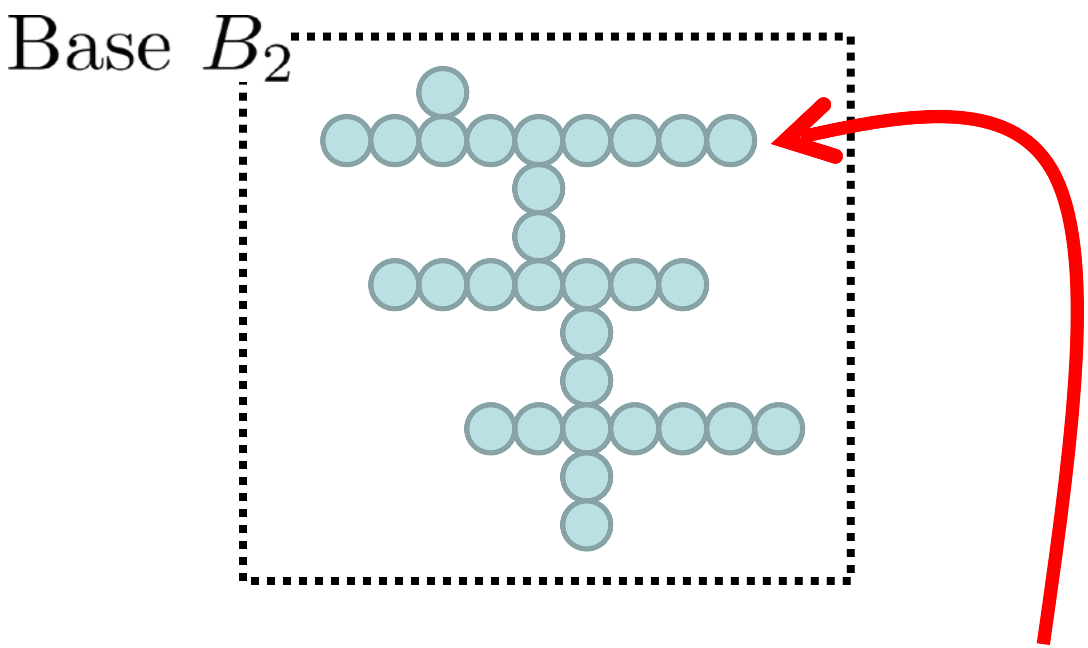
IIB: $\mathbb{R}^{5,1} \times B_2$ with pos. dep. coupling $\tau(z_B)$

F-theory on $\mathbb{R}^{5,1} \times CY_3$

$CY_3 \;=$



Geometric Picture



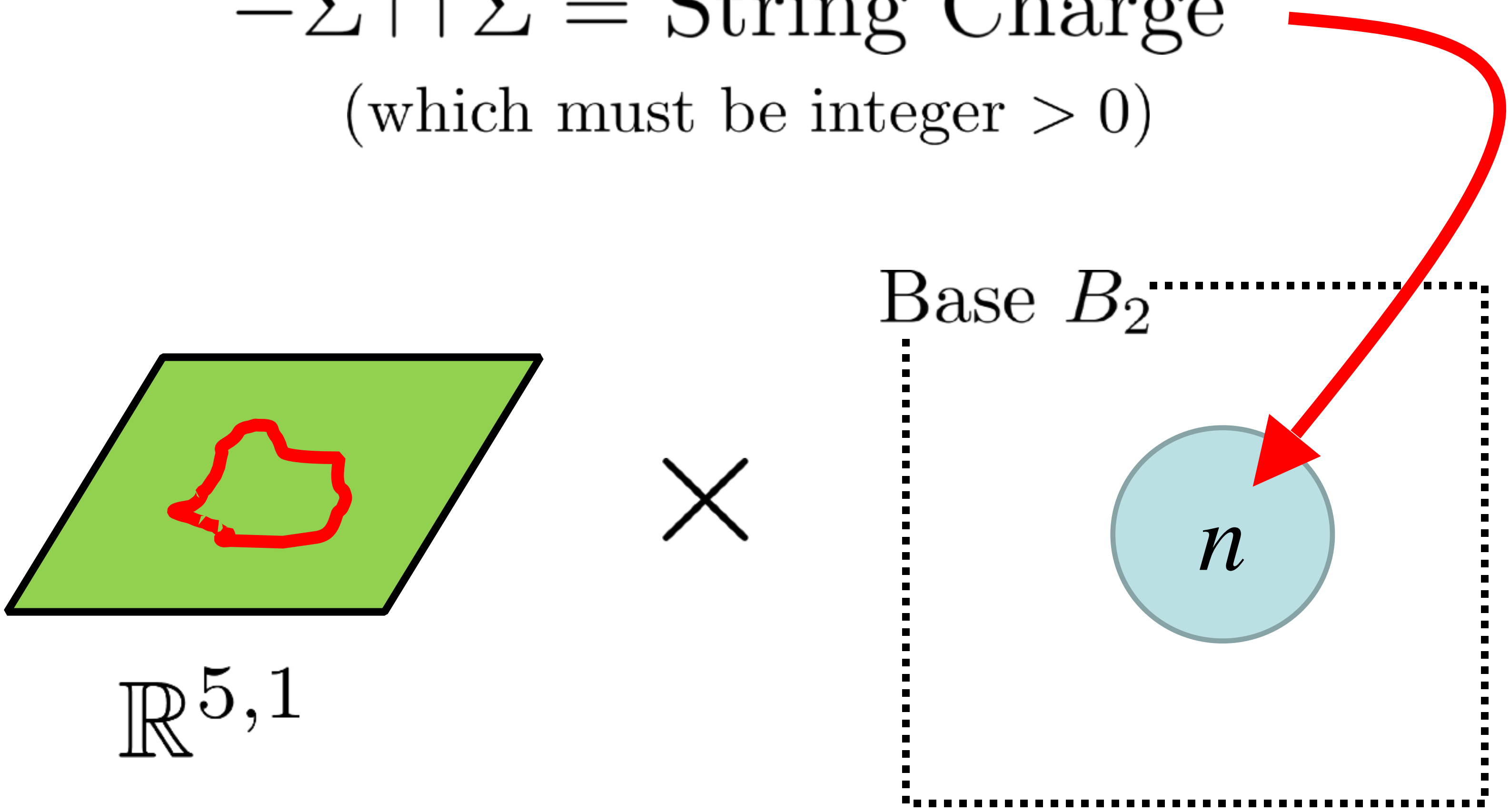
Curves in base $\Rightarrow$ strings (D3 / $\mathbb{P}^1$)

Singularities in fiber $\Rightarrow$ particles (7-brane on $\mathbb{P}^1$)

Strings from D3 on a $\mathbb{P}^1$

$$-\Sigma \cap \Sigma = \text{String Charge}$$

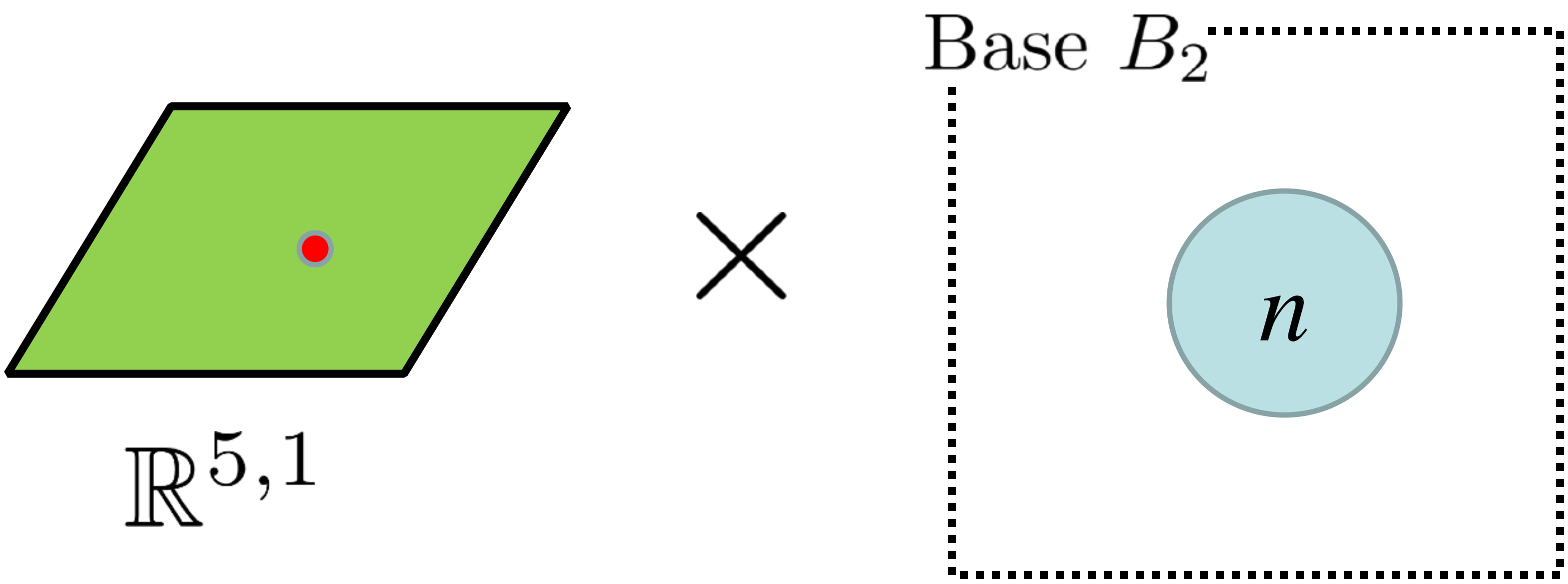
(which must be integer > 0)



Particles from D7's on a $\mathbb{P}^1$

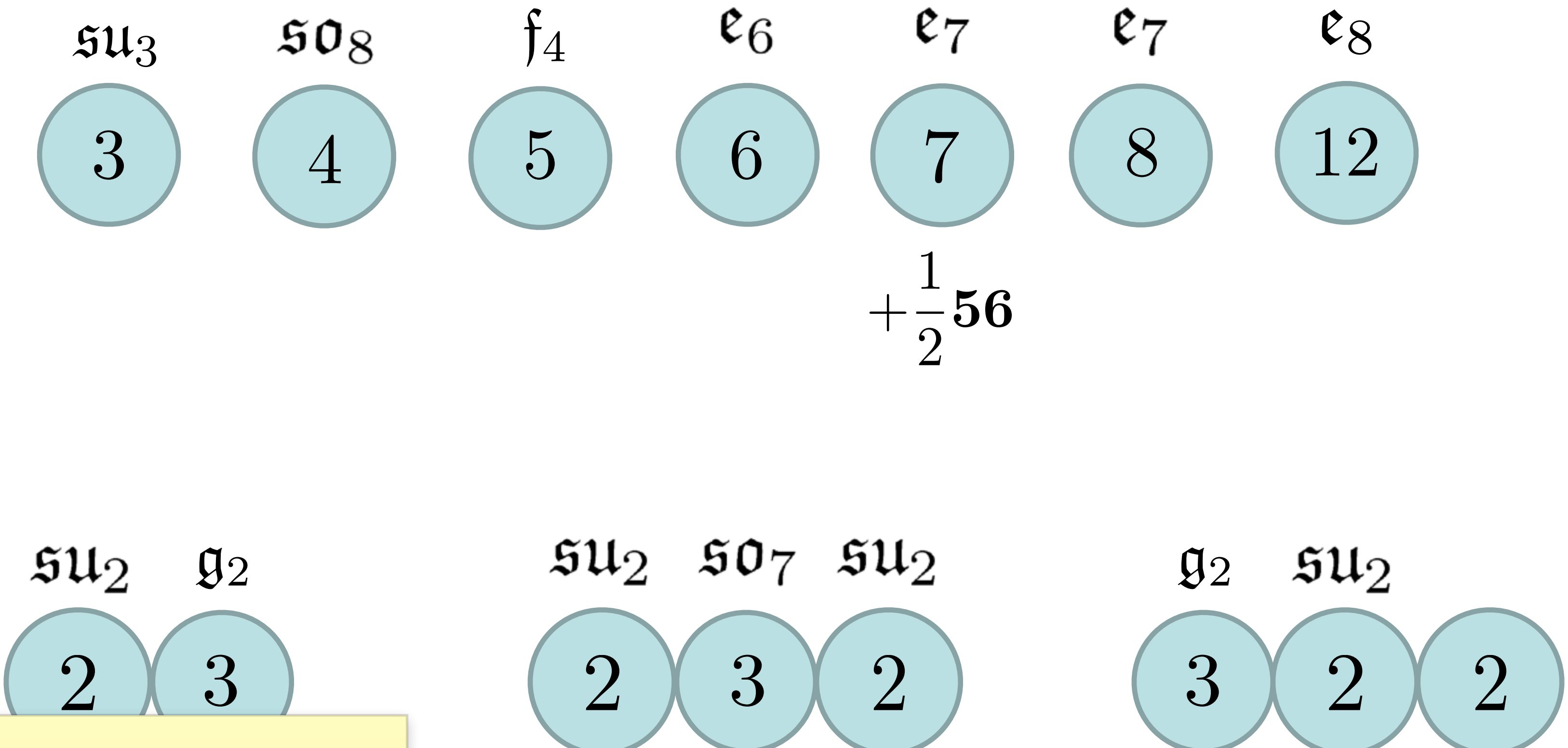
$3 \leq n \leq 12 \Rightarrow$ always have gauge fields

(elliptic fiber is singular: Morrison Taylor '12)



Non-Higgsable Clusters (NHCs)

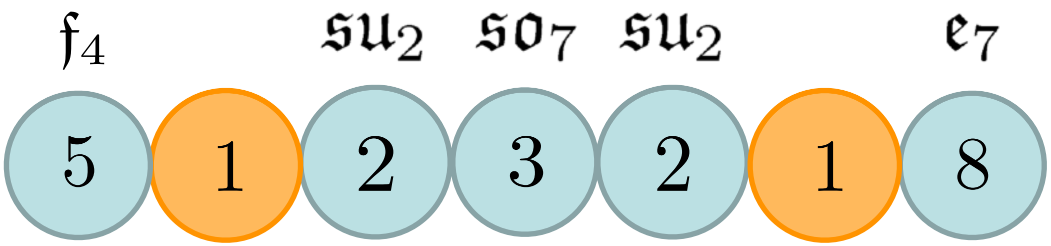
Morrison, Taylor '12



The NHCs are sort of the heroes of our story today

Rule 1: Gluing NHCs

Bases of 6D SCFTs consist of NHCs glued together by -1 curves:



Gauge algebras glued togetl

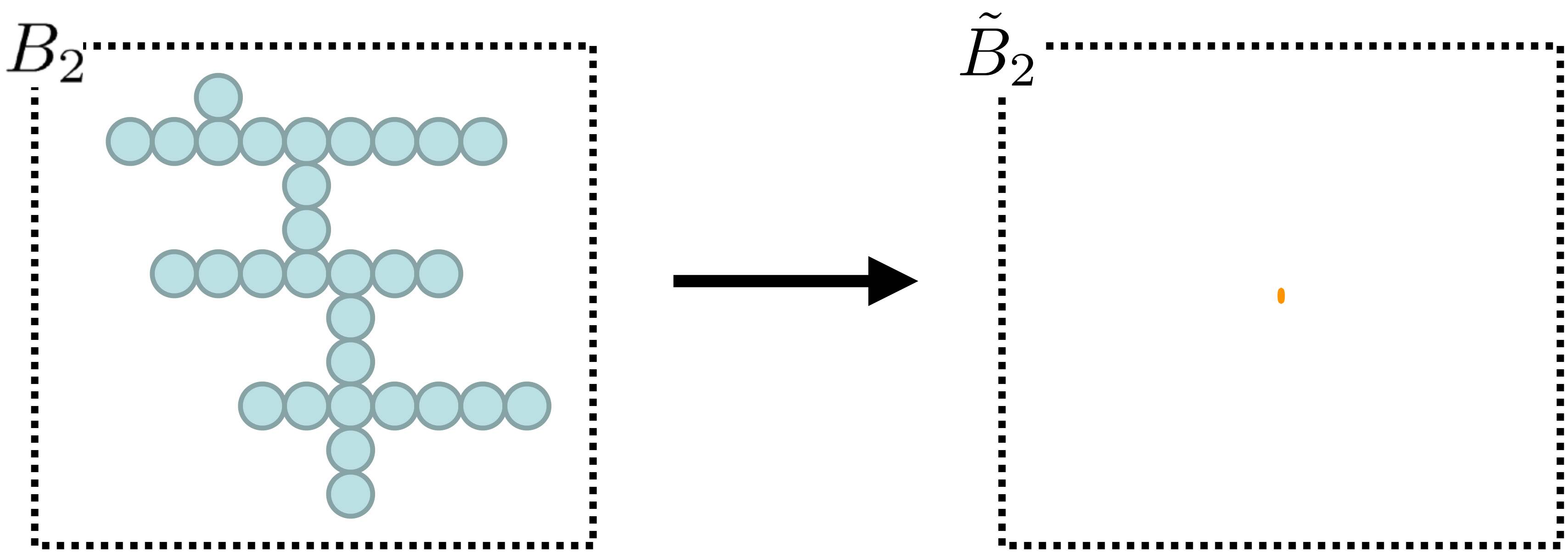
the "E_8 gauging condition":

$\mathfrak{g}_L \times \mathfrak{g}_l$

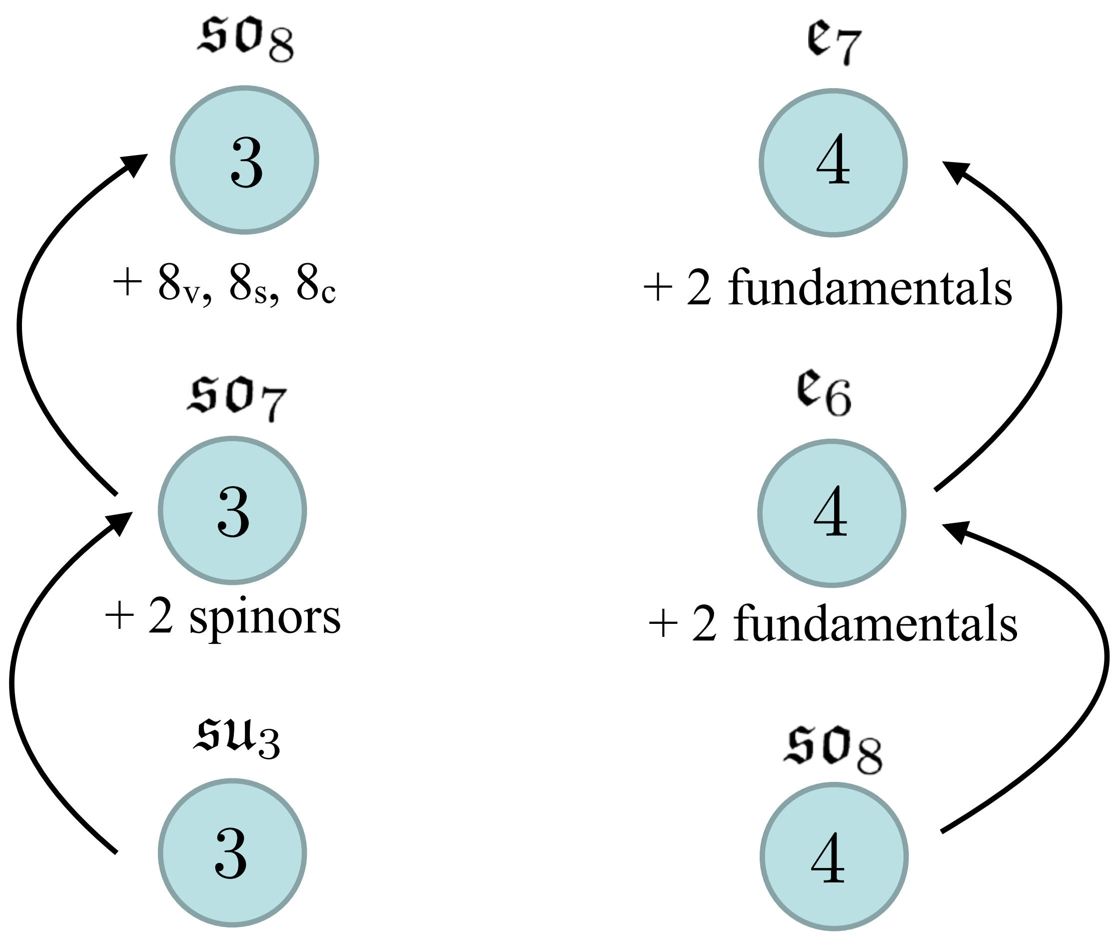
As the stars of the show, NHCs have big egos and can't play nicely together, so they need some -1 curves to come in between them and keep them from fighting. But some pairs of NHCs are too much for the -1 curves to handle, and they can't even be in the same room as

Rule 2: Contractibility of the Base

To reach the SCFT point, all curves of the base must be simultaneously contractible to a point:



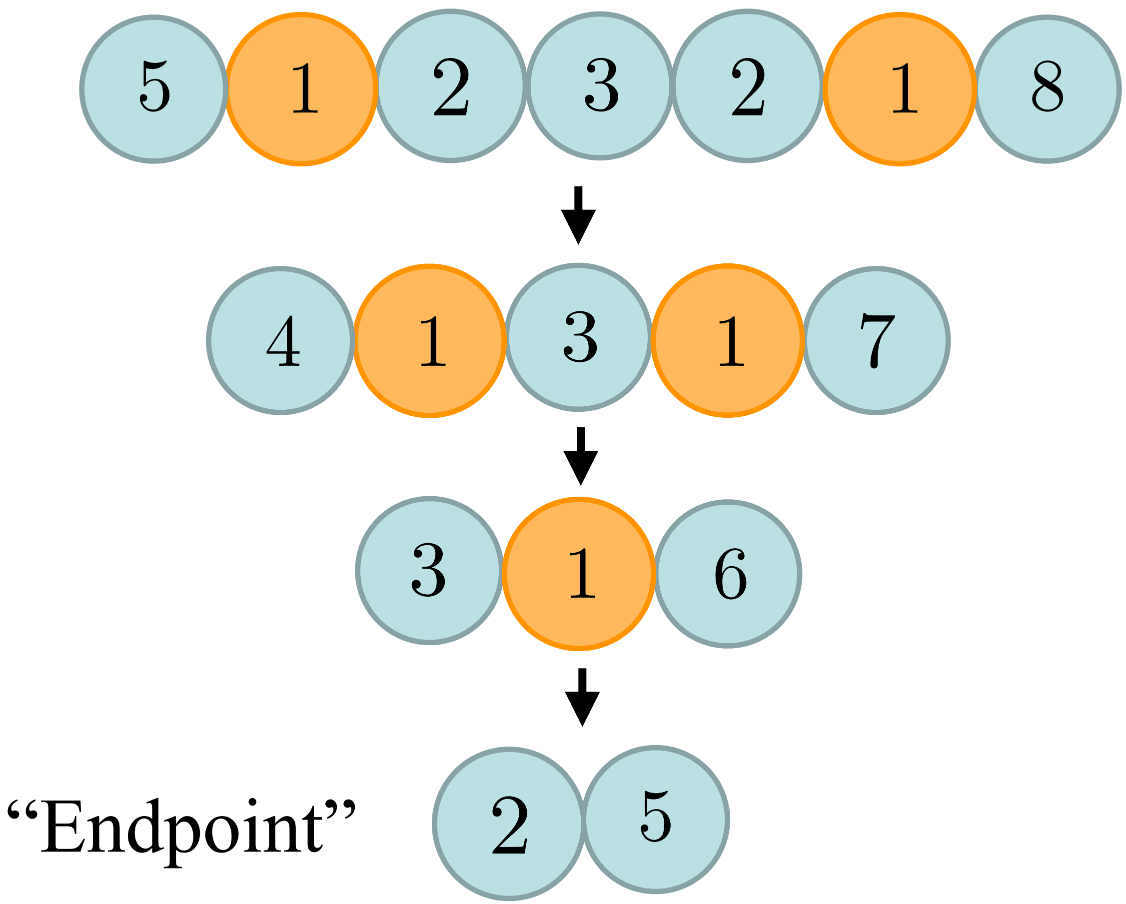
Rule 3: Anomaly Cancellation



Result 1: Classification of "Endpoint" Bases

Heckman, Morrison, Vafa '13

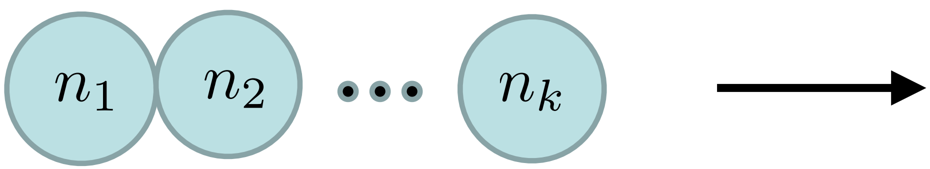
Iteratively blowdown all -1 Curves:



Result 1: Classification of "Endpoint" Bases

Heckman, Morrison, Vafa '13

Endpoints classified by discrete subgroups $\Gamma \subset U(2)$

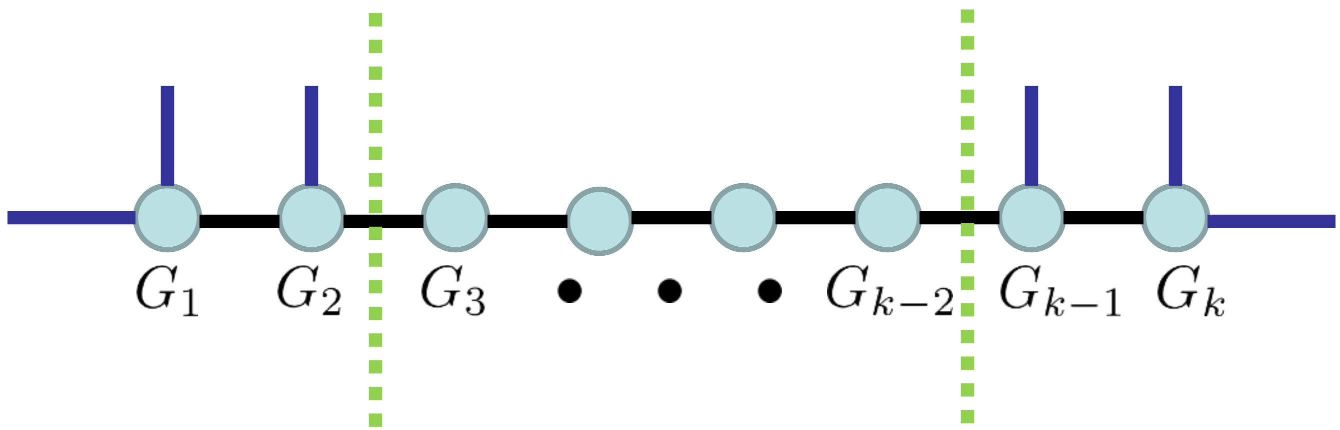


$$\frac{p}{q} = n_1 - \frac{1}{n_2 - ... - \frac{1}{n_k}}$$

$\mathbb{C}^2/\Gamma$ orbifold action: $\quad (z_1, z_2) \to (e^{2\pi i/p} z_1, e^{2\pi q i/p} z_2)$

Result 2: Linear Quivers

Heckman, Morrison, T.R., Vafa, '15

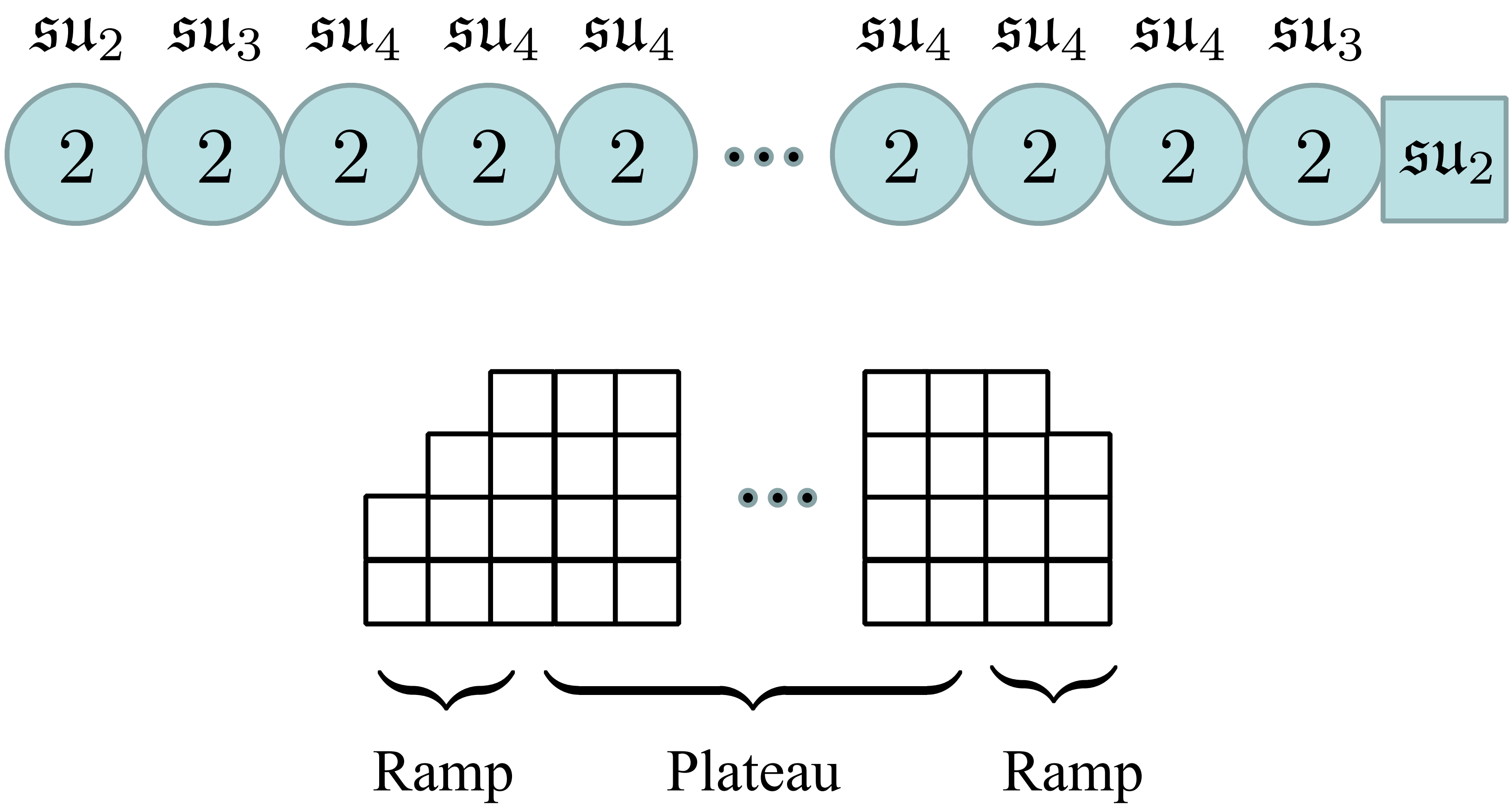


$G_i = A_n, D_n$ or E_n

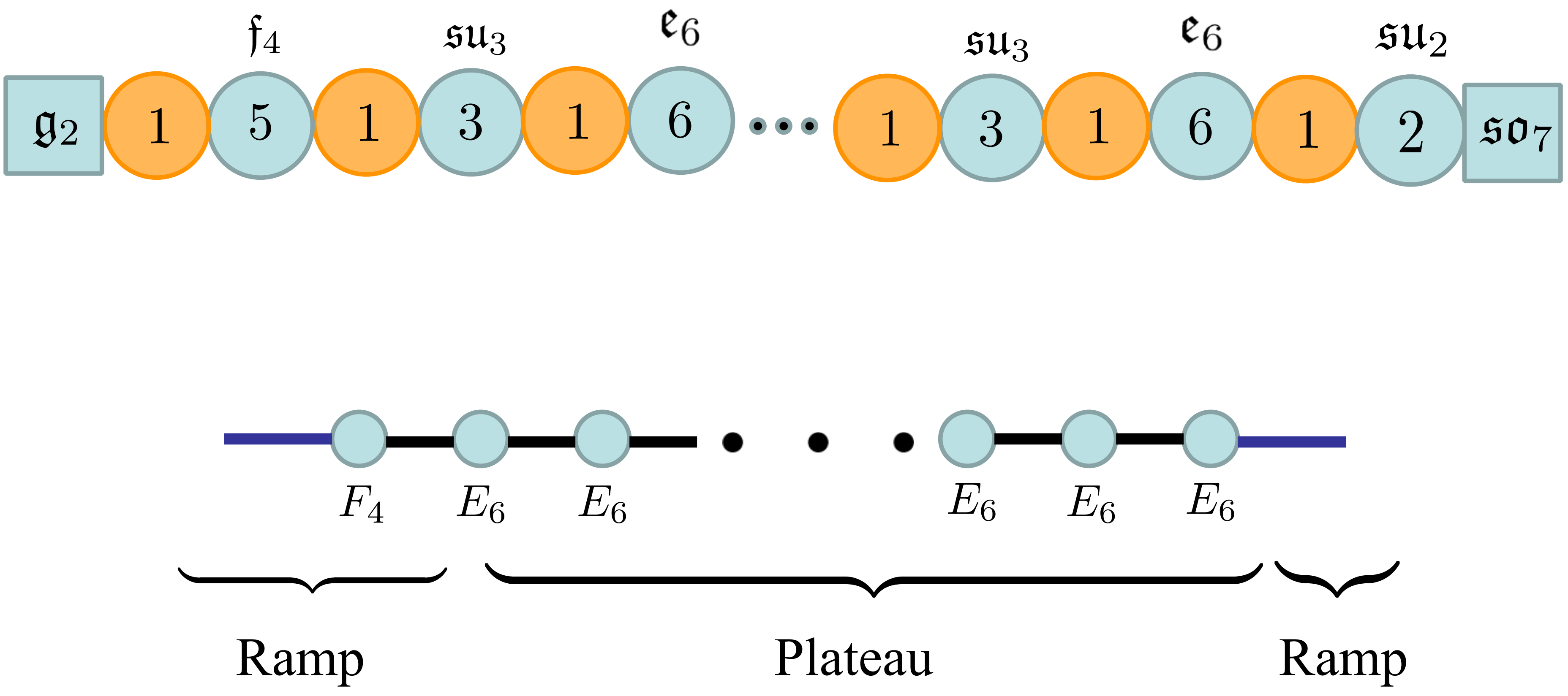
Convexity Condition:

$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

Examples


$\mathfrak{su}_2$ $\mathfrak{su}_3$ $\mathfrak{su}_4$ $\mathfrak{su}_4$ $\mathfrak{su}_4$ $\mathfrak{su}_4$ $\mathfrak{su}_4$ $\mathfrak{su}_4$ $\mathfrak{su}_3$
2 2 2 2 2 ⋯ 2 2 2 2 $\mathfrak{su}_2$
Ramp
Plateau
Ramp

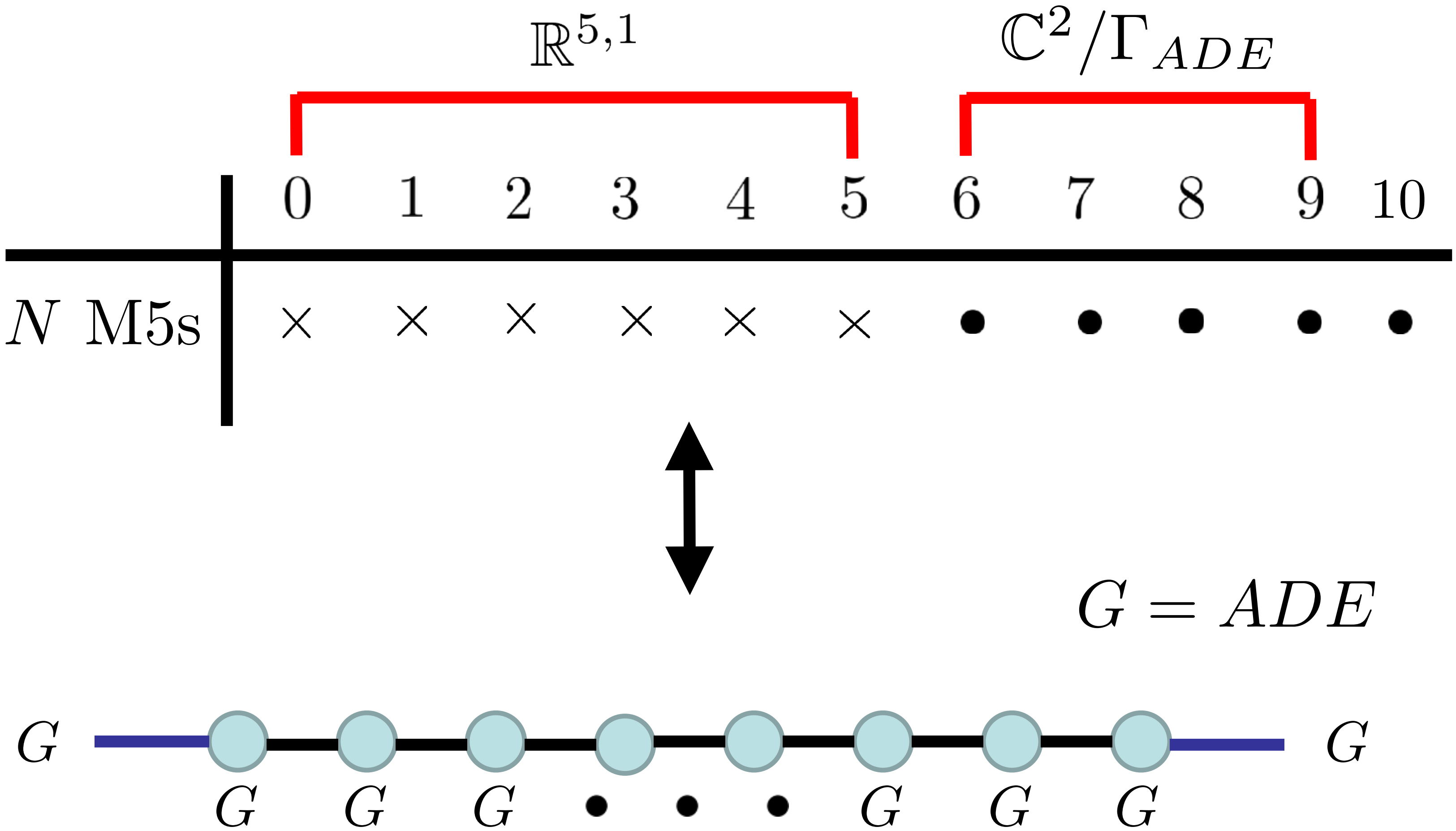
Examples



$\mathfrak{f}_4$ $\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_2$
$\mathfrak{g}_2$ 1 5 1 3 1 6 ... 1 3 1 6 1 2 $\mathfrak{so}_7$
F_4 E_6 E_6 E_6 E_6 E_6
Ramp
Plateau
Ramp

All 6D SCFTs from Group Theory

M5-Branes Probing $\mathbb{C}^2/\Gamma_{ADE}$



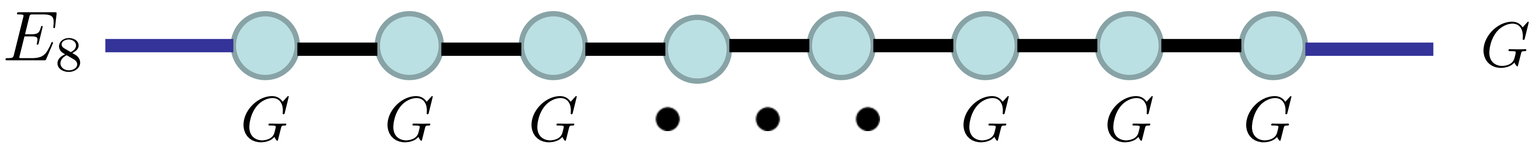
M5-Branes Probing E_8 Wall, $\mathbb{C}^2/\Gamma_{ADE}$

$\mathbb{R}^{5,1}$ (directions 0–5) $\qquad \mathbb{C}^2/\Gamma_{ADE}$ (directions 6–9)

	0	1	2	3	4	5	6	7	8	9
N M5s	×	×	×	×	×	×	•	•	•	•
E_8 Wall	×	×	×	×	×	×	×	×		•

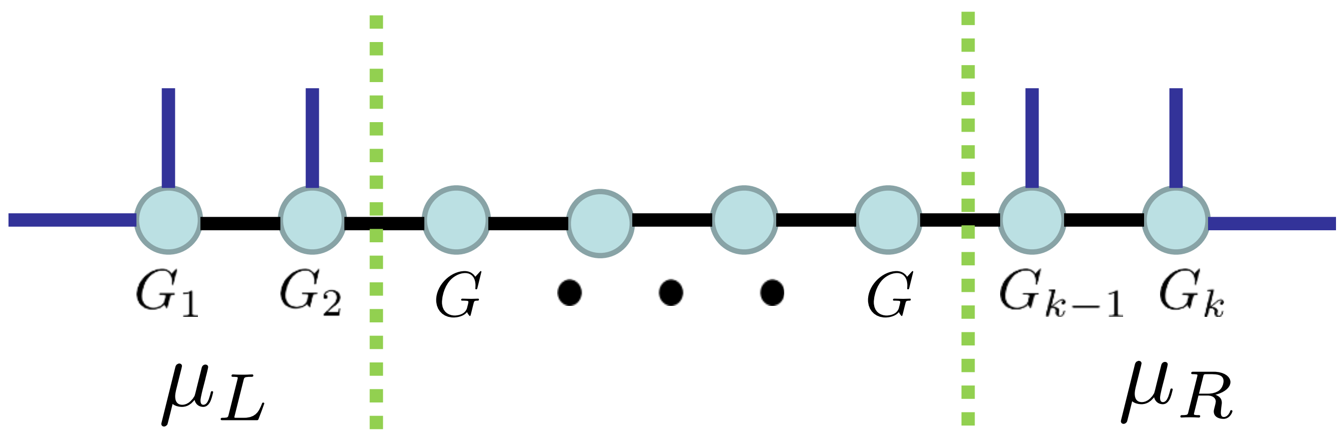
$\updownarrow$

$G = ADE$



Deformations

- Claim 1: All known 6D SCFTs with long plateau are realized as a deformation of one of these M5-brane theories
- Claim 2: Each such deformation is labeled by a pair of homomorphisms (μ_L, μ_R), characterizing the ramps on the left and right:



Homomorphisms

- For M5-branes on $\mathbb{C}^2/\Gamma_{ADE}$,

$$\mu_{L,R} \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g}_{L,R}), \quad G_{L,R} \subset G$$

$$\leftrightarrow \text{Nilpotent orbits of } \mathfrak{g}_{L,R}$$

Predicted via F-theory

- If also probing E_8 wall on left,

$$\mu_L \in \mathrm{Hom}(\Gamma_{ADE}, E_8)$$

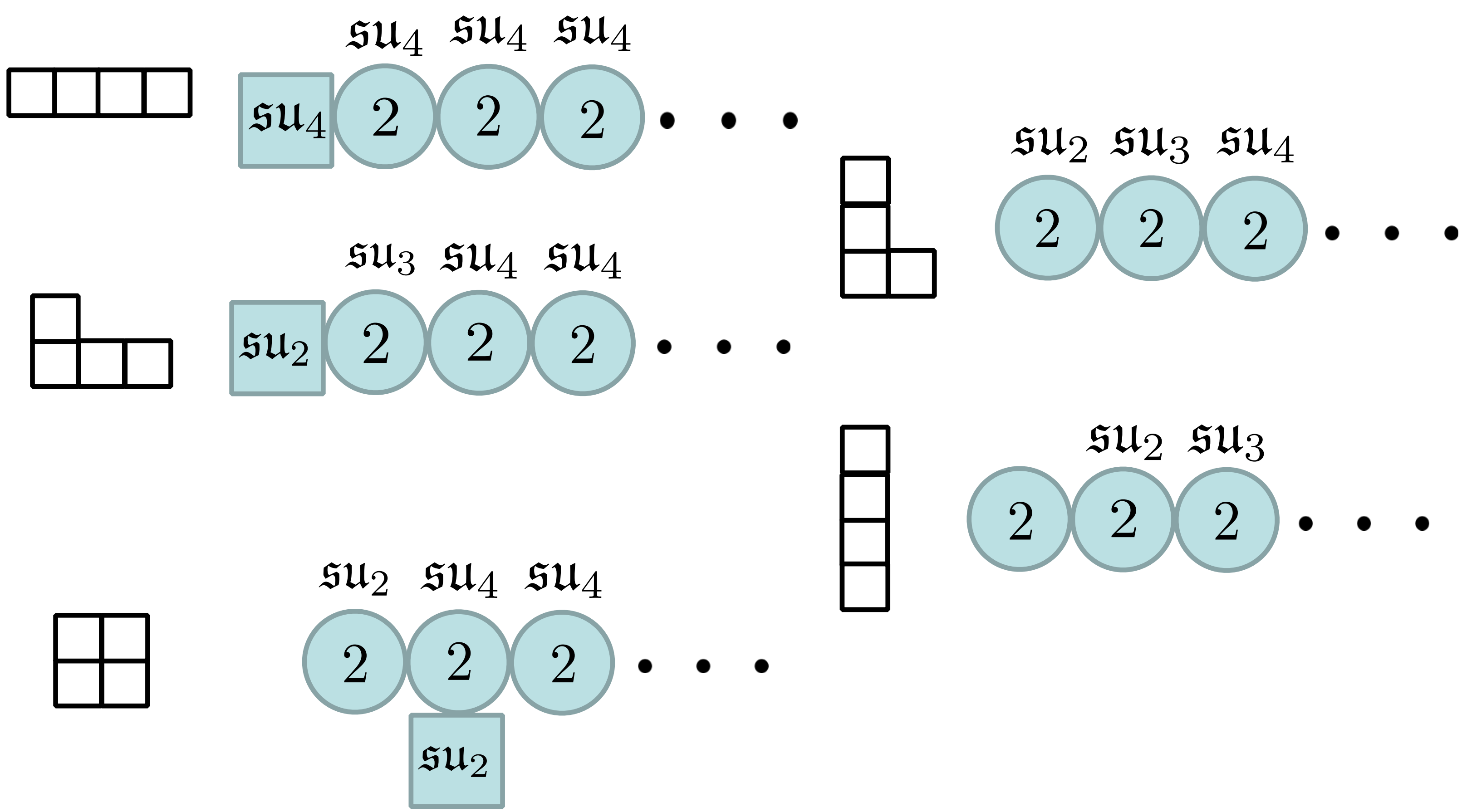
$$\mu_R \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g}_R), \quad G_R \subset G$$

Predicted via heterotic M-theory

See Del Zotto, Heckman, Tomasiello, Vafa '14 or my talk in Banff

Example: $G = SU(4)$

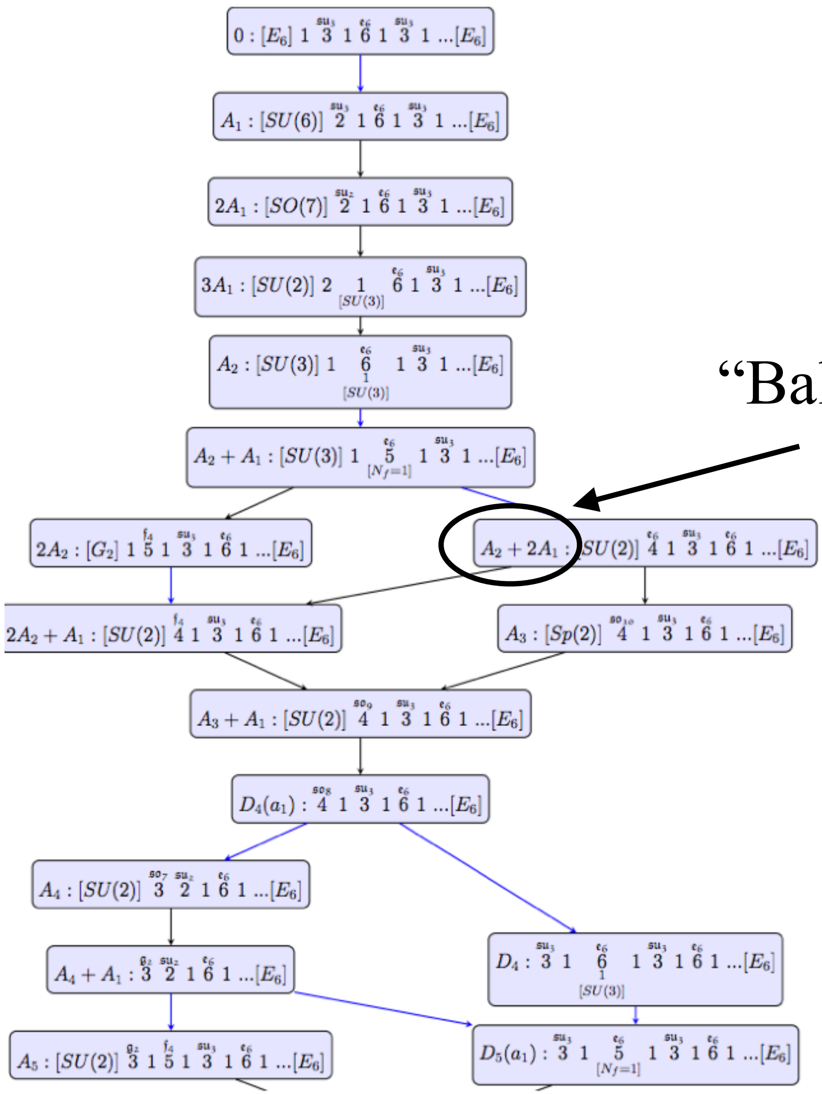
$$\mu_L \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{su}(4)) \leftrightarrow \text{Partitions of } 4$$

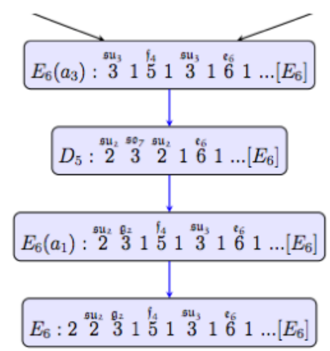


Example: $G = E_6$

$$\mu_L \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{e}_6)$$

"Bala-Carter" label





Example: $G = E_8$

$$\mu_L \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{g}_2)$$

$$1 : [G_2]\ \overset{\mathfrak{su}_2}{2}\ 2\ 1\ \overset{\mathfrak{e}_8}{12}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ ...[G_2]$$

$$\downarrow$$

$$A_1 : [SU(2)]\ 2\ 2\ 1\ \overset{\mathfrak{e}_8}{12}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ ...[G_2]$$

$$\downarrow$$

$$\widetilde{A}_1 : [SU(2)]\ 2\ 1\ \overset{\mathfrak{e}_8}{11}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ ...[G_2]$$

$$\downarrow$$

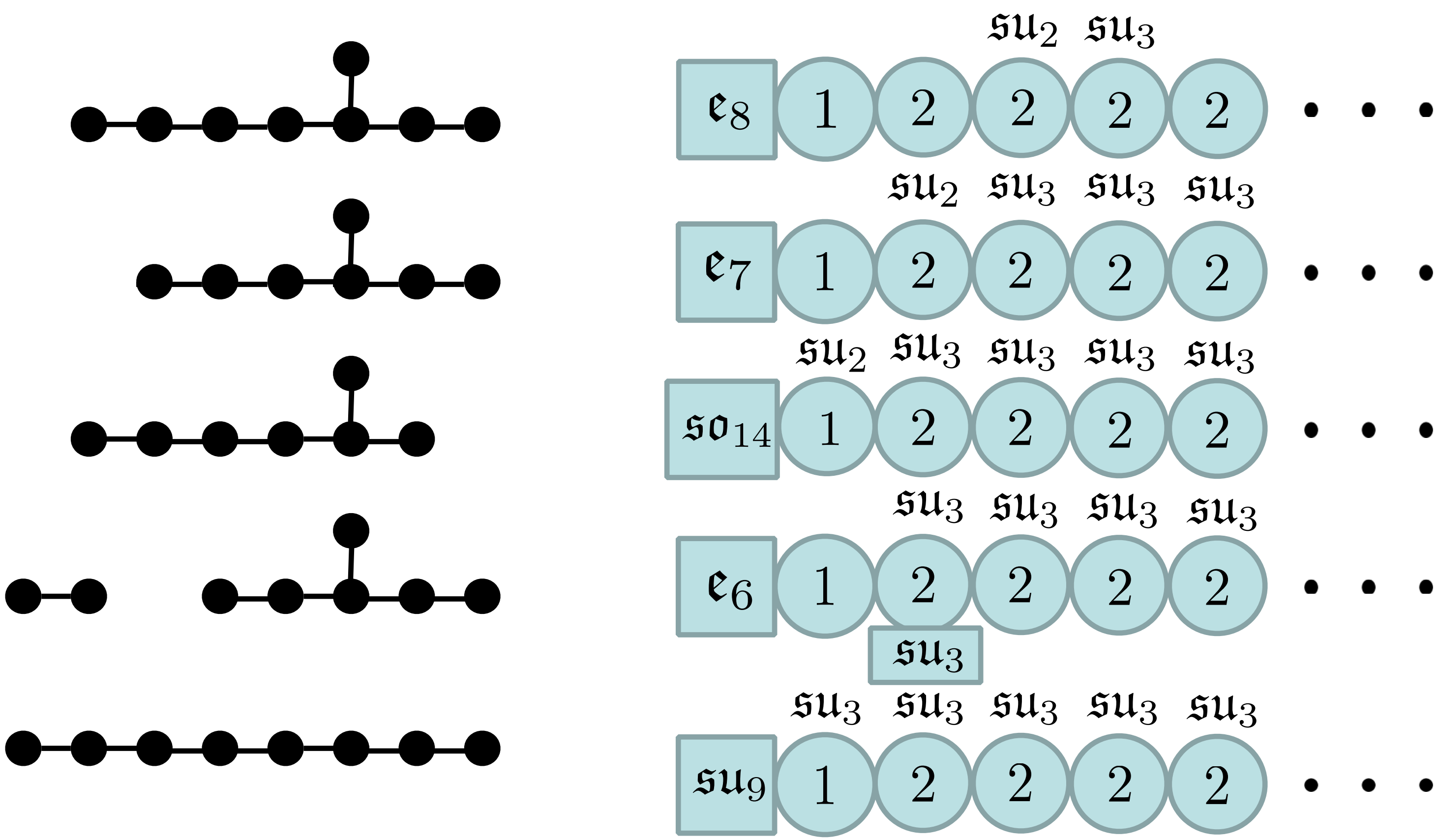
$$G_2(a_1) : \overset{\mathfrak{e}_8}{9}\ 1\ 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ ...[G_2]$$

$$\downarrow$$

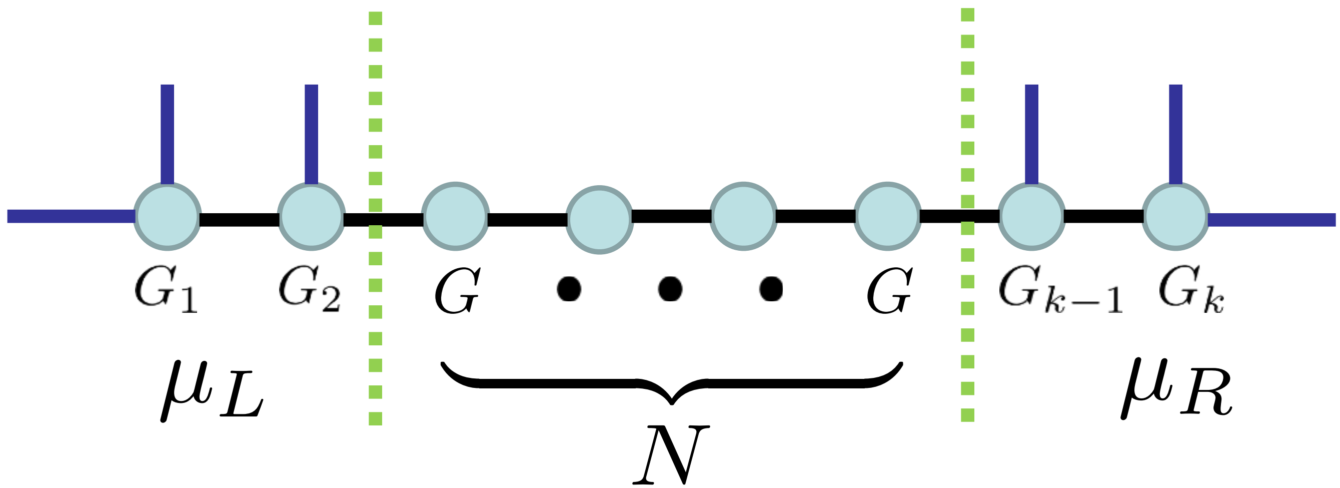
$$G_2 : \overset{\mathfrak{e}_7}{8}\ 1\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ ...[G_2]$$

Example: $G = SU(3)$

$$\mu_L \in \mathrm{Hom}(\mathbb{Z}_3, E_8)$$



Group-Theoretic Classification



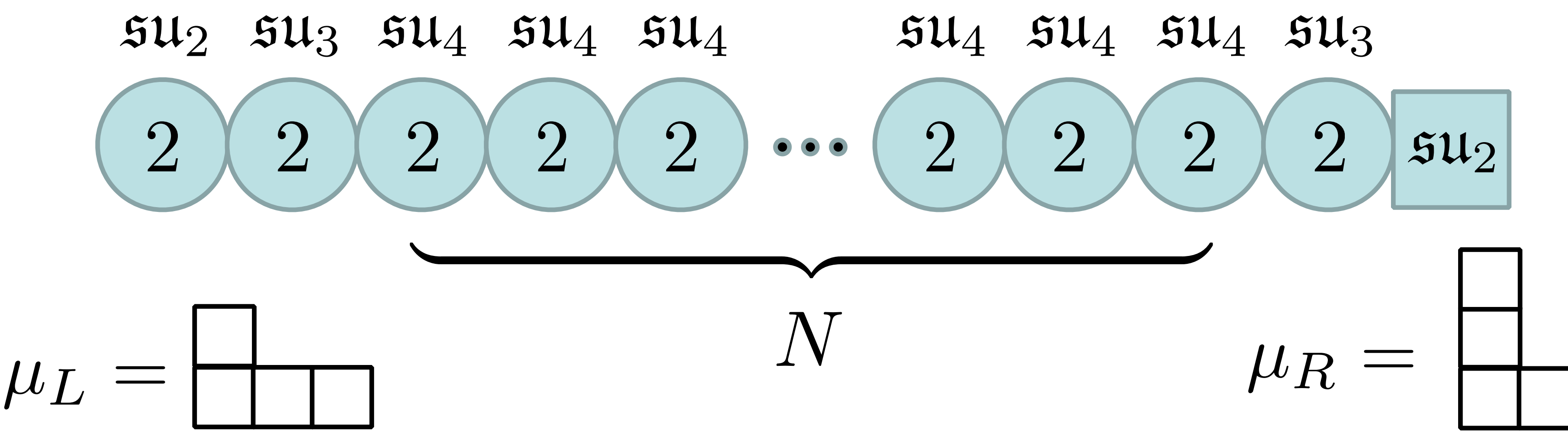
Long 6D SCFTs uniquely classified by:

- Choice of ADE group *G* in plateau
- Length *N* of plateau
- Pair of homomorphisms (μ_L, μ_R)

What about **short** SCFTs?

Example

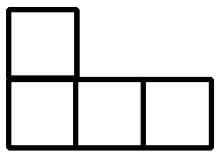
$G = \mathfrak{su}(4)$

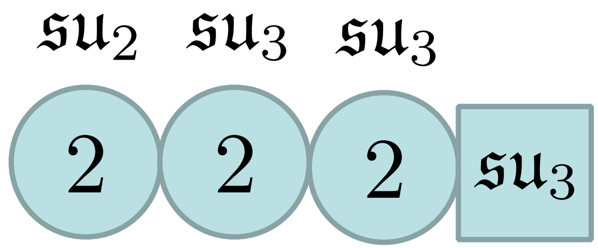


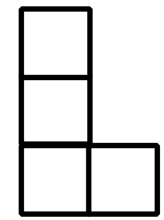
Example

Take $N \to 0$: μ_L, μ_R collide!

$G = \mathfrak{su}(4)$

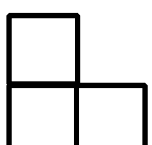
$\mu_L =$ 

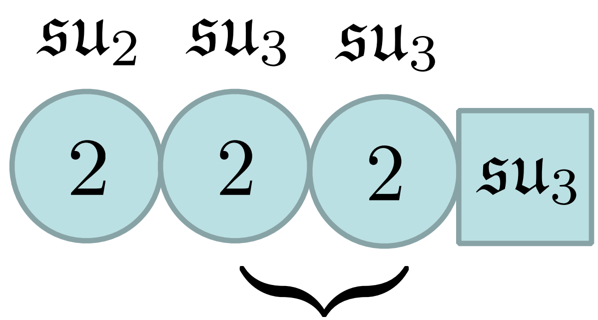


$\mu_R =$ 

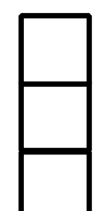
SCFT is equivalent to

$G = \mathfrak{su}(3)$

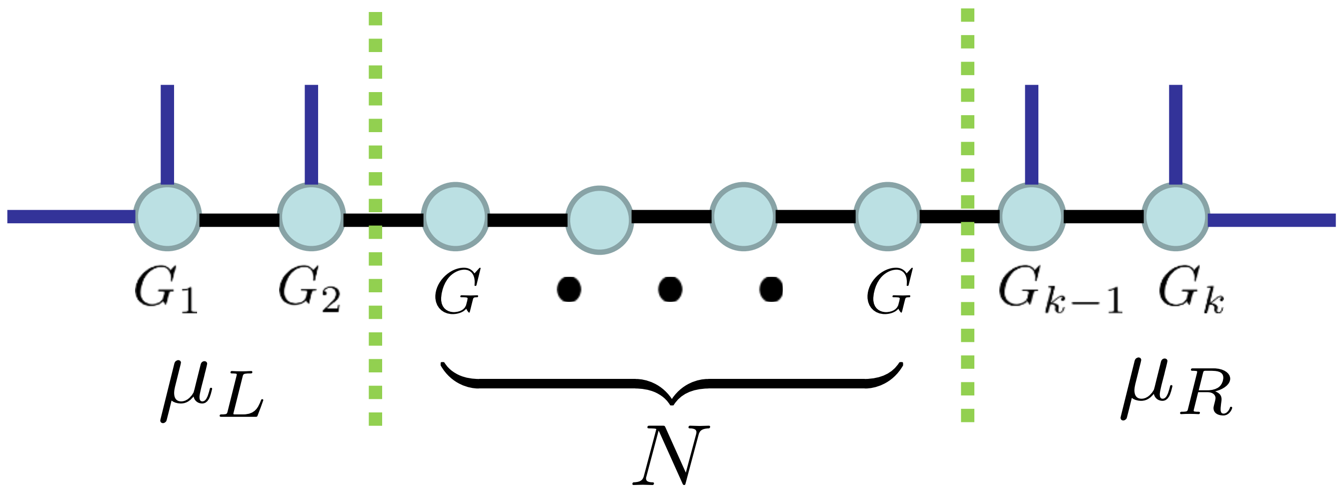
$\mu_L =$ 



$N = 2$

$\mu_R =$ 

Short 6D SCFTs



Still, almost all short 6D SCFTs can be labeled by

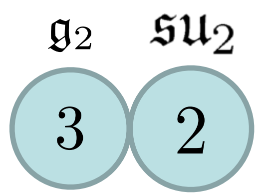
- Choice of ADE group G in plateau
- Pair of homomorphisms (μ_L, μ_R)
- Length N of plateau, **possibly** $\leq \mathbf{0}$

Not all pairs are allowed, and description is no longer unique

Example: NHCs from Group Theory

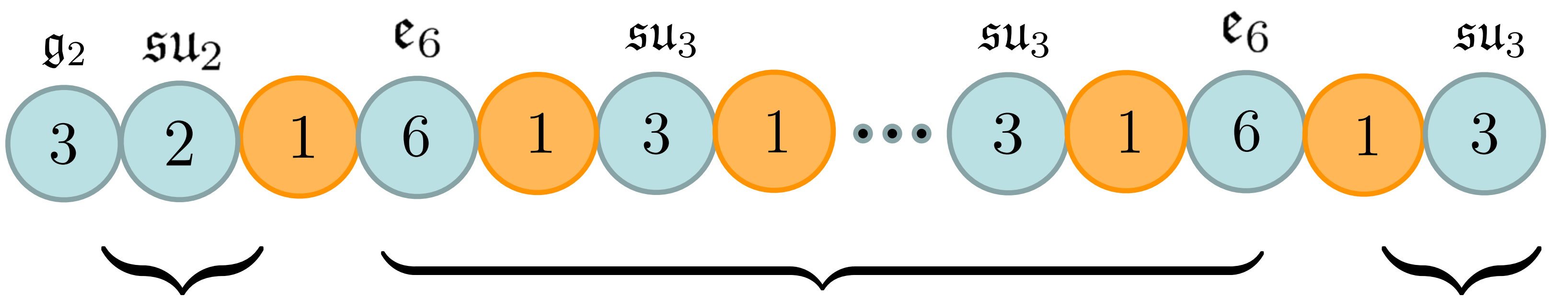
NHC	G	N	Hom_L	Hom_R
$\overset{\mathfrak{su}_3}{3}$	$\mathfrak{e}_6$	0	$\Gamma_{E_6} \to E_8$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{so}_8}{4}$	$\mathfrak{so}(8)$	1	$\mathfrak{su}(2) \to 1$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{f}_4}{5}$	$\mathfrak{e}_6$	0	$\mathfrak{su}(2) \to \mathfrak{su}(3)$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{e}_6}{6}$	$\mathfrak{e}_6$	1	$\mathfrak{su}(2) \to 1$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{e}_7}{7}$	$\mathfrak{e}_7$	1	$\mathfrak{su}(2) \to \mathfrak{su}(2)$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{e}_7}{8}$	$\mathfrak{e}_7$	1	$\mathfrak{su}(2) \to 1$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{e}_8}{12}$	$\mathfrak{e}_8$	1	$\mathfrak{su}(2) \to 1$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{g}_2}{3}$	$\mathfrak{e}_6$	0	$\mathfrak{su}(2) \to \mathfrak{e}_6$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{su}_2}{2}\,\overset{\mathfrak{so}_7}{3}\,\overset{\mathfrak{su}_2}{2}$	$\mathfrak{e}_7$	0	$\mathfrak{su}(2) \to \mathfrak{e}_7$	$\mathfrak{su}(2) \to 1$
$\overset{\mathfrak{g}_2}{3}\,\overset{\mathfrak{su}_2}{2}\,2$	$\mathfrak{e}_6$	0	$\mathfrak{su}(2) \to \mathfrak{e}_6$	$\mathfrak{su}(2) \to 1$

Example



$=$

$\lim_{N\to 0}$



$\mu_L = A_4 + A_1 \in \mathrm{Hom}(\mathfrak{su}(2), \mathfrak{e}_6)$

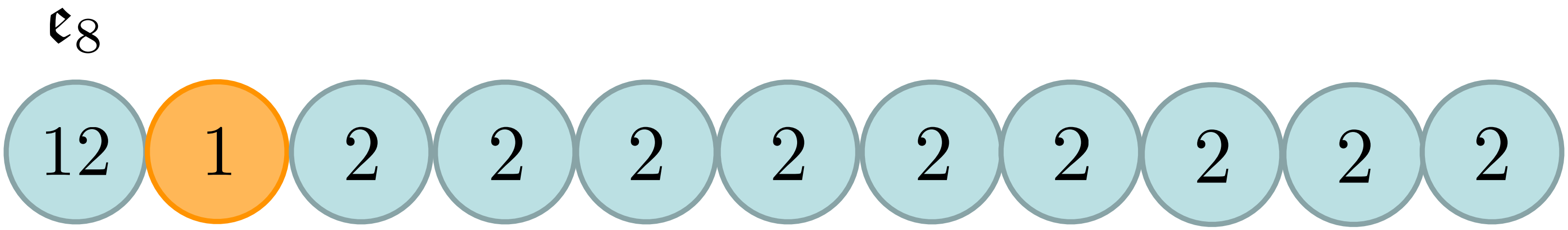
N

$\mu_R = 1$

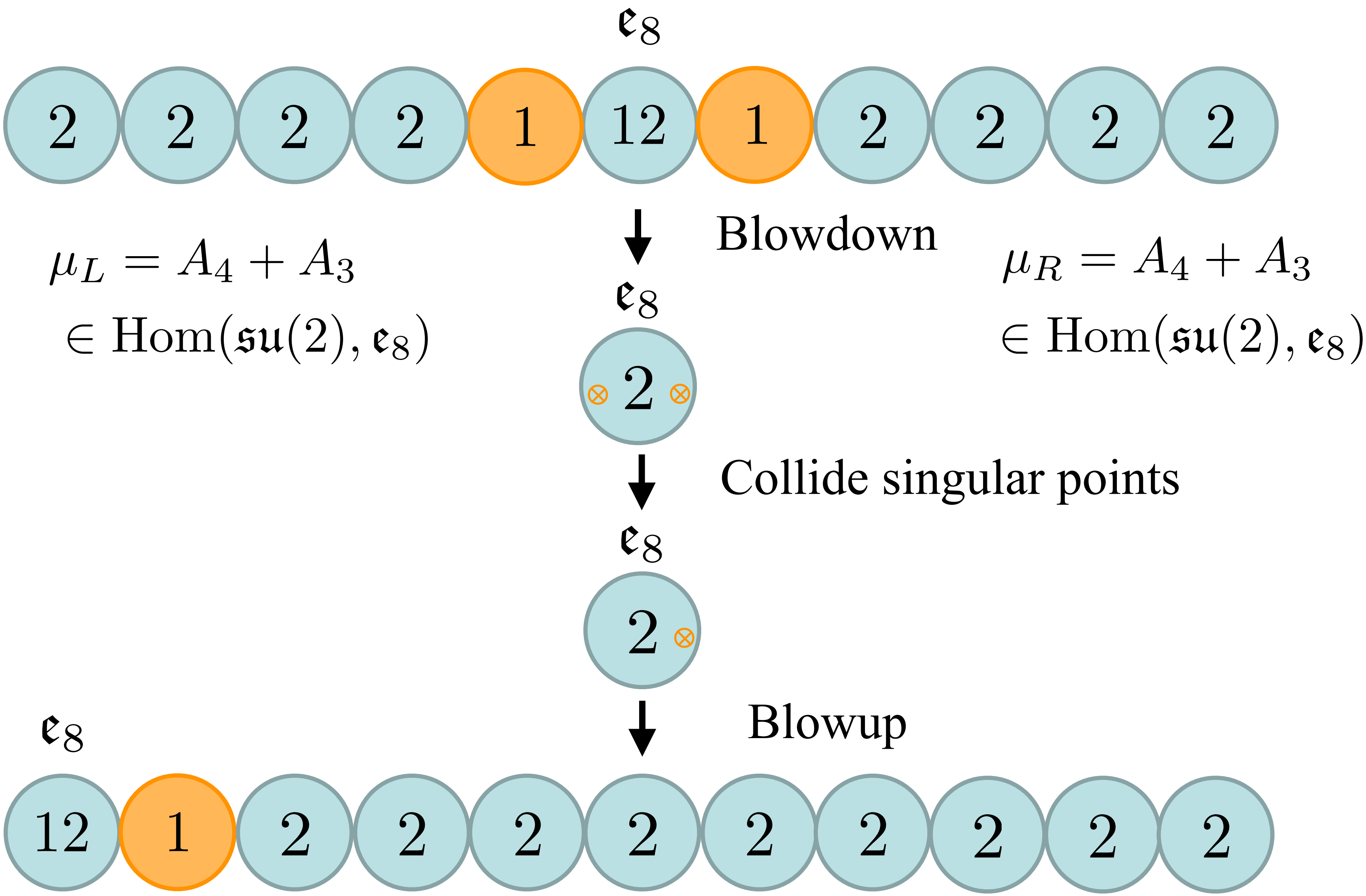
i.e. $I_8(\mathrm{LHS}) = \lim_{N\to 0} I_8^{(N)}(\mathrm{RHS})$

Do **all** 6D SCFTs admit a group-theoretic description?

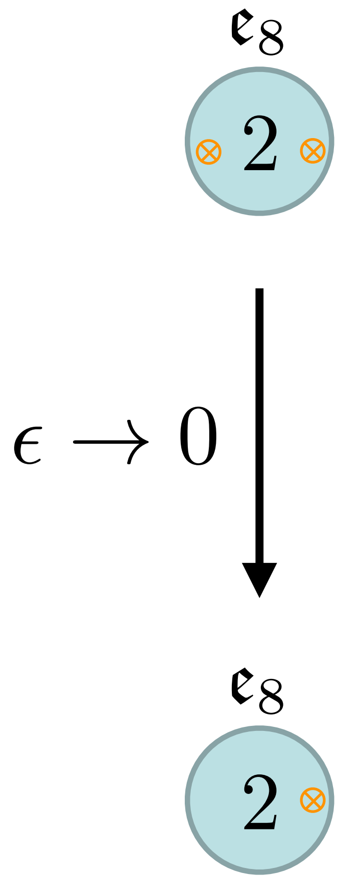
A Counterexample



A Counterexample(?)



A Counterexample(?)



$$f = \sigma^4$$

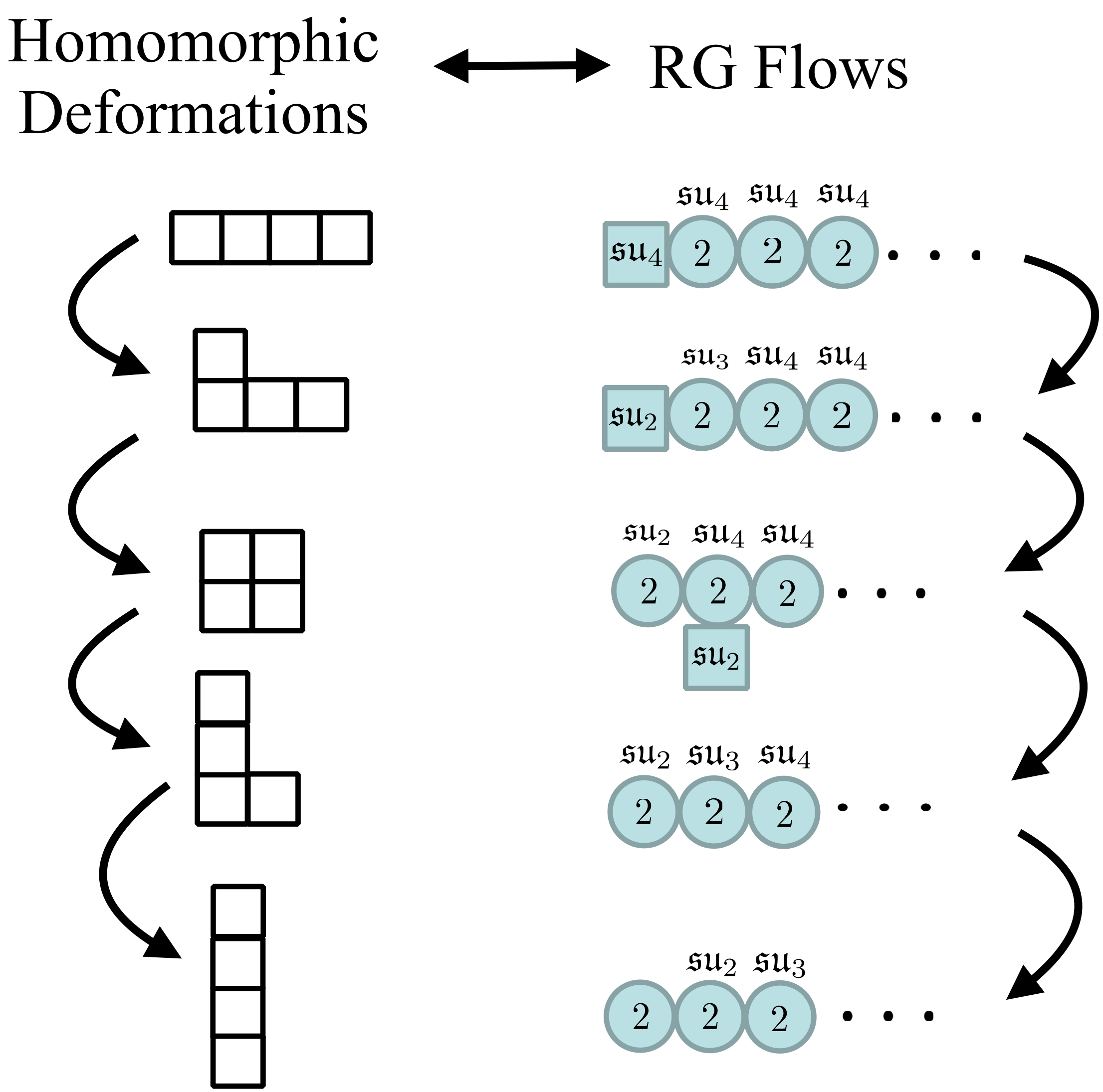
$$g = \sigma^5 z^5 (z - \epsilon)^5 + \sigma^6$$

Summary and Future Research

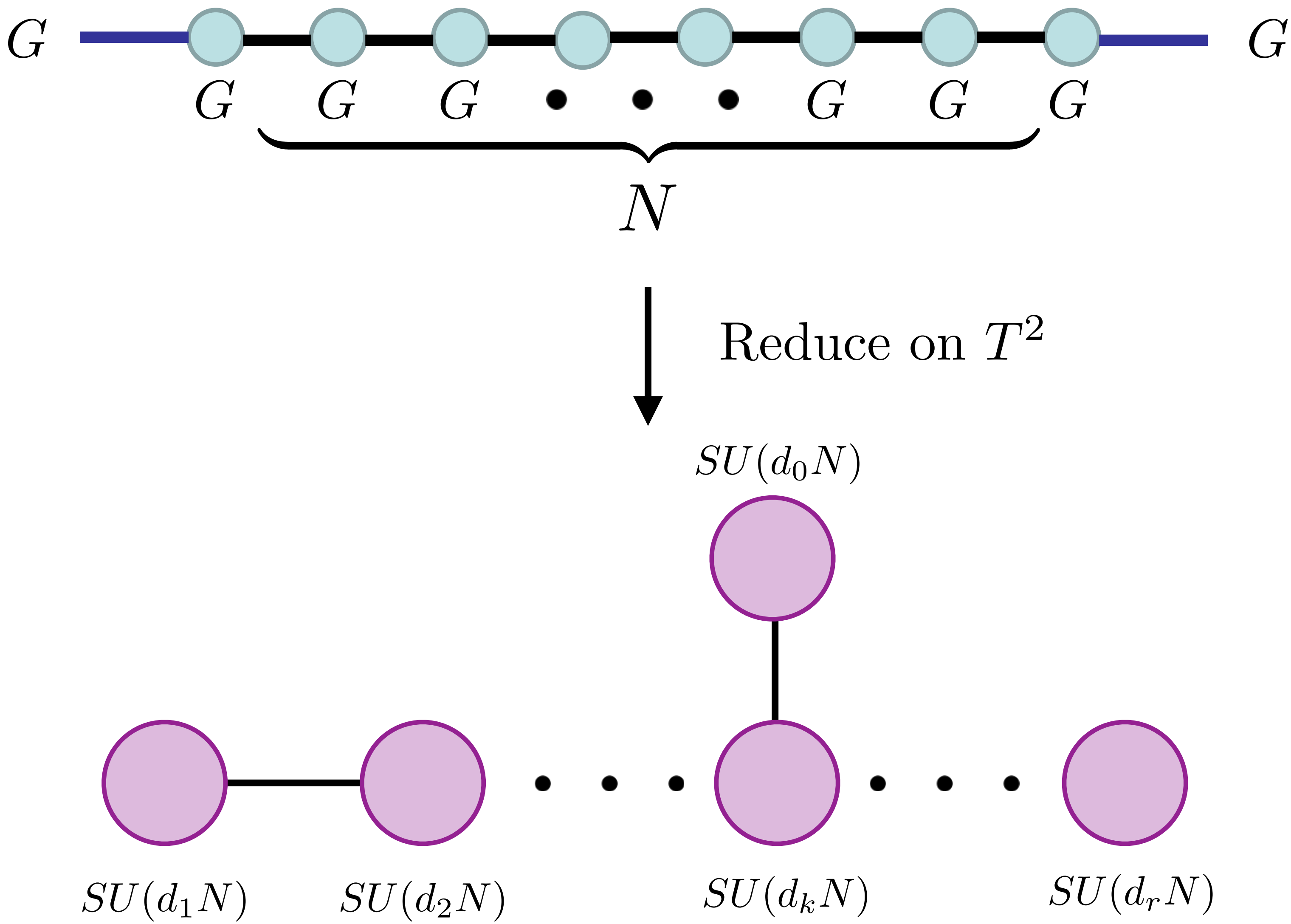
Summary

- 6D SCFTs have been atomically classified
- All long 6D SCFTs, and most short 6D SCFTs, also admit a group-theoretic classification
- There seem to be some outliers, but these might be understood via colliding homomorphisms

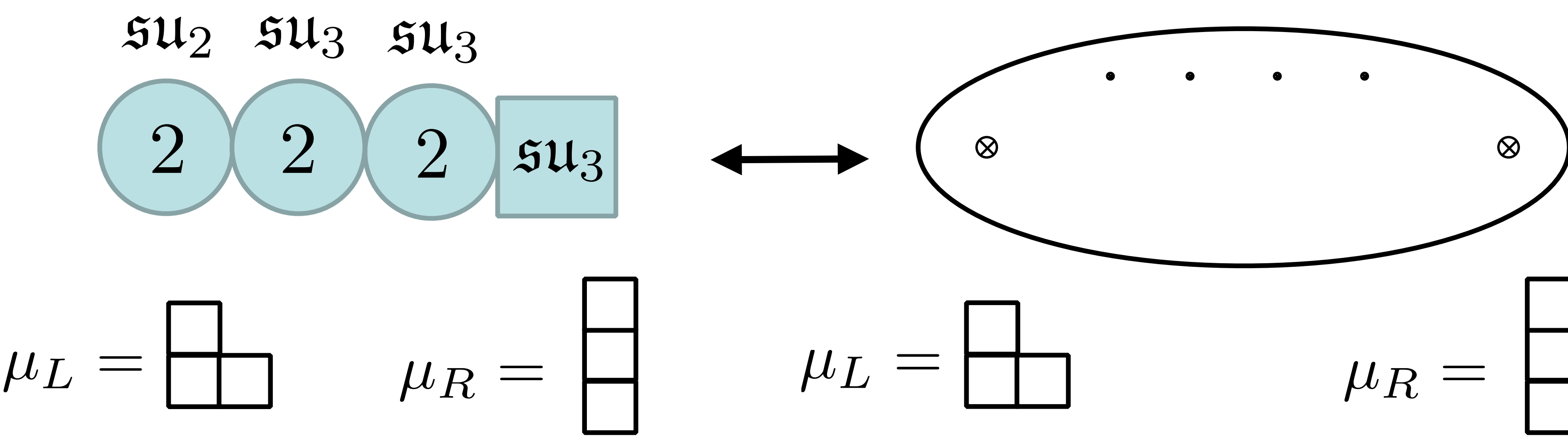
Future Direction 1: RG Flows



Future Direction 2: Compactification



Future Direction 3: Relation to Class S



$G = \mathfrak{su}(3)$ $\qquad$ $G = \mathfrak{su}(3)$

Future Direction 4: Classifying $\mathrm{Hom}(\Gamma_{ADE}, E_8)$ Frey, T.R., to appear

- A_n case: done (Kac ’83)
- E_8 case: done (Frey ’98)
- D_n case: open!
- E_6 case: open!
- E_7 case: open!

Future Direction 5: Why does this work?

Is there a deeper reason for this connection between 6D SCFTs and group theory?